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Liquid-gas phase transition in hot asymmetric nuclear matter

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We use a density-dependent relativistic mean field model to study the properties of nuclear systems at finite temperature. The liquid-gas phase transition of symmetric and asymmetric nuclear matter is discussed. A limiting pressure $p_{\rm lim}$ for hot asymmetric nuclear matter has been found because of the density dependence of the nucleon–nucleon–ρ meson coupling. It is found that the liquid-gas phase transition cannot take place if $p > p_{\rm lim}$. The binodal surface for this model is addressed. In addition, we calculated the asymmetry parameter dependence of the critical temperature.

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I. INTRODUCTION

The study of nuclear interactions under extreme conditions such as large isospin asymmetry and high temperature is of considerable interest. The van der Waals behavior of the nucleon-nucleon interaction is expected to lead to the so-called liquid-gas phase transition in nuclear matter. So the study of the liquid-gas phase transition in medium-energy heavy-ion collisions has attracted much interest for many years [1–8].

It is generally recognized that the liquid-gas phase transition of a one-component system is of first order. The chemical potential is continuous at the phase transition point but its first derivatives (entropy and volume) are discontinuous. Theoretically, much effort has been devoted to studying the equation of state for nuclear matter and to discussing the critical temperature T_C. The calculated critical temperature of symmetric nuclear matter lies in the range 13–24 MeV for various phenomenological models [2,3].

But for a multicomponent or multiple-conserved-charge system, as was pointed out by Müller and Serot [4], the liquid-gas phase transition is suggested to be of second order because of the greater dimensionality of the binodal surface. Asymmetric nuclear matter has two components, the proton and neutron, and the two conserved charges, of baryon number and the third component of isospin, will undergo a continuous second-order phase transition. Obviously, because of charge independence, the basic difference between the proton and neutron is isospin. The isospin-dependent interactions nucleon–nucleon–ρ mesons play the key role in understanding the liquid-gas phase transition since the chemical potentials of proton and neutron may depend on the third component I_3 of isospin.

In fact, the chemical potentials of the proton and neutron depend not only on I_3 but also on the $NN\rho$ coupling parameter g_ρ. Then the coupling parameter g_ρ is also essential for studying the liquid-gas phase transition because the chemical potentials determine the binodal surface directly. In Ref. [5], Qian and co-workers indicated that the effective *NN*-meson couplings are all dependent on density and temperature from the viewpoint of finite-temperature quantum field theory. By using thermofield dynamics to calculate the three-line vertex Feynman diagrams of $NN\rho$ interactions, these researchers argued that the effective coupling g_ρ decreases as the nucleon density increases. Without generality, they introduced an ansatz $g'_\rho = g_\rho(1 - \rho_B)$ to discuss the liquid-gas phase transition. Then a limiting pressure $p_{\rm lim}$ was found for a fixed temperature; the liquid-gas phase transition cannot take place in asymmetric nuclear matter if $p > p_{\rm lim}$.

It is also necessary to address the liquid-gas phase transition from different points of view, with different models and different treatments, because this may open different fields of vision. Recently relativistic mean field (RMF) theory with density-dependent (DD) meson-nucleon couplings was developed by various authors [9–11]. In the density-dependent RMF theory, the medium dependence of the nucleon-meson vertices is expressed by the baryon field operators. The nucleon-meson coupling constants in nuclear matter are adjusted to the Dirac-Brueckner self-energies. A Lorentz-invariant functional is defined to project the nuclear matter results onto the nucleon-meson vertices of the DD-ME1 model.

The organization of this paper is as follows. In the next section, we give the main formulas for the liquid-gas phase transition in the DD-ME1 model. In the third section some numerical results are presented. The last section contains a summary and discussions.

II. THE MODEL AND ASYMMETRIC NUCLEAR MATTER AT FINITE TEMPERATURE

Among the existing parametrizations for RMF theory with DD meson-nucleon coupling, the most frequently used are that of Typel and Wolter 1999 (TW99) [9] and DD-ME1 [11]. To illustrate our result, we would like to use the RMF model with DD-ME1 meson-nucleon coupling to discuss the properties of the liquid-gas phase transition in nuclear matter. This model has been proven to be successful in explaining many experimental properties of both nuclear matter and finite nuclei

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in the mean-field approximation. The Lagrangian density of the model is

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M + \Gamma_\sigma\sigma - \Gamma_\omega\gamma^\mu\omega_\mu - \frac{\Gamma_\rho}{2}\gamma^\mu\vec{\tau}\cdot\vec{\rho}^\mu\right]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}. \tag{1}$$

The Dirac spinor ψ denotes the nucleon with mass M. m_σ, m_ω, and m_ρ are the masses of the σ, ω, and ρ mesons, respectively. The antisymmetric tensors of the vector mesons take the forms $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\overrightarrow{G}_{\mu\nu} = \partial_\mu\overrightarrow{\rho}_\nu - \partial_\nu\overrightarrow{\rho}_\mu$. $\Gamma_\sigma, \Gamma_\omega$, and Γ_ρ are the coupling parameters between nucleon and σ meson, nucleon and ω meson, and nucleon and ρ meson, respectively. $\Gamma_\sigma, \Gamma_\omega$, and Γ_ρ are assumed to be vertex functions of Lorentz-scalar bilinear forms of the nucleon operators [9–11]. In most applications of the density-dependent RMF theory the meson-nucleon couplings are functions of the vector density

$$\rho_v = \sqrt{j_\mu j^\mu} \quad \text{with} \quad j_\mu = \bar{\psi}\gamma_\mu\psi. \tag{2}$$

The single-nucleon Dirac equation is derived by variation of the Lagrangian (1) with respect to $\bar{\psi}$:

$$[\gamma^\mu(i\partial_\mu - \Sigma_\mu) - (M - \Sigma)]\psi = 0 \tag{3}$$

with the nucleon self-energies defined by the following relations:

$$\Sigma = \Gamma_\sigma\sigma, \tag{4}$$

$$\Sigma_\mu = \Gamma_\omega\omega_\mu + \frac{\Gamma_\rho}{2}\vec{\tau}\cdot\vec{\rho}_\mu + \Sigma_\mu^R. \tag{5}$$

The density dependence of the coupling constants $\Gamma_\sigma, \Gamma_\omega$, and Γ_ρ produces the rearrangement contribution Σ_μ^R to the vector self-energy,

$$\Sigma_\mu^R = \frac{j_\mu}{\rho_v}\left(\frac{\partial\Gamma_\omega}{\partial\rho_v}\bar{\psi}\gamma^\nu\psi\omega_\nu + \frac{\partial\Gamma_\rho}{\partial\rho_v}\bar{\psi}\gamma^\nu\frac{\vec{\tau}}{2}\psi\cdot\vec{\rho}_\nu + \frac{\partial\Gamma_\sigma}{\partial\rho_v}\bar{\psi}\psi\sigma\right). \tag{6}$$

The inclusion of the rearrangement self-energies is essential for energy-momentum conservation and the thermodynamic consistency of the model [12].

The field equations are solved in the mean field approximation for infinite nuclear matter: the meson field operators are replaced by their expectation values. The expectation values of σ, ω, and ρ are still denoted as σ, ω, and ρ. Then by using the standard techniques of statistical mechanics, we get the thermodynamic potential Ω at finite temperature as

$$\Omega = -\frac{k_BT}{(2\pi)^3}\sum_{N=p,n}\int_0^\infty d^3\vec{k}\{\ln(1+e^{-[E_N^*(k)-\nu_N]/k_BT}) + \ln(1+e^{-[E_N^*(k)+\nu_N]/k_BT})\} + \left\{-\frac{1}{2}m_\sigma^2\sigma^2\left[1+2\frac{\rho_B}{\Gamma_\sigma}\frac{\partial\Gamma_\sigma}{\partial\rho_B}\right] + \frac{1}{2}m_\omega^2\omega^2 \times \left[1+2\frac{\rho_B}{\Gamma_\omega}\frac{\partial\Gamma_\omega}{\partial\rho_B}\right] + \frac{1}{2}m_\rho^2\rho^2\left[1+2\frac{\rho_B}{\Gamma_\rho}\frac{\partial\Gamma_\rho}{\partial\rho_B}\right]\right\}, \tag{7}$$

where $E_N^*(k) = \sqrt{M_N^{*2} + \vec{k}^2}$ and T is temperature. The quantity ν_i is related to the usual chemical potential μ_i by the equations

$$\nu_n = \mu_n - \left(\Gamma_\omega + \frac{\partial\Gamma_\omega}{\partial\rho_B}\rho_B\right)\omega + \left(\frac{\Gamma_\rho}{2} + \frac{\partial\Gamma_\rho}{\partial\rho_B}\rho_3\right)\rho + \frac{\partial\Gamma_\sigma}{\partial\rho_B}\rho_s\sigma, \tag{8}$$

$$\nu_p = \mu_p - \left(\Gamma_\omega + \frac{\partial\Gamma_\omega}{\partial\rho_B}\rho_B\right)\omega + \left(-\frac{\Gamma_\rho}{2} + \frac{\partial\Gamma_\rho}{\partial\rho_B}\rho_3\right)\rho + \frac{\partial\Gamma_\sigma}{\partial\rho_B}\rho_s\sigma, \tag{9}$$

where $\rho_3 = \rho_p - \rho_n$ and ρ_s is the scalar density. The right-hand sides of Eqs. (8) and (9) depend on ρ_3 and Γ_ρ and their derivatives with respect to ρ_B. They play the essential role in determining the liquid-gas phase transition.

Having obtained the thermodynamic potential, all other thermodynamic quantities, for example, pressure p, can be calculated as follows:

$$p = \frac{1}{3}\frac{2}{(2\pi)^3}\int d^3k\frac{k^2}{\sqrt{k^2+M^{*2}}}[n_n(k)+\bar{n}_n(k)] + \frac{1}{3}\frac{2}{(2\pi)^3}\int d^3k\frac{k^2}{\sqrt{k^2+M^{*2}}}[n_p(k)+\bar{n}_p(k)] - \frac{1}{2}m_\sigma^2\sigma^2\left[1+2\frac{\rho_B}{\Gamma_\sigma}\frac{\partial\Gamma_\sigma}{\partial\rho_B}\right] + \frac{1}{2}m_\omega^2\omega^2\left[1+2\frac{\rho_B}{\Gamma_\omega}\frac{\partial\Gamma_\omega}{\partial\rho_B}\right] + \frac{1}{2}m_\rho^2\rho^2\left[1+2\frac{\rho_B}{\Gamma_\rho}\frac{\partial\Gamma_\rho}{\partial\rho_B}\right], \tag{10}$$

where

$$n_N(k) = (\exp\{[E_N^*(k)-\nu_N]/k_BT\}+1)^{-1},$$
$$\bar{n}_N(k) = (\exp\{[E_N^*(k)+\nu_N]/k_BT\}+1)^{-1} \quad (N = n, p),$$

are the nucleon and antinucleon distributions, respectively.

The two-phase coexistence equations are

$$\mu_i^L(T,\rho_i^L) = \mu_i^V(T,\rho_i^V), \tag{11}$$

$$p^L(T,\rho_i^L) = p^V(T,\rho_i^V), \tag{12}$$

where the subscripts of each phase L and V stand for liquid and vapor, respectively. The stability conditions are given by [4]

$$\rho_B\left(\frac{\partial p}{\partial\rho_B}\right)_{T,\alpha} = \rho_B^2\left(\frac{\partial^2 F}{\partial\rho_B^2}\right)_{T,\alpha} > 0, \tag{13}$$

$$\left(\frac{\partial\mu_p}{\partial\alpha}\right)_{T,p} < 0 \quad \text{or} \quad \left(\frac{\partial\mu_n}{\partial\alpha}\right)_{T,p} > 0, \tag{14}$$

where F is the density of free energy, $\alpha = (\rho_n - \rho_p)/\rho_B$ the asymmetric parameter, and $\rho_B = \rho_n + \rho_p$.

For the parametrization of the density dependence of the coupling constants of mesons, we choose [11]

$$\Gamma_i(\rho_B) = \Gamma_i(\rho_B^{\text{sat}})f_i(x) \quad \text{for} \quad i = \sigma, \omega, \tag{15}$$

$$\Gamma_\rho(\rho_B) = \Gamma_\rho(\rho_B^{\text{sat}})\exp[-a_\rho(x-1)], \tag{16}$$

TABLE I. Parameter set in the DD-ME1 model [11].

m_σ	549.5255	c_σ	1.5342
m_ω	783.0000	d_σ	0.4661
m_ρ	763.0000	a_ω	1.3879
$\Gamma_\sigma(\rho_B^{\rm sat})$	10.4434	b_ω	0.8525
$\Gamma_\omega(\rho_B^{\rm sat})$	12.8939	c_ω	1.3566
$\Gamma_\rho(\rho_B^{\rm sat})$	7.6106	d_ω	0.4957
a_σ	1.3854	a_ρ	0.5008
b_ω	0.9781		

where

$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \tag{17}$$

is a function of $x = \rho_B/\rho_B^{\rm sat}$. This function is able to fit very well the normalized couplings derived from different Dirac-Brueckner calculations of symmetric nuclear matter. The parameters used in the DD-ME1 model are listed in Table I.

III. NUMERICAL RESULTS

We first discuss the liquid-gas phase transition of symmetric nuclear matter. In Fig. 1, we show the pressure of the system versus nucleon density at different temperatures. At low temperature, the pressure first increases and then decreases with increasing density. The p-ρ_B isotherms exhibit the form of two-phase coexistence, with an unphysical region for each. At temperature $T = 13.2$ MeV, there appears a point of inflection, where $\partial p/\partial \rho_B = 0$, $\partial^2 p/\partial^2 \rho_B = 0$. This temperature is called the critical temperature. Symmetric nuclear matter can only be in the gas phase above this temperature.

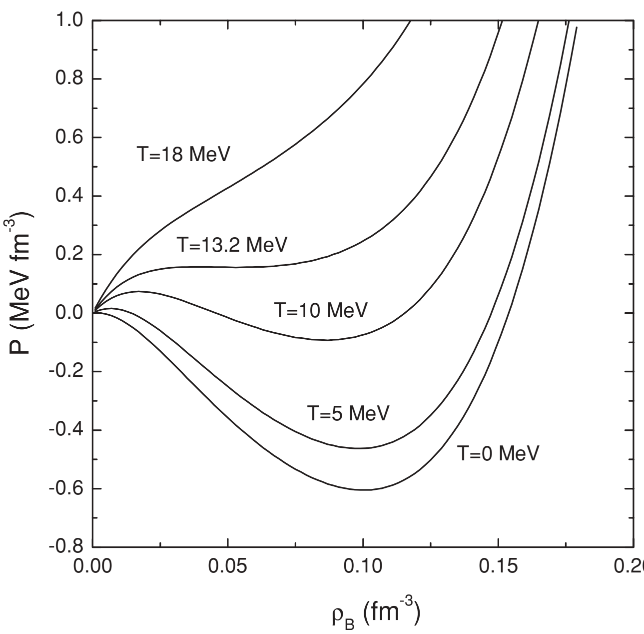

FIG. 1. The pressure of symmetric nuclear matter p versus nucleon density ρ_B at different temperatures.

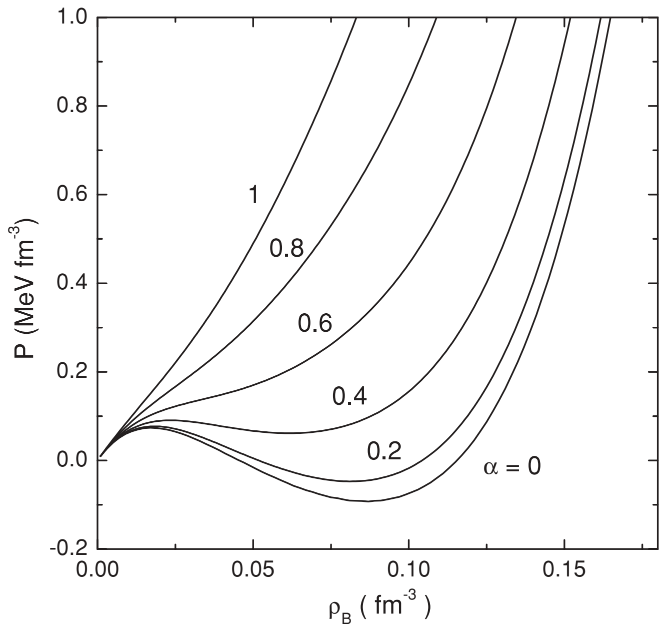

FIG. 2. Pressure of asymmetric nuclear matter versus baryon density at temperature $T = 10$ MeV with various values of α.

Figure 2 shows the pressure versus nucleon density at $T = 10$ MeV with different asymmetry parameters α. One can see that when α is small (say $\alpha < 0.4$), the pressure has a minimum. When α is large, the minimum disappears and the pressure increases monotonically with the increasing density.

Let us now turn to a disscussion of the phase transition of asymmetric nuclear matter. For the asymmetric case, the situation is more complicated. One cannot get the critical temperature from the p-ρ_B isotherms. The chemical potentials of the proton and neutron are different. In Fig. 3, we show the

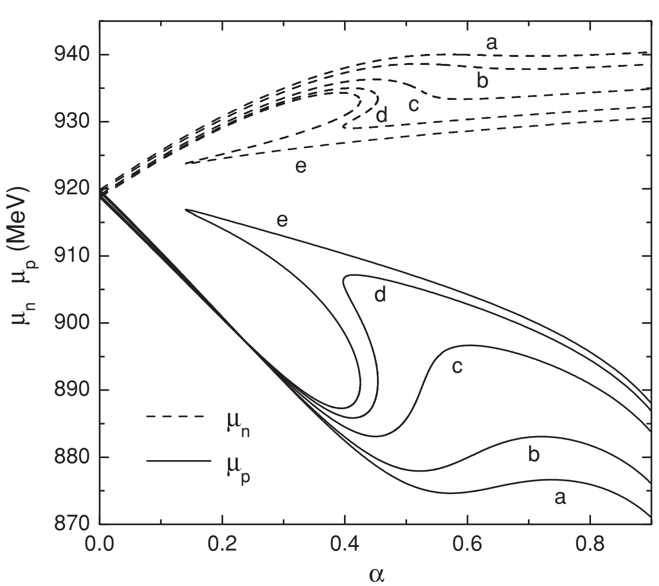

FIG. 3. The chemical isobars as a function of α at fixed temperature $T = 10$ MeV; a, b, c, d, and e refer to the pressures 0.220, 0.180, 0.120, 0.090, and 0.075 MeV fm^{-3}, respectively.

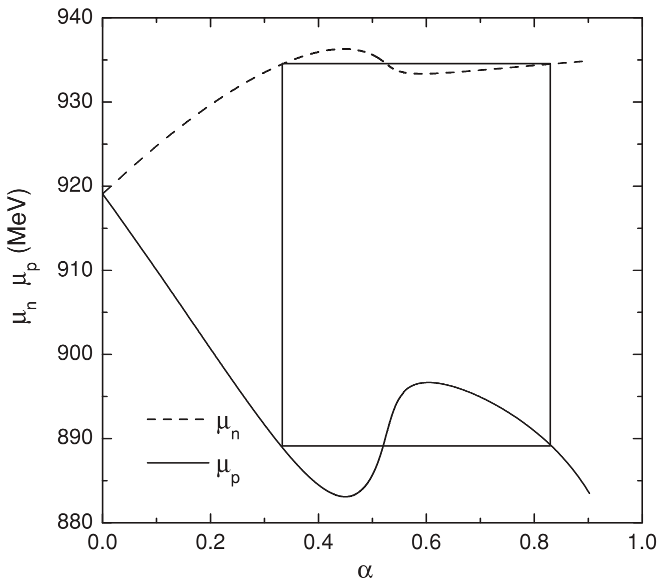

FIG. 4. Geometrical construction used to obtain the chemical potentials and asymmetry parameters in the two-phase coexistence at temperature $T = 10$ MeV and $p = 0.12$ MeV fm^{-3}.

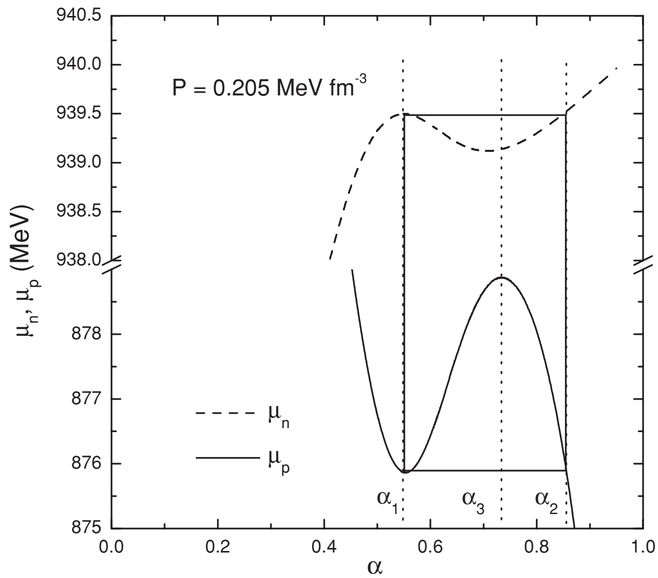

FIG. 6. Same as Fig. 4 but for $p = 0.205$ MeV fm^{-3}. For clarity, we enlarged the figure in one area.

μ_n, μ_p isobar as a function of α at temperature $T = 10$ MeV. The curves a, b, c, d, and e correspond to the pressures 0.220, 0.180, 0.120, 0.090, and 0.075 MeV fm^{-3}, respectively. The solid lines are for the proton and the dashed lines for the neutron. We see that the curves for lower pressures are more complicated than those for higher pressures. The Gibbs conditions (13) and (14) for phase equilibrium demand equal pressures and chemical potentials for two phases with different concentrations. The desired solutions can be found by means of the geometrical construction for the case of $p = 0.120$ MeV fm^{-3} shown in Fig. 4, which guarantees the same pressures and chemical potentials for protons and neutrons in the two phases with different asymmetry parameters α. The phase with higher density corresponds to the liquid phase and the lower-density one to the gas phase.

The pairs of solutions found by the geometrical method described above yield a binodal surface which is shown in Fig. 5. The binodal curve is divided into two branches. One branch corresponds to the high-density (liquid) phase, the other to the low-density (gas) phase. From Fig. 5, one can see that there exists a limiting pressure $p_{\mathrm{lim}} = 0.205$ MeV fm^{-3};

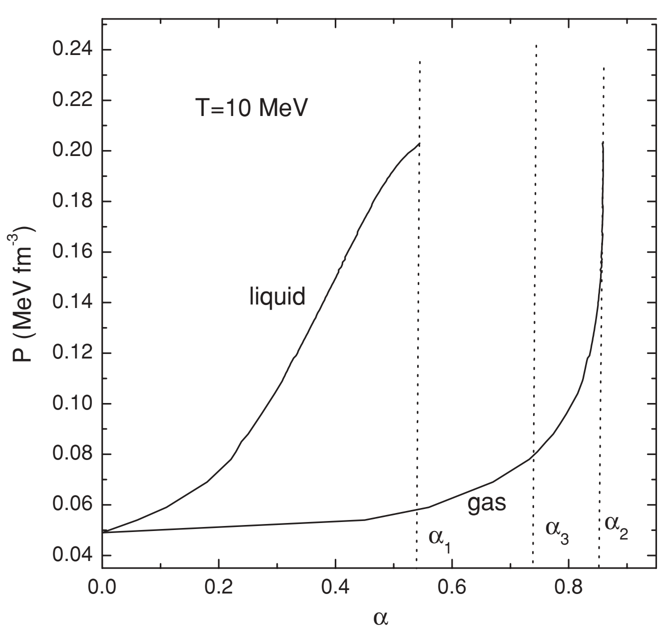

FIG. 5. Calculated variation of the relative populations of neutron stars including hyperons in the case of attractive Σ potential with respective to the total baryon density.

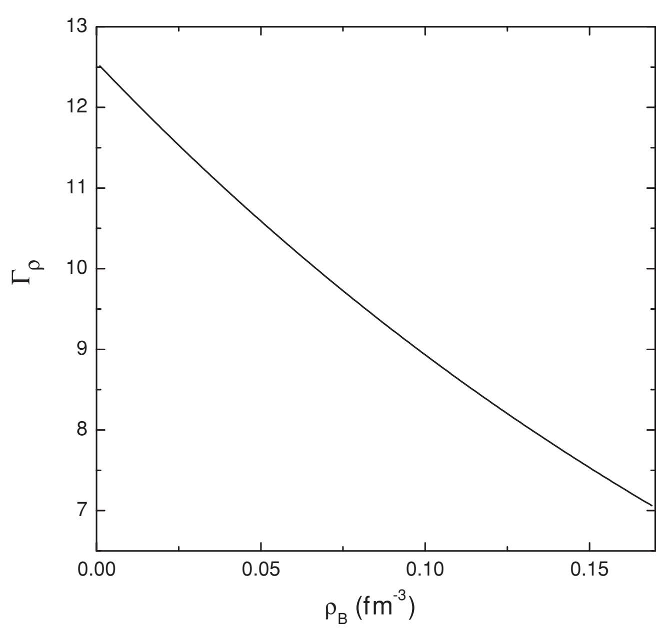

FIG. 7. Density dependence of the coupling between the ρ-meson and baryon in the density-dependent DD-ME1 parametrization.

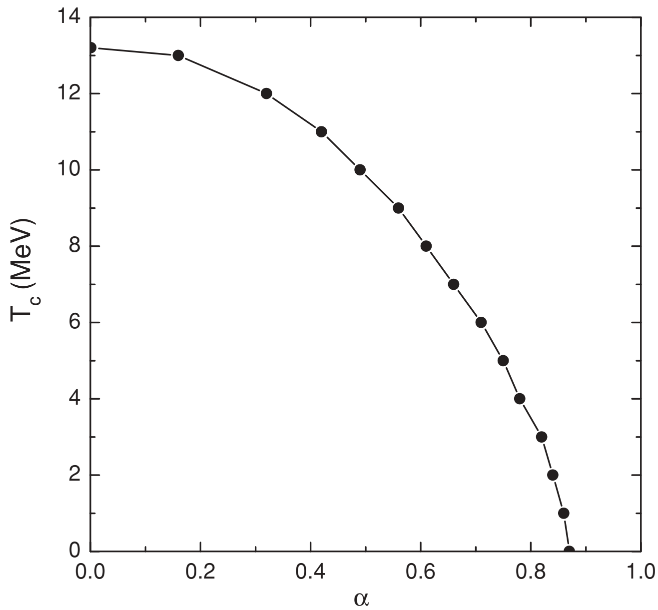

FIG. 8. The critical temperature T_c versus asymmetry parameter α.

when $p > p_{\rm lim}$ the rectangle cannot be found by means of the geometrical construction shown in Fig. 4 and the coexistence equations have no solution. The last rectangle in the chemical isobar versus α curves for $T = 10$ MeV and $p_{\rm lim} = 0.205$ MeV fm^{-3} is shown in Fig. 6, where $\alpha_1 = 0.54$ and $\alpha_3 = 0.74$ correspond to the local maximum and the local minimum of μ_n, respectively. The pair $\alpha_1 = 0.54$ and $\alpha_2 = 0.85$ form the end of the binodal surface, as shown in Fig. 5. The limiting pressure in asymmetric nuclear matter was first discovered by Qian *et al.* [5] by introducing the effective g'_ρ ansatz $g'_\rho = g_\rho(1 - \rho_B)$ in the $NN\rho$ coupling. In their calculation, the value of the limiting pressure is 0.130 MeV fm^{-3}. In the DD-ME1 model, the coupling parameter between the ρ meson and nucleon is proposed as given in Eq. (16) by Ring and co-workers [11]. In Fig. 7, we draw g_ρ as a function of ρ_B and find that g_ρ decreases monotonically with increasing baryon density. In conclusion, we have shown that the density dependence of the $NN\rho$ interaction is important to the liquid-gas phase transition. If the coupling parameter between the ρ meson and nucleon decreases as a function of ρ_B, a limiting pressure may exist.

In order to address the liquid-gas phase transition more clearly, we pay more attention to the binodal curve in Fig. 4 following Ref. [5]. The total asymmetry parameter α is divided into three regions, namely, [0,α_1], [α_1,α_3], and [α_3,α_2]. The physical behavior of isothermal compression in the different regions is different. In the first region [0,α_1], the system begins in the gas phase, experiences a liquid-gas phase transition, and ends in the liquid phase.In the second region [α_1,α_3], it begins in the gas phase, enters a two-phase region, and becomes unstable at the limiting pressure, since the chemical stability condition Eq. (14) is destroyed in this region. In the third region [α_3,α_2], the system will end in a stable phase at the limit pressure as the stability condition Eq. (14) is satisfied.

The α dependence of the critical temperature T_c is shown in Fig. 8. T_c decreases with increasing α. When α is larger than 0.87, the system can only be in the gas phase at any temperature. A liquid-gas phase transition can occur for nuclear matter with $\alpha < 0.87$ if the temperature is lower than the critical temperature.

IV. SUMMARY AND DISCUSSION

In summary, we have used a density-dependent relativistic mean field model (DD-ME1) to address the properties of the liquid-gas phase transition of symmetric and asymmetric nuclear matter. A limiting pressure $p_{\rm lim}$ for hot asymmetric nuclear matter has been found because of the density dependence of the nucleon–nucleon–ρ meson coupling. It is found that the liquid-gas phase transition cannot take place if $p > p_{\rm lim}$. However, our investigation is limited to infinite nuclear matter. It is of interest to extend the present study to finite nuclei. We leave that study for the future.

We should mention that theories not based on mean field theories find that the liquid-gas phase transition is of first order rather than second order [13]. The second-order feature seems to be a property of mean field theory which treats the matter as uniform. The matter is not uniform in Ref. [13] but it is clustered. A related study is in progress.

ACKNOWLEDGMENTS

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