

Path-length dependence of energy loss within in-medium showers

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The study of the path-length dependence of high-transverse-momentum (P_T) hadron suppression in heavy-ion collisions by measuring the dependence of hard-hadron production on the angle ϕ with the reaction plane in noncentral collisions has so far been one of the most successful tools in constraining the microscopic picture of leading-parton energy loss. With the imminent start of the Large Hadron Collider heavy-ion program and the possibility of full jet reconstruction, the focus has shifted to models that are capable of simulating full in-medium parton showers rather than tracing the leading parton only. Yet, on the level of single-inclusive hadron observables, such shower models need to reproduce the findings of leading-parton energy-loss models; in particular, they need to reproduce the correct reaction-plane-angle dependence of hadron suppression. The aim of this work is to illustrate, using the example of the Monte Carlo in-medium shower code, YAJEM (Yet another Jet Energy-loss Model), how path-length dependence arises in a Monte Carlo shower formulation, how it relates to path-length dependence of leading-parton energy loss, and how model results compare with data on the reaction-plane dependence of the nuclear suppression factor $R_{AA}(\phi)$.

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I. INTRODUCTION

The use of the modification of high-transverse-momentum (P_T) hadron production, due to the bulk medium produced in ultrarelativistic heavy-ion collisions, to study properties of the medium is known as “jet tomography” [1–5]. However, in order to gain any tomographic information about the macroscopic medium-density distribution and its evolution from measured data in practice, it is necessary to first know the microscopic mechanism by which the medium modifies high- P_T hadron spectra. A less ambitious approach is to determine under what assumptions a combination of macroscopic evolution, in terms of a bulk evolution model like fluid dynamics and a microscopic parton-medium interaction model, can describe a large body of data in a systematic way [6–9]. So far, most of these studies have been carried out for models which describe the medium modification as energy loss from the leading shower parton prior to hadronization.

A recent result is that the path-length dependence of energy loss, accessible experimentally, e.g., via the dependence of the nuclear suppression factor $R_{AA}(\phi)$ on the angle of outgoing hadrons with the reaction plane ϕ , provides a crucial constraint for microscopic physics. In particular, the path-length dependence cannot be linear in a constant medium (as is characteristic for elastic or generally incoherent processes) [10,11], but must be at least quadratic [12] (as is characteristic of coherent processes), and possibly even cubic [12,13], as found in some versions of strongly coupled QCD-like models.

On the other hand, going beyond the capability to describe single-inclusive hadron observables to full jets, Monte Carlo (MC) models capable of describing the development of a parton shower originating from a highly virtual initial parton inside a medium, followed by hadronization into jets, are being

developed. Both elastic collisions [14] and medium-induced radiation [15–18] have been considered as mechanisms for the modification.

MC models are not natural frameworks in which to incorporate the physics of quantum coherence (which appears necessary to reproduce the path-length dependence of energy loss seen in the data), but it has been demonstrated in [19] that an algorithm can be devised which consistently reproduces known eikonal infinite-energy analytical limits. Yet in the same work, it has also been shown that by including realistic effects, such as finite-energy corrections, the path-length dependence is effectively weaker than a quadratic dependence (see also [20]). This is in manifest contradiction to systematic investigations which show that the data for $R_{AA}(\phi)$ are marginally in agreement with a quadratic dependence and would be more easily reproduced with a cubic dependence [12].

The aim of this paper is to more clearly illustrate the connection between the path-length dependence of energy loss in leading-parton models and the effect of path length on an in-medium shower, and to offer a possible resolution to the apparent puzzle outlined here by suggesting that the path-length dependence observed in the data might be due to yet a different effect. For all computations in the following, the radiative-energy-loss (RAD) scenario of the MC in-medium shower code, YAJEM (Yet another Jet Energy-loss Model), is used.

II. THE MODEL

A. General description

YAJEM is described in detail in [15,16]; here we just reproduce the essential details necessary to understand how path-length dependence arises within the model.

In the absence of a medium, YAJEM utilizes the PYSHOW algorithm [21] in momentum space to model the evolution

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from a highly virtual initial parton to a shower of low-virtuality partons, and the Lund model [22] to hadronize this shower. In this framework, the QCD evolution from a highly virtual state to a parton shower is modeled as a series of branchings, $a \rightarrow bc$, where the virtuality scale decreases at each step.

In the presence of a medium, it is no longer sufficient to model in momentum space only, since the medium evolution is caused by pressure gradients in space-time and the density of matter felt by hard partons is a function of position space rather than momentum space. To make a link between the shower evolution in momentum space and the medium-density evolution in position space, YAJEM utilizes the uncertainty relation to compute the average formation time of a virtual daughter parton b , generated in the branching of a parent a , as

$$\langle \tau_b \rangle = \frac{E_b}{Q_b^2} - \frac{E_b}{Q_a^2}, \quad (1)$$

where E_i denote parton energies and Q_i denote parton virtualities. YAJEM provides the additional option to not use the average formation time but rather, for each branching, to use an actual formation time given by an exponential distribution,

$$P(\tau_b) = \exp\left[-\frac{\tau_b}{\langle \tau_b \rangle}\right], \quad (2)$$

which averages to $\langle \tau_b \rangle$. This allows one to specify the temporal structure of the shower and, combined with the assumption that all shower partons sample the medium along the eikonal trajectory of the shower-initiating parton, the complete space-time picture of the shower is fixed.

In the RAD scenario of YAJEM, the parton-medium interaction is modeled by a medium-induced virtuality gain during the parton formation time:

$$\Delta Q_a^2 = \int_{\tau_a^0}^{\tau_a^0 + \tau_a} d\zeta \hat{q}(\zeta), \quad (3)$$

where the time τ_a is given by Eq. (2), the time τ_a^0 is known in the simulation as the endpoint of the previous branching process, and the integration $d\zeta$ is along the eikonal trajectory of the shower-initiating parton. If the parton is a gluon, the virtuality transfer from the medium is increased by the ratio of their Casimir color factors, $3/4 = 2.25$. The medium is characterized via the parameter $\hat{q}(\zeta)$ where a relation such as

$$\hat{q}(\zeta) = K \cdot 2 \cdot [\epsilon(\zeta)]^{3/4} [\cosh \rho(\zeta) - \sinh \rho(\zeta) \cos \psi] \quad (4)$$

is assumed as the link to fluid-dynamical parameters, such as the energy density ϵ or the local-transverse flow rapidity ρ (at angle ψ with the parton trajectory).

One will observe that Eqs. (1) and (3) are not independent—parton virtuality determines the parton formation time, but the formation time in turn determines the virtuality gain from the medium. If $\Delta Q_a^2 \ll Q_a^2$ holds, i.e., the virtuality picked up from the medium is a correction to the initial parton virtuality, then we may add ΔQ_a^2 to the virtuality of parton a before determining the kinematics of the next branching. If the condition is not fulfilled, then the lifetime is determined by $Q_a^2 + \Delta Q_a^2$ and may be significantly shortened by virtuality picked up from the medium. In this case we iterate Eqs. (1) and (3) to determine a self-consistent pair

of $(\langle \tau_a \rangle, \Delta Q_a^2)$. This ensures that, on the level of averages, the lifetime is treated consistently with the virtuality picked up from the medium. The actual lifetime is still determined by Eq. (2). This consistency condition is fundamentally the same as the algorithm presented in [19]. It corresponds to the limit of a coherent summation of a large number of medium interactions during the formation time (in YAJEM, virtuality gain is a smooth function of path length, rather than a sum over discrete interactions transferring virtuality), and hence represents Landau Pomeranchuk Migdal (LPM) interference in this limit.

However, for technical reasons, despite the consistency condition for medium-induced virtuality gain, the in-medium shower simulated by YAJEM remains an expansion for thin media. This is because, as in the PYSHOW algorithm, parton splitting no longer occurs once a parton has reached a lower virtuality scale Q_0 [where $Q_0 = O(1)$ GeV; to be more precise, for massive partons, the limit is given by $m_{\text{eff}}^q + m_{\text{eff}}^g$, with $m_{\text{eff}}^g = \frac{1}{2} Q_0$, $m_{\text{eff}}^q = \sqrt{m_q^2 + \frac{1}{4} Q_0^2}$, and m_q is the current quark mass]. Partons falling at any point of the evolution below this scale are set on-shell and no longer branch. YAJEM inherits this condition in a medium also. This limits the distance scale L_{max} for which an in-medium shower is reliably evolved parametrically to $L_{\text{max}} \sim E/Q_0^2$, where E is the energy of the shower-initiating parton.

B. Path-length dependence

It should now be clear from where any dependence of parton distributions created by YAJEM on the in-medium path length arise.

(i) The leading effect is given by Eq. (3), which states that the amount of medium-induced virtuality is a monotonically increasing function of the path length. If \hat{q} is a constant, then the total amount of medium-induced virtuality (or the number of scattering centers probed by a parton) is linearly proportional to the path length. Since Eq. (3) holds for *every* parton in the shower, this would lead to a crescendolike scenario [23] for the virtuality transfer to the shower as a whole, in the absence of finite-energy or finite-virtuality corrections (in practice, however, such corrections actually dominate the dynamics [24]).

(ii) The consistency condition between formation time and medium-induced virtuality forms a second important ingredient. It states that low Q^2 , i.e., collinear medium-induced radiation, may not occur after arbitrarily short formation times. When applied to a single almost-on-shell parton propagating through a constant medium, it gives rise to a quadratic-path-length dependence of mean radiated energy as in [19]. However, note that since YAJEM requires $Q > Q_0$, this situation is strictly never realized. To some degree, its effect is also obscured by the randomization of formation times given by Eq. (2), as we will demonstrate.

(iii) The minimum-virtuality scale Q_0 down to which partons are evolved induces a spurious length dependence whenever a path length $L > L_{\text{max}}$ is probed; for $L \gg L_{\text{max}}$ (i.e., far outside the validity region of the model), all partons in the shower are on-shell and no branching occurs for any

density of the medium. Thus, when applied outside the region of validity, YAJEM leads to *less* modification of showers than expected in reality.

(iv) An interesting proposal, suggested in [25,26], is to *define* the minimum-virtuality scale by

$$Q_0 = \sqrt{E/L}, \quad (5)$$

where L is the actual path length that a shower is evolved in the medium and E is the energy of the shower-initiating parton, rather than leave it a constant. If this is done, it replaces the spurious Q_0 dependence with a genuine scale dependence of the shower evolution, which states that the medium cannot induce radiation below a scale Q_0 . We will test this prescription in the following.

In practical terms, it was found in [15,16] that many observables (e.g., the fragmentation function) follow, in good approximation, a scaling law with the integrated total-virtuality gain from the medium, $\Delta Q^2 = \int_0^L d\xi \hat{q}(\xi)$, inside the region of validity. This implies that the path-length dependence of such observables is approximately linear in a constant medium.

C. Implications of a dynamical minimum virtuality

We now consider the implications of Eq. (5) in more detail. The obvious problem is that Q_0 regulates the endpoint for both the vacuum- and the in-medium-parton-shower evolution (which cannot be disentangled on a branching-by-branching basis). While length dependence in the medium contribution is reasonable, there is no justification for any length effect on the vacuum-shower evolution. In contrast, in vacuum, Q_0 regulates the transition between the perturbative-partonic and nonperturbative-hadronic domain of fragmentation. This in turn is an ill-defined condition in a deconfined medium, as hadrons cannot be formed in such a medium.

Equation (5) cannot therefore be useful in general, but must be seen as an approximation for a situation in which the shower evolution is dominated by medium effects and in which the kinematics is such that the possibility of hadrons being formed in the medium is not a crucial issue. The latter condition is readily fulfilled when considering single-inclusive high- P_T pion observables (as done in this work), as the pion formation length (estimated as $\tau \sim E_\pi/m_\pi^2$) is far larger than the medium size. Note that this is not the case for the low- z part of the fragmentation function or for the production of heavier hadrons.

We find that the first condition is also fulfilled for typical in-medium path lengths, which are relevant for computing observables. Figure 1 shows vacuum and medium-modified fragmentation functions as bands, indicating their response to a change in Q_0^2 from 0.5 to 2 GeV² for typical parton paths at Relativistic Heavy Ion Collider (RHIC) kinematics. It is readily apparent from the figure that the majority of the variation with Q_0 seen in the in-medium case cannot be accounted for by the comparatively small effect of the scale change in vacuum, indicating that the path-length condition in this situation indeed dominates over the scale dependence of the transition to the nonperturbative regime. This is, in part, because Q_0 not only defines the endpoint of the perturbative shower description, but also the starting

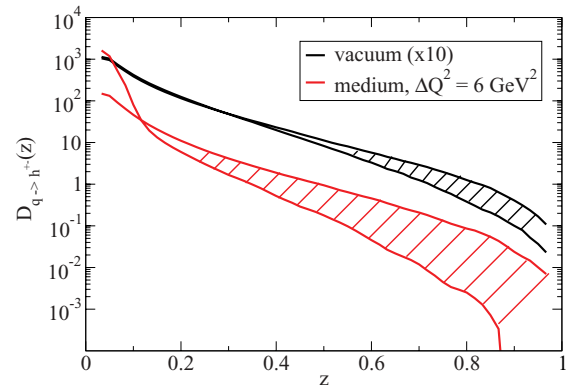


FIG. 1. (Color online) Response of the computed fragmentation function of a 20 GeV quark into charged hadrons to choices of the minimum-virtuality scale Q_0^2 from 0.5 to 2 GeV², both for the vacuum case and for a medium-modified shower for total medium-induced virtuality $\Delta Q^2 = 6$ GeV² as typical for the majority of partons in RHIC high- P_T observables within YAJEM.

point of the nonperturbative Lund hadronization model. Thus, to some degree, in the vacuum, decreased showering is compensated for by an increased action of the Lund model, whereas no such compensation occurs in the medium case (since the hadronization is, by assumption, always treated as in vacuum).

Nevertheless, Eq. (5) has to be regarded as a first approximation. Ultimately, a criterion which also takes into account the virtuality ΔQ^2 induced by the medium along the parton path would be superior. This can be understood in the limit of small path-length L . For applications to observables, ΔQ^2 and L are not independent, and small L also implies small ΔQ^2 . In this case, the appropriate limit for Q_0 is the vacuum default value $Q_0 = 1$ GeV, rather than the value given by Eq. (5), as the assumption that the medium effect dominates the dynamics of the shower is not fulfilled when ΔQ^2 is small. However, one could think of an artificial problem in which ΔQ^2 is large while L is small, in which case the limit for small path lengths is not the vacuum.

For the time being, we will nevertheless explore the consequences of the approximation given by Eq. (5) as it stands, rather than introduce a more general prescription, which brings additional model dependence. This means that the results are not reliable for small path lengths where the shower is not dominated by the medium, but the underlying assumption is justified by the vast majority of partons encountered in the computation of experimentally relevant observables.

D. Medium-modified showers and energy loss

It is not straightforward for medium-modified shower models to make contact with leading-parton energy-loss models. Typically, in an energy-loss model like [1–3], there is a clear distinction between the leading parton and radiated partons made by the *a priori* assumption that the energy of the leading parton is much larger than the energy of radiation quanta. In this limit, the medium-induced energy loss is given by the sum of the energy of all radiated quanta or, equally, by

the energy difference of the leading parton before and after the passage through the medium. *A posteriori*, one may, however, find (rare) processes in which the energy of a radiated parton is much larger than the energy of the parent—these are formally counted as large energy loss.

In a shower model, the leading (most energetic) parton can be identified at all times, but the identity of that parton may change easily during the evolution. Unlike in the energy-loss case, the radiation of a hard gluon carrying away 80% of the energy of a parent quark would not be counted as large energy loss but as change in the identity of the leading parton. No particular branching process can be attributed uniquely to vacuum or medium—differences between a medium-modified and a vacuum shower can only be defined at the level of averages over many events. This implies that the baseline to compare with is not a single parton with a definite energy, but rather a vacuum shower with a statistical distribution of hadrons at given momenta.

As a way to overcome these difficulties and make contact between in-medium shower and energy-loss modeling, in [16], c quarks as shower initiators have been considered. Due to the dead-cone effect, these have comparatively low vacuum-shower activity and a hard-fragmentation pattern, thus minimizing the distortion of the vacuum baseline from a single parton with definite energy. Moreover, since c -quark generation in branchings $g \rightarrow c\bar{c}$ is mass suppressed, tagging the c quark almost always tags the shower initiator.

Energy loss of the leading c quark can then be deduced from the ansatz

$$\frac{dN^{\text{med}}}{dE_c}(E) = \int d(\Delta E) \frac{dN^{\text{vac}}}{dE_c}(E') P(\Delta E) \delta(E' - E - \Delta E), \quad (6)$$

solving for the probability distribution of energy loss $P(\Delta E)$. Note that this ansatz contains the rather drastic assumption that there is no parametric dependence on the initial energy E . If we require $P(\Delta E)$ to be a probability distribution, the assumption may imply that, for some partons in the distribution dN/dE_c^{vac} , the energy loss ΔE is larger than their energy E , in which case they have to be considered lost to the medium.

In practice, Eq. (6) is discretized and cast into the form of a matrix equation

$$N_i(E^i) = \sum_{j=1}^n K_{ij}(E^i, \Delta E^j) P_j(\Delta E^j), \quad (7)$$

where dN/dE_c is provided at m discrete values of E labeled N_i , and $P(\Delta E)$ is probed at n discrete values of ΔE labeled P_j . The kernel K_{ij} is then the calculated dN/dE_c for all pairs $(E^i, \Delta E^j)$ where the energy loss acts as a shift of the distribution, i.e., $dN/dE_c^{\text{med}}(E) = dN/dE_c^{\text{vac}}(E + \Delta E)$. Equation (7) can, in principle, be solved for the vector P_j by inversion of K_{ij} for $m = n$. However, in general this does not guarantee that the result is a probability distribution. Especially in the face of statistical errors and finite numerical accuracy, the direct matrix inversion may permit negative P_j , which have no probabilistic interpretation. Thus, a more promising solution, which avoids these problems, is to let $m > n$, and find the vector P that minimizes $\|N - KP\|^2$ subject to the

constraints $0 \leq P_i \leq 1$ and $\sum_{i=1}^n P_i = 1$. This guarantees that the outcome can be interpreted as a probability distribution and, since the system of equations is overdetermined for $m > n$, errors on individual points R_i no longer have a critical influence on the outcome.

Note that this ansatz leads to additional complications in combination with the prescription to set the minimum-virtuality scale Q_0 in the shower given by Eq. (5). There are two reasons for this: First, in essence, for short paths, energy loss cannot be extracted reliably because the scale choice for the vacuum part becomes only irrelevant if the vacuum shower is a small correction to the medium-induced shower as discussed previously. Unfortunately, for truly long paths, the second complication becomes apparent: Unlike in the light-quark case, the charm mass m_c , rather than Eq. (5), determines the point at which the branching is stopped. In order to avoid the second limit, we use very energetic c quarks to extract energy loss, thus allowing for a larger length scale to be probed.

For these reasons, the actual effect of Eq. (5) on the path-length dependence of light-parton energy loss is only in a very limited way represented by evaluating Eq. (7), and all energy-loss results shown for this prescription need to be treated with the appropriate amount of caution. However, this is largely a complication of the way energy loss is determined from an in-medium shower result—there is no compelling reason to suspect that Eq. (5) would be as problematic for the in-medium shower-evolution result, as such, or in the computation of observables.

As a side remark, the fact that the mass of heavy quarks largely sets the scale at which the shower evolution is terminated implies that there is no substantial heavy-quark radiative energy loss in the kinematical region of RHIC experiments in YAJEM. Without an additional mechanism like elastic energy loss, YAJEM thus fails (like other radiative-energy-loss models) to account for the suppression of heavy quarks seen in single-electron spectra.

III. RESULTS

A. Path-length dependence of energy loss

In Fig. 2 the mean energy loss as a function of path length extracted from YAJEM via Eq. (7) for a 100 GeV charm quark as shower initiator is shown for different options.

In the first case, the formation time of virtual states given by Eq. (1) is not randomized by Eq. (2). The result is initially very well described by a quadratic-length dependence, until (around 3 fm) finite-energy corrections become important and the limit of applicability is reached and turn the curve over.

If randomization is taken into account, the quadratic dependence becomes very blurred, and the net result can be described well by a linear growth with path length, until finite-energy corrections and the limit of applicability turn the result over to a constant again. This agrees nicely with the scaling law identified in [16] and shows that an algorithm which in principle reproduces the analytical L^2 dependence in the right limit may not show this behavior in practically relevant situations.

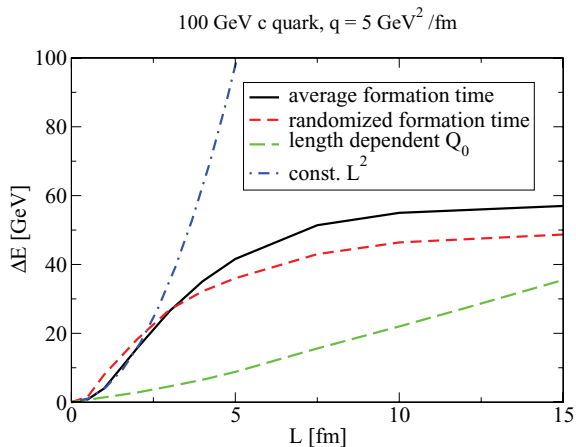


FIG. 2. (Color online) The integrated mean energy loss ΔE as a function of path length extracted from YAJEM for a 100 GeV shower-initiating c quark propagating through a constant medium with $\hat{q} = 5 \text{ GeV}^2/\text{fm}$ for different options (see text), compared with a quadratic dependence on path length.

Finally, the last case shows the effect of a variable–minimum-virtuality scale, Q_0 , according to Eq. (5). Note that, following the previous discussion, certainly beyond 10 fm, the result is not characteristic for what would happen in the light-quark case, as the charm-quark mass rather than Q_0 determines where the branching process is terminated. For a light quark, the curve would rise more strongly.

B. Path-length dependence of the nuclear suppression factor R_{AA}

In order to illustrate how path-length effects manifest in observable quantities, we use a realistic $3 + 1d$ hydrodynamic evolution model [27] for the bulk medium, and compute the dependence of the nuclear suppression factor R_{AA} on the angle with the reaction plane in this background. The straightforward computation is described in detail in [15,28]. It involves averaging the medium-modified fragmentation function (MMFF) computed from YAJEM over all possible paths through the medium, which are in turn determined by the distribution of binary collisions in the transverse plane and the orientation of the parton momentum vector with respect to the reaction plane. This result is then folded with a leading-order perturbative Quantum Chromodynamics (LO pQCD) calculation for the parton spectrum to get the yield of high- P_T hadrons. Since YAJEM obtains the MMFF at a given partonic scale, whereas LO pQCD computations require the fragmentation function to be known at a given hadronic scale, a scale-matching procedure as described in [28] has been used in the following.

In all computations, the factor K in Eq. (4) is adjusted such that the measured R_{AA} for 0%–10% central Au-Au collisions at $P_T = 6 \text{ GeV}$ is reproduced by the calculation. Both the P_T dependence of R_{AA} , the centrality dependence and the dependence on the angle ϕ with the reaction plane follow then without additional free parameters. Note that the value of K depends on the choice of Q_0 (as pointed out already in [16]).

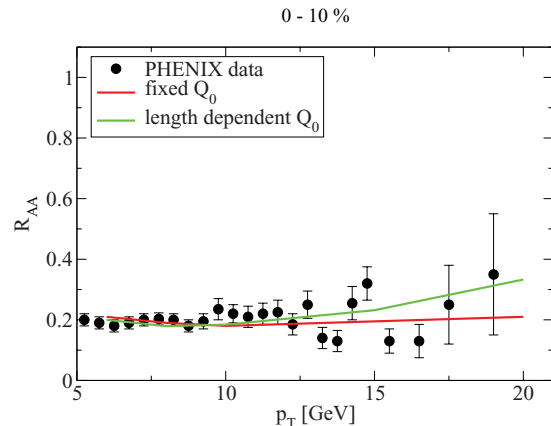


FIG. 3. (Color online) The nuclear suppression factor R_{AA} for 0%–10% central 200 AGeV Au-Au collisions computed for two different options (see text) in YAJEM, compared with PHENIX data [29].

In Fig. 3 the result for the angular average of R_{AA} is shown in comparison with PHENIX data [29] for both a fixed value of the minimum-virtuality scale $Q_0 = 0.7 \text{ GeV}$ and the dynamical choice $Q_0 = \sqrt{E/L}$, where L is the length of the parton path in the medium. As discussed in more detail in [12], there is no compelling reason to identify the Cooper-Frye freeze-out hypersurface in a hydrodynamic model with a sharp surface beyond which the medium interaction with partons is absent, and prescription to let the medium fade out gradually would be desirable. However, since the length dependence enters under the square root, the dependence on details is sufficiently weak such that one may hope to get a reasonable estimate of the effect.

As seen from the figure, both computations are able to describe R_{AA} well, however, the P_T dependence is somewhat different. This is not surprising, since the expression for Q_0 also contains a dependence on the initial parton energy E which is proportional to the final hadron P_T . Generically, for the dynamical-scale choice, a rise of R_{AA} with increasing P_T is expected. The curve for fixed Q_0 has been obtained for a randomized formation time. The result computed for average formation time is not shown, as it is indistinguishable given the averaging over many different in-medium path lengths as required in the computation of R_{AA} .

Note that the P_T dependence of R_{AA} is different from results shown in [15,16]. The reason is that, in these works, the matching procedure between hadronic and partonic scale described in [28] has not been used, which leads to an unrealistic decrease of R_{AA} with P_T .

The result for 20%–30% centrality is shown in Fig. 4, separated into in-plane and out-of-plane emission, again for a fixed value of Q_0 and a dynamically computed value, and compared with PHENIX data [30].

It is immediately apparent from the figure that a computation with fixed Q_0 results in a spread between in-plane and out-of-plane emission that is much smaller than seen in the data. This is consistent with the observation made earlier that the effective path-length dependence of energy loss in YAJEM is approximately linear, and the spread found

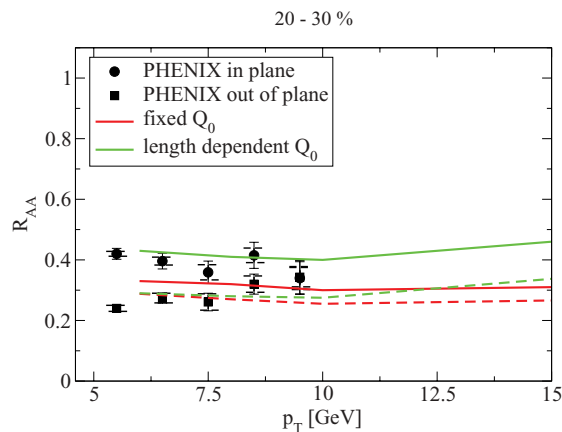


FIG. 4. (Color online) The nuclear suppression factor R_{AA} for 20%–30% central 200 AGeV Au-Au collisions computed for two different options (see text) in YAJEM, compared with PHENIX data [30]. Shown are in-plane emission (solid line) and out-of-plane emission (dashed line).

agrees with results from models in which the dependence was assumed to be linear [10,11].

On the other hand, both spread and normalization of the data are well reproduced by a dynamical choice for Q_0 which introduces, for light quarks and gluons, a strong path-length dependence of energy loss.

The scenario with dynamically computed Q_0 has thus clear observable consequences when observing hadrons at higher energies, such as is available at the Large Hadron Collider (LHC). Since the maximal in-medium path length is always limited by the nuclear diameter times a factor of order one, Eq. (5) implies that Q_0 will grow with increasing energy, thus the suppression induced by the medium will decrease, leading to a rise of R_{AA} with increasing P_T . Somewhat unfortunately, at LHC kinematics, such a rise is expected in various models for a number of reasons, among them the shape of the parton spectrum or the fact that the MMFF is, even for fixed Q_0 , energy dependent and reverts for large energies to the vacuum result [28]. However, the rise induced by the dynamically computed Q_0 occurs much faster than any of these other effects.

In contrast, the ratio of computed Q_0 at fixed energy for different path lengths is a constant. When considering in-plane and out-of-plane emission for noncentral collisions, this implies that, while the mean (angular averaged) R_{AA} changes rapidly with energy, the spread between in-plane and out-of-plane emission will be more robust and change much slower with energy. (Since, asymptotically, at high energies, the MMFF reverts to the vacuum fragmentation function, eventually the spread must vanish as well). Thus, the scenario proposed here can be tested by an observation of in-plane versus out-of-plane emission of hard hadrons in noncentral collisions for a sufficiently large range in P_T .

Detailed predictions for LHC kinematics will be postponed to a subsequent study, as they involve a significant numerical effort.

IV. DISCUSSION

From the results of the previous section, it would appear that, in the kinematical domain currently accessible experimentally, imposing a global-virtuality resolution scale given by Eq. (5) down to which the medium can influence the shower development has a much larger influence on the observable path-length dependence of high- P_T hadron production than a local-coherence condition imposed for each branching, which accounts for LPM interference.

Note that Q_0 acts, in the presence of a medium, quite differently than in the vacuum case. In a vacuum shower, Q_0 regulates the transition from a perturbative-partonic description in terms of a shower evolution to a nonperturbative-hadronization description in terms of the Lund model, and there is some transition region in which both prescriptions lead to similar results such that the final result is not strongly influenced by the detailed scale choice. However, within YAJEM, by construction (and due to a lack of insight into what physics hadronization inside the medium would entail), the medium only influences the partonic dynamics and there is no compensation of, e.g., a high-scale choice leading to weak showering by an increased action of the Lund model, as in the vacuum case. Thus, here Q_0 more directly regulates what part of the shower evolution happens in medium and what part needs to be taken care of by vacuum dynamics later.

If this result is confirmed by more-detailed investigations, it would imply that using $R_{AA}(\phi)$ to distinguish between pQCD and strongly coupled models on the basis of quadratic versus cubic path-length dependence is largely a “red herring,” as the leading effect seen in the data is something entirely different. However, since the scenario also predicts an energy dependence of the virtuality scale, translating, e.g., into a rise of R_{AA} with P_T , testing it should be comparatively easy with a large enough kinematic lever-arm as, e.g., the LHC experiments can provide.

In general, it seems that, in the context of MC shower models that can incorporate finite-energy effects and also, to some degree, quantum interference, the question of how path-length dependence arises and how to define it from observables becomes significantly more complicated than in the asymptotic limits of analytical models.

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