PHYSICAL REVIEW C **83**, 044308 (2011)

Nuclear matter symmetry energy and the symmetry energy coefficient in the mass formula

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(Received 27 January 2011; revised manuscript received 12 March 2011; published 12 April 2011)

Within the Skyrme-Hartree-Fock (SHF) approach, we show that for a fixed mass number A, both the symmetry energy coefficient $a_{\rm sym}(A)$ in the semiempirical mass formula and the nuclear matter symmetry energy $E_{\rm sym}(\rho_A)$ at a subsaturation reference density ρ_A can be determined essentially by the symmetry energy $E_{\rm sym}(\rho_0)$ and its density slope L at saturation density ρ_0. Meanwhile, we find the dependence of $a_{\rm sym}(A)$ on $E_{\rm sym}(\rho_0)$ or L is approximately linear and very similar to the corresponding linear dependence displayed by $E_{\rm sym}(\rho_A)$, providing an explanation for the relation $E_{\rm sym}(\rho_A) \approx a_{\rm sym}(A)$. Our results indicate that a value of $E_{\rm sym}(\rho_A)$ leads to a linear correlation between $E_{\rm sym}(\rho_0)$ and L and thus can put important constraints on $E_{\rm sym}(\rho_0)$ and L. Particularly, the values of $E_{\rm sym}(\rho_0) = 30.5 \pm 3$ MeV and $L = 52.5 \pm 20$ MeV are simultaneously obtained by combining the constraints from recently extracted $E_{\rm sym}(\rho_A = 0.1\ {\rm fm}^{-3})$ with those from recent analyses of neutron skin thickness of Sn isotopes in the same SHF approach.

DOI: 10.1103/PhysRevC.83.044308 PACS number(s): 21.65.Ef, 21.30.Fe, 21.10.Gv, 21.60.Jz

I. INTRODUCTION

The study of the nuclear matter symmetry energy $E_{\rm sym}(\rho)$, which essentially characterizes the isospin-dependent part of the equation of state (EOS) of asymmetric nuclear matter, is currently an exciting topic of research in nuclear physics. Knowledge about the symmetry energy is essential in understanding many aspects of nuclear physics and astrophysics [1–6] as well as some interesting issues regarding possible new physics beyond the standard model [7–10]. In recent years, significant progress has been made in determining the density dependence of $E_{\rm sym}(\rho)$ [5,6], especially its value $E_{\rm sym}(\rho_0)$ and its density slope L at saturation density ρ_0. While constraints on $E_{\rm sym}(\rho_0)$ and L from different experimental data or methods become consistently convergent [11–14], they are still far from an accuracy required for understanding enough precisely many important properties of neutron stars [3,15]. To narrow the uncertainty of the constrains on $E_{\rm sym}(\rho_0)$ and L by using more accurate data or new methods is thus of crucial importance.

Recently, based on the calculations within the droplet model and mean field models using a number of different parameter sets, Centelles *et al.* found that the symmetry energy coefficient $a_{\rm sym}(A)$ of finite nuclei with mass number A in the semiempirical mass formula can approximately equal to nuclear matter symmetry energy $E_{\rm sym}(\rho_A)$ at a reference density ρ_A in the subnormal density region, i.e., $E_{\rm sym}(\rho_A) \approx a_{\rm sym}(A)$ [16]. This relation provides the possibility to directly determine the symmetry energy at subnormal densities from the semiempirical mass formula and also has many important implications for extracting the symmetry energy from isospin-dependent observables of finite nuclei [16]. While this relation has been used to extract information on the symmetry energy around the normal density [16,17], its microscopic explanation is still missing.

Within the Skyrme-Hartree-Fock (SHF) energy density functional, we demonstrate in the present work that both $a_{\rm sym}(A)$ and $E_{\rm sym}(\rho_A)$ can be determined essentially by $E_{\rm sym}(\rho_0)$ and L and meanwhile they display very similar linear dependence on $E_{\rm sym}(\rho_0)$ or L, and thus providing an explanation for the relation $E_{\rm sym}(\rho_A) \approx a_{\rm sym}(A)$. Furthermore, we show that a value of $E_{\rm sym}(\rho_A)$ can put important constraints on $E_{\rm sym}(\rho_0)$ and L. Combining the constraints from recently extracted $E_{\rm sym}(\rho_A = 0.1\ {\rm fm}^{-3})$ with those from recent analyses of existing data on neutron skin thickness of Sn isotopes [18] within the same SHF approach leads to stringent constraints simultaneously on $E_{\rm sym}(\rho_0)$ and L.

II. $E_{\rm sym}(\rho)$ AND $a_{\rm sym}(A)$ IN THE SKYRME-HARTREE-FOCK APPROACH

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be expanded to second order in isospin asymmetry δ as

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\delta^2 + O(\delta^4), \tag{1}$$

where $\rho = \rho_n + \rho_p$ is the baryon density with ρ_n and ρ_p denoting the neutron and proton densities, respectively; $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry; $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$\begin{aligned} E_{\rm sym}(\rho) &= \frac{1}{2!}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\delta=0} \\ &= E_{\rm sym}(\rho_0) + L\chi + \frac{K_{\rm sym}}{2!}\chi^2 + O(\chi^3), \end{aligned} \tag{2}$$

with $\chi = \frac{\rho-\rho_0}{3\rho_0}$. The coefficients $L = 3\rho_0 \frac{dE_{\rm sym}(\rho)}{d\rho}|_{\rho=\rho_0}$ and $K_{\rm sym} = 9\rho_0^2 \frac{d^2E_{\rm sym}(\rho)}{d\rho^2}|_{\rho=\rho_0}$ are the slope and curvature parameters of the symmetry energy, respectively. Within the standard SHF approach, the symmetry energy can be written as (see, e.g., Ref. [19])

$$\begin{aligned} E_{\rm sym}(\rho) = &\frac{\hbar^2}{6m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{\frac{2}{3}} - \frac{1}{8}t_0(2x_0+1)\rho \\ &- \frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}\Theta_{\rm sym}\rho^{\frac{5}{3}} - \frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1}, \end{aligned} \tag{3}$$

with $\Theta_{\rm sym} = 3t_1x_1 - t_2(4+5x_2)$ and $\sigma, t_0 - t_3, x_0 - x_3$ being the Skyrme interaction parameters.

As shown in Refs. [18,20], the nine Skyrme interaction parameters, i.e., $\sigma, t_0 - t_3, x_0 - x_3$ can be expressed analytically in terms of nine macroscopic quantities ρ_0, $E_0(\rho_0)$, the incompressibility K_0, the isoscalar effective mass $m^*_{s,0}$, the isovector effective mass $m^*_{v,0}$, $E_{\rm sym}(\rho_0)$, L, gradient coefficient G_S, and symmetry-gradient coefficient G_V. In terms of these macroscopic quantities, the symmetry energy can be rewritten as

$$E_{\rm sym}(\rho) = A_1E_{\rm sym}(\rho_0) + B_1L + C_1, \quad (4)$$

with

$$A_1 = (\gamma u - u^\gamma)/(\gamma - 1), \quad (5)$$
$$B_1 = (u^\gamma - u)/[3(\gamma - 1)], \quad (6)$$
$$C_1 = E^{\rm kin}_{\rm sym}(\rho_0)u^{2/3} + Du^{5/3} - \frac{(3\gamma-2)E^{\rm kin}_{\rm sym}(\rho_0) + (3\gamma-5)D}{3(\gamma-1)}u + \frac{E^{\rm kin}_{\rm sym}(\rho_0) - 2D}{3(\gamma-1)}u^\gamma, \quad (7)$$

where $u = \rho/\rho_0$ is the reduced density; $E^{\rm kin}_{\rm sym}(\rho_0) = \frac{\hbar^2}{6m}(\frac{3\pi^2}{2}\rho_0)^{2/3}$ is the kinetic symmetry energy at ρ_0; and the parameters D and γ are defined as [21,22]

$$D = \frac{5}{9}E^0_{\rm kin}\left(4\frac{m}{m^*_{s,0}} - 3\frac{m}{m^*_{v,0}} - 1\right), \quad (8)$$
$$\gamma = \sigma + 1 = \frac{K_0 + 2E^0_{\rm kin} - 10C}{3E^0_{\rm kin} - 9E_0(\rho_0) - 6C}, \quad (9)$$

with $C = \frac{m - m^*_{s,0}}{m^*_{s,0}}E^0_{\rm kin}$ and $E^0_{\rm kin} = \frac{3\hbar^2}{10m}(\frac{3}{2}\pi^2\rho_0)^{2/3}$.

The symmetry energy coefficient $a_{\rm sym}(A)$ of finite nuclei in the semiempirical mass formula can be expressed as [16]

$$a_{\rm sym}(A) = \frac{E_{\rm sym}(\rho_0)}{1 + x_A} \quad \text{with} \quad x_A = \frac{9E_{\rm sym}(\rho_0)}{4Q}A^{-1/3}, \quad (10)$$

where the Q parameter is the so-called neutron-skin stiffness coefficient in the droplet model [23,24] and it is related to the nuclear surface symmetry energy [25,26]. Usually for a given nuclear interaction, the Q parameter can be obtained from asymmetric semi-infinite nuclear matter calculations [23,24,26–28]. As a good approximation, the Q parameter can be expressed as [27]

$$Q = \frac{9}{4}\frac{E^2_{\rm sym}(\rho_0)}{\varepsilon^e_\delta} \quad \text{with} \quad \varepsilon^e_\delta = \frac{2a}{r_{\rm nm}}\left(L - \frac{K_{\rm sym}}{12}\right), \quad (11)$$

where $r_{\rm nm} = (\frac{4}{3}\pi\rho_0)^{-1/3}$ is the radius constant of nuclear matter and a is the diffuseness parameter in the Fermi-like function from the parametrization of nuclear surface profile of symmetric semi-infinite nuclear matter. Many calculations [25–27] have indicated $a \approx 0.55$ fm and then $2a/r_{\rm nm} \approx 1$. Therefore, the x_A parameter can be approximated by

$$x_A = (L - K_{\rm sym}/12)\frac{A^{-1/3}}{E_{\rm sym}(\rho_0)}. \quad (12)$$

It should be noted that Eq. (12) is a good approximation for evaluating the $a_{\rm sym}(A)$, and within the standard SHF energy density functional the difference between the value of $a_{\rm sym}(A = 208)$ from Eq. (12) and that of using the exact Q parameter obtained from asymmetric semi-infinite nuclear matter calculations is essentially less than 1 MeV [29]. Furthermore, within the standard SHF approach, $K_{\rm sym}$ can be written in terms of the macroscopic quantites as [22]

$$K_{\rm sym} = 3\gamma L + E^{\rm kin}_{\rm sym}(\rho_0)(3\gamma - 2) + 2D(5 - 3\gamma) - 9\gamma E_{\rm sym}(\rho_0), \quad (13)$$

and thus we have

$$x_A = \left[\frac{4-\gamma}{4}L + \frac{3}{4}\gamma E_{\rm sym}(\rho_0) - \frac{(3\gamma-2)}{12}E^{\rm kin}_{\rm sym}(\rho_0) - \frac{(5-3\gamma)}{6}D\right]\frac{A^{-1/3}}{E_{\rm sym}(\rho_0)}. \quad (14)$$

For $|x_A| < 1$, $a_{\rm sym}(A)$ in Eq. (10) can be expanded as

$$a_{\rm sym}(A) = E_{\rm sym}(\rho_0)\big(1 - x_A + x_A^2 - \cdots\big). \quad (15)$$

Neglecting the x_A^2 and higher-order terms in Eq. (15) leads to

$$a_{\rm sym}(A) = A_2E_{\rm sym}(\rho_0) + B_2L + C_2, \quad (16)$$

with

$$A_2 = \left(1 - \frac{3\gamma}{4}A^{-1/3}\right), \quad (17)$$
$$B_2 = -\frac{4-\gamma}{4}A^{-1/3}, \quad (18)$$
$$C_2 = \left(\frac{(3\gamma-2)}{12}E^{\rm kin}_{\rm sym}(\rho_0) + \frac{(5-3\gamma)}{6}D\right)A^{-1/3}. \quad (19)$$

However, the convergence of the expansion in Eq. (15) is usually very slow and thus Eq. (16) is a very bad approximation to $a_{\rm sym}(A)$ even for heavy nuclei [25,30]. A much better approximation could be obtained by the two-variable Taylor expansion with respect to $E_{\rm sym}(\rho_0)$ and L at a point of $E_{\rm sym}(\rho_0) = S_0$ and $L = L_0$ as

$$a_{\rm sym}(A) = a_{\rm sym}(A)|_{E_{\rm sym}(\rho_0)=S_0, L=L_0} + [E_{\rm sym}(\rho_0) - S_0]\frac{\partial a_{\rm sym}(A)}{\partial E_{\rm sym}(\rho_0)}\bigg|_{E_{\rm sym}(\rho_0)=S_0, L=L_0} + (L - L_0)\frac{\partial a_{\rm sym}(A)}{\partial L}\bigg|_{E_{\rm sym}(\rho_0)=S_0, L=L_0} + \cdots \quad (20)$$

and keeping only the first-order terms leads to

$$a_{\rm sym}(A) = A_3E_{\rm sym}(\rho_0) + B_3L + C_3, \quad (21)$$

with

$$A_3 = \frac{1 - \frac{3}{4}\gamma A^{-1/3} + 2x^0_A}{\big(1 + x^0_A\big)^2}, \quad (22)$$

$$B_3 = \frac{\frac{\gamma-4}{4} A^{-1/3}}{\left(1 + x_A^0\right)^2}, \tag{23}$$

$$C_3 = \frac{S_0}{1 + x_A^0} - S_0 A_3 - L_0 B_3, \tag{24}$$

and

$$x_A^0 = \left[\frac{3}{4}\gamma S_0 + \frac{4-\gamma}{4} L_0 - \frac{(3\gamma-2)}{12} E_{\mathrm{sym}}^{\mathrm{kin}}(\rho_0) - \frac{(5-3\gamma)}{6} D\right] \frac{A^{-1/3}}{S_0}. \tag{25}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

One can see from Eq. (4) that $E_{\mathrm{sym}}(\rho)$ is linear functions of $E_{\mathrm{sym}}(\rho_0)$ and L with the coefficients A_1, B_1, and C_1 determined by the density ρ and nuclear matter macroscopic quantities ρ_0, $E_0(\rho_0)$, K_0, $m_{s,0}^*$, and $m_{v,0}^*$. Meanwhile, from Eqs. (10) and (14), one can see that $a_{\mathrm{sym}}(A)$ is determined by the mass number A and also the nuclear matter macroscopic quantities. In particular, $a_{\mathrm{sym}}(A)$ can also be linear functions of $E_{\mathrm{sym}}(\rho_0)$ and L if the approximation (16) or (21) is valid. As mentioned previously, the relation $a_{\mathrm{sym}}(A) \approx E_{\mathrm{sym}}(\rho_A)$ has been observed within mean field models using a number of different parameter sets for the nuclear effective interactions. Particularly, one finds $a_{\mathrm{sym}}(A = 208) \approx E_{\mathrm{sym}}(\rho_A = 0.1\ \mathrm{fm}^{-3})$, $a_{\mathrm{sym}}(A = 116) \approx E_{\mathrm{sym}}(\rho_A = 0.093\ \mathrm{fm}^{-3})$, and $a_{\mathrm{sym}}(A = 40) \approx E_{\mathrm{sym}}(\rho_A = 0.08\ \mathrm{fm}^{-3})$ [16]. This feature implies that $a_{\mathrm{sym}}(A)$ and $E_{\mathrm{sym}}(\rho_A)$ would display similar correlation with each nuclear matter macroscopic quantity among L, G_V, G_S, $E_0(\rho_0)$, $E_{\mathrm{sym}}(\rho_0)$, K_0, $m_{s,0}^*$, $m_{v,0}^*$, and ρ_0, which completely determine the nine Skyrme interaction parameters $\sigma, t_0 - t_3, x_0 - x_3$. In the following, we show that this is indeed the case by analyzing the correlations of $E_{\mathrm{sym}}(\rho = 0.1\ \mathrm{fm}^{-3})$ and $a_{\mathrm{sym}}(A = 208)$ with the nuclear matter macroscopic quantities. We have also checked the cases of $A = 116$ and 40, and obtained a similar conclusion as in the case of $A = 208$ and confirmed the relations $a_{\mathrm{sym}}(A = 116) \approx E_{\mathrm{sym}}(\rho_A = 0.093\ \mathrm{fm}^{-3})$ and $a_{\mathrm{sym}}(A = 40) \approx E_{\mathrm{sym}}(\rho_A = 0.08\ \mathrm{fm}^{-3})$.

As a reference for the correlation analyses based on the standard SHF energy density functional, we use in the present work the MSL0 parameter set [18], which is obtained by using the following empirical values for the macroscopic quantities: $\rho_0 = 0.16\ \mathrm{fm}^{-3}$, $E_0(\rho_0) = -16$ MeV, $K_0 = 230$ MeV, $m_{s,0}^* = 0.8m$, $m_{v,0}^* = 0.7m$, $E_{\mathrm{sym}}(\rho_0) = 30$ MeV, and $L = 60$ MeV, $G_V = 5\ \mathrm{MeV \cdot fm^5}$, and $G_S = 132\ \mathrm{MeV \cdot fm^5}$. And the spin-orbit coupling constant $W_0 = 133.3\ \mathrm{MeV \cdot fm^5}$ is used to fit the neutron $p_{1/2} - p_{3/2}$ splitting in ^{16}O. It has been shown [18] that the MSL0 interaction can describe reasonably the binding energies and charge rms radii for a number of closed-shell or semi-closed-shell nuclei. It should be pointed out that the MSL0 is only used here as a reference for the correlation analyses. Using other Skyrme interactions obtained from fitting measured binding energies and charge rms radii of finite nuclei as in usual Skyrme parametrization will not change our conclusion.

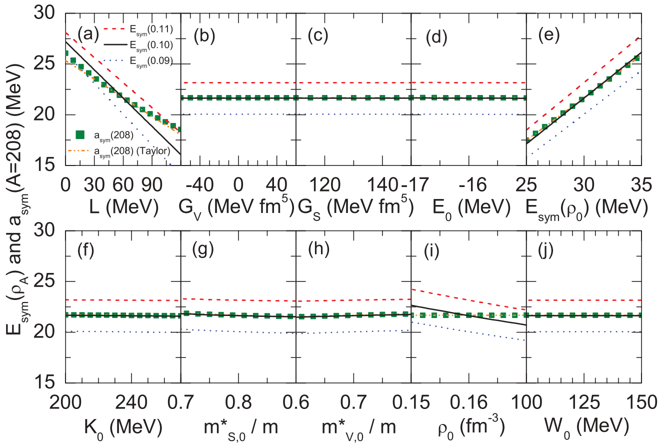

FIG. 1. (Color online) $E_{\mathrm{sym}}(\rho_A)$ with $\rho_A = 0.09, 0.10$, and $0.11\ \mathrm{fm}^{-3}$ as well as $a_{\mathrm{sym}}(A = 208)$ and its approximation with Taylor expansion in Eq. (21) (with $S_0 = 30$ MeV and $L_0 = 60$ MeV) from SHF with MSL0 by varying individually L (a), G_V (b), G_S (c), $E_0(\rho_0)$ (d), $E_{\mathrm{sym}}(\rho_0)$ (e), K_0 (f), $m_{s,0}^*$ (g), $m_{v,0}^*$ (h), ρ_0 (i), and W_0 (j).

Shown in Fig. 1 are $E_{\mathrm{sym}}(\rho_A)$ with $\rho_A = 0.09, 0.10$, and $0.11\ \mathrm{fm}^{-3}$ as well as $a_{\mathrm{sym}}(A = 208)$ [from Eq. (10) with approximation in Eq. (12)] and its approximation with Taylor expansion in Eq. (21) (with $S_0 = 30$ MeV and $L_0 = 60$ MeV) obtained from SHF with MSL0 by varying individually L, G_V, G_S, $E_0(\rho_0)$, $E_{\mathrm{sym}}(\rho_0)$, K_0, $m_{s,0}^*$, $m_{v,0}^*$, ρ_0, and W_0, namely, varying one quantity at a time while keeping all others at their default values in MSL0. We note here that using the exact Q parameter obtained from asymmetric semi-infinite nuclear matter calculations to evaluate the $a_{\mathrm{sym}}(A)$ does not change our conclusions [29]. It is interesting to see that within the uncertain ranges considered here for the macroscopic quantities, $a_{\mathrm{sym}}(A = 208)$ displays strong correlations with both $E_{\mathrm{sym}}(\rho_0)$ and L while it is almost no dependence on other macroscopic quantities [from Eqs. (10) and (14), $a_{\mathrm{sym}}(A)$ is independent of G_V, G_S, and W_0]. This is understandable since $a_{\mathrm{sym}}(A)$ is determined uniquely by the three lowest-order characteristic parameters of the symmetry energy, i.e., $E_{\mathrm{sym}}(\rho_0)$, L, and K_{sym} as seen in Eqs. (10) and (12) while K_{sym} has been found to strongly correlate with $E_{\mathrm{sym}}(\rho_0)$ and L but exhibit very weak dependence on other macroscopic quantities within the standard SHF energy density functional as shown in Ref. [20]. Furthermore, $a_{\mathrm{sym}}(A = 208)$ displays approximately linear correlations with both $E_{\mathrm{sym}}(\rho_0)$ and L which is demonstrated by the good approximation of Eq. (21) to $a_{\mathrm{sym}}(A = 208)$ observed in Fig. 1. Similarly, one can see from Fig. 1 that $E_{\mathrm{sym}}(\rho_A)$ with $\rho_A = 0.09, 0.10$, and $0.11\ \mathrm{fm}^{-3}$ display strong linear correlations with both $E_{\mathrm{sym}}(\rho_0)$ and L while they are almost independent of other macroscopic quantities except with a small dependence on ρ_0 [note $E_{\mathrm{sym}}(\rho_A)$ is independent of G_V, G_S, and W_0]. These results indicate that $a_{\mathrm{sym}}(A = 208)$ and $E_{\mathrm{sym}}(\rho_A)$ at subsaturation reference densities $\rho_A = 0.09, 0.10$, and $0.11\ \mathrm{fm}^{-3}$ are essentially determined by $E_{\mathrm{sym}}(\rho_0)$ and L. Furthermore, Fig. 1 shows that both $a_{\mathrm{sym}}(A = 208)$ and $E_{\mathrm{sym}}(\rho_A)$ display very similar linear dependence on $E_{\mathrm{sym}}(\rho_0)$ or L. Especially, it is seen from Fig. 1 that $E_{\mathrm{sym}}(\rho_A)$ with $\rho_A = 0.10\ \mathrm{fm}^{-3}$ gives the best fit to

$a_{\rm sym}(A = 208)$. In other words, for any Skyrme force determined by the nine parameters σ, $t_0 - t_3$, $x_0 - x_3$ or equivalently the nine macroscopic quantities L, G_V, G_S, $E_0(\rho_0)$, $E_{\rm sym}(\rho_0)$, K_0, $m^*_{s,0}$, $m^*_{v,0}$, ρ_0, the value of $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3})$ will be approximately equal to that of $a_{\rm sym}(A = 208)$, and this explains the relation $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) \approx a_{\rm sym}(A = 208)$ observed within mean field models using a number of different parameter sets. Furthermore, one can see that the possible small deviations between $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3})$ and $a_{\rm sym}(A = 208)$ observed for some different parameter sets [16] may be mainly due to the different values of L, $E_{\rm sym}(\rho_0)$, and/or ρ_0.

Since $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3})$ is essentially determined by $E_{\rm sym}(\rho_0)$ and L and displays linear correlations with the latter, a determination of $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3})$ will then put important constraints on $E_{\rm sym}(\rho_0)$ and L. As a matter of fact, the value of the symmetry energy around 0.1 fm^{-3} has been heavily under investigation in recent years in the literature [31–46]. For example, an analysis of the giant dipole resonance (GDR) of ^{208}Pb with Skyrme forces suggests a constraint $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) = 23.3 - 24.9$ MeV [42] while a relativistic mean-field model analysis of the GDR of ^{132}Sn leads to $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) = 21.2 - 22.5$ MeV [43]. In a recent work [17], Liu *et al.* extracted the symmetry energy coefficients $a_{\rm sym}(A)$ for nuclei with mass number $A = 20 - 250$ from more than 2000 measured nuclear masses and they obtained a value of $20.22 - 24.74$ MeV for $a_{\rm sym}(A = 208)$ within a 2σ uncertainty; thus we have $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) = 20.22 - 24.74$ MeV according to $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) \approx a_{\rm sym}(A = 208)$. In the following, as a conservative estimate, we will use $E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) = 20.22 - 24.74$ MeV since it is essentially consistent with the other two constraints from the GDR of ^{208}Pb and ^{132}Sn.

Shown in Fig. 2 are the contour curves in the $E_{\rm sym}(\rho_0)$-L plane for $E_{\rm sym}(\rho = 0.1\ {\rm fm}^{-3})$ obtained from SHF with MSL0

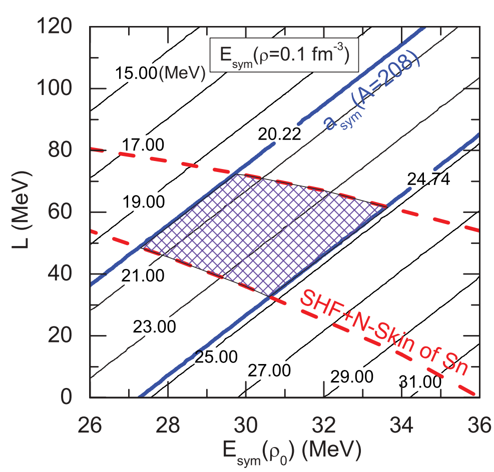

FIG. 2. (Color online) Contour curves in the $E_{\rm sym}(\rho_0)$-L plane for $E_{\rm sym}(\rho = 0.1\ {\rm fm}^{-3})$. The region between the two thick solid lines represents the constraint obtained in the present work with 20.22 MeV$\leqslant E_{\rm sym}(\rho_A = 0.10\ {\rm fm}^{-3}) \leqslant$ 24.74 MeV while the region between the two thick dashed lines is the constraint from the SHF analysis of neutron skin data of Sn isotopes within a 2σ uncertainty [18]. The shaded region represents the overlap of the two constraints.

by varying individually $E_{\rm sym}(\rho_0)$ and L. The region between the two thick solid lines in Fig. 2 represents the constraint of 20.22 MeV $\leqslant E_{\rm sym}(\rho = 0.10\ {\rm fm}^{-3}) \leqslant$ 24.74 MeV from the nuclear mass restrictions on $a_{\rm sym}(A = 208)$. Also included in Fig. 2 is a recent constraint (the region between the two thick dashed lines) from SHF analysis of neutron skin data of Sn isotopes within a 2σ uncertainty [18]. It is interesting to note that the constraint from the neutron skin data of Sn isotopes suggests L decreases with increasing $E_{\rm sym}(\rho_0)$ while the constraint from $E_{\rm sym}(\rho = 0.10\ {\rm fm}^{-3})$ displays opposite behaviors. This feature allows us to extract simultaneously the values of both $E_{\rm sym}(\rho_0)$ and L with higher accuracy, namely, $E_{\rm sym}(\rho_0) = 30.5 \pm 3$ MeV and $L = 52.5 \pm 20$ MeV by combining the two constraints as illustrated by the overlap of the two constraints represented by the shaded region in Fig. 2. Furthermore, it is seen from Fig. 2 that a value of $E_{\rm sym}(\rho = 0.10\ {\rm fm}^{-3})$ puts a strong linear correlation between $E_{\rm sym}(\rho_0)$ and L, i.e., L increases linearly with $E_{\rm sym}(\rho_0)$, which has been extensively observed in the parametrization for nuclear effective interactions in mean-field models [32–38,40,44–46].

The simultaneously extracted values of $E_{\rm sym}(\rho_0) = 30.5 \pm 3$ MeV and $L = 52.5 \pm 20$ MeV from the same SHF approach within a 2σ uncertainty are essentially overlapped with other constraints extracted from different experimental data or methods in the literature [11,12,14,16,17,23,26,42,47–49] (see Ref. [14] for a recent summary) but with higher precision. In particular, these extracted values are in remarkably good agreement with the $E_{\rm sym}(\rho_0) = 31.3 \pm 4.5$ MeV and $L = 52.7 \pm 22.5$ MeV extracted most recently from global nucleon optical potentials constrained by world data on nucleon-nucleus and (p, n) charge-exchange reactions [14]. The extracted value of $L = 52.5 \pm 20$ MeV also agrees well with the value of $L = 58 \pm 18$ MeV obtained from combining the constraint from the neutron skin data of Sn isotopes [18] with those from recent analyses of isospin diffusion and the double neutron/proton ratio in heavy-ion collisions at intermediate energies [11]. Furthermore, the extracted value of $L = 52.5 \pm 20$ MeV is consistent with the value of $L = 66.5$ MeV obtained from a recent systematic analysis of the density dependence of nuclear symmetry energy within the microscopic Brueckner-Hartree-Fock approach using the realistic Argonne V18 nucleon-nucleon potential plus a phenomenological three-body force of Urbana type [50]. It should be stressed that the simultaneously extracted values of $E_{\rm sym}(\rho_0)$ and L in the present work are obtained from the same Skyrme-Hartree-Fock energy density functional.

IV. SUMMARY

We have analyzed the correlations of the nuclear matter symmetry energy $E_{\rm sym}(\rho)$ at a subsaturation reference density ρ_A and the symmetry energy coefficient $a_{\rm sym}(A)$ of finite nuclei in the semiempirical mass formula with nuclear matter macroscopic quantities within the Skyrme-Hartree-Fock energy density functional. We have shown that $E_{\rm sym}(\rho_A)$ displays explicitly linear correlations with $E_{\rm sym}(\rho_0)$ and L and it is essentially determined by the latters but almost no dependence on other macroscopic quantities except a small dependence on the saturation density ρ_0. These features imply that a fixed value of $E_{sym}(\rho_A)$ will lead to strong linear correlation between $E_{sym}(\rho_0)$ and L. Furthermore, we have found that the two macroscopic quantities $E_{sym}(\rho_0)$ and L essentially determine the value of $a_{sym}(A)$ and the latter displays approximately linear correlations with both $E_{sym}(\rho_0)$ and L. In particular, the correlation between $a_{sym}(A)$ and $E_{sym}(\rho_0)$ (L) is found to be very similar to that between $E_{sym}(\rho_A)$ and $E_{sym}(\rho_0)$ (L), thus providing an explanation for the relation $E_{sym}(\rho_A) \approx a_{sym}(A)$ observed in mean field models using a number of different parameter sets.

Using the relation $E_{sym}(\rho_A) \approx a_{sym}(A)$, we have demonstrated that within the Skyrme-Hartree-Fock energy density functional, the value of $E_{sym}(\rho = 0.10\ \text{fm}^{-3}) = 20.22-24.74$ MeV extracted recently from nuclear masses within a 2σ uncertainty [17] can put important constraints on $E_{sym}(\rho_0)$ and L. Combining these constraints with those from recent analyses of existing data on neutron skin thickness of Sn isotopes based on the same Skyrme-Hartree-Fock approach within a 2σ uncertainty [18] allows us to extract simultaneously the values of both $E_{sym}(\rho_0)$ and L, i.e., $E_{sym}(\rho_0) = 30.5 \pm 3$ MeV and $L = 52.5 \pm 20$ MeV. These extracted values are essentially consistent with other constraints extracted from different experimental data in the literature but with higher precision.

In the present work, all analyses are based on the standard Skyrme-Hartree-Fock energy density functional. It will be interesting to see how our results change if different energy density functionals are used. On the other hand, it will be also interesting to see how our results, especially the new constraints on $E_{sym}(\rho_0)$ and L obtained in the present work, can give implications for the neutron-skin thickness of heavy nuclei, the isovector giant dipole resonance of finite nuclei, and properties of neutron stars. These studies are in progress.

ACKNOWLEDGMENTS

This work was supported in part by the NNSF of China under Grant No. 10975097, Shanghai Rising-Star Program under Grant No. 11QH1401100, and the National Basic Research Program of China (973 Program) under Contract No. 2007CB815004.

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