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# Photon decay in a strong magnetic field in heavy-ion collisions

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We calculate the photon pair production rate in a strong magnetic field created in off-central heavy-ion collisions. Photon decay leads to depletion of the photon yield by a few percent at RHIC and by as much as 20% at the LHC. It also generates a substantial azimuthal asymmetry ("elliptic flow") of the final photon distribution. We estimate $v_2 \approx 2\%$ at RHIC and $v_2 \approx 14\%$ at LHC. Photon decay measurements is an important tool for studying the magnetic fields in early stages of heavy-ion collisions.

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Ultrarelativistic heavy ions colliding at a finite impact parameter possibly create a supercritical magnetic field $B$. According to the estimates in Refs. [1] and [2], the strength of this field at $\sqrt{s} = 200$ GeV is approximately $eB \approx m_\pi^2/\hbar$, while the critical field is $eB_c = m_e^2/\hbar$. Thus, the magnetic field created in heavy-ion collisions is by many orders of magnitude stronger than any field that has been created using the state-of-the-art lasers (see, e.g., Ref. [3]). The possible existence of such fields opens a new avenue for studying the high-intensity regime of quantum electrodynamics (QED).

Various QED processes in external magnetic fields strongly depend on the time dependence of that field. Recently we argued [4] that the magnetic field is approximately stationary during the lifetime of the quark-gluon plasma (QGP) that is formed shortly after the collision. Indeed, phenomenological models describing the evolution of a QGP indicate that the thermalized medium is formed almost immediately after the collision (after ∼0.5 fm [5,6]) when the magnetic field is near the maximum of its strength. As the heavy-ion remnants recede from the collision point, the magnetic field tends to rapidly decrease with time. This induces circular currents in the QGP that, by the Faraday law, produce an induced magnetic field in the direction of the external field. Thus, the relaxation process of the external field slows down. The characteristic relaxation time is [4]

$$\tau = \frac{R^2\sigma}{4}, \tag{1}$$

where $R$ is the QGP size and $\sigma$ is its electric conductivity. In the perturbative regime one expects a high electric conductivity $\sigma \sim T/e^2$ [8]. Lattice calculations show that the electric conductivity is high even at temperatures close to $T_c$ [7]. In Ref. [4] we used the lattice data of Ref. [7] to estimate the relaxation time as $\tau \approx 160$ fm. This number is even larger if the effect of the magnetic field on the electric conductivity is taken into account [9]. It implies that the external magnetic field is a slowly varying function of time during the entire QGP lifetime. Of course, once the plasma cools down to the critical temperature and undergoes the phase transition to the hadronic gas, the conductivity becomes very small and the magnetic field cannot be sustained anymore.

In Ref. [4] we discussed the properties of the synchrotron radiation of gluons by fast quarks and argued that it has significant phenomenological implications. Indeed, the corresponding energy loss in magnetic field is comparable to that sustained by the fast quark in a hot nuclear medium. The azimuthally asymmetric form of the energy loss contributes to the "elliptic flow" phenomenon observed at RHIC. In this Brief Report we consider a cross-channel process: pair production by a photon in an external magnetic field. Specifically, we are interested to determine photon decay rate $w$ in the process $\gamma B \to f\bar{f}B$, where $f$ stands for a charged fermion, as a function of the photon's transverse momentum $k_T$, rapidity $\eta$, and azimuthal angle $\varphi$. The origin of these photons in heavy-ion collisions will not be of interest in this Brief Report.

The characteristic frequency of a fermion of species $a$ of mass $m_a$ and charge $z_a e$ ($e$ is the absolute value of the electron charge) moving in an external magnetic field $B$ (in a plane perpendicular to the field direction) is

$$\hbar\omega_B = \frac{z_a eB}{\varepsilon}, \tag{2}$$

where $\varepsilon$ is the fermion energy. Here, in the spirit of the adiabatic approximation, $B$ is a slow function of time. Calculation of the photon decay probability significantly simplifies if the motion of the electron is quasiclassical, i.e., quantization of the fermion motion in the magnetic field can be neglected. This condition is fulfilled if $\hbar\omega_B \ll \varepsilon$. This implies that

$$\varepsilon \gg \sqrt{zeB}. \tag{3}$$

For RHIC it is equivalent to $\varepsilon \gg m_\pi$, and for LHC it is equivalent to $\varepsilon \gg 4m_\pi$.

The photon decay rate was calculated in Ref. [10] and, using the quasiclassical method, in Ref. [11], it reads

$$w = -\sum_a \frac{\alpha_{\rm em} z_a^3 eB}{m_a \varkappa_a} \int_{(4/\varkappa_a)^{2/3}}^{\infty} \frac{2(x^{3/2} + 1/\varkappa_a)\mathrm{Ai}'(x)}{x^{11/4}(x^{3/2} - 4/\varkappa_a)^{3/2}}, \tag{4}$$

where the summation is over the fermion species and the invariant parameter $\varkappa$ is defined as

$$\varkappa_a^2 = -\frac{\alpha_{\rm em} z_a^2 \hbar^3}{m_a^6}(F_{\mu\nu}k^\nu)^2 = \frac{\alpha_{\rm em} z_a^2 \hbar^3}{m_a^6}(\vec{k} \times \vec{B})^2, \tag{5}$$

with the initial photon 4-momentum $k^\mu = (\hbar\omega, \vec{k})$. In heavy-ion collisions the vector of magnetic field $\vec{B}$ is orthogonal to the "reaction plane," which is spanned by the impact parameter

 

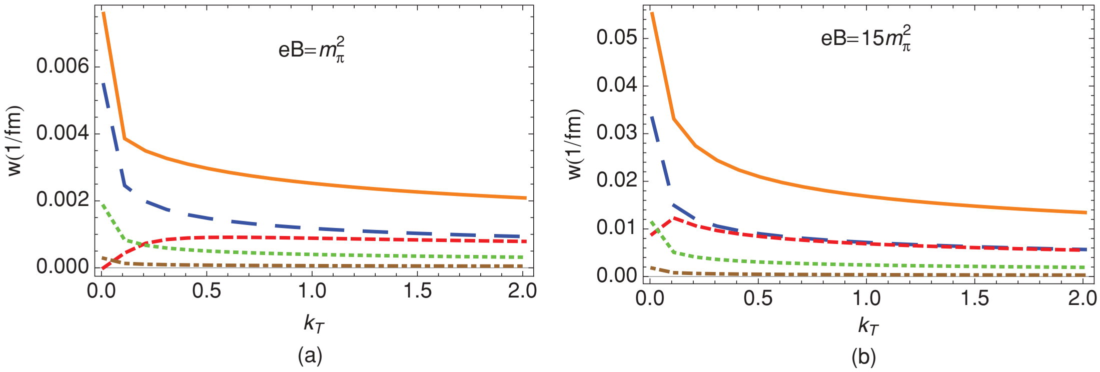


FIG. 1. (Color online) Decay rate of photons moving in a reaction plane in a magnetic field as a function of transverse momentum $k_T$: (a) at RHIC, (b) at LHC. Broken lines from bottom to top give contributions of $\gamma \to d\bar{d}$, $\gamma \to u\bar{u}$, $\gamma \to \mu^+\mu^-$, and $\gamma \to e^+e^-$ channels. The upper solid line is the total rate.

$\vec{b}$ and the collision axis $\hat{z}$. We define the polar angle $\theta$ with respect to the $z$ axis and the azimuthal angle $\varphi$ with respect to the reaction plane. In this notation, $\vec{B} = B\,\hat{\mathbf{y}}$ and $\vec{k} = k_z\hat{\mathbf{z}} + k_\perp(\hat{\mathbf{x}}\cos\varphi + \hat{\mathbf{y}}\sin\varphi)$, where $k_\perp = |\vec{k}|\sin\theta = \hbar\omega\sin\theta$. Thus, $(\vec{B}\times\vec{k})^2 = B^2(k_z^2 + k_\perp^2\cos^2\varphi)$. Introducing rapidity $\eta$ as usual, $\hbar\omega = k_\perp\cosh\eta$ and $k_z = k_\perp\sinh\eta$, we can write

$$\varkappa_a = \frac{\hbar(z_a eB)}{m_a^3}k_\perp\sqrt{\sinh^2\eta + \cos^2\varphi}. \tag{6}$$

In Fig. 1 we plotted the photon decay rate (4) for RHIC and LHC. The survival probability of photons in the magnetic field is $P = 1 - w\Delta t$, where $\Delta t$ is the time spent by a photon in plasma. We can see that for $\Delta t = 10$ fm the photon survives with probability $P_{\rm RHIC} \approx 97\%$ at RHIC, while at LHC it is $P_{\rm LHC} \approx 80\%$. Such a strong depletion certainly can be observed in heavy-ion collisions at LHC.

The azimuthal distribution of the decay rate of photons at LHC is azimuthally asymmetric, as can be seen in Fig. 2.

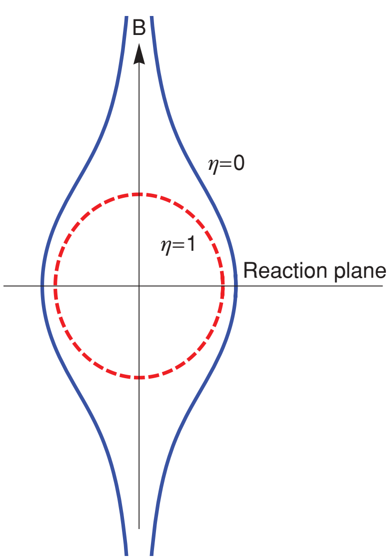


FIG. 2. (Color online) Azimuthal distribution of the decay rate of photons at different rapidities at LHC. Only the contribution of the $\gamma \to e^+e^-$ channel is shown.

The strongest suppression is in the $B$-field direction, i.e., in the direction orthogonal to the reaction plane. At $\eta \gtrsim 1$ the $\varphi$ dependence of $\varkappa_a$ is very weak, which is reflected in the nearly symmetric azimuthal shape of the dashed line in Fig. 2.

To quantify the azimuthal asymmetry it is customary to expand the decay rate in the Fourier series with respect to the azimuthal angle. Noting that $w$ is an even function of $\varphi$, we have

$$w(\varphi) = \frac{1}{2}w_0 + \sum_{n=1}^{\infty} w_n\cos(n\varphi),$$
$$w_n = \frac{1}{\pi}\int_{-\pi}^{\pi} w(\varphi)\cos(n\varphi)d\varphi. \tag{7}$$

In strong fields $\varkappa_a \gg 1$. For example, for $\gamma \to \mu^+\mu^-$ at RHIC at $\varphi = \eta = 0$ and $k_T = 1$ GeV, we get $\varkappa_\mu = 19$. Therefore, we can expand the rate (4) at large $\varkappa_a$ as [10]

$$w \approx \frac{3^{1/6}5\,\Gamma^2\left(\frac{2}{3}\right)}{2^{4/3}7\pi^{1/2}\Gamma\left(\frac{7}{6}\right)}\sum_a \frac{\alpha_{\rm em}eBz_a^3}{m_a\varkappa_a^{1/3}}$$
$$\equiv \frac{A}{(\sinh^2\eta + \cos^2\varphi)^{1/6}}, \quad \varkappa_a \gg 1. \tag{8}$$

At $\eta = 0$ the Fourier coefficients $w_n$ can be calculated analytically by using formula 3.631.9 of [12]:

$$w_{2k} = \frac{3\,2^{1/3}A}{B\left(\frac{5}{6}+k, \frac{5}{6}-k\right)}, \quad w_{2k+1} = 0, \quad k = 0, 1, 2, \ldots, \tag{9}$$

where $B$ is Euler's Beta function and $A$ is defined in (8). Substituting these expressions into (7) we find

$$w = \frac{1}{2}w_0\left[1 - \sum_{k=1}^{\infty}\frac{\sqrt{\pi}\,\Gamma\left(-\frac{1}{6}\right)}{2^{2/3}B\left(\frac{5}{6}+k, \frac{5}{6}-k\right)}\cos(2k\varphi)\right]. \tag{10}$$

The first few terms in this expansion read

$$w = \tfrac{1}{2}w_0\left(1 - \tfrac{2}{5}\cos(2\phi) + \tfrac{14}{55}\cos(4\phi) - \cdots\right). \tag{11}$$

What is measured experimentally is not the decay rate, but rather the photon spectrum. This spectrum is modified by the survival probability $P$, which is obviously azimuthally asymmetric. To quantify this asymmetry we write, using (7),

$$P = \bar{P}\left(1 + \sum_{k=1}^{\infty} v_{2k}\cos(2\varphi k)\right), \quad v_{2k} = -\frac{1-\bar{P}}{\bar{P}}\frac{2w_{2k}}{w_0}, \tag{12}$$

where $\bar{P} = \langle 1 - w\Delta t\rangle_\varphi = 1 - w_0\Delta t$ is the survival probability averaged over the azimuthal angle. Because $w_0\Delta t \ll 1$, as can be seen in Fig. 1, we can estimate, using (8) and (9),

$$\begin{aligned} v_{2k} &\approx -\frac{2w_{2k}}{w_0}\, w_0\Delta t \\ &= -\frac{2w_{2k}}{w_0}\Delta t\,\frac{5\,6^{2/3}\Gamma\left(\frac{2}{3}\right)}{7\pi}\sum_a \frac{\alpha_{\rm em}(eB)^{2/3}z_a^{8/3}}{(k_T)^{1/3}}. \end{aligned} \tag{13}$$

In particular, the "elliptic flow" coefficient is

$$v_2 = \Delta t\,\frac{2\,6^{2/3}\Gamma\left(\frac{2}{3}\right)}{7\pi}\sum_a \frac{\alpha_{\rm em}(eB)^{2/3}z_a^{8/3}}{(k_T)^{1/3}}. \tag{14}$$

For example, at $k_T = 1$ GeV and $\Delta t \sim 10$ fm/c, one expects $v_2 \simeq 2\%$ at RHIC and $v_2 \simeq 14\%$ at LHC only owing to the presence of the magnetic field. We see that the decay of photons in the external magnetic field significantly contributes to the photon asymmetry in heavy-ion collisions, along with other possible effects [13–18].

In summary, we calculated the photon pair-production rate in an external magnetic field created in off-central heavy-ion collisions. Photon decay leads to depletion of the photon yield by a few percent at RHIC and by as much as 20% at the LHC. The decay rate depends on the rapidity and azimuthal angle. At midrapidity the azimuthal asymmetry of the decay rate translates into an asymmetric photon yield and contributes to the "elliptic flow." Let us also note that photons polarized parallel to the field are 3/2 times more likely to decay than those polarized transversely [10]. Therefore, polarization of the final photon spectrum perpendicular to the field is a signature of the existence of the strong magnetic field. Finally, photon decay leads to enhancement of the dilepton yield, which will be addressed in a separate publication.

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