PHYSICAL REVIEW C **83**, 044311 (2011)

Quadrupole moment measurements for strongly deformed bands in ^{171,172}Hf

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(Received 9 February 2011; published 13 April 2011)

A lifetime experiment, using the Doppler-shift attenuation method, has been performed at Gammasphere to measure the transition quadrupole moments Q_t of strongly deformed bands in ^{171}Hf and ^{172}Hf. The measured value of $Q_t \sim 9.5$ e b for the band labeled ED in ^{171}Hf strongly supports the recent suggestion that this sequence and several structures with similar properties in neighboring Hf isotopes are associated with a near-prolate shape with a deformation enhanced relative to that of normal deformed structures. The measured values of Q_t ~ 14 e b for the bands labeled SD1 and SD3 in ^{172}Hf confirm that these sequences are associated with a prolate superdeformed shape, a property inferred in earlier work from other measured characteristics of the bands. Similar bands in $^{173-175}$Hf are also likely to be associated with superdeformed shapes. The observations are in contrast to predictions of cranking calculations performed with the ULTIMATE CRANKER code.

DOI: 10.1103/PhysRevC.83.044311 PACS number(s): 21.10.Tg, 21.60.Ev, 23.20.Lv, 27.70.+q

I. INTRODUCTION

Wobbling motion, a unique signature of nuclei with a stable triaxial deformation, was predicted about 35 years ago [1]. Only in recent years has this exotic excitation mode been established in $^{163,165,167}_{71}$Lu [2–5] and $^{167}_{73}$Ta [6] and, possibly, in ^{161}Lu [7], forming an island of triaxial strongly deformed (TSD) structures. Several theoretical investigations successfully predicted the existence of this TSD island. Among these, systematic cranking calculations by R. Bengtsson [8] using the ULTIMATE CRANKER (UC) code [9,10] predicted high-spin TSD minima with deformation parameters (ϵ_2, γ) $\sim$ (0.40, $\pm 20^\circ$) in potential energy surfaces (PES) for nuclei with $Z \sim 72$ and $N \sim 94$, where they coexist with normal deformed (ND) structures with (ϵ_2, γ) $\sim$ (0.25, 0°). These TSD minima are caused by the presence at large triaxiality of significant single-particle shell gaps for proton numbers $Z = 71, 72$ and neutron numbers $N = 94, 97$ [11,12]. According to these calculations, isotopes around $^{166}_{72}$Hf$_{94}$ should be located at the center of the predicted TSD island. However, among all the high-spin structures observed in Hf isotopes so far, TSD bands have been identified only in ^{168}Hf [13,14], and unlike in the Lu and Ta isotopes mentioned above, no evidence for wobbling was found. This feature was attributed to the presence of many TSD bands built on different quasiparticle excitations that compete favorably in excitation energy with the wobbling mode [14,15].

A number of high-spin strongly deformed bands were reported in the heavier Hf isotopes, e.g., in ^{170}Hf [16], ^{173}Hf [17], ^{174}Hf [17,18], and ^{175}Hf [19]. Some of these bands were suggested to be of TSD character [16,18,19], but such assignments raised several questions. A majority of these bands was observed in the heavier Hf nuclei, for example, eight in ^{174}Hf$_{102}$, which is rather far away from the predicted neutron shell gaps. In addition, these bands are more strongly populated than the bands in ^{168}Hf$_{96}$. Furthermore, the measured transition quadrupole moments, $Q_t \approx 13-14.5$ e b, for the bands in ^{173,174}Hf [17] are significantly larger than the values predicted for the TSD structures by UC calculations, e.g., $Q_t \approx 9.9$ e b in ^{174}Hf for a PES minimum at (ϵ_2, γ) $\sim$ (0.45, 27°). Since the Q_t moment is proportional to $\cos(30^\circ + \gamma)$ [9], the larger Q_t value implies a superdeformed nuclear shape or a smaller triaxiality than that of the predicted TSD structures or a combination of both possibilities. The larger Q_t moments could also result from the negative gamma minimum. However, this scenario is considered to be less favored because such a rotational motion requires higher excitation energy. In short, a convincing description consistent with the experimental results was not available for any of the proposed TSD structures in the heavier Hf isotopes.

To clarify the situation, a new experiment was performed where three strongly deformed bands in ^{172}Hf and one in ^{171}Hf were observed [20]. Based on a systematic investigation of the properties of *all* the strongly deformed bands in $^{170-175}$Hf, on cranking calculations employing the UC code, and on cranked relativistic mean-field (CRMF) calculations [20], it was suggested that these structures fall into two groups: the enhanced deformed (ED) and the superdeformed (SD) bands. The UC calculations are consistent with the observed properties (aligned angular momentum, excitation energy, etc.) of the ED bands, which include the new band in ^{171}Hf as well as the previously proposed TSD candidates band 1 in ^{170}Hf [16], band 1 in ^{175}Hf [19], and band ED in ^{168}Hf [14]. The calculations suggest that all these ED bands are associated with near-prolate shapes and an enhanced deformation with respect to the ND bands. Their configurations involve quasiprotons from the $i_{13/2}$ and $h_{9/2}$ orbitals. The magnitude of the calculated quadrupole moments for the ED bands is of the order of 8.5−9 e b. The CRMF calculations indicate that the SD band (band 2) in ^{175}Hf and, very likely, similar bands in $^{172-174}$Hf result from a superdeformed minimum that has little triaxial deformation and involves the $i_{13/2}$ quasiproton with the $j_{15/2}$ quasineutron. In contrast, the UC calculations with standard parameters suggest that the TSD minima lie lower in energy at the observed spins than the SD minimum for ^{171,172}Hf [20]. As mentioned above, the UC calculations failed to reproduce the experimental quadrupole moments of the SD structures in ^{173,174}Hf, and it is plausible that the same may occur in ^{172}Hf as well.

In this article, the results of quadrupole moment measurements for the strongly deformed bands in ^{171,172}Hf are reported. The measured value of the quadrupole transition moment $Q_t \sim 9.5$ e b for band ED in ^{171}Hf represents the first extensive determination of quadrupole moments for states of any ED band in these Hf isotopes. The results were obtained for six states in band ED through a detailed line-shape analysis. The Q_t values are in line with the predictions by the UC calculations. Based on the overall satisfactory agreement between the observed and calculated properties for the band ED in ^{171}Hf, the interpretation of this band, and most likely of similar sequences in ^{170,175}Hf and in ^{168}Hf, can now be viewed as well established. The measured $Q_t \sim 14$ e b value for band 1 in ^{172}Hf is very similar to those of the SD bands in ^{173,174}Hf [17] and is significantly larger than the quadrupole moment of TSD bands predicted by UC calculations. These results, together with considerations on the intensities and excitation energies of the bands, rule out the possibility of associating bands SD1–SD3 with the calculated TSD minimum. It is likely that all of the latter structures can be associated with superdeformation with little or no triaxiality.

This paper consists of three parts: the description of the line-shape analysis for band ED in ^{171}Hf, the centroid-shift analysis for the SD bands in ^{172}Hf, and, finally, the discussion of experimental results and the theoretical understandings of these bands.

II. EXPERIMENT, DATA ANALYSIS, AND RESULTS

The experiment was carried out at the Argonne Tandem Linear Accelerator System (ATLAS). The ^{128}Te(^{48}Ca, xn) reaction was employed at a beam energy of 207 MeV to populate the bands of interest in ^{171,172}Hf. The target was a 1.0-mg/cm^2-thick isotopically enriched foil, backed by a 15.81-mg/cm^2 layer of Au to slow down and stop the recoiling nuclei. A thin layer of Au of thickness 70 μg/cm^2 was evaporated on the front side of the target to prevent oxidation and sputtering. The beam energy was chosen to be the same as in the earlier measurement [20], where the population of the high-spin bands in the nuclei under investigation was optimized. Coincidence events with fold 3 and above were recorded using the Gammasphere array [21] operating in a stand-alone mode; the array was composed of 101 Compton-suppressed, large-volume germanium detectors at the time of this measurement. A total of about 6.2×10^9 events was accumulated.

Since the Doppler-shift attenuation method (DSAM) [22] involves detection of γ rays at different angles during the slowing-down process of the recoiling nuclei in the thick target, a BLUE database [23] was created. Double-gated spectra were produced from the database where the coincident γ rays were histogrammed into separate spectra based upon the angle of the detector in which the transition was observed. The spectra were background subtracted using the method outlined in Ref. [24].

A. Line-shape analysis for band ED in ^{171}Hf

Distinct line shapes, including stopped components for each transition of band ED in ^{171}Hf, were visible at forward and backward angles. This indicates that the lifetimes of the states are, indeed, comparable to the stopping time of the recoils. Wide coincidence gates corresponding to each angle were placed taking the full peak shape into account in order to avoid a possible lifetime bias. Spectra from forward, transverse, and the supplementary backward angles were then used for the line-shape analysis. For greater statistics, spectra from the rings at 69.8°, 79.2°, and 80.7° were summed for the forward angles, and those at 110.2°, 100.8°, and 99.3° were summed for the backward angles; these were found to exhibit distinct line shapes and had sufficient statistics for an analysis of transitions up to the 914-keV γ ray depopulating the $71/2^+$ state in the ED band. Velocity profiles were then calculated at the average angle for each detector group. Only the above-mentioned forward and backward rings near 90° were used because the 627-keV transition in band ED was always taken as one of the gating transitions to avoid contamination from the more dominant [521]$1/2^-$ band members. However, its moving component overlaps with the large stopped component of the 621-keV line of the [521]$1/2^-$ band in spectra recorded at backward angles greater than 110.2°, and its stopped component interferes with the moving component of the same 621-keV γ ray at forward angles smaller than 69.8°.

For the above-mentioned angle set, the lifetimes of states in band ED were extracted using the LINESHAPE analysis codes of Wells and Johnson [25]. A total of 5000 Monte Carlo simulations of the velocity history of the recoiling nuclei traversing the target and backing material was generated in time steps of 0.002 ps. Electronic stopping powers were calculated following the heavy-ion stopping theory by Ziegler *et al.* [26], and velocity profiles were generated for each angle based on the detector geometry.

Side feeding into each level and feeding into the topmost level of band ED were initially modeled as a five-transition cascade with a moment of inertia comparable to that of the in-band sequence [27,28]. The quadrupole moments of the

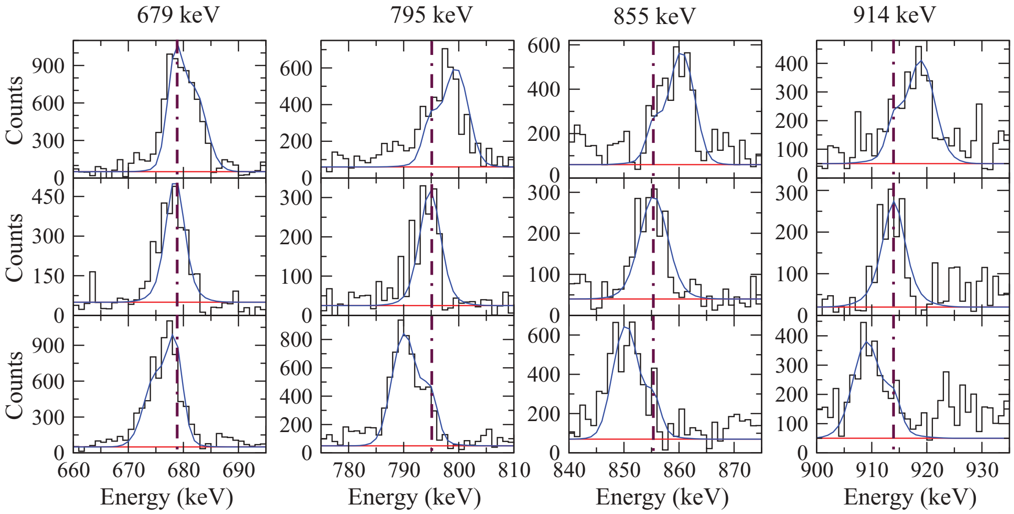

FIG. 1. (Color online) Experimental data and associated line-shape fits for γ-ray transitions in band ED of ^{171}Hf. The forward, 90°, and backward angle spectra are displayed in the top, middle, and bottom rows, respectively. Transitions are labeled by their energies on top of the figure.

side-feeding sequences were allowed to vary, which, when combined with the unknown moment of inertia, acted as the effective side-feeding lifetime parameters for each level. The relative intensities of the band members were adopted from the thin-target data obtained earlier [20].

Starting with the topmost transition in the band, the in-band and side-feeding lifetimes, background, and other parameters were rendered free to vary. The forward, transverse, and backward spectra for each transition were fitted simultaneously. The best-fit parameters for the background and stopped contaminant peaks were then fixed, and the in-band and side-feeding lifetimes were used as an effective feeding time parameter for the next lower level in the band. Each level was added and fitted in turn, until the entire band was included in a global fit with independently variable lifetimes for each in-band and side-feeding transition [27,28]. Representative examples of DSAM fits are compared with the data in Fig. 1, and the extracted lifetimes and transition quadrupole moments are listed in Table I. Uncertainties in the measurements were derived from the behavior of the χ^2 fit in the vicinity of the minimum [28,29]. Systematic errors associated with the modeling of the stopping powers are not included in the quoted errors and may be as large as 15% [30].

TABLE I. Lifetimes and the associated transition quadrupole moments for band ED in ^{171}Hf. The error bars are statistical only; see text for details.

Spin I^π ($\hbar$)	Lifetime (fs)	Q_t (e b)	Q_{sf} (e b)
$\frac{55}{2}^+$	130(14)	11.0(6)	10.7(5)
$\frac{59}{2}^+$	104(14)	10.0(6)	8.1(5)
$\frac{63}{2}^+$	76(12)	9.7(6)	7.6(5)
$\frac{67}{2}^+$	58(7)	9.2(6)	5.1(3)
$\frac{71}{2}^+$	39(5)	9.5(7)	5.7(4)
$\frac{75}{2}^+$	28(5)	9.6(10)	7.1(7)

B. Centroid-shift analysis for the strongly deformed bands in ^{172}Hf

In contrast to band ED in ^{171}Hf, the transitions of all three strongly deformed bands studied in ^{172}Hf did not contain any stopped component; rather, the peaks were shifted. As a result, a centroid-shift analysis was undertaken. Because in this case the time scale for depopulating a level by γ-ray emission is significantly smaller than the stopping time of the recoiling nuclei, the γ rays of interest were always emitted from a moving source. In order to locate the transitions of interest at different angles, the so-called fractional Doppler shift $F(\tau)$ for each γ ray was taken into account in the gating conditions set in BLUE when sorting spectra for different rings of Gammasphere. As the amount of shift at different angles was unknown at the onset, $F(\tau)$ values close to those observed in similar bands of ^{174}Hf [17] were assumed initially. Double-gated, background-subtracted spectra were obtained for each ring with those assumed $F(\tau)$ values in order to search for the transitions in the bands. Through an iterative procedure the $F(\tau)$ values were adjusted for each γ ray until the γ-ray spectrum with the best peak to total was achieved. In this way, proper gating conditions were ensured, and the final spectrum for a particular band was constructed at each angle by summing up all optimized double-gated spectra of the band members. Representative spectra for band SD1 at forward and backward angles can be found in Fig. 2.

Fits of shifted peak centroids versus cos(θ) were subsequently performed with the first-order Doppler shift expression $E_{\gamma s} = E_{\gamma 0}[1 + \beta_0 F(\tau)\cos(\theta)]$ to extract the fractional Doppler shift values $F(\tau)$. Here θ is the detector angle, and $\beta_0 = v_0/c = 0.0255$ is the calculated initial recoil velocity of a residue formed at the center of the target. The measured $F(\tau)$ values for the transitions in bands SD1, SD2, and SD3 as a function of γ-ray energy are presented in Fig. 3.

The transition quadrupole moments Q_t for the bands were extracted from the experimental $F(\tau)$ values with the code

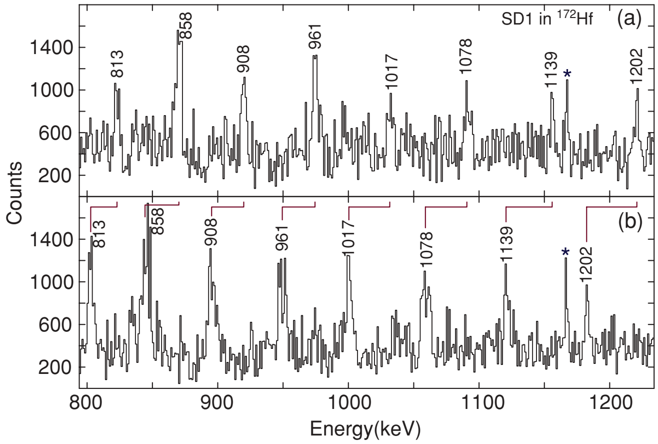

FIG. 2. Representative, summed coincidence γ-ray spectra for band SD1 in ^{172}Hf. (a) spectrum from forward-angle detectors in two rings of Gammasphere corresponding to an average angle $\theta \sim 54.2^\circ$; (b) spectrum from backward-angle detectors with an average angle $\theta \sim 125.8^\circ$. The peaks are labeled by their unshifted γ-ray energies. The asterisks indicate a stopped transition that depopulates a low-spin level in ^{172}Hf with a long lifetime.

FITFTAU [31], which calculates the average recoil velocity at which the decay from a particular level occurs. The version of the code used in the present work had the following usual assumptions: (i) The Q_t moment remains constant within a band. (ii) The side feeding into each level in the main band is modeled as a single cascade; the energy spacing in the cascade was assumed to be similar to that of the main band, and the number of side-feeding transitions in the cascade is equal to the number of in-band transitions above the level of interest in the main band. (iii) The side-feeding quadrupole moment Q_{sf}, which may have values different from the Q_t moment in the main band, also remains constant for a cascade. The code calculates an $F(\tau)$ curve corresponding to certain values of Q_t and Q_{sf} and compares these to the experimental $F(\tau)$ curve; these values are then adjusted until the χ^2 is minimized.

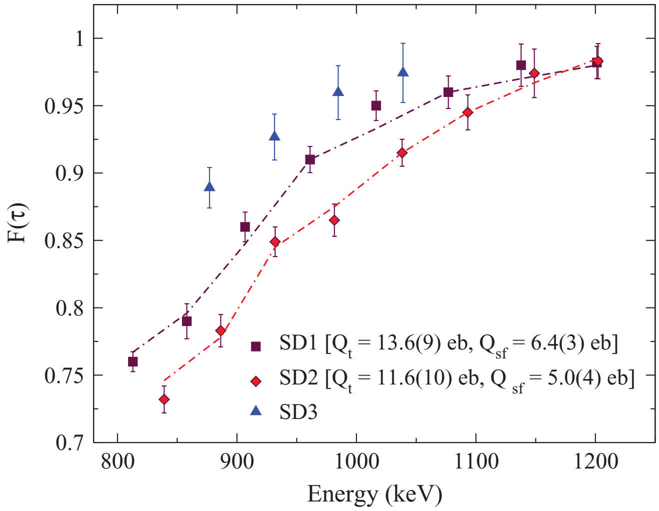

FIG. 3. (Color online) Experimental $F(\tau)$ values (data points) for transitions in bands SD1, SD2, and SD3 in ^{172}Hf. The dashed lines connect the corresponding $F(\tau)$ values calculated by the FITFTAU code.

The measured $F(\tau)$ values for the transitions in bands SD1, SD2, and SD3 can be seen in Fig. 3, together with the calculated curves for the first two bands. The extracted values for the quadrupole moments are given as well, with the associated statistical errors. A calculated curve was not obtained for band SD3 as only four experimental data points are available for this sequence, which has very low intensity and, as a result, could not be fitted well.

III. DISCUSSION

A. Band ED in ^{171}Hf

In all coincidence spectra, the transitions in bands SD1–SD3 of ^{172}Hf appear as shifted, sharp peaks. This is a characteristic feature of bands associated with very short level lifetimes and large quadrupole moments, such as those typically seen in SD bands. The transitions of band ED in ^{171}Hf, on the other hand, exhibit broadened line shapes. In principle, the same centroid-shift analysis can be used for the bands in ^{171}Hf and ^{172}Hf. However, the Doppler-shifted line shapes of the transitions in the band ED are rather broad, and a detailed analysis of the line shapes provides more precise information. The line-shape analysis also gives an opportunity to compare more extensively the band ED and the ND bands since the Q_t values of the latter in ^{171}Hf were measured previously by Cullen *et al.* [32] with a similar approach. Until the present work, little information was available on the deformation of ED bands in the Hf isotopes, as only a preliminary value of $Q_t \sim 9$ e b was quoted for band 1 in ^{175}Hf [19] from an unpublished work. The presently measured quadrupole moments of $Q_t \sim 9.5$ e b for band ED are larger than those of ND bands, e.g., $Q_t \sim 6.5$ e b for band [521]1/2 in ^{171}Hf [32], confirming that band ED is associated with an enhanced deformation. The ED minimum predicted by UC calculations is characterized by deformation parameters $(\epsilon_2, \gamma) \sim (0.3, 4^\circ)$, corresponding to an enhanced deformation with little triaxiality, and a Q_t moment ~8.5 e b. The structure is associated with an intrinsic configuration of $\pi(i_{13/2}h_{9/2}) \otimes \nu h_{9/2}$, where the $\pi i_{13/2}$ intruder orbital drives the nucleus to larger deformation [20]. The relationship between Q_t and deformation parameters ϵ_2 and γ can be estimated using the following expression [9]:

$$Q_t = \frac{8}{5\sqrt{3}} Zer_0^2 A^{2/3} \epsilon_2 \left(1 + \frac{1}{2}\epsilon_2\right) \cos(\gamma + 30^\circ) \qquad (1)$$

The measured Q_t value is in line with the calculated one. For example, a combination of $(\epsilon_2, \gamma) \sim (0.32, 0^\circ)$ would lead to $Q_t \sim 9.5$ e b. Hence, the present measurements not only provide the first firm value of quadrupole moments for the proposed ED bands that exist systematically in Hf isotopes of this mass region but also appear to validate the proposed interpretation. However, the UC calculations also predicted a TSD minimum near $(\epsilon_2, \gamma) \sim (0.43, 20^\circ)$ in ^{171}Hf, and the latter needs to be considered. At lower spins, this PES minimum is built on a three-quasiparticle configuration $\pi(i_{13/2}h_{9/2}) \otimes \nu(j_{15/2})$, or $\pi 6^1 \nu 7^1$ in a short-handed notation for the high-j intruder orbitals involved, with an associated moment $Q_t \sim 8$ e b. For $I \gtrsim 35.5\hbar$, a five-quasiparticle structure $\pi(i_{13/2})^2 \otimes \nu(i_{13/2}h_{9/2}j_{15/2})$, or $\pi 6^2 \nu 7^1$, with $Q_t \sim 9.7$ e b, becomes favored. Obviously, it is not possible to discriminate between the ED or TSD nature of the new band based solely on the measured quadrupole moment; other experimental observables must be considered. A detailed comparison in the previous work has revealed good agreement between band ED and the calculations [20]. Specifically, the observed aligned angular momenta, which were about $5\hbar$ larger than in the ND bands over a large frequency range, agree with those expected for the predicted quasiparticle configurations. The experimental and calculated excitation energies, subtracting a rigid-rotor reference, match well also. The predicted $\pi 6^2 \nu 7^1$ TSD band, as depicted in Fig. 4, is located ~3 MeV above the yrast line at spin $\sim 35\hbar$; it crosses band ED at ~55 $\hbar$. The predicted TSD bands remain unobserved in the experiment. This is in contrast to the situation in ^{168}Hf, where both ED and TSD bands have been observed. However, the latter nucleus is closer to the predicted TSD shell gaps of $N = 94$, 97, and as a result, the TSD band is lower in excitation energy than would be the case in ^{171}Hf (see Fig. 4). Hence, one would expect a more favorable feeding of a TSD band in ^{168}Hf than in ^{171}Hf. In summary, all the arguments above, together with the value of the measured quadrupole moments for the band ED presented here, indicate that this structure is associated with an enhanced deformation and little triaxiality.

As pointed out in previous work [14,20], band ED in ^{171}Hf has properties very similar to those of several previously reported candidate TSD bands in neighboring Hf isotopes; e.g., TSD1 in ^{170}Hf [16], band 1 in ^{175}Hf [19], and band ED in ^{168}Hf [14]. These bands all have similar kinematic moments of inertia [$J^{(1)}$], dynamic moments of inertia [$J^{(2)}$], and aligned angular momenta (i_x). Furthermore, the excitation energies of these bands all agree well with those predicted for bands built on ED minima in the UC calculations. These facts suggest that the $i_{13/2}h_{9/2}$ proton configuration is common to all these sequences. This configuration, coupled with different neutron configurations in the various isotopes, is then largely responsible for the enhanced, axial deformation. Based on the discussion above and the present quadrupole moment measurements for the band ED in ^{171}Hf, the interpretation for this group of bands can now be considered as well established.

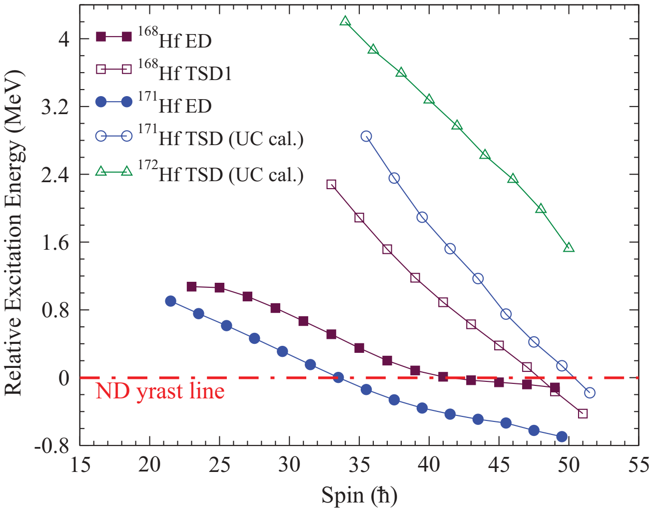

FIG. 4. (Color online) Observed and calculated relative excitation energies of strongly deformed bands with respect to their ND yrast lines in a few Hf isotopes.

B. Bands SD1–SD3 in ^{172}Hf

Based on the measured $F(\tau)$ values of the transitions in the strongly deformed bands in ^{172}Hf and the calculations with the FITFTAU code, a value of 13.6(9) e b was obtained for the quadrupole transition moment in band SD1. The systematic error associated with the uncertainties in the stopping powers is not included. However, confidence in the present results also comes from direct observation of clear differences between the measured Doppler shifts for transitions in the ND, ED, and SD bands in the same data set, which are not affected by these systematic errors. The shifts and line shapes are large and better defined for transitions in band ED than for those in the ND bands within the same spin region, implying that band ED is associated with shorter level lifetimes and larger quadrupole deformation. Furthermore, transitions in bands SD1–SD3 consistently appear as shifted sharp peaks, instead of Doppler-broadened line shapes, indicating that these sequences are associated with substantially shorter level lifetimes and larger deformations than band ED. In addition, the experimental F(τ) values of band SD1 are similar to those of the bands in ^{173,174}Hf, and the extracted value of $Q_t = 13.6(9)$ e b for band SD1 is close to that of $Q_t = 13.8(4)$ e b in band SD1 of ^{174}Hf, where slightly different target material (^{130}Te on Au backing) was used [17]. Hence, a consistent picture appears to emerge. As stated above, the $F(\tau)$ values for the first four transitions in band SD3 were obtained, but the Q_t value could not be extracted for the band. However, the higher experimental $F(\tau)$ values seen in Fig. 3 clearly point toward a larger deformation for this band when compared to the SD1 sequence. On the other hand, the quadrupole moment of $Q_t = 11.6(10)$ e b for band SD2 is lower than those in bands SD1 and SD3. This could *a priori* be viewed as somewhat of a surprise since the $J^{(2)}$ moment of inertia of band SD2 is larger by ~10% than those for bands SD1 and SD3 over the frequency range of interest (see Fig. 5). However, the $J^{(2)}$ curve, as a function of rotational frequency, for band SD2 displays irregularities at $\hbar\omega \sim 0.47$ MeV and at $\hbar\omega \sim 0.62$ MeV, with the latter being more pronounced. Such irregularities are often the signature of band crossings, and it is likely that the large $J^{(2)}$ values find their origin in such crossings.

The purpose of quadrupole moment measurements for bands SD1–SD3 in ^{172}Hf is to understand whether these correspond to TSD or SD bands. These three bands are not linked to the lower-spin ND states [20]. So the spins, parities, and excitation energies of the levels in the three sequences remain unknown, and a definitive interpretation in terms of intrinsic configurations, based on a comparison between

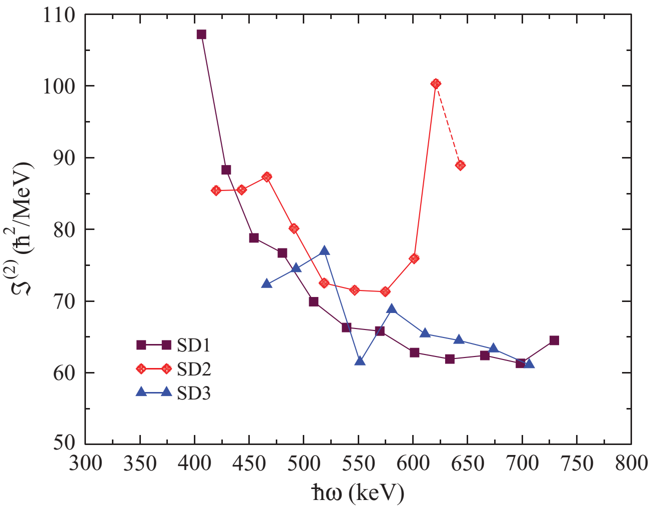

FIG. 5. (Color online) Dynamic moments of inertia $J^{(2)}$ as a function of rotational frequency for the three strongly deformed bands in ^{172}Hf.

experiment and theory, cannot be obtained. Nevertheless, as indicated in Ref. [20], some striking similarities exist between these bands and others in $^{173-175}$Hf [17,19]: (i) their $J^{(2)}$ moments resemble each other; (ii) band SD1 in ^{174}Hf and band 2 in ^{175}Hf are isospectral, and similarly, band SD1 in ^{172}Hf and the band in ^{173}Hf are isospectral as well; (iii) the quadrupole moment $Q_t = 13.6(9)$ e b for band SD1 in ^{172}Hf is very similar to the values of $Q_t \approx 13$–14.5 e b for the bands in $^{173-174}$Hf [17,19]. Isospectral bands, especially those with large deformation, are often associated with closely related configurations. In particular, they often contain the same high-j shape-driving orbitals [33], and it is likely that the SD bands in ^{172}Hf have configurations similar to those of their heavier neighbors. Among all the bands in the SD group, only band 2 in ^{175}Hf has been linked to known levels [19]. The CRMF calculations suggest that this band and, very likely, the similar bands in $^{172-174}$Hf are built on a prolate superdeformed minimum with a configuration involving $\pi i_{13/2} \otimes \nu j_{15/2}$ (or $\pi 6^1 \nu 7^1$) high-j intruder orbitals [20]. Recent UC calculations led to a similar conclusion for band 2 in ^{175}Hf [14]. The calculated quadrupole moment, $Q_t \approx 12\,e$ b, agrees reasonably well with a preliminary value of $Q_t \approx 13$ e b quoted from unpublished work for the band in Ref. [19]. On the other hand, the UC calculations predicted TSD minima, rather than prolate, axially deformed SD structures at high spins in ^{172}Hf [20] and ^{173,174}Hf [17]. However, the measured transition quadrupole moments, $Q_t \approx 13$–14 e b, for the bands in ^{174}Hf are significantly larger than the predicted values, $Q_t \approx 9.9$ e b, for the TSD structures at $(\epsilon_2, \gamma) \sim (0.45, 27^\circ)$ in this nucleus [17]. The predicted TSD minimum in ^{172}Hf is around $(\epsilon_2, \gamma) \approx (0.43, 16^\circ)$, above spin $I \sim 34\,\hbar$, with an associated moment $Q_t \sim 11$ e b [20]. The presently measured value of $Q_t = 13.6(9)$ e b for band SD1 is substantially larger. For a band with such a large Q_t value, the estimated ϵ_2 deformation would be 0.52 (or $\beta_2 = 0.6$) if $\gamma = 16^\circ$ and would be 0.43 (or $\beta_2 = 0.48$) if $\gamma = 0^\circ$. Thus, the available evidence favors an interpretation in terms of a prolate superdeformed shape for the bands discussed above in $^{172-175}$Hf. The calculated PES minima and measured quadrupole moments for several isotopes in this region are summarized in Table II.

TABLE II. Calculated and experimental deformations of strongly deformed bands in some Lu and Hf isotopes.

	Calculations	Experiment	
Isotope	(ϵ_2, γ), Q_t (e b) [Ref.]	Band	Q_t (e b) [Ref.]
^{163}Lu	(0.4, 20°), 9.2 [34]	TSD1	8.4(4) [35]
^{168}Hf	(0.43, 20°), 10.5 [14]	TSD1	11.4(12) [13]
^{171}Hf	(0.3, 4°), 8.5 [20]	ED	9.5(6) [this work]
^{172}Hf	(0.43, 16°), 11 [20]	SD1	13.6(9) [this work]
^{173}Hf		SD	14.5(8) [17]
^{174}Hf	(0.45, 27°), 9.9 [17]	SD1	13.8(4) [17]
^{175}Hf	(0.4, ~0°), 12 [14,20]	Band 2	~13 [19]

Further consideration about the excitation energies and the intensities of bands SD1–SD3 in ^{172}Hf provides additional support for this SD interpretation. The predicted TSD bands are calculated to lie 2–4 MeV above the ND yrast line (see Fig. 4), i.e., at excitation energies where feeding would be expected to be very small. Indeed, among all the known ND bands in ^{172}Hf [36], band 13 has the highest excitation energy above the yrast line (~0.9 MeV) and an intensity of only 0.1% relative to the total γ-ray flux into the ground state. The TSD bands, lying at higher energy, would be expected to be characterized by intensities of the order of 0.01% or lower; i.e., values that do not compare well with the measured relative intensities of bands SD1 [0.7(2)%], SD2 [0.5(1)%], and SD3 [0.4(1)%] [20]. While this argument is valid for the case of SD bands in ^{172}Hf, it does not necessarily hold for the weakest bands of the region that are labeled as SD bands. In those cases, further experimental and theoretical work is desirable.

IV. CONCLUSIONS

In summary, transition quadrupole moments were measured for strongly deformed bands in ^{171}Hf and ^{172}Hf. The measured value of $Q_t \sim 9.5$ e b for band ED in ^{171}Hf strongly supports the recent suggestion that this sequence and similar bands in the ^{168,170,175}Hf isotopes are associated with little triaxiality and deformations enhanced relative to that of normal deformed structures. Theoretical calculations indicate that these structures involve an $i_{13/2}h_{9/2}$ proton configuration, which is largely responsible for the enhanced deformation. The measured values of $Q_t \sim 14$ e b for bands SD1 and SD3 in ^{172}Hf confirm that these sequences are associated with a prolate superdeformed shape. Further consideration about the excitation energies and the intensities of bands SD1–SD3 in ^{172}Hf provides additional support for this SD interpretation. Similar bands in $^{173-175}$Hf are also likely to be associated with superdeformed shapes. The observations are in contrast to predictions of cranking calculations performed with the ULTIMATE CRANKER code, which suggest a triaxial strongly deformed shape for $^{172-174}$Hf at high spins.

ACKNOWLEDGMENTS

The authors thank the ANL operation staff at Gammasphere. Special thanks also go to J. P. Greene for target preparation. This work was supported by the US Department of Energy, Office of Nuclear Physics, under Grants Nos. DE-FG02-95ER40939 (MSU), DE-AC02-06CH11357 (ANL), DE-FG02-96ER40983 (UT), and DE-FG02-94ER40848 (UML), as well as by the National Science Foundation under Grant No. PHY-0854815 (USNA).

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