Balance functions reexamined

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(Received 2 November 2010; revised manuscript received 22 December 2010; published 28 February 2011)

The idea of glue clusters, i.e., short-range correlations in the quark-gluon plasma close to freeze-out, is used to estimate the width of balance functions in momentum space. A good agreement is found with the recent measurements of the STAR Collaboration for central Au-Au collisions.

DOI: 10.1103/PhysRevC.83.024914 PACS number(s): 25.75.Gz

I. INTRODUCTION

It was observed by Bass, Danielewicz, and Pratt [1] that measurements of balance functions in nucleus-nucleus collisions provide a "clock" which allows determination of the time when the charges observed in the final state are created. The measurements performed by the STAR [2–4] and NA49 [5,6] Collaborations proved (or at least strongly suggested) that the charges appear at the late stage of the production process, close to the freeze-out [7], thus implying that the quark-gluon plasma phase is dominated by gluons.

In addition, these experiments showed that the measured widths of the balance functions in central AA collisions are substantially smaller than those seen in pp and in peripheral AA collisions. The natural conclusion from this observation is that the presence of the quark-gluon plasma phase induces additional correlations in the system. The nature of these correlations is of course a matter of debate. They were extensively discussed [7] in the framework of the thermal blast wave model [8]. It was argued that, when the quark-gluon plasma is sufficiently cool, pairs of positive and negative charges are created, with thermal momentum distribution, in certain restricted domains in space (the existence of these domains is derived from Hanbury-Brown–Twiss measurements [9]). The resulting momentum separation is measured through the balance function. Since the creation of charges happens close to the end of the evolution, their separation induced by expansion of the system is not effective and thus a small width of the balance function is maintained.

In [10], on the other hand, the additional correlations were interpreted as evidence for clustering in the quark-gluon plasma. It was shown that production of uncorrelated isotropic clusters of gluons decaying into $q\bar{q}$ pairs, when supplemented by their coalescence into hadrons [11], explains—in a natural way—the correct width of the balance function in pseudorapidity.

In the coalescence model [11] it is assumed that hadrons are created by coalescence of "constituent" quarks and antiquarks into $q\bar{q}$, qqq, and $\bar{q}\bar{q}\bar{q}$ color singlets. If there are no correlations in the system before coalescence, then the only correlations that can appear between hadrons are those induced by resonances (which also result from coalescence). This picture describes reasonably well the hadron-hadron data [12]. The additional short-range correlations, needed to account for the experimental measurements of the STAR [2–4] and NA49 [5,6] Collaborations, may only appear if the constituent quarks and antiquarks are correlated between themselves before the coalescence process takes place. The postulated glue clusters supply these additional correlations.

Since pseudorapidity is determined solely by the particle production angle, distributions in pseudorapidity are only marginally sensitive to particle momenta. It is therefore interesting to verify if the hypothesis of glue clusters can explain also the recently measured [3] balance functions in momentum space. This is the subject of the present investigation. Our main conclusion is that, indeed, the presence of glue clusters at the last stages of the evolution of the quark-gluon plasma can account for the observed width of balance funtions in q_{side}, q_{long} and q_{out}. This result requires that the average momentum $\langle q\rangle$ carried by a quark and antiquark in the decay of a glue cluster be located around ∼120 MeV.

In the next section we derive the momentum dependence of the balance function. Comparison with data is presented in Sec. III. Our conclusions and comments are listed in the last section. The Appendix summarizes our treatment of acceptance corrections.

II. BALANCE FUNCTION FROM GLUE CLUSTERS

Consider a two-body decay of a glue cluster into a $q\bar{q}$ pair. The distribution of the decay products is

$$\frac{dn}{dp_1dp_2} = v_c(p_1 - p_2)\delta(p_1 + p_2 - P), \tag{1}$$

where P is the momentum of the cluster, p_1 and p_2 are the momenta of the decay products, and the function v_c is responsible for the details of the decay (dp stands for three-dimensional relativistic phase space, $dp = d^3p/E = d^2p_\perp dy$).

Consequently, the distribution of quarks and antiquarks arising from two clusters is

$$\frac{dN(p_u, p_{\bar{u}}, p_d, p_{\bar{d}})}{dp_udp_{\bar{u}}dp_ddp_{\bar{d}}} = \int dP_UdP_D\rho_c(P_U)\rho_c(P_D) \times \delta(p_u + p_{\bar{u}} - P_U)v_c(p_u - p_{\bar{u}}) \times \delta(p_d + p_{\bar{d}} - P_D)v_c(p_d - p_{\bar{d}}), \tag{2}$$

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where ρ_c is the distribution of clusters in momentum space. The subscripts U, D refer to the flavor of the quarks to which the cluster decays, and $u, d, \bar{u}, \bar{d}$ to the flavor of the quarks and antiquarks themselves.

For the balance function we obtain[1]

$$B(p_+, p_-) \sim \int dp_u dp_{\bar{u}} dp_d dp_{\bar{d}} \delta(p_u + p_{\bar{d}} - p_+) \times \delta(p_d + p_{\bar{u}} - p_-) \Phi(p_u - p_{\bar{d}}) \times \Phi(p_d - p_{\bar{u}}) \frac{dN(p_u, p_{\bar{u}}, p_d, p_{\bar{d}})}{dp_u dp_{\bar{u}} dp_d dp_{\bar{d}}}, \quad (3)$$

where $\Phi(x)$ describes the coalescence process, and $p_\pm$ are the momenta of the observed particles.

Introducing

$$\delta_+ = p_u - p_{\bar{d}}, \qquad \delta_- = p_d - p_{\bar{u}}, \quad (4)$$

we have

$$B(p_+, p_-) \sim \int d\delta_+ d\delta_- \rho_c(p_u + p_{\bar{u}}) \rho_c(p_d + p_{\bar{d}}) \Phi(\delta_+) \times \Phi(\delta_-) v_c(p_u - p_{\bar{u}}) v_c(p_d - p_{\bar{d}}), \quad (5)$$

$$[p_+ = p_u + p_{\bar{d}}, \quad p_- = p_d + p_{\bar{u}}],$$

and thus

$$B(\delta) \sim \int d\delta^+ d\delta^- \Phi[\delta^+ + \delta^-)/2] \Phi[\delta^+ - \delta^-)/2] \times \int dp \rho_c[(p + \delta^-)/2] \rho_c[(p - \delta^-)/2] \times v_c[(\delta + \delta^+)/2] v_c[(\delta - \delta^+)/2], \quad (6)$$

where

$$p = p_+ + p_-, \quad \delta = p_+ - p_-, \quad \delta^\pm = \delta_+ \pm \delta_-. \quad (7)$$

Equation (6) represents our final result. Its consequences for the widths of the balance functions in various components of the momentum are discussed in the next section.

III. The widths of balance functions

To simplify the discussion and reduce the number of parameters, we assume in this section that the functions $v_c(x)$ and $\rho_c(x)$ are Gaussians:

$$v_c(x) \sim e^{-x^2/v^2}, \quad \rho_c(x) = e^{-x_\parallel^2/r_\parallel^2 - x_\perp^2/r^2}. \quad (8)$$

Then the dependence on δ^+ and δ^- in (6) factorizes out and we have

$$B(\delta) \sim e^{-\delta^2/2v^2} \Omega(\delta), \quad \Omega(\delta) = \int dp e^{-p_\parallel^2/2r_\parallel^2} e^{-p_\perp^2/2r^2}. \quad (9)$$

If there are no acceptance restrictions, the integral extends over full phase space, $\Omega(\delta)$ is a constant, and the balance function is isotropic: its width does not depend on the chosen direction[2] and is entirely determined by the parameter v^2. In the STAR experiment [3], however, the acceptance corrections are important. They were estimated and are summarized in the Appendix.

Equation (9) contains three parameters: v, r, and $r_\parallel$. This number can still be reduced, using the available information on the distribution of transverse and longitudinal momenta. Indeed, the single-particle transverse momentum distribution can be expressed as

$$D(p_{+\perp}) = e^{-2p_{+\perp}^2/(r^2+v^2)} d^2 p_{+\perp} \to \langle p_{+\perp}^2 \rangle = (r^2 + v^2)/2 \quad (10)$$

with an analogous formula for $p_\parallel$. Consequently we have

$$r^2 = 2\langle p_\perp^2 \rangle - v^2, \quad r_\parallel^2 = 4\langle p_\parallel^2 \rangle - v^2 \quad (11)$$

and we are left with one parameter, v^2, which should be adjusted to data.

One sees from these formulas that $r_\parallel$ is very large and therefore its exact value is not important for estimate of $\delta_{\rm long}$. Therefore, to determine v, we used the value $\langle \delta_{\rm long} \rangle =$ 190 MeV, measured in the central Au-Au collisions [3]. Using Eq. (A10) from the Appendix, one obtains $v = 276$ MeV. With this value and $\langle p_\perp \rangle = 400$ MeV [13], Eqs. (A8) and (A5) give

$$\langle \delta_{\rm side} \rangle = 284 \text{ MeV}, \quad \langle \delta_{\rm out} \rangle = 126 \text{ MeV}. \quad (12)$$

This should be compared with

$$\langle \delta_{\rm side} \rangle = 280 \pm 10 \text{ MeV}, \quad \langle \delta_{\rm out} \rangle = 0.110 \pm 10 \text{ MeV}, \quad (13)$$

measured in [3]. Given the crudeness of our (Gaussian) approximation, the agreement is more than satisfactory. The width uncorrected for acceptance is $\langle \delta \rangle = 220$ MeV.

From the obtained value $v = 276$ MeV and using (1) one can evaluate the average value $\langle q \rangle$ of the momentum carried by the quark and antiquark in the decay of the cluster. One obtains $\langle q \rangle = v\sqrt{\pi}/4 \approx 122$ MeV and $\sqrt{\langle q^2 \rangle} = v/2 \approx$ 138 MeV.

IV. CONCLUSIONS AND COMMENTS

In conclusion, we verified that the hypothesis of glue clusters, i.e., positive short-range correlations in the quark-gluon plasma [10], can account for the small widths of the balance functions in momentum space, observed recently by the STAR collaboration [3] for central Au-Au collisions. In particular, it was shown that when the parameters of the model are determined from the observed $\langle \delta_{\rm long} \rangle$, one obtains reasonable values of $\langle \delta_{\rm side} \rangle$ and of $\langle \delta_{\rm out} \rangle$. This result confirms the existence of correlations in the plasma. It also shows that they can be effectively studied by the method of balance functions [1].

Several comments are in order.

(i) It should be emphasized that the observed small width of the balance functions in momentum space simply *implies* the existence of additional short-range correlations between particles produced in central nucleus-nucleus collisions (as compared to *pp* collisions). The nature and origin of these correlations remain a subject of debate but their very existence is beyond doubt. For

[1] Since we are only interested in the width of the balance function, henceforth we shall ignore all normalizing factors.

[2] This was already observed in [7].

example, the explanation of the balance functions in the blast wave model [8], presented in [7], also exploits the correlations in the plasma (in the form of "domains" from which the balancing charges are emitted).

(ii) The existence of short-range correlations in the quark-gluon plasma close to the phase transition should not be surprising. Indeed, the lattice calculations indicate strong deviations from the Stefan-Boltzmann limit even at temperatures greatly exceeding T_c [14]. Thus the presence of quasiparticles is likely. The nature of these quasiparticles is an open question. Our results suggest that they may take the form of gluonic clusters.

(iii) Although our semianalytic estimates are admittedly rather crude, it is interesting to observe that the average momentum characterizing the decay of a glue cluster (120 MeV) seems close to the breakup temperature in the blast wave model, as discussed in [7]. This may suggest that the approach of [7] and our interpretation may not be impossible to reconcile. Precise data on the balance functions for KK and $K\pi$ pairs may throw some light on this problem.

(iv) In our argument we have used the coalescence model [11]. Although the model is rather successful in explaining, e.g., the particle content [15] and v_2 scaling [16], its basic idea is often questioned since it is difficult to reconcile with the picture of the pion as the Goldstone boson of chiral symmetry breaking [17]. A possible way out is to admit that the constituent quark mass depends on the temperature of the system (or rather on its distance from the temperature of the chiral transition). If true, then also the mass of the glue clusters will depend on the temperature. Although our results do not depend on the constituent quark mass, it may be interesting to investigate this problem in more detail.

(v) It was noticed in [7] that corrections due to production of hadronic resonances may change the results by 10%–20%. This is not dramatic for our semiquantitative approach but must be eventually improved if serious comparison with data is attempted.

It is well known that resonance production is essential in description of the short-range correlations in pp collisions [12] and that it allows us to explain the single-particle spectra in nucleus-nucleus collisions (see, e.g., [18]). Furthermore, it has been shown [19] that thermal, uncorrelated production of resonances cannot reproduce the small widths of balance functions in central nucleus-nucleus collisions. It should be noticed, however, that if hadronic resonances are formed by coalescence of q and $\bar{q}$ that are decay products of glue clusters, they are correlated. This—in turn—should produce additional correlations between their decay products and thus reduce the widths of the balance functions. It would be interesting to investigate such a possibility in detail. This, however, requires more sophisticated analysis than the one presented here.

(vi) The present work represents only a first-order approximation to the problem. We considered only the simplest case when the hadron pairs are created by coalescence of quarks and antiquarks which are decay products of just two clusters. The contributions from more than two clusters should certainly be included in a more precise analysis. It seems likely, however, that they are suppressed because (i) the probability of having more clusters very close in space (as required by the coalescence mechanism) is small and (ii) matching the color of quarks from two different clusters reduces this contribution even further. At present we can only mention that (a) for three clusters the width of the balance function depends on details of the coalescence process and therefore its estimate involves more parameters than hitherto considered, and (b) if clusters are uncorrelated, contributions from four clusters cancel in the balance function (see e.g. [10]).

Let us also add that the Gaussian approximation is certainly rather crude and our treatment of the acceptance corrections is at best approximate. This can of course be improved, although we feel that it would not be justified at the present level of understanding.

In summary, the present investigation shows that the hypothesis of glue clusters in quark-gluon plasma can account for the recently measured widths of the balance functions in central Au-Au collisions. This may have important consequences for the phenomenology of the quark-gluon plasma. It remains an open question if this result is confirmed when data in larger acceptance regions are available.

V. ACKNOWLEDGMENTS

Thanks are due to J.-P. Blaizot, K. Fialkowski, M. Praszalowicz, and K. Redlich for interesting comments. This investigation was supported in part by Grant No. N202 125437 of the Polish Ministry of Science and Higher Education (2009-2012).

APPENDIX: ACCEPTANCE CORRECTIONS

We first consider transverse directions, i.e. two-dimensional vectors p_+ and p_-. The distribution is given by (9) with the acceptance limits

$$D^2 \leqslant p_{+x}^2 + p_{+y}^2 \leqslant \Delta^2, \quad D^2 \leqslant p_{-x}^2 + p_{-y}^2 \leqslant \Delta^2, \tag{A1}$$

where $D = 200$ MeV and $\Delta = 600$ MeV.

We are interested in the distribution of δ_{out} and δ_{side}, defined with respect to the total momentum of the pair. Since the system is invariant with respect to common rotation of all vectors, one can select the y axis as "out" and the x axis as "side." Thus we are looking for the configuration in which

$$p_{x+} + p_{x-} = 0 \rightarrow \delta_{\text{side}} = \delta_x = p_{x+} - p_{x-} = 2p_{x+}, \quad p_{\pm y} \geqslant 0. \tag{A2}$$

Consider first $\delta_{\text{out}} = \delta_y$. This is the most complicated case because in this direction there is a significant effect of the flow, which strongly distorts the spectrum and must be taken into account. Flow implies that the cluster moves (on average) with the velocity V of the fluid. Therefore the laboratory system in which it is observed moves with the velocity $-V$ and the distribution of the decay products is

$$e^{-(\delta'_{\rm out})^2/2v^2}e^{-(p'_{\rm out})^2/2r^2},\quad \delta'_{\rm out}=p'_+-p'_-,\quad p'_{\rm out}=p'_++p'_-, \tag{A3}$$

where

$$p'_\pm=\gamma[p_\pm-VE_\pm],\quad E_\pm=\sqrt{p_\pm^2+x^2+m_\pi^2}. \tag{A4}$$

Consequently, the observed distribution of the balance function is (we abbreviate $p_{+x}\equiv x$)

$$P(\delta_{\rm out})=\int_0^\Delta dx e^{-2x^2/v^2}\int_{p_{\min}}^{p_{\max}}e^{-\delta_{\rm out}^{\prime 2}/2v^2}e^{-p_{\rm out}^{\prime 2}/2r^2}, \tag{A5}$$

where $p_{\max}=2\Delta(x)-\delta_{\rm out}$, $p_{\min}=2D(x)-\delta_{\rm out}$ for $\delta_{\rm out}\leqslant 2D(x)$, $p_{\min}=0$ for $\delta_{\rm out}\geqslant 2D(x)$, and

$$\Delta(x)=\sqrt{\Delta^2-x^2},\quad D(x)=\sqrt{D^2-x^2}. \tag{A6}$$

Numerical evaluation of (A5) shows that, with $V=0.6$ (which we used in our estimates), the effect of flow reduces the width significantly.

For $\delta_{\rm side}=\delta_x$ the effect of flow is largely canceled and we neglect it. Consequently, the distribution is

$$P(\delta_{\rm side})=e^{-\delta_{\rm side}^2/2v^2}\int_{D(x)}^{\Delta(x)}dp_{+y}dp_{-y}e^{-(p_{+y}+p_{-y})^2/2r^2}\times e^{-(p_{+y}-p_{-y})^2/2v^2} \tag{A7}$$

where $x=\delta_{\rm side}/2$. One Gaussian integration can be expressed in terms of error functions and one obtains

$$P(\delta_{\rm side})=e^{-\delta_{\rm side}^2/2v^2}\int_{D(x)}^{\Delta(x)}dy_+e^{-2y_+^2/(r^2+v^2)}\times\left\{\mathrm{erf}(s|B_+|)+\mathrm{erf}(s|B_-|)-\left[\frac{A_+}{|A_+|}\mathrm{erf}(s|A_+|)-\frac{A_-}{|A_-|}\mathrm{erf}(s|A_-|)\right]\right\} \tag{A8}$$

with

$$A_\pm=\pm D(x)+w,\quad B_\pm=\pm\Delta(x)+w,\quad w=y_+\frac{r^2-v^2}{r^2+v^2},\quad s^2=\frac{r^2+v^2}{2r^2v^2}. \tag{A9}$$

Note that we always have $B_+\geqslant 0$ and $B_-\leqslant 0$.

Finally, let us consider $\delta_{\rm long}$. In this case there is no lower limit on particle momenta. Consequently, the distribution becomes

$$P(\delta_{\rm long})\sim e^{-\delta_{\rm long}^2/2v^2}\int_0^{2\Delta-\delta_{\rm long}}dp_{\rm out}e^{-p_{\rm out}^2/2r_\parallel^2}=e^{-\delta_{\rm long}^2/2v^2}\mathrm{erf}\left(\frac{2\Delta-\delta_{\rm long}}{r\sqrt{2}}\right). \tag{A10}$$

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