Binding energy per nucleon and hadron properties in nuclear matter

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We investigate the binding energy per nucleon and hadron properties in infinite and homogeneous nuclear matter within the framework of the in-medium modified Skyrme model. We first consider the medium modifications of the single hadron properties by introducing the optical potential for pion fields into the original Lagrangian of the Skyrme model. The parameters of the optical potential are well fitted to the low-energy phenomenology of pion-nucleus scattering. Furthermore, the Skyrme term is also modified in such a way that the model reproduces the bulk properties of nuclear matter, in particular, the binding energy per nucleon.

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The equation of state (EOS), which gives the density dependence of the binding energy per nucleon for a given nucleus, has been one of the most importance issues in nuclear many-body problems. There is a great amount of different theoretical approaches in trying to describe the EOS and, clearly, we can mention only some of a few representatives [1–7]. In general, those approaches and corresponding representatives can be classified into three classes as microscopic many-body approaches [1–4], effective field theories [5,6], and phenomenological methods [7]. These approaches provide very useful tools for understanding properties of dense and hot matter.

On the other hand, the Skyrme model [8,9] also presents a simple but good framework for investigating the bulk properties of nuclear matter. One can classify various Skyrmion approaches into subclasses: In the first one, investigations are mainly devoted to the classical crystalline structure and its behavior under the extreme conditions [10,11]. In the second, the properties of exotic many-baryon systems were treated [12–14]. There are also some early attempts to explain many-body systems considering the single skyrmion in hypersphere [15,16].

Moreover, there is an another alternative way to study the properties of the single skyrmion in nuclear matter [17,18] and its connection with quantum-mechanical variational methods [19], in order to analyze the bulk properties of nuclear matter [20]. To perform this analysis, it is essential to know the properties of the single hadron and the NN interaction in a symmetric [17,18] as well as asymmetric [21] nuclear environment.

As a more realistic approach, the in-medium modified Skyrme model [17] itself can be used so as to reproduce the properties of the single hadron in nuclear matter as well as those of matter in bulk. To carry out the proposed goal, we consider not only the changes of the kinetic and mass terms of the standard Skyrme Lagrangian as done in Ref. [17] but also possible modifications of the Skyrme term. It is well known that Skyrme's quartic stabilizing term may be related to vector mesons [22] that can be realized in implicit gauge symmetry of the nonlinear sigma model Lagrangian [23]. In this sense, the Skyrme parameter modification may be pertinent to the changes of the vector mesons in nuclear matter.

In Ref. [17], the in-medium modified Skyrme Lagrangian was presented, the mass term being modified based on the phenomenology of low-energy pion-nucleus scattering [24]. The modified mass term led also to the change of the kinetic term. In the present work, we additionally consider the modification of the Skyrme stabilizing term in the Lagrangian,

$$\begin{aligned}\mathcal{L}^* = {}& \frac{F_\pi^2}{16}\,\mathrm{Tr}\left[\left(\frac{\partial U}{\partial t}\right)\left(\frac{\partial U^\dagger}{\partial t}\right) + \alpha_p(\boldsymbol{r})(\nabla U)\cdot(\nabla U^\dagger)\right] \\ &+ \frac{1}{32e^2\gamma(\boldsymbol{r})}\,\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2 \\ &+ \frac{F_\pi^2 m_\pi^2}{16}\,\alpha_s(\boldsymbol{r})\mathrm{Tr}(U+U^\dagger-2),\end{aligned} \tag{1}$$

where $F_\pi = 108.78$ MeV denotes the pion decay constant, $e = 4.85$ is the Skyrme parameter, and $m_\pi = 134.98$ MeV stands for the pion mass.

The expressions of medium functionals $\alpha_s(\rho(\boldsymbol{r}))$ and $\alpha_p(\rho(\boldsymbol{r}))$ can be found in Ref. [17]. Here we introduce the new density-dependent functional $\gamma(\boldsymbol{r}) = \gamma(\rho(\boldsymbol{r}))$ which provides the in-medium dependence of the Skyrme parameter (i.e., $e^2 \to e^{*2} = e^2\gamma$). To fix this additional dependence, we will concentrate on the bulk properties of infinite and homogenous nuclear matter with a constant density ($\rho = \mathrm{const}$). Consequently, one can choose the spherically symmetric "hedgehog" form for the boson field $U = \exp\{i\boldsymbol{n}\cdot\boldsymbol{\tau}F(r)\}$, where $\boldsymbol{n}$ denotes the unit vector in coordinate space, $\boldsymbol{\tau}$ are the usual Pauli matrices, and $F(r)$ stands for the profile function of the pion field. The pertinent field equation is obtained by minimizing the medium-modified mass of the static skyrmion,

$$\begin{aligned}M_S^* = {}& \frac{\pi F_\pi}{e}\int_0^\infty dx\,\left\{\alpha_p\left(\frac{x^2F'^2}{2}+s^2\right)\right. \\ &\left. + \frac{4s^2F}{\gamma}\left(F'^2+\frac{s^2}{2x^2}\right) + \alpha_s\beta^2x^2(1-\cos F)\right\},\end{aligned} \tag{2}$$

where $x = eF_\pi r$, $s = \sin F$, and $\beta = m_\pi/(eF_\pi)$.

The collective quantization of the classical skyrmion [9] yields the in-medium modified nucleon mass and the

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corresponding $\Delta - N$ mass splitting, respectively, as

$$m_N^* = M_S^* + \frac{3}{8\lambda^*}, \qquad m_{\Delta-N}^* = \frac{3}{2\lambda^*},$$
$$\lambda^* = \frac{2\pi}{3e^3 F_\pi}\int_0^\infty dx\, x^2 s^2 \left\{1 + \frac{4}{\gamma}\left(F'^2 + \frac{s^2}{x^2}\right)\right\}. \quad (3)$$

The effective pion-nucleon coupling can be derived by calculating the in-medium modified πNN form factor [18]:

$$G_{\pi NN}^*(q^2) = \frac{4\pi M_N^*}{3e^2 F_\pi}\alpha_p \int_0^\infty \frac{j_1(\tilde{q}x)}{\tilde{q}x} S_\pi x^3 dx, \quad (4)$$

where $\tilde{q} = q/eF_\pi$, j_1 is the spherical Bessel function and $S_\pi(x)$ is defined as

$$S_\pi = -\left(\frac{2F'}{x} + F''\right)\cos F + \left(F'^2 + \frac{2}{x^2} + \frac{\alpha_s}{\alpha_p} m_\pi^2\right)s. \quad (5)$$

If the modification of the Skyrme term is ignored, the values of all input parameters are fitted to the phenomenology or taken from it [17]. The additional functional $\gamma(\rho)$, in general, may be related to the vector meson properties in nuclear matter. In this context, the lessening value of the Skyrme parameter in nuclear medium may correspond to a decrease of the $g_{\rho\pi\pi}$ coupling and, therefore, to the change of the rho meson width in nuclear matter or to a diminishing value of its mass in medium (i.e., $m_\rho^*/m_\rho < 1$). There are experimental indications to those changes of the ρ meson properties [25–27] and the theoretical predictions [28,29]. Following the ideas presented in those theoretical approaches, one may be able to fit γ. However, it is still under debate how the properties of the ρ meson undergo in medium both theoretically and experimentally, and the conclusions are model dependent. Thus, in the present work, we will proceed to fit the form of the functional γ to the bulk properties of nuclear matter rather than following a specific model.

As a first step, we explicitly choose its form to reproduce the first coefficient (in volume term) in the semiempirical Weizsäcker-Bethe-Bacher mass formula. Then the binding energy per nucleon at a given density can be defined simply as

$$\Delta E_{B=1} = m_N^*(\rho) - m_N^{\text{free}}. \quad (6)$$

This is somehow a crude approximation but a comprehensive one. One could even fit the form of γ by investigating different terms in the mass formula and by examining the interplay between them. However, within the present work, the approximation defined in Eq. (6) will be enough for the qualitative analysis of the changes due to the modification of the Skyrme parameter.

We have tried various forms of the dependence of γ on the nuclear density ρ such as linear, quadratic, polynomial, exponential forms, etc. It turns out that the best fit to the ground state of nuclear matter is achieved by the following form:

$$\gamma(\rho) = \exp\left(-\frac{\gamma_{\text{num}}\rho}{1 + \gamma_{\text{den}}\rho}\right), \quad (7)$$

where γ_{num} and γ_{den} are variational parameters. Let us first discuss the behavior of the binding energy when the Skyrme term is intact in nuclear matter [i.e., $\gamma(\rho) = 1$]. The corresponding binding energy is depicted as the dotted curve in Fig. 1 with

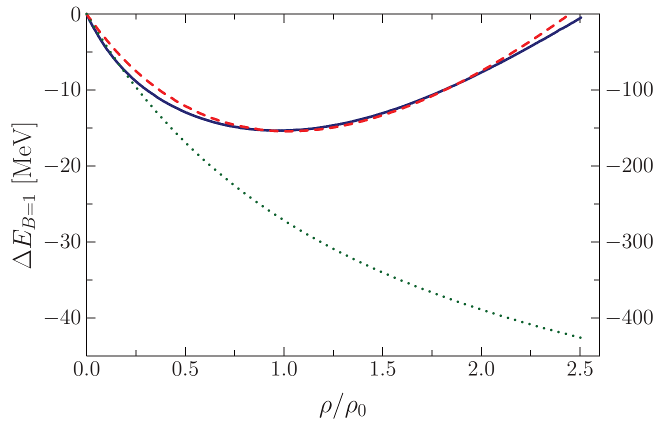

FIG. 1. (Color online) The binding energy per nucleon as a function of ρ/ρ_0. The solid curve (left scale) corresponds to the parametrization of γ in Eq. (7), with $\gamma_{\text{num}} = 2.1m_\pi^{-3}$, $\gamma_{\text{den}} = 1.45m_\pi^{-3}$, and P-wave scattering volume $c_0 = 0.21m_\pi^{-3}$ used. The dashed one (left scale) draws the case when $\gamma_{\text{num}} = 0.8m_\pi^{-3}$, $\gamma_{\text{den}} = 0.5m_\pi^{-3}$, and P-wave scattering volume $c_0 = 0.09m_\pi^{-3}$. S-wave scattering length is fixed at $b_0 = -0.024m_\pi^{-1}$ and the correlation parameter has value $g' = 0.7$. The dotted one (right scale) shows the case that the Skyrme term is intact in nuclear matter and consequently $\gamma(\rho) = 1$. The normal nuclear density is given as $\rho_0 = 0.5m_\pi^3$.

the energy scale drawn at the right vertical axis. One can note that in this case the binding energy monotonically falls off as the density increases. In the language of the single skyrmion, it indicates that the skyrmion swells to a larger volume and all skyrmions of the system start to overlap. Thus, the density of the system continuously increases. This is not surprising, because the medium modification in this case can be simply related to that of the pion decay constant $F_\pi \to F_\pi^* = F_\pi\sqrt{\alpha_p}$. For the moment, one can ignore the explicit chiral symmetry breaking term in the Lagrangian, because its influence to the stability is rather small in comparison with the effects coming from the first two terms. The decreasing value of the pion decay constant changes the contribution from the nonlinear kinetic term. As a result, the skyrmions swell to the larger volume and it is necessary to prevent this by some mechanism. It implies that one must introduce either strong repulsive NN interactions at short distances or some mean-field mechanism as in the Walecka model [6,7]. However, one interesting way to avoid this collapse may be to modify the Skyrme term and it is also physically motivated.

We also present two different results with the modified Skyrme term in Fig. 1: The solid and dashed curves draw the parametrization of Eq. (7) with the energy scale depicted at the left vertical axis. Note that the values of the variational parameters, γ_{num} and γ_{den}, are chosen in such a way that the minimum of the binding energy occurs at the normal nuclear matter density and reproduces correctly the first coefficient in the empirical formula for the binding energy. The results show that the dependence on the density is rather insensitive to the changes of input parameters from pion-nucleus scattering phenomenology.

Let us discuss thermodynamic properties of nuclear matter. The pressure is given as $p = \rho^2\partial\Delta E_{B=1}/\partial\rho$. It

TABLE I. Compressibility of nuclear matter K and an effective Δ-nucleon mass difference $m^*_{N-\Delta}$ at the normal nuclear matter density $\rho_0 = 0.5m_\pi^3$. The variational parameters $\gamma_{\rm num}$ and $\gamma_{\rm den}$ are fitted to reproduce the minimum of the binding energy per nucleon ~ 15.7 MeV at the normal nuclear matter density. The correlation parameter is taken to be $g' = 0.7$.

b_0 (m_π^{-1})	c_0 (m_π^{-3})	$\gamma_{\rm num}$ (m_π^{-3})	$\gamma_{\rm den}$ (m_π^{-3})	K (MeV)	$m^*_{N-\Delta}$ (MeV)
−0.024	0.21	2.098	1.451	1647.47	105.21
−0.024	0.15	1.448	0.998	1148.18	129.39
−0.024	0.09	0.797	0.496	582.79	170.34
−0.029	0.21	2.106	1.506	1637.16	107.13
−0.029	0.15	1.444	1.031	1142.00	131.59
−0.029	0.09	0.785	0.502	580.03	172.91

vanishes naturally at the equilibrium point for the parametrization of Eq. (7). We want to emphasize that the pressure is always decreasing with the Skyrme term intact. Another important quantity is the compressibility of nuclear matter:

$$K = 9\rho_0^2 \left. \frac{\partial^2 \Delta E_{B=1}}{\partial \rho^2} \right|_{\rho=\rho_0}. \tag{8}$$

The corresponding results are listed in Table I with two different values of the S-wave scattering length b_0 [24].

The results show that the compressibility of nuclear matter and the effective $\Delta - N$ mass difference are rather stable under the change of b_0. On the contrary, they are quite sensitive to the value of P-wave scattering volume c_0. At the empirical value of $c_0 = 0.21m_\pi^{-3}$, the compressibility turns out to be very large ($K \sim 1640$ MeV) in comparison with those obtained in relativistic Dirac-Brueckner-Hartree-Fock approaches [3,4] and in the Walecka model [7]. We find that as lower values of c_0 are used K is noticeably decreased. For example, for $c_0 = 0.09m_\pi^{-3}$ the compressibility is already consistent with that of the Walecka model ($K \sim 580$ MeV). If one uses even a smaller value of c_0 such as $c_0 = 0.06m_\pi^{-3}$, the result of K is further brought down to be comparable with that in Dirac-Brueckner-Hartree-Fock approaches ($K \sim 300$ MeV), which is close to the empirical value. It indicates that the present work prefers smaller values of c_0 than that used in the pionic atom analysis as far as the compressibility is concerned. We are reminded that K is sensitive to the position of the saturation point. Fitting the saturation point at slightly lower densities, we see that the compressibility decreases drastically. However, the situation may change if one considers a more accurate approximation with the surface and symmetry energy terms explicitly taken into account in Eq. (6).

In Fig. 2, the dependence of the $\Delta - N$ mass difference on the nuclear matter density is drawn. The results show that the modified Skyrme term leads to rather different results from those without the modification. With the Skyrme term modified (see solid and dashed curves), the results of $m^*_{\Delta-N}$ fall off faster as the density increases in comparison with that with the original Skyrme term (see dotted curve). Of course, this is due to the explicit density dependence of the moment of inertia λ^* [see Eq. (3)] through the additional density functional γ. It implies that it is easier to make the nucleon excited to the Δ state in nuclear matter, which seems more realistic than that without the modification of the Skyrme term.

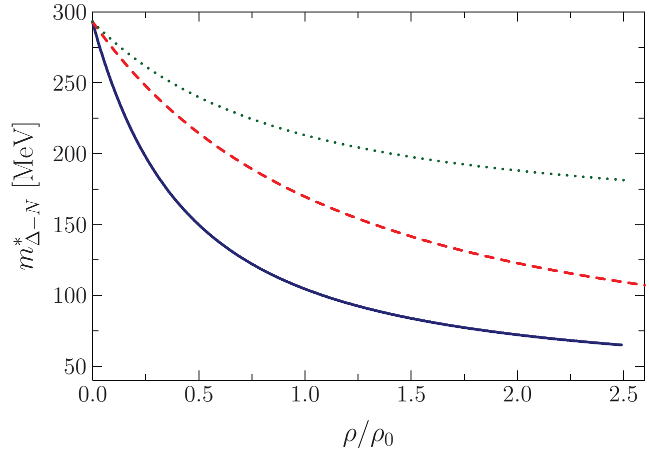

FIG. 2. (Color online) The density dependence of the $\Delta - N$ mass difference in nuclear matter. The notations and input parameters are the same as those in Fig. 1.

In Fig. 3, the changes of the πNN coupling constant are depicted. Here the modifications of the Skyrme term bring about more dramatic results. When the Skyrme term is intact (i.e., $\gamma = 1$), $g^*_{\pi NN}$ monotonically decreases as the density increases (see the dotted curve in Fig. 3). However, when one introduces the density dependence of the Skyrme term, the results are noticeably changed. Using the parametrization of Eq. (7) with the different values of input parameter c_0, we find that the πNN coupling in medium changes drastically. For example, with the value of $c_0 = 0.09m_\pi^{-3}$ the in-medium pion-nucleon coupling constant $g^*_{\pi NN}$ starts to increase monotonically up to high ($\rho \sim 5\rho_0$) densities as drawn in the dashed curve. The $g^*_{\pi NN}$ with $c_0 = 0.09m_\pi^{-3}$ will disappear at around $\rho \sim 5\rho_0$. On the other hand, if

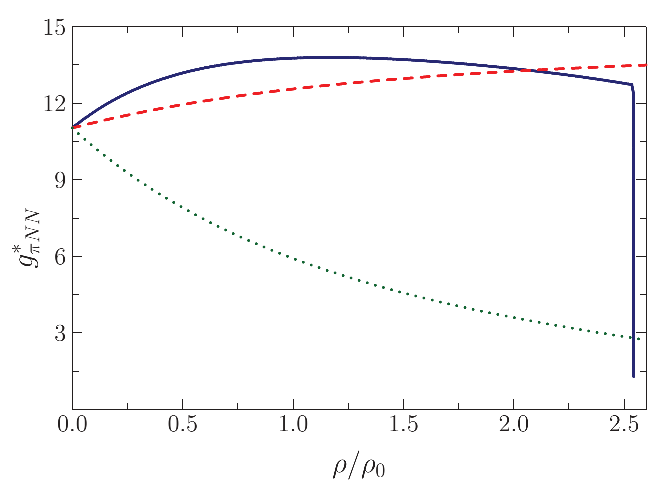

FIG. 3. (Color online) The dependence of the πNN coupling constant on the density. Notations are similar to those in Fig. 1.

one uses $c_0 = 0.21 m_\pi^{-3}$, it is getting increased up to the normal nuclear matter density and stays more or less constant. Then it slowly falls off as the density increases. When it approaches the critical point $\rho \sim 2.54\rho_0$, it drops sharply and goes to zero (see the solid curve in Fig. 3). Above the critical point ($\rho > \rho_{\rm crit} \approx 2.54\rho_0$), the skyrmion does not exist.

Let us draw the attention again to the bulk properties. Following Klebanov [10], quantizing the skyrmion [9], and using the formula presented in Ref. [30], one can estimate the symmetry energy in the semiempirical formula for the nuclear binding energy: $E_{\rm sym} = m^*_{\Delta-N}/12$. This crude formula of the symmetry energy already provides the enlightening results. For example, γ parameterized as in Eq. (7), $E_{\rm sym}(\rho_0) \approx 14.19$ MeV for $c_0 = 0.09 m_\pi^{-3}$ whereas $E_{\rm sym}(\rho_0) \approx 8.71$ MeV for $c_0 = 0.21 m_\pi^{-3}$. These results for the symmetry energy must be compared with the experimental one $E_{\rm sym} \sim 20 - 30$ MeV. The order of the symmetry energy calculated within the Skyrme model is comparable to the experimental data. To estimate the symmetry energy more accurately, however, one should consider the minimization of the whole binding energy, taking into account the interplay between the different terms in the mass formula as we stated already. Moreover, one should consider the effects of finite nuclei and explicit isospin-breaking effects as in Refs. [21,31], with the Skyrme term modified additionally.

In summary, we aimed at studying the modifications of the quartic term in the Skyrme model. The results from this work show that the additional modifications change dramatically the whole picture and allow one to understand the role of the modifications in a more comprehensive way. One can note that an alternative approach to many-body systems within the Skyrme model [12,13] points to the changes of the input parameters (so-called "calibration") according to the number of baryons in the system. Within our approach these changes were shown in a more realistic and transparent way and were treated not only at the level of the system but also at the level of its constituents.

Finally, we assert that the present approach is self-consistent: It treats the single hadron properties, the hadron-hadron interactions, and the bulk matter properties on an equal footing. Moreover, it is closely related to phenomenological low-energy data at the single-hadron level as well as at the level of hadronic systems.

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