Strange hadronic stars in relativistic mean-field theory with the FSUGold parameter set

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Relativistic mean-field theory with parameter set FSUGold that includes the isoscalar-isovector cross interaction term is extended to study the properties of neutron star matter in β equilibrium by including hyperons. The influence of the attractive and repulsive Σ potential on the properties of neutron star matter and the maximum mass of neutron stars is examined. We also investigate the equations of state for pure neutron matter and for nonstrange hadronic matter for comparison. For a pure neutron star, the maximum mass is about $1.8 M_{\rm sun}$, while for a strange (nonstrange) hadronic star in β equilibrium, the maximum mass is around $1.35 M_{\rm sun}$ ($1.7 M_{\rm sun}$).

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I. INTRODUCTION

Hadronic matter under extreme conditions has attracted a lot of interest for many years. On the one hand, many theoretical and experimental efforts have been devoted to the study of heavy ion collision. Modern high-energy ion collisions accelerators enable to produce nuclei with high isospin asymmetry and nuclear densities higher than those in normal nuclei. On the other hand, the nuclear physics of neutron stars has become a hot topic that connects astrophysics with extreme high-density nuclear physics. The equation of state (EOS) of nuclear matter under a high density condition is one of the main objects of nuclear physics with direct astrophysical implications. With the progress of astronomical observation and nuclear experiment, astrophysics phenomena and nuclear physics are combined more and more tightly. In this article, we will concentrate our investigation on the neutron star matter and neutron stars.

Neutron stars are born as a result of supernova explosions. These stars are highly condensed since their masses are of the order of the solar mass, but their radii are only 10–12 km [1]. Neutron star matter is charge neutral and in the β-equilibrium condition. Since the matter density in the neutron star interior can exceed several times the nuclear saturation density, neutron star matter provides an interesting possibility to study the strong interaction effects that are poorly understood at high density. In fact, the structure and composition of a neutron star is determined by the EOS of the strongly interacting constituents. The classical view of normal nuclear matter consisting of neutrons, protons, and leptons is insufficient for neutron star matter and a more realistic composition is needed. At high density, kaon condensation, quark deconfinement, and/or hyperons are possible to appear and much attention has been paid to these issues (see Refs. [2–20]).

Owing to the nonperturbative nature of quantum chromodynamics (QCD) in low-energy regions, it is very difficult to study a nuclear system by using QCD directly. Phenomenological models reflecting the characteristic of the strong interaction are widely used in the studying of the properties of hadrons and nuclear matter. The relativistic mean field (RMF) is a pioneering framework to describe the nuclear system as a relativistic many-body system of baryons and mesons [21]. Along this direction, many important extensions of RMF theory have been made, for example, adding a nonlinear scalar field to improve the compressibility value of nuclear matter, adding pions to investigate the chiral symmetry and partial conservation of the axial current, considering the quark degrees of freedom within baryons [quark-meson coupling (QMC) model [22] and improved quark mass density dependent model [23]], and so on.

Recently, RMF theory with the parameter set FSUGold was proposed by Todd-Rutel and Piekarewicz [24]. In this parameter set, the additional isoscalar-isovector coupling (Λ_v) term was introduced to soften the symmetry energy of nuclear matter at high densities. Consequently, the neutron skin thickness in ^{208}Pb is reduced to be 0.21 fm. This new parameter set can successfully reproduce the properties of nuclear matter and the ground-state properties of some spherical nuclei [25].

In Refs. [25,26], the FSUGold model is applied to investigate the properties of neutron stars successfully. However, the composition of the baryons considered in Refs. [25,26] is only protons and neutrons. To discuss the properties of neutron stars more realistically, one usually should take into account not only nucleons but also hyperons. The main aim of this article is then to extend the FSUGold model to include all baryon octets and then calculate the properties of the neutron star matter and the neutron stars. As for hyperons, the experimental effort has resulted in some significant data. The recent Nagara event [27] provided a definite identification of $^{6}_{\Lambda\Lambda}$He production with the precise $\Lambda\Lambda$ binding energy value $B_{\Lambda\Lambda} = 7.25 \pm 1.19^{+0.18}_{-0.11}$ MeV, which suggests that the effective $\Lambda\Lambda$ interaction should be considerably weaker ($\Delta B_{\Lambda\Lambda} \simeq 1$ MeV) than that deduced from the earlier measurement ($\Delta B_{\Lambda\Lambda} \simeq 5$ MeV) [28]. However, we still lack accurate knowledge about the Σ-N interaction, even though there have been some hints of a high-density repulsion of the Σ-N interaction as indicated by some experiments [29–31]. As for hyperon-meson coupling constants, they are usually derived from the SU(6) quark

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model or constrained by reasonable hyperon potentials. The nucleon-meson coupling constants are generally determined by fitting the properties of nuclear matter at the saturation point and ground-state properties of finite nuclei.

In this article, we will take both the attractive and the repulsive forms of the Σ potential in our calculation and examine the effect of the Σ potential on the properties of neutron star matter and neutron stars. The organization of this article is as follows. In the next section, we give the main formulas of the extended FSUGold model. The main formulas of neutron stars are also included. In the third section, some numerical results are presented. The last section contains a summary and discussions.

II. THE MODEL

To describe the properties of hadronic matter, the RMF theory is usually implemented, in which baryons interact via the exchange of mesons. The baryons considered here include nucleons (N:p and n) and hyperons (Λ, Σ, and Ξ) investigated for the first time by Glendenning [32]. The exchanged mesons include the isoscalar scalar meson (σ), the isoscalar vector meson (ω), the isovector vector meson (ρ), and the cross-interaction term $\omega^2\rho^2$ introduced in Ref. [24]. For neutron star matter in the β equilibrium, the effective Lagrangian can be written as

$$
\begin{aligned}
\mathcal{L} = {} & \sum_B \bar{\psi}_B \left[i\gamma^\mu \partial_\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma^\mu \omega_\mu - \frac{g_{\rho B}}{2}\gamma^\mu \vec{\tau}\cdot\vec{\rho}^{\mu} \right] \psi_B + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma \\
& - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{\kappa}{3!}(g_{\sigma N}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma N}\sigma)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu + \frac{\zeta}{4!}\left(g_{\omega N}^2 \omega_\mu \omega^\mu\right)^2 \\
& + \frac{1}{2}m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^{\mu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu} + \Lambda_v \left(g_{\rho N}^2 \vec{\rho}_\mu \cdot \vec{\rho}^{\mu}\right)\left(g_{\omega N}^2 \omega_\mu \omega^\mu\right) + \sum_l \bar{\psi}_l [i\gamma^\mu \partial_\mu - m_l]\psi_l, \qquad (1)
\end{aligned}
$$

where the symbol B includes the entire baryon octet (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-) and l represents e^- and μ^-; m_B denotes the baryon free mass. m_σ, m_ω, and m_ρ are the masses of σ, ω, and ρ mesons, respectively. The antisymmetric tensors of vector mesons take the forms $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\overrightarrow{G}_{\mu\nu} = \partial_\mu \overrightarrow{\rho}_\nu - \partial_\nu \overrightarrow{\rho}_\mu$. The isoscalar meson self-interactions (via κ, λ, and ζ terms) are necessary for the appropriate EOS of symmetric nuclear matter [33]. The new additional isoscalar-isovector coupling (Λ_v) term is used to modify the density dependence of the symmetry energy and the neutron skin thicknesses of heavy nuclei [24]. $g_{\sigma B}$, $g_{\omega B}$, and $g_{\rho B}$ are the coupling constants between the baryon and σ meson, baryon and ω meson, and baryon and ρ meson, respectively.

With the mean-field approximation by which the operators of meson fields are replaced by their expectation values, we obtain the meson field equations as

$$m_\sigma^2 \sigma + \frac{1}{2}\kappa g_{\sigma N}^3 \sigma^2 + \frac{1}{6}\lambda g_{\sigma N}^4 \sigma^3 = \sum_B g_{\sigma B}\rho_B^S, \qquad (2)$$

$$m_\omega^2 \omega + \frac{\zeta}{6} g_{\omega N}^4 \omega^3 + 2\Lambda_v g_{\rho N}^2 g_{\omega N}^2 \rho^2 \omega = \sum_B g_{\omega B}\rho_B, \qquad (3)$$

$$m_\rho^2 \rho + 2\Lambda_v g_{\rho N}^2 g_{\omega N}^2 \omega^2 \rho = \sum_B g_{\rho B}\tau_{3B}\rho_B, \qquad (4)$$

where ρ_B and ρ_B^S are baryon density and scalar density, respectively, with

$$\rho_B = \frac{2}{(2\pi)^3}\int_0^{k_F^B} d^3k, \qquad (5)$$

$$\rho_B^S = \frac{2}{(2\pi)^3}\int_0^{k_F^B} d^3k \frac{m_B^*}{\sqrt{k^2 + m_B^{*2}}}. \qquad (6)$$

In the last two equations, k_F^B is the Fermi momentum and m_B^* is the effective mass of baryon B, which can be related to the scalar meson field as $m_B^* = m_B - g_{\sigma B}\sigma$. With the requirement of translational invariance and rotational symmetry of static, homogenous, infinite nuclear matter, only zero components (ω_0 and ρ_{03}) of the vector fields survive and they are still denoted as ω and ρ in the above meson equations.

For the neutron star matter with baryons and charged leptons, the β-equilibrium conditions are guaranteed with the following relations of chemical potentials for different particles:

$$\mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e, \qquad (7)$$

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n, \qquad (8)$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e, \qquad (9)$$

$$\mu_\mu = \mu_e, \qquad (10)$$

and the charge neutrality condition is fulfilled by

$$n_p + n_{\Sigma^+} = n_e + n_{\mu^-} + n_{\Sigma^-} + n_{\Xi^-}, \qquad (11)$$

where n_i is the number density of particle i. The chemical potentials of baryons and leptons read

$$\mu_B = \sqrt{K_F^{B\,2} + m_B^{*\,2}} + g_{\omega B}\omega + g_{\rho B}\tau_{3B}\rho, \qquad (12)$$

$$\mu_l = \sqrt{K_F^{l\,2} + m_l^2}, \qquad (13)$$

where K_F^l is the Fermi momentum of the lepton $l(e, \mu)$. Once the solution has been found, the EOS of the neutron star matter can be calculated from

$$\varepsilon = \sum_B \frac{\gamma_B}{(2\pi)^3}\int_0^{K_F^B}\sqrt{m_B^{*\,2}+k^2}d^3k + \frac{1}{2}m_\omega^2\omega^2 + \frac{\zeta}{8}g_{\omega N}^4\omega^4 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\kappa}{6}g_{\sigma N}^3\sigma^3 + \frac{\lambda}{24}g_{\sigma N}^4\sigma^4 + \frac{1}{2}m_\rho^2\rho^2 + 3\Lambda_v g_{\rho N}^2 g_{\omega N}^2\omega^2\rho^2 + \frac{1}{\pi^2}\sum_l\int_0^{k_F^l}\sqrt{k^2+m_l^2}k^2dk, \tag{14}$$

$$p = \sum_B \frac{1}{3}\frac{\gamma_B}{(2\pi)^3}\int_0^{K_F^B}\frac{k^2}{\sqrt{m_B^{*\,2}+k^2}}dk^3 + \frac{1}{2}m_\omega^2\omega^2 + \frac{\zeta}{24}g_{\omega N}^4\omega^4 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{\kappa}{6}g_{\sigma N}^3\sigma^3 - \frac{\lambda}{24}g_{\sigma N}^4\sigma^4 + \frac{1}{2}m_\rho^2\rho^2 + \Lambda_v g_{\rho N}^2 g_{\omega N}^2\omega^2\rho^2 + + \frac{1}{3\pi^2}\sum_l\int_0^{k_F^l}\frac{k^4}{\sqrt{k^2+m_l^2}}dk. \tag{15}$$

With the obtained EOS, the mass-radius relation and other relevant quantities of the neutron star can be derived by solving the Oppenheimer and Volkoff equation [1]

$$\frac{dp(r)}{dr} = -\frac{Gm(r)\varepsilon}{r^2}\left(1+\frac{p}{\varepsilon C^2}\right)\left(1+\frac{4\pi r^3 p}{m(r)C^2}\right) \times\left(1-\frac{2Gm(r)}{rC^2}\right)^{-1}, \tag{16}$$

$$dM(r) = 4\pi r^2\varepsilon(r)\,dr, \tag{17}$$

where G is the gravitational constant and C is the velocity of light, and the EOS for neutron matter is given by Eqs. (14) and (15), we can study the physical behavior of neutron stars for the extended model.

In Table I, we list the parameters of the original FSUGold model. This parameter set can reproduce that nuclear matter saturates at a Fermi momentum of $k_F = 1.3\ \text{fm}^{-1}$ (i.e., saturation density at $\rho_0 = 0.148\ \text{fm}^{-3}$) with a binding energy per particle $E/A = -16.3$ MeV at zero temperature, and the compression constant to be about $K(\rho_0) = 230$ MeV. The mixed isoscalar-isovector coupling (Λ_v) modifies the density dependence of the symmetry energy and the neutron skin thickness of heavy nuclei.

For the meson-hyperon couplings, we take those in the SU(6) quark model for the vector coupling constants

$$g_{\rho\Lambda} = 0, \quad g_{\rho\Sigma} = 2g_{\rho\Xi} = 2g_{\rho N}, \tag{18}$$

$$g_{\omega\Lambda} = g_{\omega\Sigma} = 2g_{\omega\Xi} = \tfrac{2}{3}g_{\omega N}. \tag{19}$$

The scalar couplings are usually fixed by fitting hyperon potentials with $U_Y^{(N)} = g_{\omega Y}\omega_0 - g_{\sigma Y}\sigma_0$, where σ_0 and ω_0 are

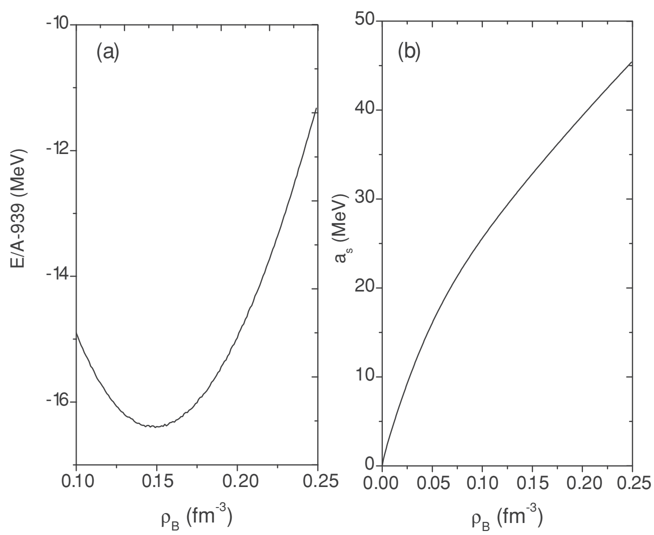

FIG. 1. Binding energy per nucleon in symmetric nuclear matter (left panel) and symmetry energy (right panel) in the FSUGold model. The compression modulus of nuclear matter for the FSUGold model is 230 MeV [24]. The asymmetry energy of saturated matter is about 32.5 MeV in the calculation.

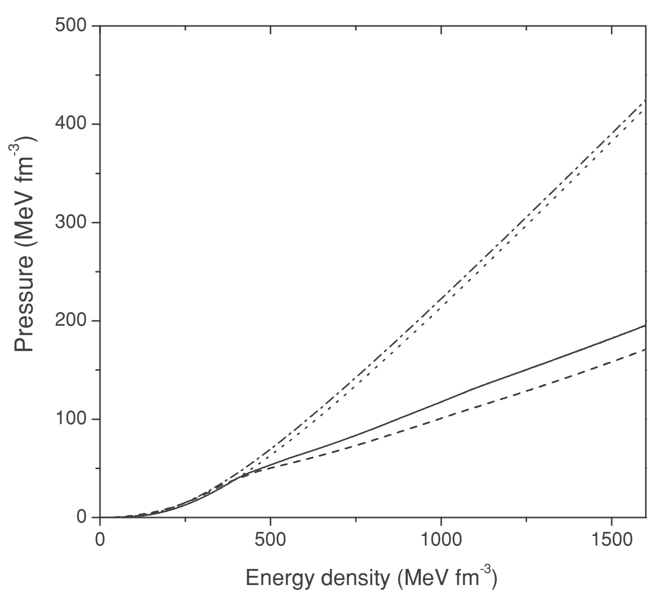

FIG. 2. Equation of state for four cases: only neutrons (dash-dotted line); $n + p$ nucleons in β equilibrium with leptons (dotted line); and $n + p +$ hyperons with $U_\Sigma^{(N)} = -30$ MeV (solid line); and $n + p +$ hyperons with $U_\Sigma^{(N)} = 30$ MeV (dashed line).

TABLE I. The parameter set GSUGold.

m_σ (MeV)	m_ω (MeV)	m_ρ (MeV)	$g^2_{\sigma N}$	$g^2_{\omega N}$	$g^2_{\rho N}$	κ	λ	ζ	Λ_v
491.5	783	763	112.2	204.5	138.5	1.42	0.0238	0.06	0.03

the values of the scalar and vector meson strengths at saturation density [34]. The Λ-N interaction has been well studied and $U_\Lambda^{(N)} = -28$ MeV was obtained with bound Λ hypernuclear states [35]. One of the unsettled issues in hypernuclear physics is the Σ-N interaction in nuclear matter. An attractive potential was generally used in the past for Σ to be bounded in nuclear matter [36]. However, a detailed scan for Σ hypernuclear states turned out to give negative results [30,31]. The study of Σ^- atoms also showed strong evidence for a sizable repulsive

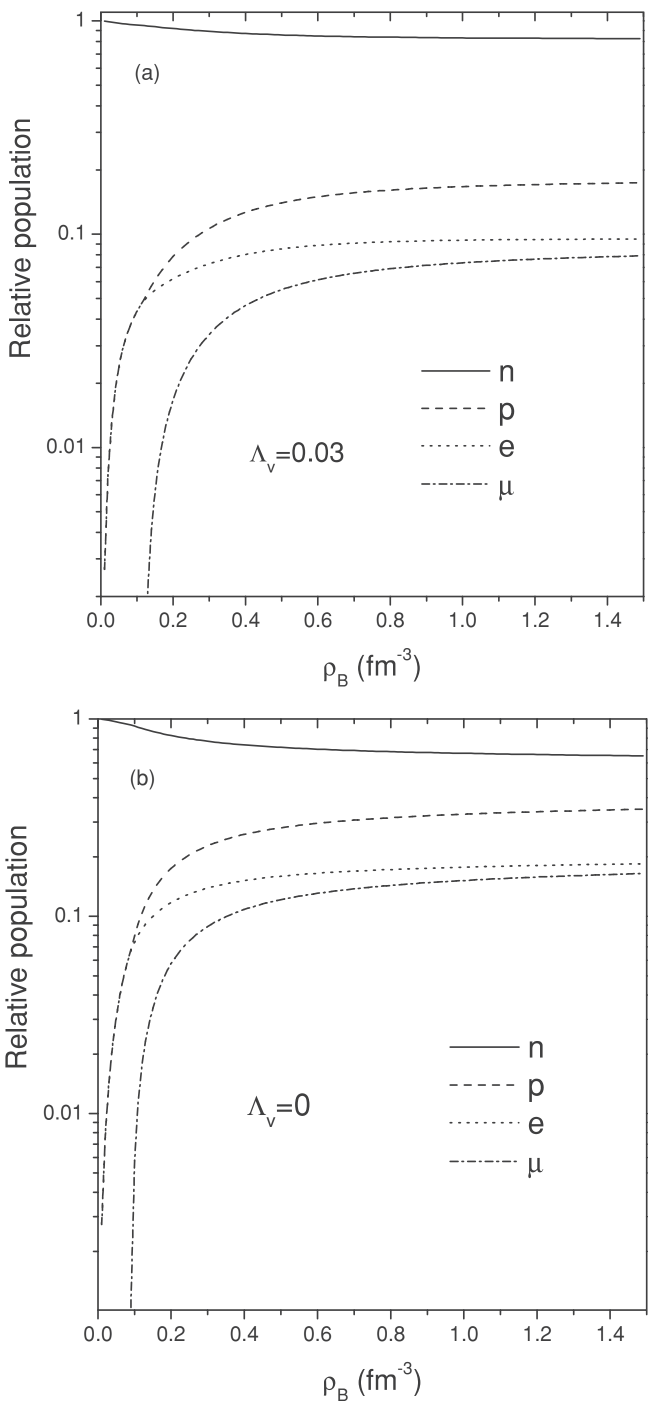

FIG. 3. Populations in neutron star matter without hyperons as a function of density.

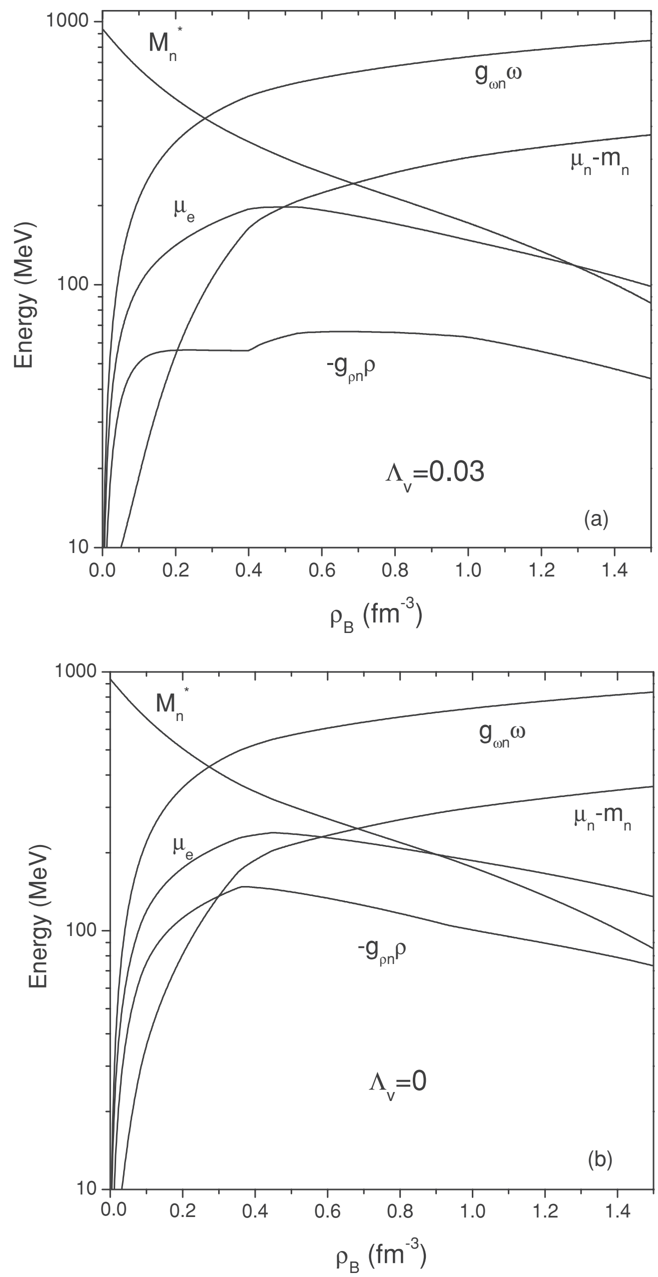

FIG. 4. Field amplitudes and chemical potentials in the case of full β equilibrium among all octet baryons for $U_\Sigma^{(N)} = -30$ MeV.

TABLE II. Calculated results of the maximum masses of neutron stars including hyperons. The upper row is the result with $U_\Sigma^N = -30$ MeV, and the lower row is that with $U_\Sigma^N = 30$ MeV.

Λ_V	0	0 (with σ^*, ϕ)	0.03	0.03 (with σ^*, ϕ)
$M_{\max}(M_{\text{sun}})$	1.36	1.33	1.34	1.32
$M_{\max}(M_{\text{sun}})$	1.32	1.30	1.31	1.29

potential in the nuclear core at $\rho = \rho_0$ [37–39]. A recent review again confirmed the repulsive nature of the Σ^- potential with a new geometric analysis of the Σ^- atom data [40]. Therefore, for the Σ-N interaction, we consider two cases: $U_\Sigma^{(N)} = -30$ MeV, as used in Ref. [36], and $U_\Sigma^{(N)} = 30$ MeV, as used in Ref. [41]. Besides, the Ξ-N interaction in nuclear matter is attractive with the potential $U_\Xi^{(N)} = -18$ MeV [41]. We take then such a value in our calculation.

III. NUMERICAL RESULT

Before taking into account the neutron star matter, in Fig. 1 we draw the curve of binding energy per nucleon versus baryon number density for symmetric nuclear matter (left panel) and symmetry energy of nuclear matter (right panel) in the FSUGold model. From this figure, we can find that the FSUGold model can reproduce the saturation curve of symmetric nuclear matter.

Then we first investigate the most simple neutron star whose baryon composition includes only neutrons and protons. For the description of such simple neutron stars, the discussion in the second section still works with the exclusion of hyperons from the Lagrangian and the meson field equations. The EOS is given by Eqs. (14) and (15) for the FSUGold model when the neutron star matter reaches β equilibrium. The curve of the EOS of such simple neutron stars is shown by the dotted line in Fig. 2. In addition, we show the EOS for a pure neutron star (dash-dotted line). We find that the EOS of the pure neutron star is slightly stiffer than that of the neutron star composed of protons and neutrons.

In Fig. 3, we show the particle population including n, p, e, and μ for different densities by solid (n), dashed (p), dotted (e), and dash-dotted (μ) curves, respectively. The upper graph is for $\Lambda_v = 0.03$ and the lower graph is for $\Lambda_v = 0$.

Now we are in a position to show the results of the neutron star matter including hyperons in the FSUGold model. In our calculation, we take $U_\Lambda^{(N)} = -28$ MeV and $U_\Xi^{(N)} = -18$ MeV to determine the scalar coupling constants $g_{\sigma\Lambda}$ and $g_{\sigma\Xi}$. As for the Σ potential, we consider the attractive potential $U_\Sigma^{(N)} = -30$ MeV and the repulsive potential $U_\Sigma^{(N)} = 30$ MeV, respectively. To explain the above data, to supplement Table I, the relevant scalar coupling constants are $g_{\sigma\Lambda} = 6.31$, $g_{\sigma\Xi} = 3.27$, and $g_{\sigma\Sigma} = 6.36(4.60)$ derived for the attractive (repulsive) Σ potential. In Fig. 4, we show the meson field amplitudes and chemical potentials of the neutron and electron for the case of $U_\Sigma^{(N)} = -30$ MeV. One finds that the neutron chemical potential and the ω mean field increase monotonically as a function of density. The electron chemical and $-\rho_{03}$ mean field increase at low baryon density and then decrease when the baryon density becomes too high. Meanwhile, the effective mass of the nucleon decreases monotonically as a function of density. In Fig. 2, the results of the EOS of the neutron star matter, including hyperons for the attractive and repulsive Σ potential, are displayed by the solid and dashed lines, respectively. It was shown that the appearance of hyperon degrees of freedom has a significant effect on the global properties of hadron matter and neutron stars, lowering the total pressure of the system and softening the EOS because it suppresses the overall Fermi energy and momentum of baryons and leptons [1].

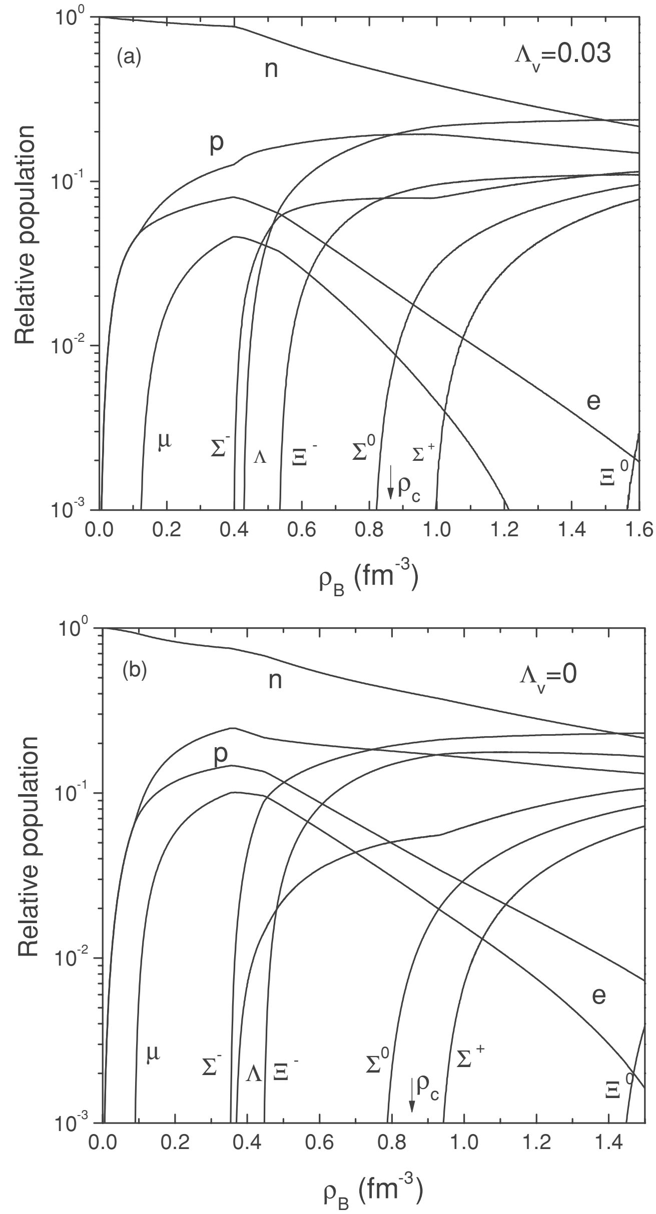

FIG. 5. Calculated variation behavior of the relative populations of the compositions of neutron stars including hyperons in the case of attractive Σ potential with respect to the total baryon density. ρ_c denotes the baryon density at the center of the neutron star.

TABLE III. The masses of strange mesons and strange meson-hyperon coupling constants.

m_{σ^*}(MeV)	m_ϕ (MeV)	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Sigma}$	$g_{\sigma^*\Xi}$	$g_{\phi\Lambda}$	$g_{\phi\Sigma}$	$g_{\phi\Xi}$
975	1020	4.75	4.75	9.12	−6.73	−6.73	−13.4

The calculated results of the variation behavior of the relative populations of all compositions with respect to the total baryon density are demonstrated in Figs. 5 and 6 for the attractive and repulsive Σ potentials, respectively. Figure 5

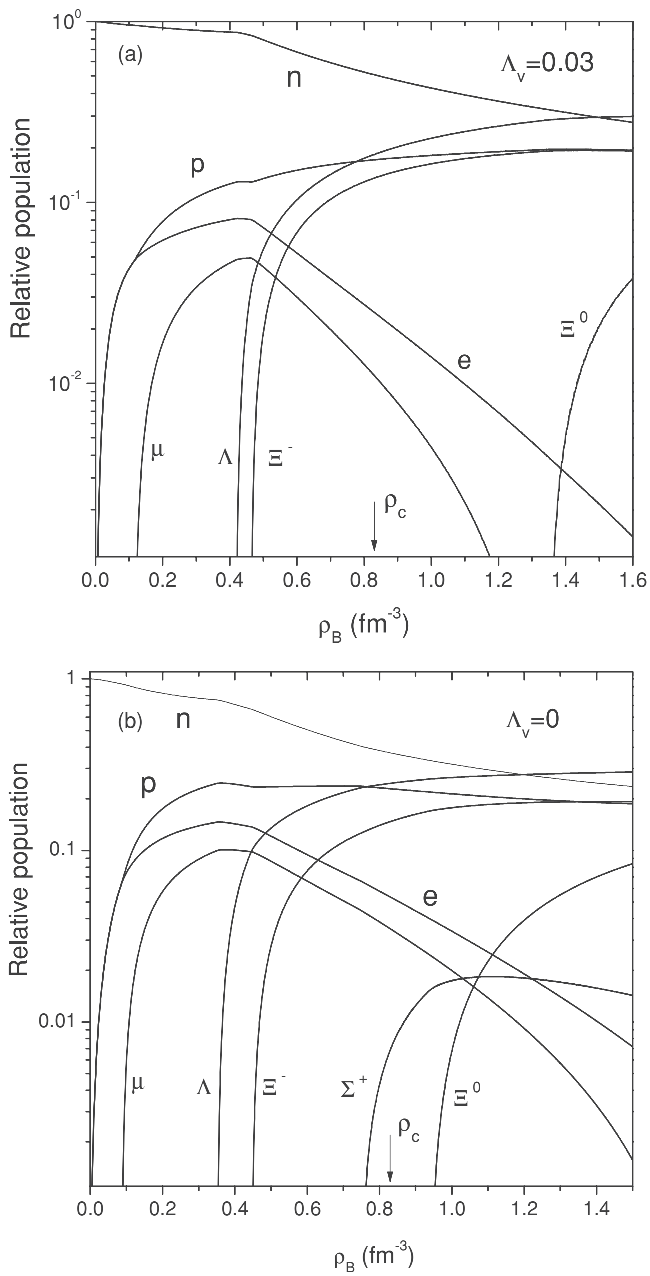

FIG. 6. Calculated variation behavior of the relative populations of the compositions of neutron stars including hyperons in the case of the repulsive Σ potential with respect to the total baryon density. ρ_c denotes the baryon density at the center of the neutron star.

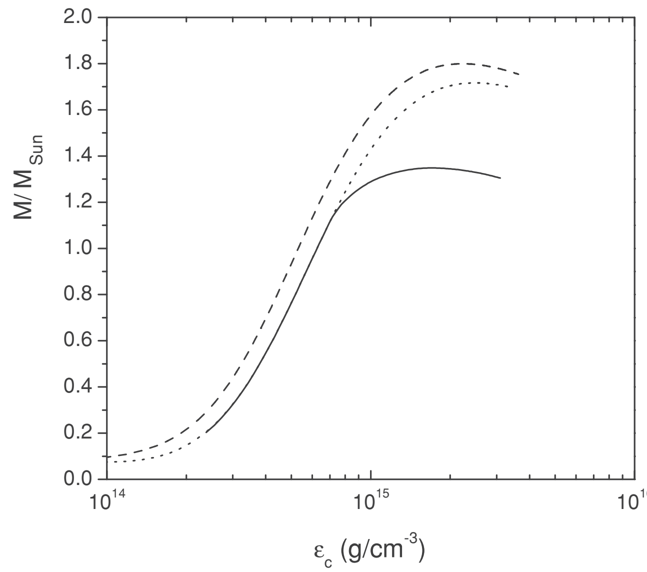

FIG. 7. Neutron star mass as a function of the central density for three cases: only neutrons (dashed line); $n + p$ nucleons in β equilibrium with leptons (dotted line); and $n + p +$ hyperons with $U_\Sigma^{(N)} = -30$ MeV (solid line).

shows that, for the attractive Σ potential $U_\Sigma^{(N)} = -30$ MeV, hyperons Σ^- and Λ appear at 2–3ρ_0. The figure also indicates that the Ξ^- hyperons with negative charge also appear at a lower baryon density. Then Σ^0, Σ^+, Ξ^0 appears in turn at high density. As for leptons, their relative populations increase with the ascent of the density in the low density region. Then they decrease with the increase of the baryon density and disappear at some critical density. Figure 6 presents the relative populations with the repulsive Σ potential. Compared with Fig. 5 for the attractive Σ potential, the main difference is that Σ hyperons do not appear up to the maximum density considered here, $\sim 10\rho_0$, beyond the central density of the neutron star. Because Σ hyperons do not appear in the reasonable range of baryon density for neutron star matter, Ξ^- hyperons emerge at the relatively lower baryon density region to keep the charge neutrality of the whole system.

Given a central density ε_c of the neutron star, we find the corresponding central pressure density P_c from the EOS (14) and (15). We substitute Eqs. (14) and (15) into the Oppenheimer-Volkoff Eq. (16) and integrate from P_c to zero because the pressure at the surface of the neutron star is zero, $P(R) = 0$. Then we obtain the mass M (in units of sun mass) of the neutron star. the outcome is shown in Fig. 7 and Table II. We show the mass of the neutron star in units of sun mass $M/M_{\rm sun}$ as a function of the central density ε_c in Fig. 7 for three cases. The maximum mass of pure neutron stars is about $1.8M_{\rm sun}$ with a central density 1.2 fm^{-3}. The maximum mass changes to $1.7M_{\rm sun}$ and $1.35M_{\rm sun}$ when proton and hyperons are included, respectively.

Since the strange mesons (σ^* and ϕ) play an important role in describing the interaction between hyperons, we also take the contribution of strange mesons into account in the calculation by fitting the coupling constants between strange mesons and hyperons according to a weak Y-Y interaction [20]. The coupling constants between strange mesons and hyperons and the masses of strange mesons are listed in Table III. The calculated results of the maximum masses of neutron stars are summarized in Table II. We find that adding the strange mesons in the calculation can soften the EOS of neutron star matter and make the maximum masses slightly smaller.

IV. SUMMARY AND DISCUSSION

In this article, RMF theory with parameter set FSUGold that includes the isoscalar-isovector cross-interaction term is extended to study the properties of neutron star matter in β equilibrium by including hyperons. The calculated results of the variation behavior of the relative populations of all compositions with respect to the total baryon density are demonstrated. The influence of the attractive and repulsive Σ potential on the properties of neutron star matter and the maximum mass of neutron stars is examined. We also investigate the EOS's for pure neutron matter and for non-strange hadronic matter for comparison. For a pure neutron star, the maximum mass is about $1.8M_{\rm sun}$, while for a strange (nonstrange) hadronic star in β equilibrium, the maximum mass is around $1.35M_{\rm sun}$ ($1.7M_{\rm sun}$). We should indicate that the maximum masses of strange hadronic stars are clearly too low, which means the parametrization in this framework may not be very appealing.

The Bose-Einstein condensate of negatively charged kaons is not taken into account in this work. The reasons are twofold as follows. On the one hand, the kaon-nucleon interaction has not been very clear recently. Waas and Weise found an attractive potential for the K^- at the saturation nuclear density of about $U_K(\rho_0) = -120$ MeV [42]. Coupled channel calculations at finite density have yielded a value of $U_K(\rho_0) = -100$ MeV [43]. More recent, self-consistent calculations with a chiral Lagrangian [44] and coupled channel calculations including a modified self-energy of the kaon [45] indicated that the kaon may experience an attractive potential with potential depths of -80 even -50 MeV at the saturation density. On the other hand, augmenting the calculation by including the kaon condensate will reduce the maximum masses of stars even more, which may be excluded in our work.

For more complex consideration, the interactions by exchanging the δ meson should be taken into account in the calculation since the δ meson plays a very important role in describing the neutron star matter. In addition, the hadron quark phase transition at high density is also important for the further study of neutron star properties. Related investigations are in progress.

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