Nuclear pairing reduction due to rotation and blocking

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Nuclear pairing gaps of normally deformed and superdeformed nuclei are investigated using the particle-number-conserving (PNC) formalism for the cranked shell model, in which the blocking effects are treated exactly. Both rotational frequency ω dependence and seniority (number of unpaired particles) ν dependence of the pairing gap $\tilde{\Delta}$ are investigated. For the ground-state bands of even-even nuclei, PNC calculations show that, in general, $\tilde{\Delta}$ decreases with increasing ω, but the ω dependence is much weaker than that calculated by the number-projected Hartree-Fock-Bogolyubov approach. For the multiquasiparticle bands (seniority $\nu > 2$), the pairing gaps stay almost ω independent. As a function of the seniority ν, the bandhead pairing gaps $\tilde{\Delta}(\nu, \omega = 0)$ decrease slowly with increasing ν. Even for the highest seniority ν bands identified so far, $\tilde{\Delta}(\nu, \omega = 0)$ remains greater than 70% of $\tilde{\Delta}(\nu = 0, \omega = 0)$.

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I. INTRODUCTION

Since the seminal article by Bohr, Mottelson, and Pines [1], significant effects of nuclear pairing were established in fundamental nuclear properties [2]. Soon afterward, the Bardeen-Cooper-Schrieffer (BCS) theory for metallic superconductivity and quasiparticle (qp) formalism were transplanted into the nuclear structure literature to treat the nuclear pairing correlation [3–5]. Now the BCS or the more elaborate Hartree-Fock-Bogolyubov (HFB) approximations are the standard methods in nuclear physics. However, along with their great successes, both BCS and HFB approximations for nuclear pairing raise some concerns [6,7]. One of them is the nonconservation of the particle number. Because the number of nucleons in a nucleus is not very large ($n \sim 10^2$), and the number of valence nucleons ($n \sim 10$) dominating the nuclear low-lying excited states is very limited, the relative particle-number fluctuation, $\delta n/n$, is not negligible. Indeed, it was found that in all self-consistent solutions to the cranked HFB equation a pairing collapsing occurs for angular momentum I greater than a critical value I_c [8].

Much work has been done to restore this broken symmetry. The Lipkin-Nogami (LN) method [9–11] was established in early 1960s. After using this approximate particle-number-projection method, the pairing phase transition disappears [12,13]. However, earlier studies showed that the LN method broke down in the weak-pairing limit [14,15]. At the same time, various particle-number-projection approaches of pairing interaction in BCS or HFB formalism were developed [16–19]. In these approaches, the ideal treatment is variation after projection, but when spin goes higher, this method becomes very complicated and computationally expensive. All these methods tried to solve the problem of the particle-number nonconservation. However, when it was achieved, no pairing phase transition was found [16,17]. This proves that the occurrence of nuclear pairing collapsing originates from particle-number nonconservation. Other than the variational approach, some methods that directly solve the corresponding Schrödinger equation have been developed [6,20]. In these methods, the particle number is strictly conserved. The particle-number-conserving method used in Ref. [20], in which the single-particle states stem from the Hartree-Fock mean field, is a little different from the method used in our work, in which the single-particle states stem from the Nilsson model.

Another problem related to the violation of particle-number conservation is the occurrence of spurious states in the BCS (HFB) qp formalism. As pointed out by Richardson [21], an important class of low-lying excitations in nuclei cannot be described in the standard BCS- or HFB-like theories. The remedy in terms of the particle-number projection considerably complicates the algorithm, yet fails to properly describe the higher energy spectrum of the pairing Hamiltonian [7].

The issue of most concern is the proper treatment of the Pauli blocking effect on pairing, which is responsible for the odd-even differences in nuclear properties (binding energies, moments of inertia, etc.). As emphasized by Rowe [22], although the blocking effects are straightforward, it is very difficult to treat them consistently in the qp formalism because they introduce different qp bases for different blocked orbitals. Indeed, it was shown that the properties of a rotational band are very sensitive to the Coriolis response of the blocked single-particle orbitals [23].

In this paper, in order to investigate the pairing reduction due to rotation and blocking, we use the particle-number-conserving (PNC) formalism for treating the cranked shell model (CSM) with pairing interaction, in which the particle number is conserved and the blocking effects are treated exactly. The details of the PNC formalism for calculating the moment of inertia (MOI) have already been given by Zeng *et al.* [24]. Only the PNC formalism for calculating the nuclear pairing gap is given in Sec. II. Section III gives the PNC calculations for nuclear pairing gaps of various types of pair-broken rotational bands in normally deformed (ND)

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and superdeformed (SD) nuclei [seniority (number of unpaired particles) $\nu \geqslant 2$ for even-even nuclei, $\nu > 1$ for odd-A nuclei], as well as the rotational frequency ω dependence and seniority ν dependence of pairing gaps. A brief summary is given in Sec. IV.

II. PNC FORMALISM FOR NUCLEAR PAIRING GAP

The CSM Hamiltonian of an axially deformed nucleus in the rotating frame is

$$H_{\rm CSM} = H_0 + H_{\rm P} = H_{\rm Nil} - \omega J_x + H_{\rm P}, \tag{1}$$

where $H_{\rm Nil}$ is the Nilsson Hamiltonian, $-\omega J_x$ is the Coriolis interaction with cranking frequency ω about the x axis, $H_{\rm P} = H_{\rm P}(0) + H_{\rm P}(2)$ is the pairing interaction,

$$H_{\rm P}(0) = -G_0 \sum_{\xi\eta} a_\xi^+ a_{\bar\xi}^+ a_{\bar\eta} a_\eta = -G_0 \sum_{\xi\eta} s_\xi^+ s_\eta, \tag{2}$$

$$H_{\rm P}(2) = -G_2 \sum_{\xi\eta} q_2(\xi) q_2(\eta) a_\xi^+ a_{\bar\xi}^+ a_{\bar\eta} a_\eta = -G_2 \sum_{\xi\eta} q_2(\xi) q_2(\eta) s_\xi^+ s_\eta, \tag{3}$$

where $\bar\xi$ ($\bar\eta$) labels the time-reversed state of a Nilsson state $\xi(\eta)$, $q_2(\xi) = \sqrt{16\pi/5}\langle\xi|r^2 Y_{20}|\xi\rangle$ is the diagonal element of the stretched quadrupole operator, G_0 and G_2 are the effective strengths of monopole and quadrupole pairing interactions, respectively, and $s_\xi^+ = a_\xi^+ a_{\bar\xi}^+$ ($s_\eta = a_{\bar\eta} a_\eta$) is the pair creation (annihilation) operator.

In the PNC calculation, $H_{\rm CSM}$ is diagonalized in a sufficiently large cranked many-particle configuration (CMPC) space [24] and $|\Psi\rangle$ is expressed as

$$|\Psi\rangle = \sum_i C_i |i\rangle \quad (C_i \text{ real}), \tag{4}$$

where $|i\rangle$ is an eigenstate of H_0 with configuration energy $E_i^{(0)}$, characterized by the particle number N, parity π, signature r ($=e^{-i\pi\alpha}$), and seniority ν (number of unpaired particles). For the seniority $\nu = 0$ ground-state band ($K^\pi = 0^+$) of an even-even nucleus (qp vacuum in the BCS formalism), each $|i\rangle$ in Eq. (4) is of the product form

$$s_\xi^+ s_\eta^+ \cdots |0\rangle, \quad \xi \neq \eta \neq \cdots . \tag{5}$$

For the seniority $\nu = 1$ band ($\sim$1-qp band in the BCS formalism) in an odd-even nucleus, $|i\rangle$ is of the form

$$a_\lambda^+ s_\xi^+ s_\eta^+ \cdots |0\rangle, \quad \xi \neq \eta \neq \cdots (\neq \lambda), \tag{6}$$

where λ is the blocked single-particle state, $\xi \neq \eta \neq \cdots (\neq \lambda)$ (Pauli blocking effect) and the angular momentum projection along nuclear symmetry z axis $K = \Omega_\lambda$. For the seniority $\nu = 2$ band ($\sim$2-qp band in the BCS formalism) in an even-even nucleus, $|i\rangle$ is of the form

$$a_\lambda^+ a_\sigma^+ s_\xi^+ s_\eta^+ \cdots |0\rangle, \quad \xi \neq \eta \neq \cdots (\neq \lambda \neq \sigma), \tag{7}$$

where $\lambda \neq \sigma$ are two blocked single-particle states ($K = \Omega_\lambda + \Omega_\sigma$). The PNC forms of the $\nu > 2$ (multiquasiparticle) bands are similar. Strictly speaking, due to the Coriolis interaction, $-\omega J_x$, ν, and K are not exactly conserved for $\omega \neq 0$. Walker and Dracoulis [25] pointed out that some forms of K-mixing must exist to enable the K-forbidden transition observed in many low-lying rotational bands of axially symmetric nuclei. However, in the low-ω region, ν and K may be served as useful quantum numbers characterizing a low-lying excited rotational band.

The kinematic and dynamic MOIs for the state $|\Psi\rangle$ are as follows [24,26]:

$$J^{(1)} = \frac{1}{\omega}\langle\Psi|J_x|\Psi\rangle, \quad J^{(2)} = \frac{d}{d\omega}\langle\Psi|J_x|\Psi\rangle, \tag{8}$$

where

$$\langle\Psi|J_x|\Psi\rangle = \sum_i C_i^2 \langle i|J_x|i\rangle + 2\sum_{i<j} C_i C_j \langle i|J_x|j\rangle \tag{9}$$

is the angular momentum alignment of the state $|\Psi\rangle$.

In the PNC formalism, the nuclear pairing gap may be reasonably defined as [16,17,27]

$$\tilde\Delta = G_0\left[-\frac{1}{G_0}\langle\Psi|H_{\rm P}|\Psi\rangle\right]^{1/2}, \tag{10}$$

where $|\Psi\rangle$ is a PNC eigenstate [Eq. (4)] of $H_{\rm CSM}$ with eigenvalue E. In the BCS formalism for $H_{\rm CSM}$ with the monopole pairing interaction only, $H_{\rm P} = -G_0 S^+ S$, where $S^+ = \sum_\xi s_\xi^+$, $S = \sum_\eta s_\eta$, and for the qp vacuum band $|0\rangle$,

$$|0\rangle = \Pi_\xi (U_\xi + V_\xi s_\xi^+)|0\rangle, \quad U_\xi^2 + V_\xi^2 = 1, \tag{11}$$

$\tilde\Delta$ is reduced to the usual definition of nuclear pairing gap Δ,

$$\Delta = G_0 \langle\langle 0|S^+|0\rangle\rangle = G_0 \sum_\xi U_\xi V_\xi. \tag{12}$$

Calculations show that for the low-lying excited eigenstates of $H_{\rm CSM}$, the number of important CMPCs (with weight $\geqslant 1\%$, say) is very limited (usually $<$20 for the ND rare-earth nuclei); thus it is not difficult to get sufficiently accurate solutions to the low-lying excited eigenstates of $H_{\rm CSM}$ by diagonalizing $H_{\rm CSM}$ in a sufficiently large CMPC space [23,24]. To ensure the PNC calculations for nuclear low-lying excited states are both workable and accurate [7,28], it is essential to adopt a CMPC truncation (Fock-space truncation) in the PNC calculation in place of the usual single-particle level (SPL) truncation in shell-model calculations. This is understandable from the perturbation expansion of $H_{\rm CSM}$ (1), as it refers to a many-particle system with pairing interactions. In general, the lower the configuration energy of the many-particle configuration (MPC), the larger the weight of the corresponding MPC in low-lying excited eigenstates of $H_{\rm CSM}$ will be. The stability of the final results with respect to the basis cutoff has been illustrated in detail by Molique and Dudek [7], as well as by Liu *et al.* [29].

In the following PNC calculations, $H_{\rm CSM}$ is diagonalized in the CMPC space with dimension 1500 for both protons and neutrons. The corresponding effective proton and neutron pairing strength are adopted to reproduce the experimental odd-even differences in nuclear binding energies. Proper Nilsson level schemes are adopted to reproduce the experimental bandhead energies and MOIs of the low-lying excited seniority

$\nu = 1$ (1-qp) bands. Thus, the pairing gaps $\tilde{\Delta}$ of various low-lying excited bands can be convincingly extracted by the PNC calculations without any free parameter.

III. CALCULATIONS AND DISCUSSIONS

In this section the PNC calculations for nuclear pairing gaps of some typical bands in ND and SD nuclei are presented. The rotational frequency ω dependence and seniority ν dependence of the pairing gaps are discussed in detail.

A. Ground-state bands of ^{168}Yb and ^{168}Hf

The angular momentum dependence of pairing gaps $\tilde{\Delta}_n$ (neutrons) and $\tilde{\Delta}_p$ (protons) for the ground-state band (gsb) of ^{168}Yb and ^{168}Hf have been calculated in the number-projected HFB (NHFB) approach in Ref. [17]. The pairing gap reductions in the observed angular momentum range $I = 0 \to 44\hbar$ ($\approx \omega = 0.61$ MeV/$\hbar$) for ^{168}Yb(gsb) and $I = 0 \to 34\hbar$ ($\approx \omega = 0.52$ MeV/$\hbar$) for ^{168}Hf(gsb) calculated by NHFB are [16,17]

$$\begin{aligned} &\frac{\tilde{\Delta}_n(I = 44\hbar)}{\tilde{\Delta}_n(I = 0)} \approx 48\%, \\ &\frac{\tilde{\Delta}_p(I = 44\hbar)}{\tilde{\Delta}_p(I = 0)} \approx 63\% \quad \text{for} \quad {}^{168}\text{Yb(gsb)}, \\ &\frac{\tilde{\Delta}_n(I = 34\hbar)}{\tilde{\Delta}_n(I = 0)} \approx 38\% \quad \text{for} \quad {}^{168}\text{Hf(gsb)}. \end{aligned} \tag{13}$$

For comparison, in this section the ω dependence of pairing gaps of ^{168}Yb(gsb) and ^{168}Hf(gsb) are calculated using the PNC formalism. To validate the PNC calculations of $\tilde{\Delta}_p$ (proton) and $\tilde{\Delta}_n$ (neutron) Fig. 1(b), the kinematic MOIs $J^{(1)}$ are also calculated under the PNC formalism and compared with the experiments [30,31] [see Fig. 1(a)]. The experimental MOIs $J^{(1)}$ are very well reproduced by the PNC calculations (except in the band-crossing region). Thus, we believe the PNC calculations of pairing gaps (ω dependence, ν dependence, etc.) are trustworthy. In the observed rotational frequency range, the pairing gap reductions calculated in the PNC formalism are

$$\begin{aligned} &\frac{\tilde{\Delta}_n(\omega = 0.61\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 70\%, \\ &\frac{\tilde{\Delta}_p(\omega = 0.61\ \text{MeV}/\hbar)}{\tilde{\Delta}_p(\omega = 0)} \approx 80\% \quad \text{for} \quad {}^{168}\text{Yb}, \\ &\frac{\tilde{\Delta}_n(\omega = 0.52\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 70\%, \\ &\frac{\tilde{\Delta}_p(\omega = 0.52\ \text{MeV}/\hbar)}{\tilde{\Delta}_p(\omega = 0)} \approx 83\% \quad \text{for} \quad {}^{168}\text{Hf}, \end{aligned} \tag{14}$$

which remains more than 70% of the bandhead value throughout the experimental ω range. As expected, in both the NHFB and the PNC formalisms no pairing phase transition from superfluidity to normal motion ($\tilde{\Delta} \to 0$) is found with increasing ω. However, the ω dependence of $\tilde{\Delta}$ in the PNC calculations is weaker than that calculated by the NHFB approach. In addition, it is noted that due to the neutron subshell effect at $N = 98$, in both PNC and NHFB calculations for ^{168}Yb, the pairing gap reduction of the neutron is larger than that of the proton. It was noted by Hamamoto [34] that an inherent issue of the CSM is the violation of rotational symmetry, and the reliability of calculations in the CSM, particularly in the band-crossing region, is questionable. Afterward, angular momentum projection techniques were developed [35]. It is interesting to note that the ω dependence of the pairing gaps for the gsb of ^{168}Yb calculated by the angular momentum projection technique [36] are similar to that of the PNC calculations.

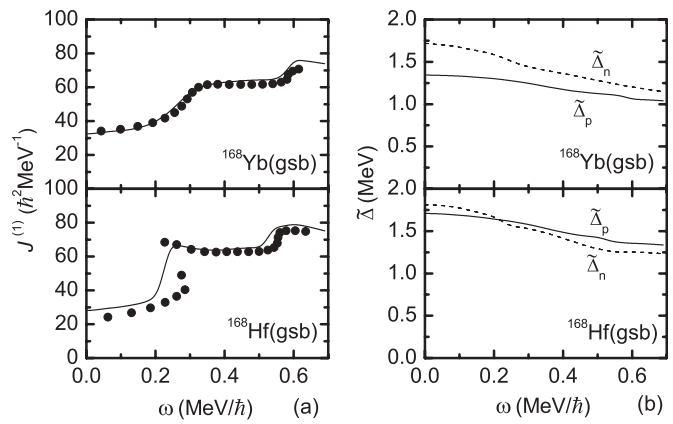

FIG. 1. The MOIs and pairing gaps $\tilde{\Delta}$ for the ground-state bands of ^{168}Yb and ^{168}Hf. (a) The experimental MOIs [30,31] are denoted by the solid circle •, and the calculated MOIs by the PNC method are denoted by solid lines. The Nilsson parameters (κ, μ) and deformation (ε_2, ε_4) are taken from [32,33]. The monopole and quadrupole pairing strengths for protons and neutrons are adopted to reproduce the odd-even differences in nuclear binding energies, $G_n = 0.30$, $G_{2n} = 0.010$, $G_p = 0.29$ for ^{168}Yb; $G_n = 0.39$, $G_p = 0.35$ for ^{168}Hf. (b) The PNC calculated pairing gaps for protons (neutrons) are denoted by solid (dashed) lines.

B. Multiquasiparticle bands of the heavier rare-earth nuclei ($A \sim 178$)

The seniority ν dependence of nuclear pairing gaps has been investigated by Dracoulis *et al.*, using the LN method [37]. They showed that the bandhead pairing gap $\Delta(\nu, \omega = 0)$ decreases approximately by

$$\Delta(\nu, \omega = 0) = (0.75)^{\nu/2}\Delta(\nu = 0, \omega = 0). \tag{15}$$

In this section we will investigate the ν dependence of the $\tilde{\Delta}$ using the PNC formalism. To get pairing gaps for these multiquasiparticle bands, a proper Nilsson level scheme for the deformed heavier rare-earth nuclei ($A \sim 178$) is necessary. However, the level scheme (Lund systematics) [32,33] is unable to properly reproduce the experimental bandhead energies of the low-lying excited 1-qp bands of ^{177}Ta, particularly the gsb, $\pi 7/2^+[404]$. So the Nilsson parameters (κ, μ) in [32] are slightly adjusted (see the caption of Fig. 2). Figure 2 shows the experimental [38] and calculated MOIs of four 1-qp bands in ^{177}Ta.

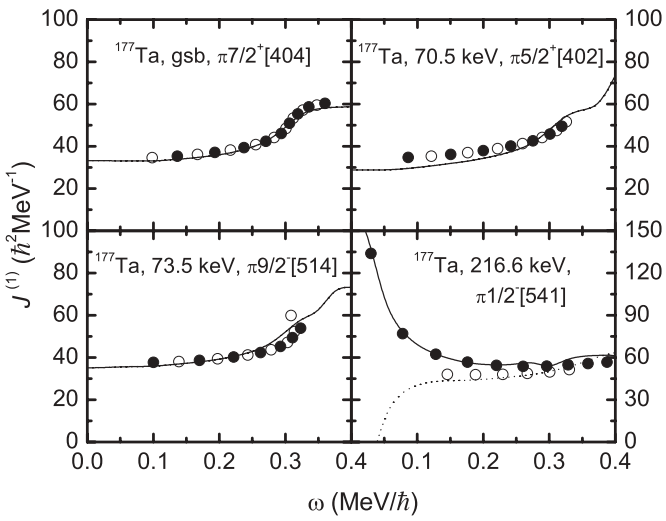

FIG. 2. MOIs of four low-lying seniority $\nu = 1$ bands in ^{177}Ta. The experimental MOIs [38] are denoted by • ($\alpha = 1/2$) and ○ ($\alpha = -1/2$), respectively. The calculated MOIs by the PNC method are denoted by solid lines ($\alpha = 1/2$) and dotted lines ($\alpha = -1/2$), respectively. The Nilsson parameters (κ, μ) in [32] are slightly adjusted to reproduce the bandhead energies of the 1-qp bands. For protons, $\kappa_4 = 0.060$ ($N = 4$), $\kappa_5 = 0.061$ ($N = 5$), $\mu_4 = 0.55$, and $\mu_5 = 0.69$. For neutrons, $\kappa_5 = 0.066$, $\kappa_6 = 0.058$, $\mu_5 = 0.49$, and $\mu_6 = 0.40$. The deformation parameters $(\varepsilon_2, \varepsilon_4) = (0.24, 0.04)$ are from [33], i.e., an average of the neighboring even-even Hf and W isotopes. The effective pairing interaction strengths for both protons and neutrons, $G_n = 0.26$ MeV, $G_p = 0.26$ MeV, are determined by the experimental odd-even differences in nuclear binding energies.

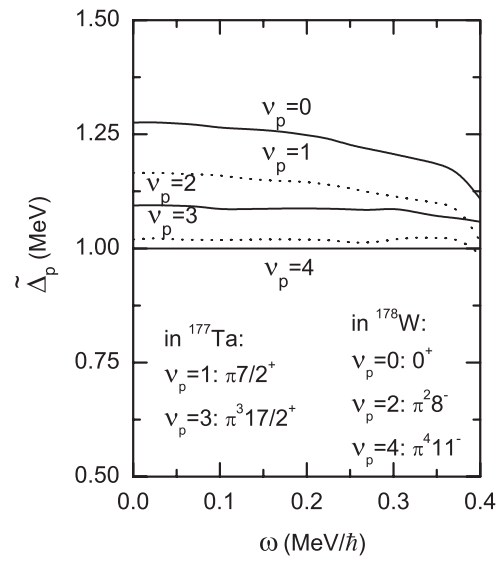

FIG. 3. The proton pairing gaps $\tilde{\Delta}_p$ for the $\nu_p = 1, 3$ bands in ^{177}Ta (dotted lines) and $\nu_p = 0, 2, 4$ configurations in ^{178}W (solid lines).^{177}Ta: $\nu_p = 1$ band (gsb), $\pi 7/2^+[404]$; $\nu_p = 3$, $K^\pi = 17/2^+$ band at 1523 keV [38], $\pi^3 17/2^+(7/2^+[404] \otimes 9/2^-[514] \otimes 1/2^-[541])$. ^{178}W: gsb, $\nu_p = 0$, $K^\pi = 0^+$; $\nu_p = 2$ configuration $\pi^2 8^-(7/2^+[404] \otimes 9/2^-[514])$ in $K^\pi = 15^+$ $(\pi^2 8^- \otimes \nu^2 7^-)$ band at 3653 keV; $\nu_p = 4$ configuration $\pi^4 11^-(7/2^+[404] \otimes 5/2^+[402] \otimes 9/2^-[514] \otimes 1/2^-[541])$ in $K^\pi = 18^+$ $(\pi^4 11^- \otimes \nu^2 7^-)$ band at 4878 keV [37,39].

In Fig. 3, we show the PNC calculations of the proton pairing gaps for $\nu_p = 1, 3$ bands in ^{177}Ta and the gsb and multiquasiparticle bands with $\nu_p = 2$ and 4 proton configurations in ^{178}W [37–39]. The ω dependence of $\tilde{\Delta}_p$ for ^{178}W(gsb) is similar to that of ^{168}Yb(gsb) and ^{168}Hf(gsb) [see Fig. 1(b)]. The ω dependence of $\tilde{\Delta}_p$'s of the four low-lying excited 1-quasiproton bands of ^{177}Ta is similar to ^{178}W(gsb), so only the $\tilde{\Delta}_p$ for the gsb ($\pi 7/2^+[404]$) of ^{177}Ta is shown in Fig. 3. The ω dependence of the $\nu_p = 2$ configuration $\pi^2 8^-$ $(7/2^+[404] \otimes 9/2^-[514])$ in ^{178}W is weaker. For the low-lying excited multiquasiparticle (seniority $\nu_p > 2$) bands, the $\tilde{\Delta}_p$'s stay *nearly ω independent*. In fact, for realistic nuclei, the blocking effects on pairing are significant only for a few orbitals nearest the Fermi surface. For low-lying excited multiquasiparticle bands, a few orbitals nearest the Fermi surface are almost blocked, and for orbitals far from the Fermi surface the ω dependence of the blocking effects on pairing are quite small.

As a function of seniority ν, in general, the pairing gap $\tilde{\Delta}(\nu)$ gradually decreases with increasing ν. The pairing gap reductions at the bandhead ($\omega = 0$) calculated by the PNC method are

$$\begin{aligned} &\frac{\tilde{\Delta}_p(\nu=1)}{\tilde{\Delta}_p(\nu=0)} \approx 91\%, \quad \frac{\tilde{\Delta}_p(\nu=2)}{\tilde{\Delta}_p(\nu=0)} \approx 86\%, \\ &\frac{\tilde{\Delta}_p(\nu=3)}{\tilde{\Delta}_p(\nu=0)} \approx 80\%, \quad \frac{\tilde{\Delta}_p(\nu=4)}{\tilde{\Delta}_p(\nu=0)} \approx 78\%, \end{aligned} \tag{16}$$

which are weaker than those given in Eq. (15) [37]. Even for the highest seniority ν bands identified so far, the pairing gap $\tilde{\Delta}(\nu, \omega = 0)$ is always larger than 70% of $\tilde{\Delta}(\nu = 0, \omega = 0)$.

C. Ground-state bands of ^{238}U and ^{253}No

The PNC calculations for the ground-state bands of the actinide nuclei ^{238}U and ^{253}No are shown in Fig. 4. The experimental data of the MOIs are taken from [40,41]. The PNC calculations for the pairing gap reduction show

$$\begin{aligned} &\frac{\tilde{\Delta}_n(\omega = 0.30\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 85\%, \\ &\frac{\tilde{\Delta}_p(\omega = 0.30\ \text{MeV}/\hbar)}{\tilde{\Delta}_p(\omega = 0)} \approx 84\% \quad \text{for} \quad {}^{238}\text{U(gsb)}, \\ &\frac{\tilde{\Delta}_n(\omega = 0.30\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 84\%, \\ &\frac{\tilde{\Delta}_p(\omega = 0.30\ \text{MeV}/\hbar)}{\tilde{\Delta}_p(\omega = 0)} \approx 76\% \quad \text{for} \quad {}^{253}\text{No(gsb)}, \end{aligned} \tag{17}$$

i.e., $\tilde{\Delta}_p$'s and $\tilde{\Delta}_n$'s decrease very slowly with increasing ω, quite similar to the rare-earth nuclei.

D. Pairing gaps of SD bands

Chasman [43] pointed out that BCS treatment of nuclear pairing is not appropriate for SD states because the single-particle level density near the Fermi surface is low and the

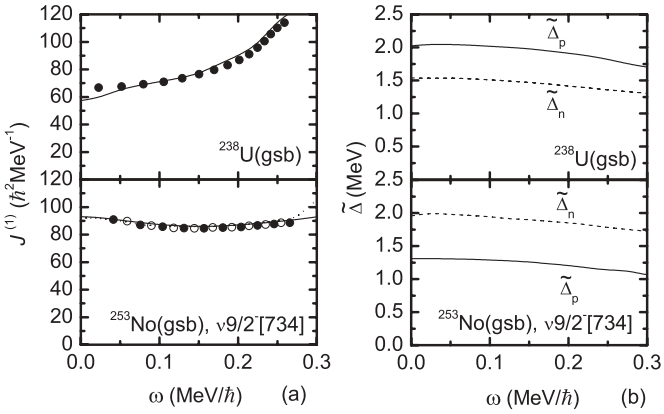

FIG. 4. The MOIs and pairing gaps of the gsb of ^{238}U and ^{253}No. (a) The experimental MOIs [40,41] are denoted by • ($\alpha = 0, 1/2$) and ○ ($\alpha = -1/2$). The calculated MOIs by the PNC method are denoted by solid lines ($\alpha = 0, 1/2$) and dotted lines ($\alpha = -1/2$). The Nilsson parameters (κ, μ) are taken from [32] for both ^{238}U and ^{253}No. Deformations ($\varepsilon_2, \varepsilon_4$) = (0.21, −0.04) of ^{238}U are taken from [33]. For ^{253}No, the quadrupole deformation parameters for the neighbor even-even nuclei are deduced as $\beta_2 = 0.28 \pm 0.02$ from the experiment [42]. Here we choose $\varepsilon_2 = 0.26$ and $\varepsilon_4 = 0.01$ in our calculation for ^{253}No. To reproduce the experimental odd-even difference in binding energies and the ω dependence of MOIs, it seems that a small amount of quadrupole pairing interaction is necessary, i.e., $G_n = 0.29$ MeV, $G_{2n} = 0.015$ MeV, $G_p = 0.32$ MeV, $G_{2p} = 0.040$ MeV for ^{238}U; $G_n = 0.22$ MeV, $G_{2n} = 0.010$ MeV, $G_p = 0.26$ MeV, $G_{2p} = 0.010$ MeV for ^{253}No. (b) The PNC calculated pairing gaps $\tilde{\Delta}_p$'s ($\tilde{\Delta}_n$'s) for the ground-state bands of ^{238}U and ^{253}No are denoted by solid (dashed) lines.

BCS method is not correct in this limit. In most cases, the linkage between the SD bands and low-lying excited states have not yet been established, and thus the actual spins of some SD bands are not determined, but the dynamic MOIs $J^{(2)}$ can be extracted from the observed differences in subsequent γ transition energies:

$$J^{(2)}(I) = 4\hbar^2/[E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)]. \quad (18)$$

Obviously, the accuracy of $J^{(2)}$ is lower than $J^{(1)}(I) = (2I+1)\hbar^2/E_\gamma(I+1 \to I-1)$. However, the actual spins of some SD bands have been established experimentally (e.g., see Ref. [44]), and thus the $J^{(1)}$'s can be accurately extracted. The ω dependence of experimental MOIs for a series of SD bands [45–47] was reproduced very well by the PNC calculations for the CSM with both monopole and quadrupole pairing interactions [29,48,49]. Figure 5 ahows the PNC calculations of the pairing gaps $\tilde{\Delta}$ for the $\nu_n = 1$ SD bands ^{193}Hg(1) ($\nu 5/2^-[512], \alpha = -1/2$), ^{193}Hg(2b) ($\nu 9/2^+[624], \alpha = 1/2$), the $\nu = 0$ SD band ^{194}Hg(1), and the $\nu_n = 2$ SD band ^{194}Hg(2) ($\nu^2 7^-, 5/2^-[512] \otimes 9/2^+[624], \alpha = 0$). The Nilsson level schemes are taken from [50]. PNC calculations show the following:

(i) For SD bands in Hg isotopes, $\tilde{\Delta}_p$(proton)$\gg$ $\tilde{\Delta}_n$(neutron), which is caused by the large gap at $Z = 80$ in the proton Nilsson level scheme of SD Hg isotopes.

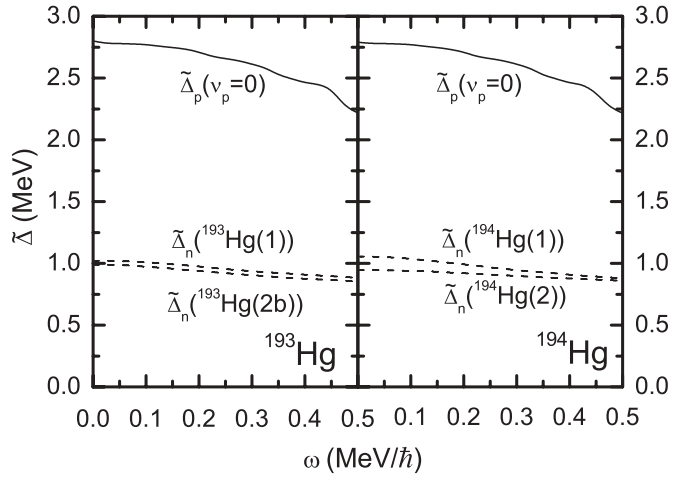

FIG. 5. The pairing gaps of the SD bands in Hg isotopes calculated by the PNC formalism. The Nilsson level schemes are taken from [50]. ^{193}Hg(1): 1-quasineutron SD band, $\nu 5/2^-[512], \alpha = -1/2$; ^{193}Hg(2b): 1-quasineutron SD band, $\nu 9/2^+[624], \alpha = 1/2$. ^{194}Hg(1): quasivacuum SD band ($\alpha = 0$); ^{194}Hg(2): 2-quasineutron SD band $\nu^2 7^-(5/2^-[512] \otimes 9/2^+[624]), \alpha = 0$.

(ii) For SD bands, no pairing collapsing is found with increasing ω either. For ^{194}Hg(1) ($\nu = 0$, SD band), the pairing gap reduction with increasing ω is

$$\frac{\tilde{\Delta}_n(\omega = 0.50\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 83\%, \qquad \frac{\tilde{\Delta}_p(\omega = 0.50\ \text{MeV}/\hbar)}{\tilde{\Delta}_p(\omega = 0)} \approx 80\%. \quad (19)$$

For both the $\nu_n = 1$, SD bands ^{193}Hg(1) and ^{193}Hg(2b)

$$\frac{\tilde{\Delta}_n(\omega = 0.50\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 86\%. \quad (20)$$

For the $\nu_n = 2$, SD band ^{194}Hg(2)

$$\frac{\tilde{\Delta}_n(\omega = 0.50\ \text{MeV}/\hbar)}{\tilde{\Delta}_n(\omega = 0)} \approx 90\%. \quad (21)$$

IV. SUMMARY

The ω and ν dependencies of the nuclear pairing gaps of multiquasiparticle bands in well-deformed and SD nuclei are calculated under the PNC formalism, in which the blocking effects on pairing are exactly taken into account. PNC calculations show that the ω dependence of pairing gaps $\tilde{\Delta}$ for the $\nu = 0$ (qp-vacuum) bands is weaker than that predicted in the particle-number-projected HFB formalism. For the low-lying excited $\nu > 2$ (∼multiquasiparticle) bands, $\tilde{\Delta}_p$'s and $\tilde{\Delta}_n$'s stay almost ω independent. As a function of seniority ν, the bandhead pairing gaps $\tilde{\Delta}(\omega = 0, \nu)$ decrease slowly with

increasing ν. Even for the highest seniority bands identified so far, the pairing gaps $\tilde{\Delta}_p(\omega = 0, \nu)$ and $\tilde{\Delta}_n(\omega = 0, \nu)$ remain larger than 70% of the bandhead value of the qp-vacuum band.

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