Role of the coupling between neck and radial degrees of freedom in evolution from dinucleus to mononucleus

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The coupled Langevin equations in two-dimensional collective space are used to study the dynamics of nuclear neck growth. Special attention is paid to the effects of coupling between neck and radial degrees of freedom on the evolution from dinucleus to mononuleus. The dynamic model is applied for the study of neck evolution of the mass asymmetric system ^{50}Ti + ^{249}Cf. In order to estimate the effects of the coupling, we use the equations in the coupled and uncoupled cases. Our results show that the coupling between neck and radial motions reduces the neck growth velocity and delays the transition from dinuclear to mononuclear regimes. In addition, by solving these dynamic equations we get the probability distributions of radial degree of freedom at the injection point $s_{\rm in}$ in the asymmetric fission valley. In this way, $s_{\rm in}$ is no longer an adjustable parameter in the fusion-by-diffusion model. The distributions obtained are located at positions very close to $s = 0$. Moreover, the coupling significantly reduces the fluctuation in the $s_{\rm in}$ space. Our results show that the quasifissionlike events are unlikely to take place during the transition period for the mass asymmetric systems. Based on the $s_{\rm in}$ distribution obtained, the evaporation residue (ER) cross sections for $3n$ and $4n$ evaporation channels in the ^{50}Ti +^{249}Cf reaction leading to formation of ^{296}120 and ^{295}120 isotopes are calculated. The maximum ER cross sections in $3n$ and $4n$ channels with the $s_{\rm in}$ distributions are equal to 0.1 and 0.065 pb, respectively, which are more than two times larger than those obtained by the fusion-by-diffusion model with the $s_{\rm in} = 0$ assumption.

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I. INTRODUCTION

In our previous work [1], we demonstrated that evolution from dinucleus to mononucleus plays an important role in the formation of a superheavy nucleus. In that work, the dynamics of neck growth for the reaction system ^{136}Xe + ^{136}Xe was simulated in terms of the two-dimensional Langevin equation. The simulation results have shown that for the ^{136}Xe + ^{136}Xe reaction, more than 80% of quasifission events occur during the transition from dinuclear to mononuclear regimes. Incorporating this essential physical ingredient in the calculations, the fusion-by-diffusion model [2,3] nicely accounts for the experiment of the ^{136}Xe + ^{136}Xe reaction performed in Dubna [4].

However, in our previous work we used a schematic version of the liquid drop model [5,6] to describe the dynamic process. In that model, the geometrical shape of the system is parametrized in terms of two spheres, representing the approaching nuclei, joined by a conical or cylindrical neck. This description of the configuration allows for a simple polynomial approximation to the liquid-drop potential energy and analytic formula for the one-body dissipation function [6–8]. However, a conical or cylindrical neck may be not so realistic in shape. In this connection, both the potential energy and the dissipation function of this schematic model are only approximately valid. In order to overcome this deficiency, in this work we use a more realistic dynamic model based on the nuclear deformation potential energy calculated in the framework of the finite range liquid-drop model (FRLDM) [9,10], the one-body dissipation [11–14], and the shape of nuclear surface specified by two spheres smoothly joined with a hyperboloidal neck. With this model we make a detailed analysis of mass asymmetric systems that are more favorable for synthesis of superheavy elements because the fusion probability increases with the decrease of the product of the charge numbers Z_p and Z_t of projectile and target nuclei, respectively. For mass symmetric systems, the neck degree of freedom is weakly coupled to the others. However, for mass asymmetric systems, usually the effects of the coupling between different degrees of freedom should be important. In the following, we will pay special attention to the influence of the coupling between the neck and radial degrees of freedom in the dynamical evolution of the nuclear systems. The coupling between the neck and mass asymmetric degrees of freedom is not taken into account, mostly due to the fact that motions of the neck and mass asymmetry happen in different time scales. As shown below, the neck grows very rapidly, therefore the transition from dinucleus to mononucleus is realized in a short time scale. On the other hand, the mass asymmetry of the system changes very slowly, especially at the early stages of neck growth, the flow of nuclear mass through the window between two contact nuclei is strongly damped because the friction of the asymmetric degree of freedom is approximately in inverse proportion to the radius of the neck [6].

The main motivation of this work concerns a modification of the fusion-by-diffusion model [2,3]. This model brings out the basic physics of the observed hindrance with an elementary formula. However, in formulating the model, numerous approximations were introduced. One of these is that the dynamics of the neck growth phase was bypassed by introducing an adjustable parameter $s_{\rm in}$, the separation between

the surfaces of the approaching nuclei at which injection into the asymmetric fission valley takes place. This may bring about uncertainty in the fusion hindrance factor (or the fusion probability) because it sensitively depends on the parameter $s_{\rm in}$ for the reaction systems with large Coulomb parameter $z = Z_p Z_t/(A_p^{1/3} + A_t^{1/3})$. In our approach, the dynamics of neck growth is taken into account in terms of the multidimensional Langevin equations. By solving these dynamic equations event by event, the probability distribution of $s_{\rm in}$ is obtained. Therefore, $s_{\rm in}$ is no longer a free parameter in the modified fusion-by-diffusion model. In order to estimate the effects of this modification, we will calculate the formation cross sections of superheavy nuclide $^{295,296}120$ via a hot fusion reaction of ^{50}Ti + ^{249}Cf and compare the results with those published in Ref. [15].

The paper is arranged as follows. In Sec. II, dynamic evolution from dinucleus to mononucleus is simulated in terms of the multidimensional Langevin equations. This is followed by a presentation and discussion of the coupling effects on the neck growth. With the $s_{\rm in}$ distributions obtained in the simulations, the formation cross sections of superheavy nuclei $^{295,296}120$ are calculated and the results are presented in Sec. III. Finally, the summary and conclusion are given in Sec. IV.

II. DYNAMICS OF NECK GROWTH

The orientation effects of deformed nuclei play an important role in the subbarrier fusion. However, it is not so easy to define the potential energy of deformation for the case of two touching arbitrary oriented deformed nuclei. The main difficulty here stems from the lack of knowledge about the subsequent evolution of the nuclear shapes after two nuclei come into contact. For simplicity, target deformation is not taken into account in the dynamic simulations, and the shape of the system is specified in terms of two spheres with radii R_1 and R_2 smoothly connected by a hyperboloidal neck. An example of a shape described in this way is shown in Fig. 1. Three variables may be defined for a given shape: elongation $L = (l_2 - l_1) + R_1 + R_2$, mass asymmetry $\eta = (A_2 - A_1)/(A_1 + A_2)$, and neck size n. Here $l_{1,2}$ are the center positions of two outside spheres and $A_{1,2}$ denote the mass numbers of two portions left and right of the middle vertical plane located at $z = l_3$. Instead of elongation, we use $s = L - 2(R_1 + R_2)$, i.e., the separation between the surfaces of the approaching nuclei in the Monte Carlo simulations and neck size n is defined as neck radius at $z = l_3$ with l_3 the center position of the neck portion.

The coupled Langevin equations of motion in multidimensional collective space have the form

$$\begin{aligned} \frac{dq_i}{dt} &= \mu_{ij} p_j, \\ \frac{dp_i}{dt} &= -\frac{1}{2} p_j p_k \frac{\partial \mu_{jk}}{\partial q_i} - \frac{\partial V(q)}{\partial q_i} - \gamma_{ij}\mu_{jk} p_k + \theta_{ij}\xi_j(t), \end{aligned} \tag{1}$$

where $q_i \equiv s, n$ stand for the collective coordinates, p_i its conjugate momenta, $V(q)$ is the (FRLMD) potential energy of deformation, μ_{ij} denotes the inverse matrix elements of the inertia tensor m_{ij}, and γ_{ij} is the friction tensor. The normalized random variables ξ_j $(j = 1, 2)$ are assumed to be independent white noises. The strength θ_{ij} of the random force is given by $\theta_{ik}\theta_{kj} = T\gamma_{ij}$ with T the temperature of the heat bath. We make the Werner-Wheeler approximation [16,17] for incompressible and irrotational flow to calculate the collective inertia. The following expression is used for one-body dissipation [11–14]:

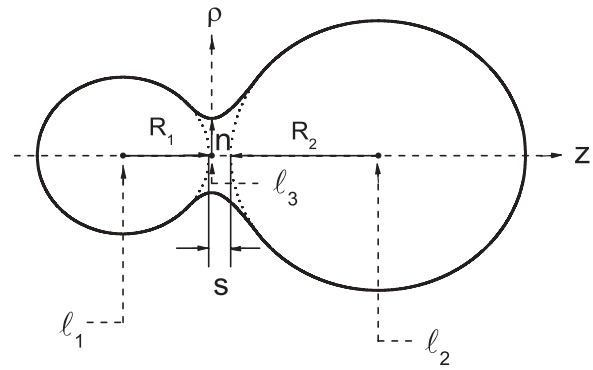

FIG. 1. A shape described by two spheres with radii R_1 and R_2 smoothly connected with a hyperboloidal neck. l_i (i=1,2,3) denote the center positions of three portions. The surface of the hyperboloidal neck is specified by a_3 and c_3. In the case of hyperboloidal neck shape, the value of a_3 is equal to the neck radius at l_3, i.e., $a_3 = n$. The quantity c_3 is imaginary for this shape and hence not shown.

$$\begin{aligned} \gamma_{ij} = {} & \rho_m \bar{v}\pi\bar{n}^2 f(n)\delta_{sj} + 2\pi\rho_m\bar{v}\int_{l_1-R_1}^{l_2+R_2} \frac{\rho_s}{\sqrt{1+\rho_s'^2}} \\ & \times[(A_i\rho_s' + A_i'\rho_s/2)(A_j\rho_s' + A_j'\rho_s/2)]dz, \end{aligned} \tag{2}$$

where the primes denote differentiation with respect to z, and ρ_m is the nuclear mass density, $\bar{v}$ is the average nucleon speed, $\bar{n}$ is the average neck radius, and ρ_s stands for the radius of nuclear surface in a cylindrical coordinate; the quantities A_i and A_j are defined in Ref. [17] as a function of z and q. The first and second terms in Eq. (2) are the window ($\gamma_{ij}^{\rm window}$) and wall ($\gamma_{ij}^{\rm wall}$) dissipation tensors, respectively. For the window dissipation, only the radial element $\gamma_{ss}^{\rm window}$ is important; the others are negligible in magnitude. In order to make a smooth transition from a dinuclear regime to a mononuclear one, we introduce a weight function $f(n)$ in the window term with an expression similar to the one used in Ref. [6],

$$f(n) = \cos^2\left[\frac{1}{2}\pi\left(\frac{n}{R_i}\right)^2\right], \tag{3}$$

with $R_i = R_1$ if $R_1 \leqslant R_2$; otherwise $R_i = R_2$.

We set the initial coordinate and momentum values of radial motion, $s(0) = 0$ and $p_s(0) = p_r$, based on an assumption that the results of the first step may give initial conditions of the second step [18]. Here p_r is the radial momentum at the contact point with its distribution $f(p_r)$ calculated in terms of the surface friction model (SFM) [1,19–21] during the capture process.

As an example, the calculated radial momentum distributions at the contact point are presented in Fig. 2 for the system ^{50}Ti + ^{249}Cf at the incident energies of 235 and 245 MeV, respectively. For the neck motion, the initial conditions may be approximately estimated as follows. As two nuclei approach

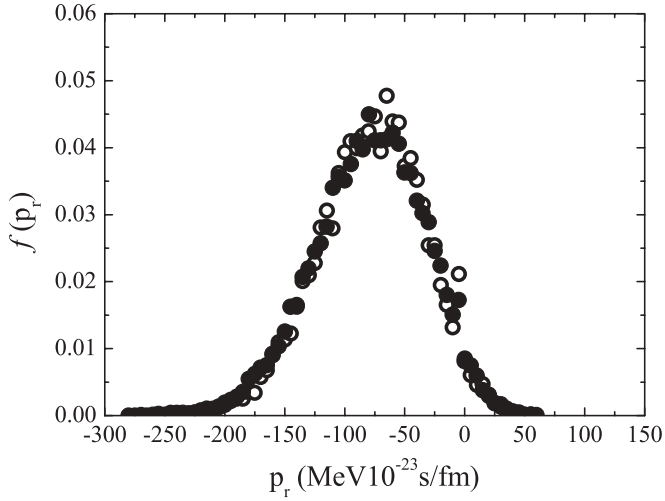

FIG. 2. The radial momentum distributions at the contact point for the system ^{50}Ti + ^{249}Cf at the incident energies of 235 (open circles) and 245 MeV (filled circles), respectively. The distributions are calculated in terms of the surface friction model (SFM).

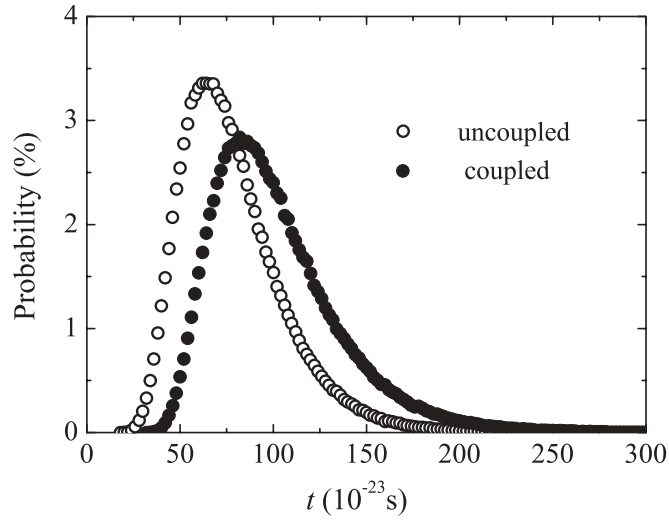

FIG. 3. Distributions of transition time for the system ^{50}Ti + ^{249}Cf with two approaches of the multidimensional Langevin equations. The open and filled circles represent the results of uncoupled and coupled calculations, respectively.

the contact point, the diffused surfaces of the two nuclei overlap. The nucleons in the overlap most probably fill in the neck region because of the incompressibility of nuclear matter. As a consequence, the dinuclear system has already developed a neck at the contact configuration. The number of nucleons in the neck region can be estimated with [22]

$$A_{\text{neck}} = \int d\mathbf{r}\, \varrho(\mathbf{r}) \exp\left[-\frac{(z-l_3)^2}{b^2}\right], \tag{4}$$

where $\varrho(\mathbf{r})$ is the distribution function of nucleon density and the parameter b is set to be 0.8 fm. With the number of nucleons in the neck (A_{neck}) calculated, one can figure out the initial neck radius $n(0)$ at the contact point by means of the nuclear shape as shown in Fig. 1. As the neck size increases from zero to $n(0)$, the potential energy decreases by an amount of $\Delta V(q)$. We assume that this part of the potential energy transforms into the kinetic energy of the neck motion. Based on these assumptions, we set approximately the initial momentum of neck degree of freedom:

$$p_n(0) = \sqrt{2m_{nn}\Delta V(q)}, \tag{5}$$

with the inertia of neck m_{nn} calculated in terms of the Werner-Wheeler method [16,17]. We define $n = \sqrt{0.5}R_i$ with $R_i = \min(R_1, R_2)$ to be the boundary between the dinuclear and mononuclear regimes, which is similar to the criterion suggested by Swiatecki [5].

The Langevin equations are applied for the study of neck dynamics of the mass asymmetric system ^{50}Ti + ^{249}Cf. We apply the equations in the coupled and uncoupled (by switching off the coupling terms) approaches. Figure 3 shows the transition time distributions for the system ^{50}Ti + ^{249}Cf with the two approaches. Here the transition time is defined as the time at which the reaction system arrives at the mononuclear regime, i.e., injection into the asymmetric fission valley takes place. It may be seen from the figure that in the coupled case the transition time increases as compared to the uncoupled results. The deterministic (i.e., in the absence of the stochastic force) velocity of neck growth in a unit of light velocity c is shown in Fig. 4. As shown from the figure, the coupling between different degrees of freedom reduces the velocity of neck growth before $t = 2 \times 10^{-22}$ s. This may turn out to retard the system injecting into the asymmetric fission valley.

Displayed in Figs. 5 and 6 are the s_{in} probability distributions, $f(s_{\text{in}})$ calculated by the uncoupled and coupled Langevin equations at two incident energies. For both cases, the distributions are located at positions very close to $s = 0$. However, for the uncoupled mode the injecting events are much more spread out in the s_{in} space with the distributions deviated from the Gaussian function. Besides, our results show that the quasifissionlike events are unlikely to take place during

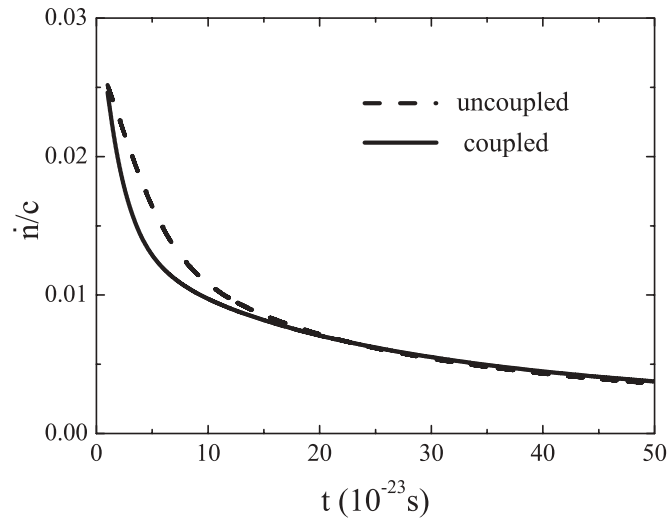

FIG. 4. The deterministic velocities of neck growth in a unit of light velocity c as a function of time for the system ^{50}Ti + ^{249}Cf. The dashed and solid lines stand for the results of uncoupled and coupled approaches, respectively.

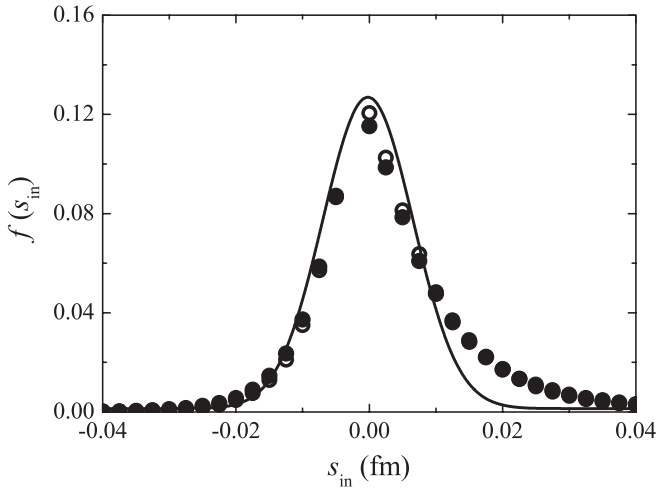

FIG. 5. Probability distributions of s_{in} for the system ^{50}Ti + ^{249}Cf at the incident energies of 235 and 245 MeV, which are represented by the open and filled circles, respectively. They are calculated with the uncoupled Langevin equations. The line in the figure is the Gaussian fit of the data.

the transition period for the mass asymmetric systems such as ^{50}Ti + ^{249}Cf.

III. SYNTHESIS OF SUPERHEAVY NUCLEI ^{295,296}120

Based on the s_{in} distributions obtained, we calculate the formation cross sections of superheavy nuclei ^{295,296}120. The models and parameters used here are the same as used in our previous work [15] except adopting the s_{in} distributions instead of setting $s_{in} = 0$. For the sake of completeness, we will repeat briefly the main points of the formalisms used.

The cross section of a superheavy nucleus produced in a heavy ion fusion-evaporation reaction is calculated as follows:

$$\sigma_{ER}(E) = \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) P_{capt}(E,l) P_{CN}(E,l) P_{xn}(E,l). \quad (6)$$

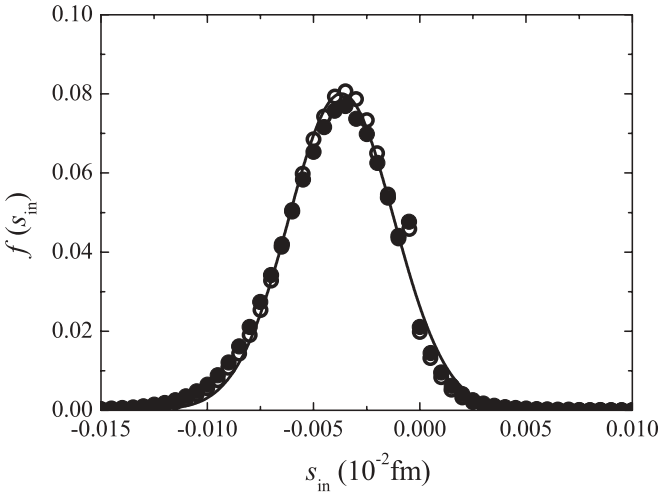

FIG. 6. Same as Fig. 5, but calculated by the coupled Langevin equations.

Here P_{capt} is the capture probability of the colliding nuclei after overcoming the Coulomb barrier and moving up to the contact point. We calculate P_{capt} by means of the approach proposed by Zagrebaev *et al.* [23,24]. In their approach, the coupling between the relative motion of the nuclei and their dynamic as well as static deformation are taken into account in terms of a semiphenomenological barrier distribution function method. The last factor P_{xn} represents the survival probability of the excited compound nucleus after evaporation of x neutrons in the cooling process. We calculate the last factor with a more or less convenient method; for details see Ref. [25]. Starting from the injection point, the system diffuses over the saddle-point hill, and with some probability reaches the compound nucleus configuration due to the thermal fluctuation. P_{CN} defines this probability. According to the fusion-by-diffusion model [2,3], the probability managed to overcome the saddle-point barrier [2,3] is given by

$$P_{CN} = \tfrac{1}{2}\mathrm{erfc}\left(\sqrt{B(s_{in})/T}\right), \quad (7)$$

where $B(s_{in})$ is the height of the barrier opposing fusion along the asymmetric fission valley on the way from the injection point to the saddle and T is an effective temperature. In our approach, the thermal fluctuation in the neck growth process results in a s_{in} distribution. This brings about the height of the barrier $B(s_{in})$ as a relevant distribution. Therefore, the fusion probability should be given by a convolution of function erfc and the probability $f(s_{in})$,

$$P_{CN} = \frac{1}{2}\int \mathrm{erfc}\left(\sqrt{B(s_{in})/T}\right) f(s_{in}) ds_{in}. \quad (8)$$

In Fig. 7, we plot the predictions of evaporation residue (ER) cross sections for 3n and 4n evaporation channels in the ^{50}Ti + ^{249}Cf reaction leading to the formation of ^{296}120 and ^{295}120

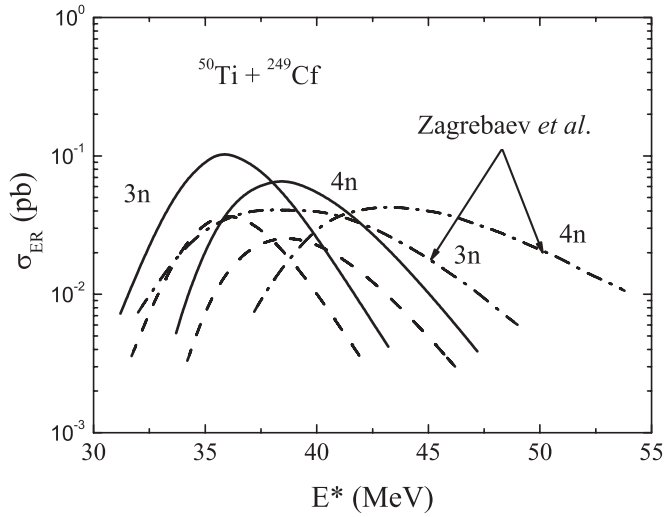

FIG. 7. Predicted evaporation residue cross sections for the 3n and 4n evaporation channels in the ^{50}Ti + ^{249}Cf reaction leading to the formation of ^{296}120 and ^{295}120 isotopes. The dashed and solid lines are the results of our work calculated by Eqs. (7) and (8), respectively. The dash-dotted lines stand for the predictions of Zagrebaev and Greiner [26].

isotopes. The results of Zagrebaev and Greiner [26] are also presented in this figure as dash-dotted lines. The dashed lines in the figure are obtained from the calculations with Eq. (7) assuming the injection point at the separation distance $s_{in} = 0$, while the solid lines are the results calculated by using Eq. (8) with the s_{in} probability distributions predicted by the coupled and uncoupled Langevin equations. The results obtained with these two approaches are indistinguishable in the figure. Our calculated maximum ER cross sections in $3n$ and $4n$ channels with the s_{in} distributions are 0.1 and 0.065 pb, respectively, which are more than two times larger than those obtained with the $s_{in} = 0$ assumption.

IV. SUMMARY AND CONCLUSIONS

The coupled Langevin equations in two-dimensional collective space have been used to study the dynamics of neck growth. Special attention is paid to the effects of coupling between the neck and radial degrees of freedom on the evolution from dinucleus to mononuleus. In this dynamic model, the shape of the nuclear surface is specified by two spheres smoothly joined with a hyperboloidal neck, the nuclear potential energy of deformation, the inertia, and the dissipation tensors are given by the finite range liquid-drop model (FRLDM) [9,10], the Werner-Wheeler approximation [16,17] for incompressible and irrotational flow, and the wall-plus-window model [11,12].

The dynamic model is applied for the study of neck evolution of the mass asymmetric system ^{50}Ti + ^{249}Cf. We have used the equations in the coupled and uncoupled approaches. By calculating the velocity of the neck growth and the transition time distributions, we have found that the coupling between neck and radial motions reduces the neck growth velocity and delays the transition from dinuclear to mononuclear regimes. In addition, by solving these dynamic equations, we obtained the s_{in} probability distribution at the injection point in the asymmetric fission valley. In this way, s_{in} is no longer an adjustable parameter in the fusion-by-diffusion model. The distributions obtained are located at positions very close to $s_{in} = 0$. In addition, our results have shown that the quasifissionlike events are unlikely to take place during the transition period for the mass asymmetric systems.

Based on the s_{in} distributions obtained, the evaporation residue cross sections for $3n$ and $4n$ evaporation channels in the ^{50}Ti + ^{249}Cf reaction leading to the formation of ^{296}120 and ^{295}120 isotopes have been calculated. The maximum ER cross sections in $3n$ and $4n$ channels with the s_{in} distributions are equal to 0.1 and 0.065 pb, respectively, which are more than two times larger than those obtained by the original fusion-by-diffusion model with the $s_{in} = 0$ assumption [15]. However, the ER cross sections calculated by the coupled and uncoupled approaches are practically the same.

Along with the elements $Z = 118$ and 117 produced at Dubna [27,28], superheavy element $Z = 120$ may become the next element to be synthesized. Therefore, the theoretical predictions of its production cross sections are highly required. The cross section of 0.1 pb is too small to synthesize superheavy elements under the present experimental limit for the registration of the evaporation residual nuclei. However, as pointed out by Oganessian [29] it is technically possible to increase the intensity of the projectile beams and use more effective "on-line" mass separation methods with thicker target materials. By means of the improved experimental conditions, superheavy element 120 may be synthesized in the near future.

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