Single-energy amplitudes for pion photoproduction in the first resonance region

R. L. Workman
Center for Nuclear Studies, Department of Physics The George Washington University, Washington, DC 20052, USA
(Received 25 July 2010; published 4 March 2011)

We consider multipole amplitudes for low-energy pion photoproduction, constructed with minimal model dependence, at single energies. A previous fit making minimal use of Watson's theorem has been reexamined in light of more recent measurements. Problems associated with the choice of a unique solution are discussed. Comparisons with more recent fits to the full resonance region are made. Explanations are suggested for the discrepancies and more precise measurements of the recoil polarization, P, are motivated.

DOI: 10.1103/PhysRevC.83.035201 PACS number(s): 25.20.Lj, 11.80.Et, 11.55.Bq, 13.60.Le

I. INTRODUCTION AND MOTIVATION

In a series of papers, Grushin and collaborators [1] extracted multipole amplitudes for $\pi^+ n$ photoproduction, $\pi^0 p$ photoproduction, and combined these to produce isospin components, from 280 to 420 MeV, without employing Watson's theorem. A number of subsequent studies [2] took this set to be the least biased determination of multipoles over the delta resonance region. As the amplitudes were obtained in the early 1980's, prior to a number of recent high-precision measurements, we have reexamined these results and the methods used in their determination.

Apart from checking old values, this exercise is relevant to experimental programs now measuring complete, or nearly complete, experiments for pion and kaon photoproduction. The relative model independence of this method allows checks of database consistency that we will use to suggest further measurements. Below we also briefly compare the methods associated with amplitude reconstruction and multipole fitting.

In Ref. [1], multipoles were extracted from $\pi^+ n$ photoproduction data of type S only. In the language of Ref. [3], type-S data include the unpolarized cross section and single-polarization asymmetries (P, Σ, T). As these do not constitute a complete experiment, in the strict sense of Ref. [4], some assumptions are required. The fits were performed between 280 and 420 MeV, using a truncated multipole expansion, including E_{0+}, M_{1-}, E_{1+}, and M_{1+}, the remaining terms assumed to be real and given by the electric Born terms.

The multipoles and helicity amplitudes are related by

$$
\begin{aligned}
H_1 &= \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} \sin\theta \sum_{\ell=1}^{\infty} \\
&\quad \times [E_{\ell+} - M_{\ell+} - E_{(\ell+1)-} - M_{(\ell+1)-}]\,(P''_\ell - P''_{\ell+1}), \\
H_2 &= \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} \sum_{\ell=0}^{\infty} [(\ell+2)E_{\ell+} + \ell M_{\ell+} + \ell E_{(\ell+1)-} \\
&\quad - (\ell+2)M_{(\ell+1)-}]\,(P'_\ell - P'_{\ell+1}), \\
H_3 &= \frac{1}{\sqrt{2}} \sin\frac{\theta}{2} \sin\theta \sum_{\ell=1}^{\infty} [(E_{\ell+} - M_{\ell+} \\
&\quad + E_{(\ell+1)-} + M_{(\ell+1)-}]\,(P''_\ell + P''_{\ell+1}), \\
H_4 &= \frac{1}{\sqrt{2}} \sin\frac{\theta}{2} \sum_{\ell=0}^{\infty} [(\ell+2)E_{\ell+} + \ell M_{\ell+} \\
&\quad - \ell E_{(\ell+1)-} + (\ell+2)M_{(\ell+1)-}]\,(P'_\ell + P'_{\ell+1}).
\end{aligned} \tag{1}
$$

From these one can construct the transversity amplitudes [3],

$$
\begin{aligned}
b_1 &= \tfrac{1}{2}[(H_1 + H_4) + i(H_2 - H_3)], \\
b_2 &= \tfrac{1}{2}[(H_1 + H_4) - i(H_2 - H_3)], \\
b_3 &= \tfrac{1}{2}[(H_1 - H_4) - i(H_2 + H_3)], \\
b_4 &= \tfrac{1}{2}[(H_1 - H_4) + i(H_2 + H_3)],
\end{aligned} \tag{2}
$$

which simplify the discussion of amplitude reconstruction, as the type-S observables determine their moduli,

$$
\begin{aligned}
\frac{d\sigma}{dt} &= |b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2, \\
P\frac{d\sigma}{dt} &= |b_1|^2 - |b_2|^2 + |b_3|^2 - |b_4|^2, \\
\Sigma\frac{d\sigma}{dt} &= |b_1|^2 + |b_2|^2 - |b_3|^2 - |b_4|^2, \\
T\frac{d\sigma}{dt} &= |b_1|^2 - |b_2|^2 - |b_3|^2 + |b_4|^2.
\end{aligned} \tag{3}
$$

Information on relative phases comes from double-polarization measurements. For example, the beam-target set of observables, tabulated in Ref. [3], is given by [5]

$$
\begin{aligned}
G\frac{d\sigma}{dt} &= 2\,\mathrm{Im}(b_1 b_3^* + b_2 b_4^*), \\
H\frac{d\sigma}{dt} &= 2\,\mathrm{Re}(b_1 b_3^* - b_2 b_4^*), \\
E\frac{d\sigma}{dt} &= -2\,\mathrm{Re}(b_1 b_3^* + b_2 b_4^*), \\
F\frac{d\sigma}{dt} &= 2\,\mathrm{Im}(b_1 b_3^* - b_2 b_4^*).
\end{aligned} \tag{4}
$$

For $\pi^+ n$ photoproduction, the interference between (complex) fitted multipoles and a given (real) high-ℓ contribution fixes the overall phase between transversity amplitudes [3]. An amplitude reconstruction requires more observables [4], and is the most model-independent method, but results in transversity amplitudes, for each energy-angle pair, only up to an unknown phase. If multipoles are the goal, an angular integral is required, and this cannot be performed without determining the phase.

TABLE I. Single-energy fits to π^+n data at 280 MeV (see text). Multipoles given in $10^{-3}/m_\pi$ units.

Multipole	Grushin [1]	SES	Fit 1	Fit 2
Re E_{0+}	17.18(0.29)	16.2	16.72(0.18)	16.17(0.23)
Im E_{0+}	−3.10(0.98)	0.57	−3.41(0.87)	0.5
Re M_{1-}	3.84(0.19)	3.46	3.74(0.18)	3.75(0.29)
Im M_{1-}	−0.70(0.84)	−0.13	−2.02(0.87)	0.33(0.58)
Re E_{1+}	2.64(0.08)	2.96	2.99(0.06)	2.70(0.11)
Im E_{1+}	0.00(0.26)	0.70	−0.08(0.29)	0.78(0.19)
Re M_{1+}	−16.00(0.30)	−14.85	−16.24(0.24)	−14.76(0.18)
Im M_{1+}	−6.76(1.10)	−9.63	−5.96(0.98)	−10.06(0.35)

Therefore, at some point, every multipole analysis requires constraints beyond the experimental data.

II. FITTING π^+n DATA

In order to check the results of Ref. [1], data from 280 to 420 MeV were fitted using the above prescription and a more recent database [6–8]. The higher-ℓ multipoles were taken from the MAID analysis [9], which includes vector-meson exchange, rather than a simple electric Born term. This modification had a negligible effect on the fits. The fitted multipoles were then compared to the original determinations of Ref. [1] and single-energy solutions (SESs) tied to the SAID energy-dependent multipole analysis [10].

The present and original fits of Ref. [1] were generally consistent, except in cases where more recent data contradicted older measurements. However, some very large deviations from the SAID SES values were found at the lowest energy and at the resonance energy (340 MeV). Comparisons are given in Tables I and II. The SES values were obtained assuming Watson's theorem and fitting both neutral- and charged-pion data over narrow energy bins, assuming a linear energy dependence given by the energy-dependent fit. Errors on the fitted isospin multipoles were generally in the 2%−5% range.

The 280-MeV fit (fit 1) deviates from the trend shown in the 300–420 MeV results, and this was noticed in Ref. [1] where an inconsistency in the data was suggested. The large negative fitted value for Im E_{0+} at this energy contradicts results, $(0.4 \pm 0.2)\ 10^{-3}/m_\pi$, found in the SAID [10], MAID [9], and Bonn-Gatchina [11] fits. As a test, this parameter was fixed and the remaining multipoles varied. The result (fit 2) is consistent with the SES and is plotted, along with fit 1 and the SES, in Fig. 1. Note that the modified value for Im E_{0+} has an effect noticeable mainly in the recoil polarization, the remaining quantities having been remeasured with greater precision. The 66 included data are fitted with a χ^2 of 80 in fit 1, which is essentially equivalent to the Grushin result. The modified E_{0+} multipole (fit 2) produces a χ^2 of 102. In comparison, the SES fit to both π^+n and π^0p data, between 278 and 282 MeV, produces a χ^2 of 205 for 115 data.

In Table II, a similar comparison is made at 340 MeV. In this case, however, a precise remeasurement [8] of Σ found values shifted from the set available to Grushin [1]. As a result, the refit (fit 1) did not confirm the original set of multipoles. Here too a large value for Im E_{0+} was found, contradicting the SAID [10], MAID [9], and Bonn-Gatchina [11] results, $(0 \pm 0.4)\ 10^{-3}/m_\pi$. Again, fixing this parameter to zero and refitting the remaining multipoles resulted in a solution (fit 2) more compatible with the SES result. In Fig. 2 this readjustment is expressed mainly in a different shape for P, which has sizable error bars. The change to E_{0+} in fit 2 has χ^2 increasing to 30 for 39 data, compared to a χ^2 of 14 in fit 1.

In summary, consistency between the SES results and the method of Ref. [1] is sensitive to the rather poorly determined P data. More precise P data would test the assumptions used in Ref. [1]. It should be realized that almost every existing fit assumes the high-ℓ multipoles are real and given by the Born plus vector-meson exchange terms. Predictions for the beam-target observables [3], given by fit 2, are compared to SAID and available G data [12] in Fig. 3.

III. FITTING π^0p DATA

If π^+n multipoles are available, they can be used to perform a similarly model-independent fit to π^0p photoproduction data. Unfortunately, the existing P data for this channel are even worse over the delta resonance region. The set at 350 MeV has the clearest trend and has been fitted, again assuming a truncated multipole expansion, ignoring higher-ℓ terms. Results are compared in Table III.

Here, neglecting higher-ℓ multipoles leaves an undetermined overall phase. In a fit from Ref. [1], this phase was determined by setting Re M_{1+} to a fixed value. For this fit we find a χ^2/data of ∼0.9 for the considered set of 62 type-S

TABLE II. Single-energy fits to π^+n data at 340 MeV (see text). Multipoles given in $10^{-3}/m_\pi$ units.

Multipole	Grushin [1]	SES	Fit 1	Fit 2
Re E_{0+}	10.29(0.42)	11.36	11.19(0.41)	12.42(0.30)
Im E_{0+}	2.00(0.52)	−0.14	2.15(0.55)	0.0
Re M_{1-}	1.82(1.40)	4.53	2.89(1.45)	4.32(1.27)
Im M_{1-}	−0.11(0.22)	−0.17	1.17(0.30)	0.50(0.31)
Re E_{1+}	0.30(0.35)	1.79	0.69(0.38)	1.22(0.29)
Im E_{1+}	−0.41(0.10)	0.30	0.47(0.14)	0.18(0.16)
Re M_{1+}	1.34(0.98)	−1.82	1.11(0.24)	−1.66(0.61)
Im M_{1+}	−19.26(0.46)	−18.29	−18.84(0.22)	−18.31(0.21)

TABLE III. Single-energy fits to π^0p data at 350 MeV (see text). Multipoles given in $10^{-3}/m_\pi$ units.

Multipole	Grushin [1]	SES	Fit 1	Fit 2	CRS
Re E_{0+}	−1.64(0.46)	−2.69	−2.33(0.46)	−1.58(0.42)	−1.20
Im E_{0+}	1.03(0.24)	2.81	1.27(0.24)	2.14(0.31)	2.36
Re M_{1-}	−2.97(1.99)	−2.89	−2.84(1.84)	−2.73(1.85)	18.67
Im M_{1-}	0.57(0.17)	0.51	−0.33(0.45)	0.90(0.40)	−4.41
Re E_{1+}	0.70(0.62)	1.34	0.63(0.58)	0.38(0.57)	−7.74
Im E_{1+}	−0.78(0.08)	−0.30	−0.47(0.14)	−0.70(0.15)	1.44
Re M_{1+}	−1.3	−5.70	−6.41(0.40)	−4.13(0.01)	15.50
Im M_{1+}	23.89(0.10)	22.81	23.0	23.56(0.11)	−6.36

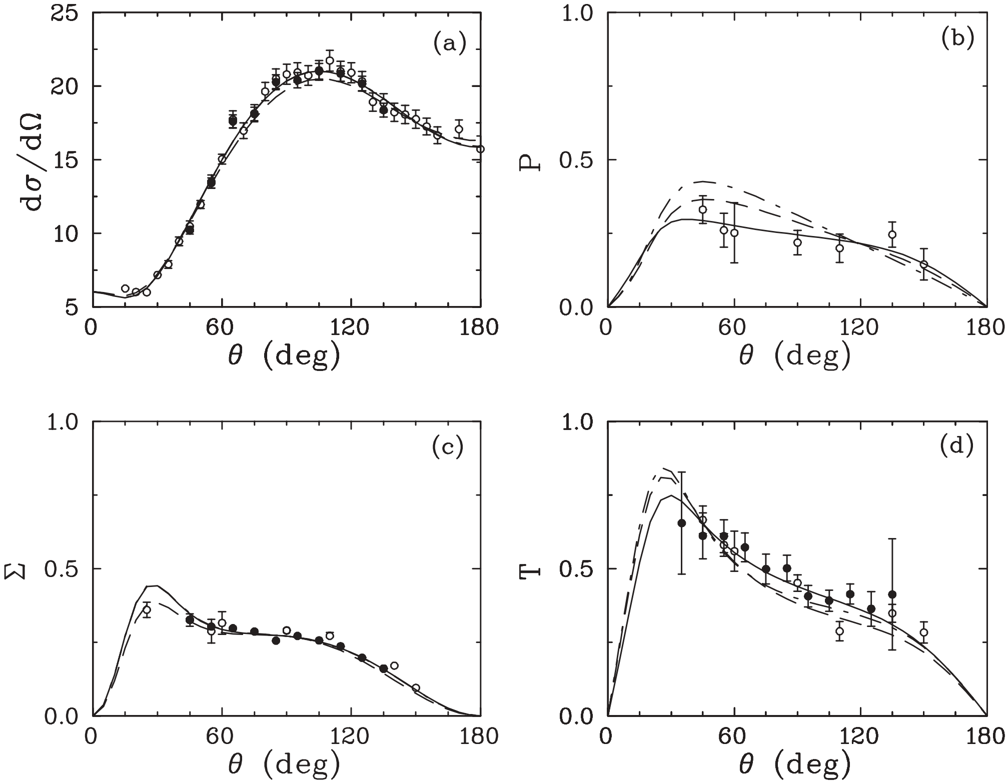

FIG. 1. Fits to π^+n type-S observables at 280 MeV. Fit 1 (solid), SES (dashed), fit 2 (dotted-dashed). Post-1990 data [7,8] (solid symbols), pre-1990 data [6] (open symbols).

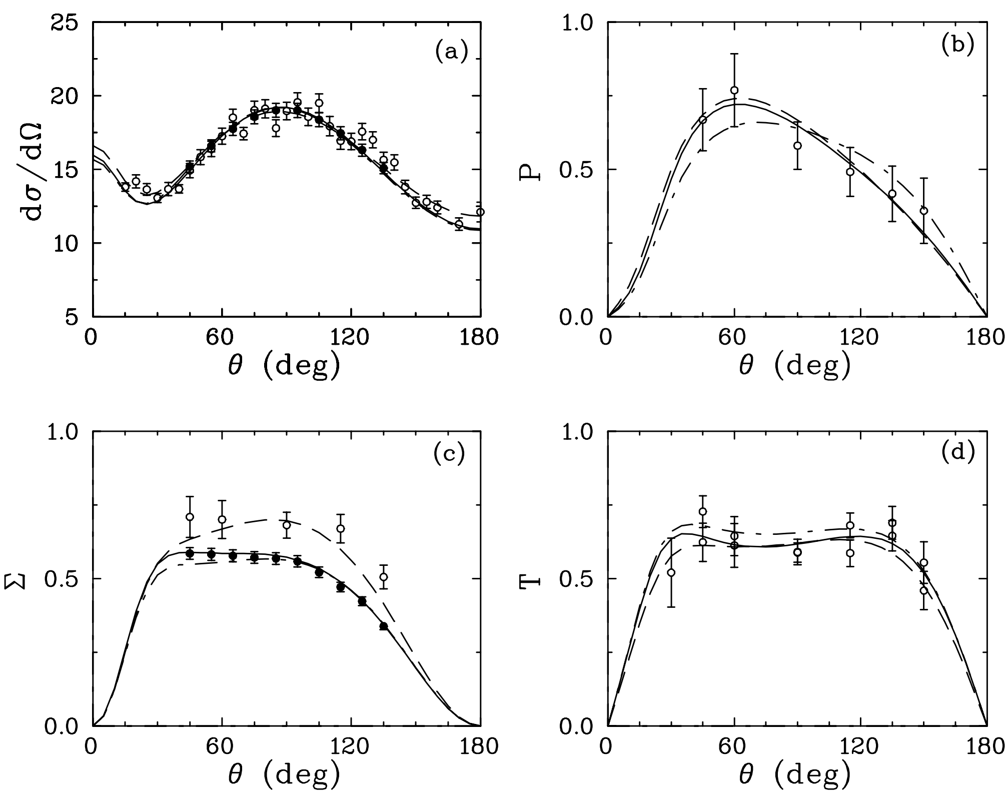

FIG. 2. Fits to π^+n type-S observables at 340 MeV. Fit 1 (solid), Ref. [1] (dashed), fit 2 (dotted-dashed). Data as in Fig. 1.

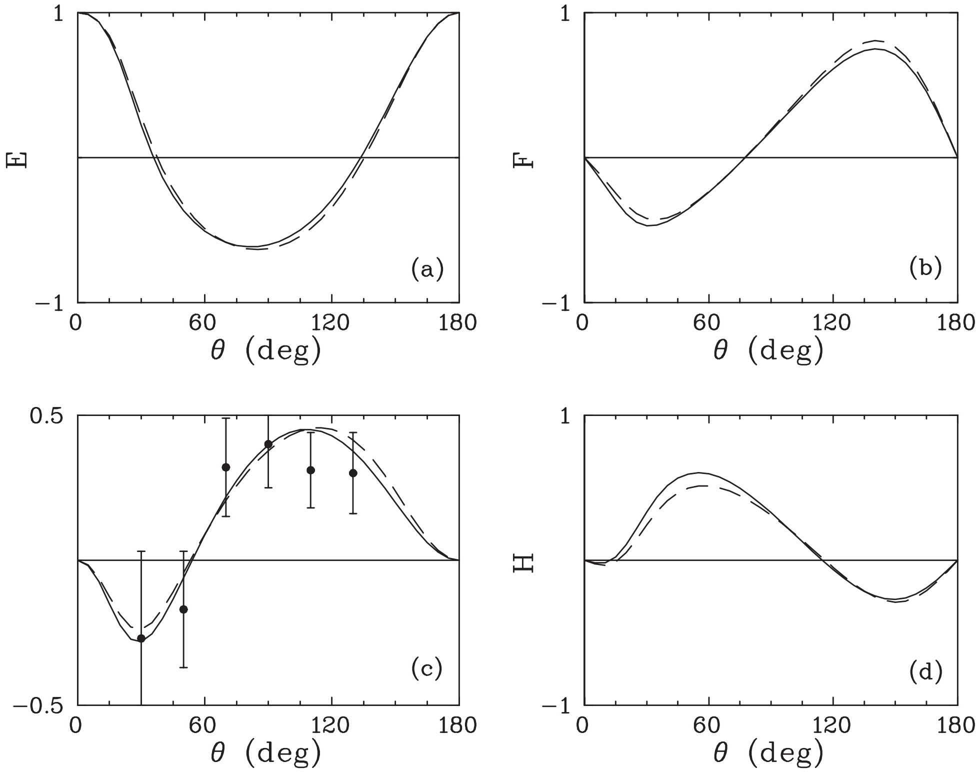

FIG. 3. Prediction of π^+n beam-target observables at 340 MeV from fit 2 (solid) compared to the SAID energy-dependent fit (dashed). Data from Ref. [12].

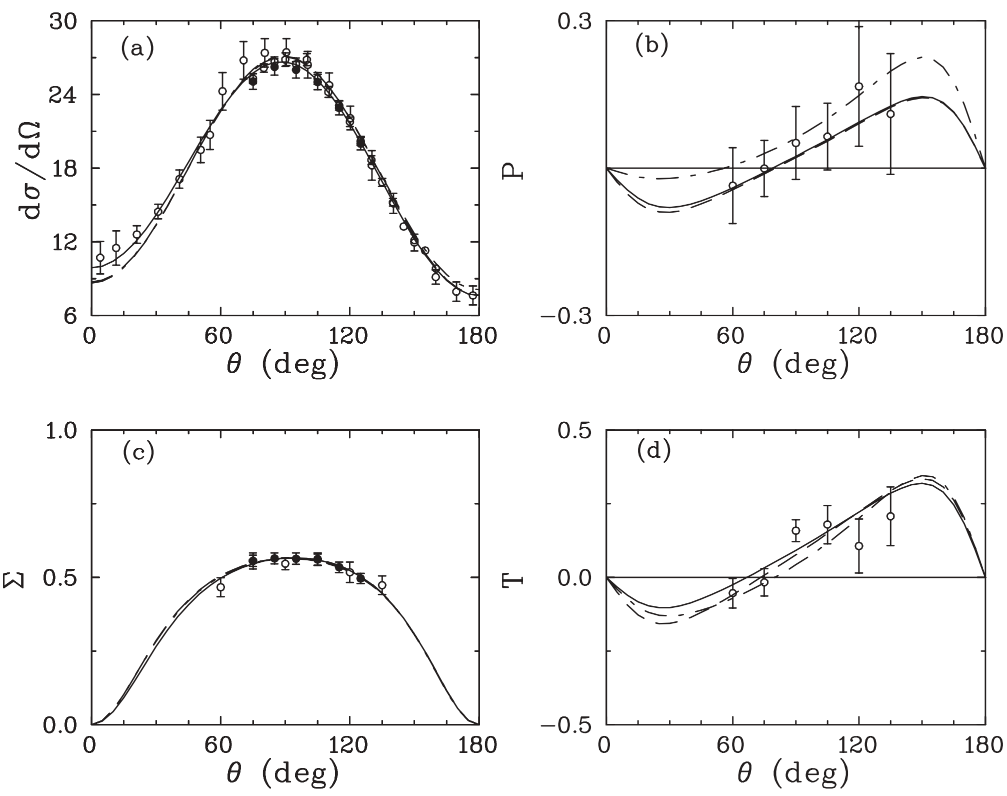

FIG. 4. Fits to π^0p type-S observables at 350 MeV; Fit 1 (solid), Ref. [1] (dashed), Ref. [1] (Born for $\ell > 1$; dotted-dashed). Post-1990 data [8] (solid symbols), pre-1990 data [6] (open symbols).

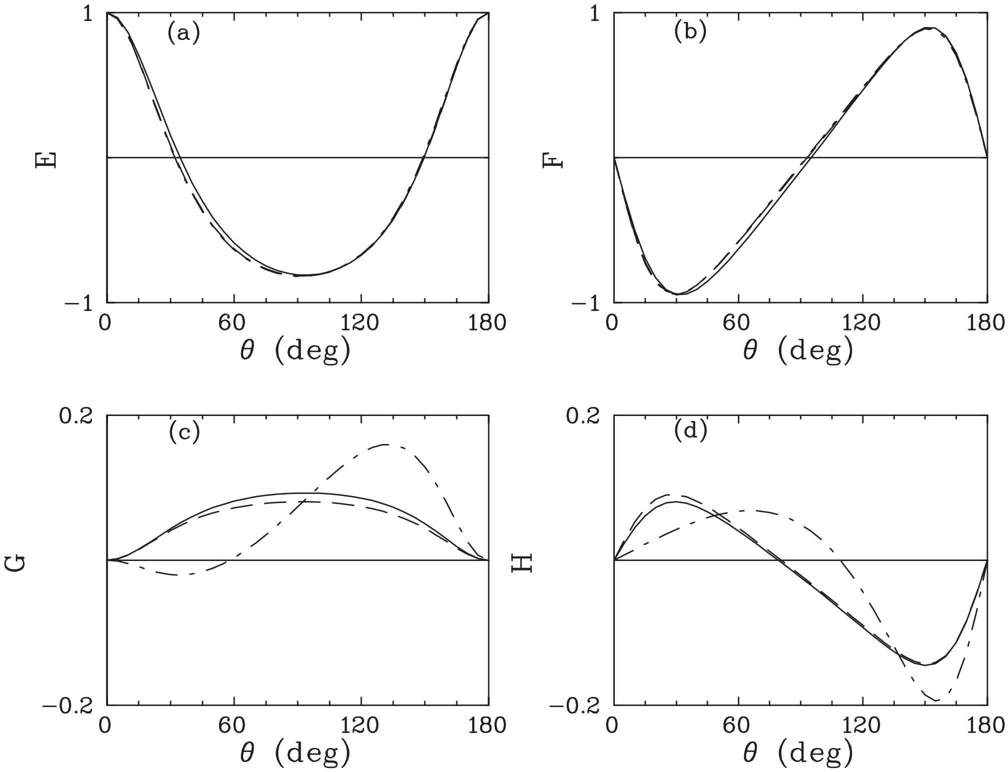

FIG. 5. Prediction of $\pi^0 p$ beam-target observables at 350 MeV; Fit 1 (solid), Ref. [1] (dashed), Ref. [1] (Born for $\ell > 1$; dotted-dashed).

measurements. In fit 1, we have fixed instead Im M_{1+}. In fit 2, a value from the SAID energy-dependent fit was assumed for $M_{1+}^{\pi^+ n}$ and a parameter α was fitted using Watson's theorem [13],

$$M_{1+}^{\pi^0 p} = \alpha e^{i\delta_{33}} + \frac{1}{\sqrt{2}} M_{1+}^{\pi^+ n} \tag{5}$$

with δ_{33} being the P_{33} phase from elastic πN scattering. Two values for α were found, positive and negative, the positive value being chosen above to conform with the phase of the tabulated SES results. In fit 1, a second solution with Re M_{1+} positive was also found. Fits 1 and 2 produce exactly the same observables, leading to transversity amplitudes with fixed relative phases but different overall phases. This is true also of the variants having different signs for α and Re M_{1+}. The result labeled CRS was obtained by conjugating the roots of the complex polynomials for each transversity amplitude [14] from fit 1. This is a symmetry of the type-S observables and half of the double-polarization quantities. The resulting solution is therefore not related to fits 1 and 2 by a rotation of the multipoles. As fits 1 and 2, and the conjugated-root solution (CRS), give identical results for type-S observables, further information is required to select the correct solution. If the multipoles of CRS are rotated to have a phase for $M_{1+}^{\pi^0 p}$ matching fit 2, the resulting values for $E_{1+}^{\pi^0 p}$ will not combine, via Eq. (5), to give the proper phase for $E_{1+}^{3/2}$. In Ref. [1], the neutral- and charged-pion results were combined in an isospin analysis assuming that the $E_{1+}^{3/2}$ and $M_{1+}^{3/2}$ amplitudes had the same phase, without fixing this to be the phase from πN elastic scattering. However, given the sizable errors found for E_{1+}, a direct application of Watson's theorem seemed more effective.

In Fig. 4, the fit from Ref. [1] is compared to fit 1 and data for type-S observables. We also show the effect of adding the MAID Born contribution, for waves with $\ell > 1$, to the Grushin multipoles in Table III. The effect is minimal except for P, which changes significantly, but not outside the large uncertainties of these data. These comparisons are carried over to the beam-target set in Fig. 5. As in Fig. 3, for $\pi^+ n$, the quantities E and F are quite stable, while G and H change significantly with the addition of the higher-ℓ contributions, given by vector-meson exchange. The curves with this addition look more like the MAID result. Fit 1 and CRS give identical results for E and H, but have opposite signs for G and F.

Somewhat different results for P and G were also found when the Σ data [8], used in the fit, were replaced by a measurement with wider angular coverage [15]. In addition, preliminary measurements of a quantity proportional to G appear to have a shape unlike that predicted by fit 1 [16]. Precise measurements of P and G would clearly help to stabilize the fit.

IV. CONCLUSIONS

We have reexamined the extraction of pion photoproduction multipoles from type-S data with minimal model input. In the process, we have suggested that deviations from recent fits covering the resonance region may be owing to problems in the database. This study should also give some qualitative guidance to those who plan to extract multipoles from the present generation of polarized photoproduction experiments. The results given here suggest that very precise data will be required for a reliable extraction of all but the dominant multipoles. This is particularly evident it Table II, where a sizable change in Im E_{0+} and a wrong sign for Re M_{1+} are linked to modest changes in the fit to P data.

The procedure for $\pi^+ n$ photoproduction could be continued up to higher energies, if the real high-ℓ multipole assumption remains valid. For $\pi^0 p$, the existence of multiple solutions makes an isospin decomposition more challenging. The use of Eq. (4) is also restricted to energies where the P_{33} phase is elastic. Finally, we note the possibility of accidental symmetries, generating solutions beyond those considered here. This possibility was considered in Ref. [14].

ACKNOWLEDGMENTS

This work was supported in part by the US Department of Energy Grant No. DE-FG02-99ER41110. The author thanks M. Paris and L. Tiator for discussions regarding the general amplitude reconstruction problem.

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