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Azimuthal correlations from transverse momentum conservation and possible local parity violation

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We analytically calculate the contribution of transverse momentum conservation to the azimuthal correlations that have been proposed as signals for possible local strong parity violation and recently have been measured in heavy ion collisions. These corrections are on the order of the inverse of the total final-state particle multiplicity and, thus, are on the same order as the observed signal. The corrections contribute with the same sign to both like-sign and opposite-sign pair correlations. Their dependence on the momentum is in qualitative agreement with the measurements by the solenoidal tracker at the BNL Relativistic Heavy Ion Collider Collaboration, while the pseudorapidity dependence differs from the data.

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I. INTRODUCTION

Topological configurations occur generically in non-Abelian gauge theories and are known to be essential for understanding the vacuum structure and hadron properties in quantum chromodynamics (QCD) (for reviews, see, e.g., Ref. [1]). Also, they have been shown to play important roles in hot QCD matter (i.e., quark-gluon plasma), which existed in the early universe and now is created in heavy ion collisions [2]. Despite much indirect evidence, a direct experimental manifestation of the topological effects has not been achieved and, therefore, is of great interest. One salient feature of the topological configurations is the $\mathcal{P}$- and $\mathcal{CP}$-odd effects they may induce. Based on that, it has been suggested [3–6] to look for the possible occurrence of $\mathcal{P}$- and $\mathcal{CP}$-odd domains with local strong parity violation for a direct detection of topological effects. Such domains may naturally arise in a heavy ion collision caused by the so-called sphaleron transitions in the created hot QCD matter. In particular, the so-called chiral magnetic effect (CME) predicts that, in the presence of the strong external (electrodynamic) magnetic field at the early stage after a (noncentral) collision, sphaleron transitions induce a separation of negatively and positively charged particles along the direction of the magnetic field, which is perpendicular to the reaction plane defined by the impact parameter and the beam axis. Such an out-of-plane charge separation, however, varies its orientation from event to event, either parallel or antiparallel to the magnetic field (depending on whether the CME is caused by sphaleron or antisphaleron transition). As a result, the expectation value of any $\mathcal{P}$-odd observable vanishes, and only the variance of such an observable may be detected, which makes the measurement of CME rather challenging. Recently, the solenoidal tracker at the BNL Relativistic Heavy Ion Collider (STAR) Collaboration, in Ref. [7], has published measurements of charged-particle azimuthal correlations proposed in Ref. [8] as a CME signal and found interesting patterns partly consistent with CME expectations. The STAR data have generated considerable interest, and many subsequent works have appeared, which propose alternative explanations [9–12], which suggest data interpretations and new observations [13–16], and study further consequences of CME [17–19].

We start with a discussion of the proposed CME signal as measured by STAR. In Ref. [8], it has been suggested that the CME may be approached indirectly by the measurement of the following two-particle correlation:

$$\begin{aligned}\gamma &= \langle\cos(\phi_1+\phi_2-2\Psi_{RP})\rangle \\ &= \langle\cos(\phi_1-\Psi_{RP})\cos(\phi_2-\Psi_{RP})\rangle - \langle\sin(\phi_1-\Psi_{RP}) \\ &\quad\times \sin(\phi_2-\Psi_{RP})\rangle,\end{aligned} \tag{1}$$

where Ψ_{RP}, ϕ_1, and ϕ_2 denote the azimuthal angles of the reaction plane and produced charged particles, respectively. Because this observable measures the difference between the in-plane and out-of-plane projected azimuthal correlations, it has been argued that this observable is particularly suited for revealing the CME signal, which is an out-of-plane charge separation. Specifically, the CME predicts $\gamma > 0$ for opposite-sign pairs and $\gamma < 0$ for same-sign pairs. However, as has been pointed out in Refs. [9–11,14–16], in noncentral collisions, the presence of elliptic flow [20] already differentiates between in and out of plane. As a consequence, essentially any two-particle correlation will contribute to the preceding observable, although the correlations' dynamical mechanism may generally be reaction-plane independent. These so-called background correlations need to be well understood theoretically and possibly need to be determined by independent measurements.[1] The STAR publication [7] presents data for the correlator γ together with the reaction-plane-independent correlator,

$$\delta = \langle\cos(\phi_1-\phi_2)\rangle, \tag{2}$$

[1]It is worth emphasizing that the contributions to γ from elliptic-flow-induced correlations and from the CME have similar centrality trends: Both the elliptic flow and the magnetic field, necessary for the CME, increase from central to peripheral collisions.

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in the midrapidity region for both same- and opposite-sign pairs in AuAu and CuCu collisions at two energies $\sqrt{s_{NN}} = 200$ and 62 GeV. The data are encouraging: At first sight, the results for γ seem to be qualitatively consistent with the CME expectations. However, as shown in Ref. [14], when the correlator δ is taken into account as well, the interpretation of the data, in terms of the CME, requires almost exact cancellation of the CME and all possible background correlations. Consequently, to extract a possible signal for the CME, the understanding of these background correlations becomes crucial at this stage.

One well-known possible source of azimuthal correlation is the conservation of transverse momentum, which has been qualitatively discussed in Ref. [11] and has been suggested to be a significant contribution to the measured observable γ. The argument goes as follows. Consider, for a moment, *all* particles in the final state, charged and neutral over all phase space. Next, rewrite the correlator Eq. (1) as (for simplicity, we set $\Psi_{RP} = 0$ here and in the rest of the paper)

$$\gamma = \left\langle \frac{\sum_{i\neq j}\cos(\phi_i+\phi_j)}{\sum_{i\neq j}1}\right\rangle = \left\langle \frac{\left(\sum_i \cos(\phi_i)\right)^2 - \left(\sum_i \sin(\phi_i)\right)^2 - \sum_i \cos(2\phi_i)}{\sum_{i\neq j}1}\right\rangle, \quad (3)$$

where i and j are summed over all particles. If we further assume that all particles have exactly the same magnitude of transverse momentum p_t, the conservation of transverse momentum implies

$$\sum_i \cos(\phi_i) = \sum_i \sin(\phi_i) = 0, \quad (4)$$

and, in consequence, one obtains for sufficiently large N,

$$\gamma = \frac{-v_2}{N}. \quad (5)$$

Here, v_2 is the elliptic-flow coefficient measured for all produced particles, and N is the total number of all produced particles (in full phase space). This contribution to the azimuthal correlations from transverse momentum conservation (TMC) turns out to be on the order of the data measured by STAR and, therefore, bears interest and importance.

However, the previous argument relies on two assumptions, which are not realized in the actual measurement. First, STAR measures only the charged particles in a small pseudorapidity region $|\eta| < 1$, which accounts for only $\sim$15% of the total number of produced particles. Second, the magnitude of the transverse momentum is not a constant but, rather, is distributed, more or less, according to a thermal distribution. Therefore, a more realistic estimate for the contribution of TMC to the foregoing correlation functions is required. The influence of momentum conservation on observables in heavy ion collisions has been discussed in the literature in the context of spectra [21], elliptic flow [21,22], directed flow [23], and certain two-particle densities [24], and corrections of various importance have been established. In the present paper, we will address the effect of TMC on the correlation functions relevant for the potential measurement of the CME. To this end, we derive the necessary formalism, which allows us to quantify the azimuthal correlations caused by TMC.

Before going into detail, let us discuss a few qualitative features, which are to be expected from TMC, and which will be demonstrated in detail in the following. First, TMC introduces a back-to-back correlation for particle pairs because they tend to balance each other in momentum. Second, the expected correction should scale inversely with the total number of particles because more particles provide more ways to balance the momentum and, thus, to dilute the effect on two-particle correlations. Furthermore, the correlation should be stronger in plane than out of plane because of the presence of elliptic flow. As a result, we expect that TMC results in a negative contribution to the observable γ, which increases with the strength of the elliptic flow v_2. Finally, we note that TMC is blind to particle charge, which leads to identical contributions to same-sign and opposite-sign pair correlations. Because of these features, TMC *alone* cannot be expected as a full account for the observed charged-particle azimuthal correlation patterns. Rather, it should be considered as an important background effect that contributes significantly and, therefore, necessitates quantitative studies for establishing any final interpretation of the data.

The paper is organized as follows. In Sec. II, by assuming only TMC, we present analytical calculations of the correlators γ and δ, In Sec. III, we take the STAR acceptance into account and compare our results with the available data (integrated over transverse momentum). In Sec. IV, we discuss specific differential observables, such as transverse momentum and rapidity-dependent correlations. Our conclusions are listed in Sec. V, where some comments also are included.

II. GENERAL FORMULAS

Let us assume that there are N particles in total produced in a given heavy ion collision event with individual momenta $\vec{p}_1, \ldots, \vec{p}_N$. We denote the particles' transverse momenta by $\vec{p}_{1,t}, \ldots, \vec{p}_{N,t}$ and their magnitudes by $p_{1,t}, \ldots, p_{N,t}$. The N-particle density f_N (normalized to unity) with enforced TMC can be written as

$$f_N(\vec{p}_1,\ldots,\vec{p}_N) = \frac{\delta^2(\vec{p}_{1,t}+\cdots+\vec{p}_{N,t})f(\vec{p}_1)\cdots f(\vec{p}_N)}{\int_F \delta^2(\vec{p}_{1,t}+\cdots+\vec{p}_{N,t})f(\vec{p}_1)\cdots f(\vec{p}_N)d^3\vec{p}_1\cdots d^3\vec{p}_N}, \quad (6)$$

where $f(\vec{p}_i)$ is the normalized $[\int_F f(\vec{p})d^3\vec{p} = 1]$ single-particle distribution. Note that the preceding integrals are taken over the full phase space (denoted by F) rather than the region where the particles are actually measured. In Eq. (6), we explicitly assume that all produced particles are governed by the same single-particle distribution.[2] We also ignore any other two-particle correlations because, in this paper, we only focus on the effects of TMC. To calculate a two-particle

[2]This assumption is reasonably well satisfied in heavy ion collisions in which the final-state particles are mostly pions.

correlator, such as $\langle\cos(\phi_1+\phi_2)\rangle$, we need the two-particle density, which can be obtained from Eq. (6) by integrating out $N-2$ momenta,

$$f_2(\vec{p}_1,\vec{p}_2)=f(\vec{p}_1)f(\vec{p}_2)\frac{\int_F \delta^2\left(\sum_{i=1}^{N}\vec{p}_{i,t}\right)\prod_{i=3}^{N}[f(\vec{p}_i)d^3\vec{p}_i]}{\int_F \delta^2\left(\sum_{i=1}^{N}\vec{p}_{i,t}\right)\prod_{i=1}^{N}[f(\vec{p}_i)d^3\vec{p}_i]}. \tag{7}$$

To perform the integrals in Eq. (7), we follow the techniques of Refs. [21,22,25] and make use of the central limit theorem. The sum of M uncorrelated transverse momenta $\sum_{i=1}^{M}\vec{p}_{i,t}=\vec{K}_t$ has a Gaussian distribution if M is sufficiently large,[3] that is,

$$\begin{aligned}G_M(\vec{K}_t)&=\int_F \delta^2\left(\sum_{i=1}^{M}\vec{p}_{i,t}-\vec{K}_t\right)\prod_{i=1}^{M}[f(\vec{p}_i)d^3\vec{p}_i]\\&=\frac{1}{2\pi M\sqrt{\langle p_x^2\rangle_F\langle p_y^2\rangle_F}}\\&\quad\times\exp\left(-\frac{K_x^2}{2M\langle p_x^2\rangle_F}-\frac{K_y^2}{2M\langle p_y^2\rangle_F}\right).\end{aligned} \tag{8}$$

Here, x and y denote the two components of transverse momentum and

$$\langle p_x^2\rangle_F=\int_F f(\vec{p})p_x^2d^3\vec{p},\qquad \langle p_y^2\rangle_F=\int_F f(\vec{p})p_y^2d^3\vec{p}, \tag{9}$$

where the integrations are over full phase space F. By using Eq. (8), we can express Eq. (7) in the following manner:

$$\begin{aligned}f_2(\vec{p}_1,\vec{p}_2)&=f(\vec{p}_1)f(\vec{p}_2)\frac{G_{N-2}(-\vec{p}_{1,t}-\vec{p}_{2,t})}{G_N(0)}\\&=f(\vec{p}_1)f(\vec{p}_2)\frac{N}{N-2}\\&\quad\times\exp\left(-\frac{(p_{1,x}+p_{2,x})^2}{2(N-2)\langle p_x^2\rangle_F}-\frac{(p_{1,y}+p_{2,y})^2}{2(N-2)\langle p_y^2\rangle_F}\right).\end{aligned} \tag{10}$$

By expanding in powers of $1/N$ and by restricting ourselves to pairs with $|\vec{p}_{1,t}+\vec{p}_{2,t}|\ll\sqrt{2N\langle p_t^2\rangle}$,[4] we obtain

$$\begin{aligned}f_2(\vec{p}_1,\vec{p}_2)&\simeq f(\vec{p}_1)f(\vec{p}_2)\\&\quad\times\left(1+\frac{2}{N}-\frac{(p_{1,x}+p_{2,x})^2}{2N\langle p_x^2\rangle_F}-\frac{(p_{1,y}+p_{2,y})^2}{2N\langle p_y^2\rangle_F}\right).\end{aligned} \tag{11}$$

[3]We have checked, by simple Monte Carlo calculations, that the central limit theorem applies for $M>10$, see also Ref. [21]. This condition is very well satisfied in heavy ion collisions, even in the most peripheral ones.

[4]In practice, this is hardly any restriction because, for sufficiently large N, only pairs with very large transverse momentum violate the condition $|\vec{p}_{1,t}+\vec{p}_{2,t}|\ll\sqrt{2N\langle p_t^2\rangle}$, which are strongly suppressed by the rapidly decreasing single-particle distributions $f(\vec{p}_1)f(\vec{p}_2)$.

As already mentioned, the earlier correlation function does not distinguish between same- and opposite-sign pairs (or pairs that involve neutral particles) because the TMC involves all particles without discrimination on the charge of particles. We also note that the two-particle density $f_2(\vec{p}_1,\vec{p}_2)$ is maximum for back-to-back configurations, as can easily be seen from the dependence on the combinations $-(p_{1,x}+p_{2,x})^2$ and $-(p_{1,y}+p_{2,y})^2$.

Given the two-particle density Eqs. (7) and (11), we can proceed to evaluate various two-particle azimuthal correlations in any given kinematic region, for example, the one introduced in Eq. (1),

$$\langle\cos(\phi_1+\phi_2)\rangle=\frac{\int_\Omega f_2(\vec{p}_1,\vec{p}_2)\cos(\phi_1+\phi_2)d^3\vec{p}_1d^3\vec{p}_2}{\int_\Omega f_2(\vec{p}_1,\vec{p}_2)d^3\vec{p}_1d^3\vec{p}_2}, \tag{12}$$

and analogously for $\langle\cos(\phi_1-\phi_2)\rangle$. Here, we denote, by Ω, the part of the phase space covered by the actual experiment.

Finally, we need the single-particle distribution, for which we assume the following rather general form ($\Psi_{RP}=0$):

$$f(\vec{p})d^3\vec{p}=\frac{g(p_t,\eta)}{2\pi}[1+2v_2(p_t,\eta)\cos(2\phi)]d^2\vec{p}_td\eta, \tag{13}$$

where v_2 is the p_t- and η-dependent elliptic-flow coefficient (with η as the pseudorapidity). By taking Eq. (11) into account and by performing elementary calculations, we obtain our main result:[5]

$$\langle\cos(\phi_1+\phi_2)\rangle=-\frac{1}{N}\frac{\langle p_t\rangle_\Omega^2}{\langle p_t^2\rangle_F}\frac{2\bar{v}_{2,\Omega}-\bar{\bar{v}}_{2,F}-\bar{\bar{v}}_{2,F}(\bar{v}_{2,\Omega})^2}{1-(\bar{\bar{v}}_{2,F})^2}, \tag{14}$$

where we have introduced certain weighted moments of v_2,

$$\bar{v}_2=\frac{\langle v_2(p_t,\eta)p_t\rangle}{\langle p_t\rangle}=\frac{\int g(p_t,\eta)v_2(p_t,\eta)p_td^2\vec{p}_td\eta}{\int g(p_t,\eta)p_td^2\vec{p}_td\eta}, \tag{15}$$

and

$$\bar{\bar{v}}_2=\frac{\langle v_2(p_t,\eta)p_t^2\rangle}{\langle p_t^2\rangle}=\frac{\int g(p_t,\eta)v_2(p_t,\eta)p_t^2d^2\vec{p}_td\eta}{\int g(p_t,\eta)p_t^2d^2\vec{p}_td\eta}. \tag{16}$$

Again, the indices F and Ω indicate that all integrations in Eqs. (15) and (16) are performed over full phase space (F) or the phase space in which particles are measured (Ω), respectively. For completeness, let us add that N denotes the total number of produced particles (charged and neutral), and $\langle p_t\rangle_\Omega$ is the average transverse momentum of the *measured* particles.

One important lesson from the previous result is that, even if we measure particles in a limited fraction of the full phase

[5]Here, and in the following, we assume that the azimuthal part of Ω covers full 2π, that is, $\phi\in[0,2\pi)$. This is the only restriction we impose on Ω.

space (e.g., a narrow pseudorapidity bin), the effect of TMC on $\langle\cos(\phi_1+\phi_2)\rangle$ is *not* suppressed.[6] This may be roughly understood in the following way: If each particle is generated in an independent manner except the constraint from overall TMC, then, for each given particle, its p_t effectively has a chance of $1/N$ (in the large N limit) to be balanced by every other particle. That is equivalent to say that each pair has a back-to-back correlation of strength $1/N$, which is preserved despite what fraction of particles is selected for measurement. Furthermore, while not changing the order of magnitude of the effect, the details of the p_t and η dependence of single-particle distribution and v_2 may slightly affect the quantitative results.

Next, we examine two approximations to the main result, Eq. (14).

(i) If all the produced particles are measured (i.e., $\Omega = F$) and all have the same magnitude of the transverse momentum [i.e., $g(p_t,\eta) \propto \delta(p_t - p_0)h(\eta)$], we recover the result Eq. (5) discussed in Sec. I.

(ii) If one allows for a finite acceptance but neglects the p_t and η dependence of v_2 (i.e., $\bar{\bar{v}}_{2,F/\Omega} = \bar{v}_{2,F/\Omega} = v_{2,F/\Omega}$), Eq. (14) reduces to

$$\langle\cos(\phi_1+\phi_2)\rangle = -\frac{v_2}{N}\frac{\langle p_t\rangle_\Omega^2}{\langle p_t^2\rangle_F}. \tag{17}$$

Because $\frac{\langle p_t\rangle_\Omega^2}{\langle p_t^2\rangle_F}$ only depends weakly on the acceptance Ω, the corrections caused by TMC are, as already pointed out, more or less, independent of the number of observed particles.

Next, let us calculate the contribution from TMC to the correlation function $\delta = \langle\cos(\phi_1-\phi_2)\rangle$, which has also been measured by STAR. Following similar procedures, we obtain the following result:

$$\langle\cos(\phi_1-\phi_2)\rangle = -\frac{1}{N}\frac{\langle p_t\rangle_\Omega^2}{\langle p_t^2\rangle_F}\frac{1+(\bar{v}_{2,\Omega})^2 - 2\bar{\bar{v}}_{2,F}\bar{v}_{2,\Omega}}{1-(\bar{\bar{v}}_{2,F})^2}. \tag{18}$$

We notice a few interesting features. First, the correlation scales like $-1/N$ as expected. Second, the effect does not depend on elliptic flow in leading order and, thus, is much stronger than that in $\langle\cos(\phi_1+\phi_2)\rangle$, which is of order $\hat{O}(v_2/N)$. Let us again examine the same two approximations for the foregoing result.

(i) If all the produced particles are measured (i.e., $\Omega = F$) and all have the same magnitude of transverse momentum [i.e., $g(p_t,\eta) \propto \delta(p_t - p_0)h(\eta)$], we obtain $\langle\cos(\phi_1-\phi_2)\rangle = -\frac{1}{N}$.

(ii) If one allows for a finite acceptance but neglects the p_t and η dependence of v_2 (i.e., $\bar{\bar{v}}_{2,F/\Omega} = \bar{v}_{2,F/\Omega} = v_{2,F/\Omega}$), we obtain

$$\langle\cos(\phi_1-\phi_2)\rangle = -\frac{1}{N}\frac{\langle p_t\rangle_\Omega^2}{\langle p_t^2\rangle_F}. \tag{19}$$

Thus, the main difference between the two correlators, γ [Eq. (14)] and δ [Eq. (18)] is the presence of v_2 in the former. Therefore, effects of TMC will be much more visible in the correlator $\langle\cos(\phi_1-\phi_2)\rangle$.[7]

III. COMPARISON WITH DATA

In this section, we compare our results Eqs. (14) and (18) with recently published STAR data [7], where charged particles have been measured in the pseudorapidity interval $-1 < \eta < 1$ and the transverse momentum region $p_t > 0.15$ GeV. For the results integrated over transverse momentum, an additional cut of $p_t < 2$ GeV was imposed. As seen from Eqs. (14)–(16) and (18), to calculate the contribution of the TMC to $\langle\cos(\phi_1+\phi_2)\rangle$ and $\langle\cos(\phi_1-\phi_2)\rangle$, we need full information about single-particle distribution $g(p_t,\eta)$ and elliptic flow $v_2(p_t,\eta)$ in the full phase space. Unfortunately, such complete information is not currently available. Thus, we will make some reasonable assumptions that hopefully allow us to obtain an approximate insight into the discussed effect.

First, let us estimate the total number of produced particles N. From the PHOBOS measurement [26], we know that the total number of charged particles $N_{\rm ch}$ grows linearly[8] with the number of participants $N_{\rm part}$ (or, equivalently, the number of wounded nucleons [27]). At $\sqrt{s_{NN}} = 200$ GeV, we find that $N_{\rm ch} \approx 14N_{\rm part}$ [26]; thus, the total number of particles can reasonably be approximated by

$$N \approx (3/2)N_{\rm ch} = 21N_{\rm part}. \tag{20}$$

This result allows us to roughly estimate the contribution of TMC. We simply assume that $\langle p_t\rangle_\Omega^2 = \langle p_t^2\rangle_F$ and $\bar{v}_{2,\Omega} = \bar{\bar{v}}_{2,F}$, which leads to

$$\langle\cos(\phi_1+\phi_2)\rangle \cdot N_{\rm part} = -\bar{v}_{2,\Omega}/21 \approx -0.004, \tag{21}$$

and

$$\langle\cos(\phi_1-\phi_2)\rangle \cdot N_{\rm part} = 1/21 \approx -0.05, \tag{22}$$

where we take $\bar{v}_{2,\Omega} = 0.08$ to be slightly larger than the elliptic-flow parameter $v_{2,\Omega} = 0.06$ to account for the momentum dependence of v_2. Later, we will show that these simple assumptions are well reproduced in a more detailed calculation.

While this effect gives a contribution with the same sign and order of magnitude for the same-sign pair correlation data, it is a factor of 3–5 (very peripheral-midcentral) less in magnitude for $\langle\cos(\phi_1+\phi_2)\rangle$ and a factor of 1.5–4 (midcentral-very peripheral) larger than the STAR data. It also gives the same

[6]This statement is, of course, limited by the applicability of the central limit theorem, which, however, works for as little as ten particles.

[7]In Ref. [23], the effect of TMC for the determination of the directed flow parameter v_1 has been carried out. The authors find that the correction scales with the number of observed particles. While this may appear as a contradiction to our finding that the corrections to $\langle\cos(\phi_1-\phi_2)\rangle$ are essentially independent of the observed multiplicity, straightforward algebra shows that the results of Ref. [23] are fully consistent with our results. Indeed, the authors of Ref. [23] also use the formalism of Ref. [22] to derive their results.

[8]In contrast to the number of charged particles at midrapidity that grows slightly faster than $N_{\rm part}$.

contribution to the opposite-sign pair correlation for which the data are positive.

To perform more precise calculations, we further assume that the single-particle distribution $g(p_t, \eta)$ can be expressed in the following manner:

$$g(p_t, \eta) = \frac{1}{T^2} \exp\left(\frac{-p_t}{T}\right) h(\eta), \tag{23}$$

where, for simplicity, we take a thermal distribution and assume factorization of the momentum and pseudorapidity dependence. To introduce a slight pseudorapidity dependence of T [28], we will distinguish between T_F and $T_{\rm mid}$, which depends upon whether we integrate over the full pseudorapidity range or the midrapidity region. In the following, we take $T_{\rm mid} = 0.225$ GeV and $T_{\rm mid}/T_F = 1.1$ [28]. In Eq. (23), $h(\eta)$ is the normalized pseudorapidity single-particle distribution, which can be well represented by a double Gaussian form

$$h(\eta) = \frac{1}{2\sqrt{2\pi\sigma^2}}\left[\exp\left(-\frac{(\eta-\eta_0)^2}{2\sigma^2}\right) + \exp\left(-\frac{(\eta+\eta_0)^2}{2\sigma^2}\right)\right], \tag{24}$$

with $\eta_0 = 2$ and $\sigma = 1.9$ for $\sqrt{s_{NN}} = 200$ GeV.[9] Finally, we have to specify the momentum and pseudorapidity dependence of elliptic flow. For simplicity, we represent this dependence with a factorized liner ansatz for both the transverse momentum and the pseudorapidity, which represents the presently available data reasonably well up to $p_t \simeq 2$ GeV [30–32],[10]

$$v_2(p_t, \eta) = C p_t \frac{7 - |\eta|}{7}, \tag{25}$$

and $v_2 = 0$ for $|\eta| > 7$. Here, $C = 0.14$ so that, for the midrapidity region $|\eta| < 1$, the calculated elliptic flow equals 0.06 [30–32].

With these parametrizations, we find that: $\langle p_t\rangle_\Omega^2 \approx 0.257$, $\langle p_t^2\rangle_F \approx 0.252$, $\bar{v}_{2,\Omega} \approx 0.0886$, and $\bar{\bar{v}}_{2,F} \approx 0.0773$. By substituting these numbers into Eqs. (14) and (18), we obtain

$$\langle\cos(\phi_1 + \phi_2)\rangle \cdot N_{\rm part} \approx -0.005, \tag{26}$$

and

$$\langle\cos(\phi_1 - \phi_2)\rangle \cdot N_{\rm part} \approx -0.05, \tag{27}$$

which are very close to the numbers estimated at the beginning of this section. While it is difficult to estimate the precise uncertainty of our calculation, however, we expect our results to be correct within a few tens of percent.

IV. DIFFERENTIAL DISTRIBUTIONS

The STAR Collaboration also presented data for $\langle\cos(\phi_1 + \phi_2)\rangle$ as a function of $p_+ = (p_{1,t} + p_{2,t})/2$ and $p_- = |p_{1,t} - p_{2,t}|$. Both distributions are very informative: The azimuthal correlation $\langle\cos(\phi_1 + \phi_2)\rangle$ increases roughly linearly with p_+, while it only depends weakly on p_-. In Ref. [14], we showed that such behavior is not inconsistent with the CME and can be understood if we assume that correlated pairs have slightly larger momenta than uncorrelated ones.

As we will show in the following, a qualitatively similar behavior can be obtained from TMC. With the two-particle distribution Eq. (11), the p_+ differential distribution reads

$$\langle\cos(\phi_1+\phi_2)\rangle_{p_+} = \frac{\int_\Omega f_2(\vec{p}_1, \vec{p}_2)\delta(2p_+ - p_{1,t} - p_{2,t})\cos(\phi_1+\phi_2) d^3\vec{p}_1 d^3\vec{p}_2}{\int_\Omega f_2(\vec{p}_1, \vec{p}_2)\delta(2p_+ - p_{1,t} - p_{2,t}) d^3\vec{p}_1 d^3\vec{p}_2}, \tag{28}$$

and analogously for $p_- = |p_{1,t} - p_{2,t}|$.

Let us first discuss the simplified case in which $v_2(p_t, \eta)$ is replaced by its average value v_2. By taking Eqs. (11) and (23) into account, we obtain[11]

$$\langle\cos(\phi_1+\phi_2)\rangle_{p_+} = -v_2 \frac{2p_+^2}{15NT^2}, \tag{29}$$

and

$$\langle\cos(\phi_1+\phi_2)\rangle_{p_-} = -v_2 \frac{3T^2 + 3Tp_- + p_-^2}{6NT(T+p_-)}, \tag{30}$$

where, for simplicity, we set $T_F = T_{\rm mid} = T$. As can be seen, the correlation shows a strong quadratic growth with increasing p_+, in qualitative agreement with the data. The dependence on p_-, on the other hand, is essentially constant for $p_- \ll T$ before it exhibits a linear increase. By performing analogous calculations for δ, we obtain

$$\langle\cos(\phi_1-\phi_2)\rangle_{p_+/p_-} = \frac{1}{v_2}\langle\cos(\phi_1+\phi_2)\rangle_{p_+/p_-}, \tag{31}$$

thus, the p_+ and p_- dependence is identical; however, the signal is significantly stronger.

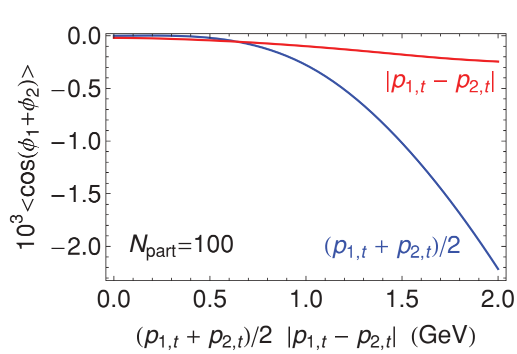

FIG. 1. (Color online) The two-particle azimuthal correlation $\langle\cos(\phi_1 + \phi_2)\rangle$ vs $p_+ = (p_{1,t} + p_{2,t})/2$ (blue line) and $p_- = |p_{1,t} - p_{2,t}|$ (red line) for $N_{\rm part} = 100$.

[9] We performed the fit to the PHOBOS 35%–45% centrality AuAu data [29].

[10] We have checked the influence of constant v_2 for $p_t > 2$ GeV and found it negligible.

[11] In this section, for simplicity, we integrate from $p_t = 0$ despite the finite cut $p_t > 0.15$ GeV in the STAR experiments.

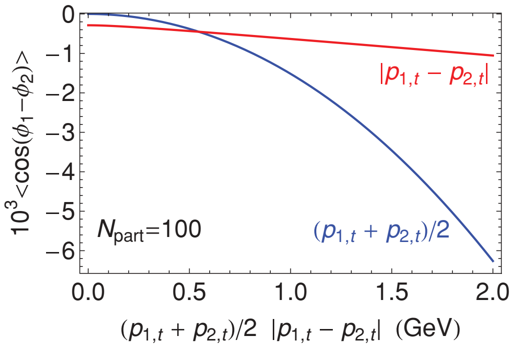

FIG. 2. (Color online) The two-particle azimuthal correlation $\langle\cos(\phi_1-\phi_2)\rangle$ vs $p_+ = (p_{1,t}+p_{2,t})/2$ (blue line) and $p_- = |p_{1,t} - p_{2,t}|$ (red line) for $N_{\rm part} = 100$.

We also performed full calculations with $g(p_t,\eta)$, $h(\eta)$, and $v_2(p_t,\eta)$ discussed in Sec. III and given by Eqs. (23), (24), and (25), respectively. However, in this calculation, we correct $v_2(p_t,\eta)$ and assume a constant value for v_2 for transverse momenta $p_t > 2$ GeV. The results for $N_{\rm part} = 100$ are presented in Figs. 1 and 2 for $\langle\cos(\phi_1+\phi_2)\rangle$ and $\langle\cos(\phi_1-\phi_2)\rangle$, respectively.

Again, similar to what has been observed in the STAR data, the correlation $\langle\cos(\phi_1+\phi_2)\rangle$ grows rapidly with increasing p_+, while it appears much flatter with increasing p_-. By comparing our results with the STAR data, we see that the TMC gives a comparable signal for p_+ and p_- larger than 1 GeV and underestimates the data for lower values of p_+ and p_-.

Finally, let us discuss the pseudorapidity dependence of the correlation $\langle\cos(\phi_1+\phi_2)\rangle$. The dependence of $\langle\cos(\phi_1+\phi_2)\rangle$ on $|\eta_1-\eta_2|$ has been measured by STAR and has been found to be dominated by $|\eta_1-\eta_2| < 2$, which is consistent with the CME expectation. It is quite clear that such dependence cannot be obtained in the present calculation because no pseudorapidity dependence appears in the nontrivial part of the two-particle correlation function shown in Eq. (11). Consequently, TMC predicts the correlator $\langle\cos(\phi_1+\phi_2)\rangle$ to be essentially flat as a function of $|\eta_1-\eta_2|$ in the midrapidity region except for a very mild dependence caused by the slight dependence of $f(\vec{p})$ and v_2 on η. However, here, we have assumed that the transverse momentum is balanced over the entire rapidity interval. In an actual heavy ion reaction, it is not unreasonable to expect that the transverse momentum is balanced over a shorter rapidity interval. If this were the case, we would predict not only a stronger rapidity dependence of the signal, but also a considerably stronger signal at midrapidity. Therefore, it would be worthwhile to construct and to measure an equivalent of the charge balance function [33] for the transverse momentum. This problem is currently under consideration.

V. CONCLUSIONS AND COMMENTS

We have quantitatively investigated the contribution of TMC to the azimuthal correlation observables $\gamma = \langle\cos(\phi_1+\phi_2)\rangle$ and $\delta = \langle\cos(\phi_1-\phi_2)\rangle$ measured by the STAR Collaboration as motivated by the possible strong local parity violation and CME. Our conclusions can be summarized as follows.

(i) The contribution caused by TMC is comparable in magnitude to the prediction of the CME as well as the data. In the STAR acceptance, we find this contribution to be approximately equal to $\gamma \approx -1.7 v_2/N$, where v_2 is the elliptic-flow coefficient at midrapidity and N is the total number of produced particles (neutral and charged). This result suggests rather weak energy dependence at RHIC because v_2 and N scale similarly with energy.

(ii) By taking $v_2 = 0.06$ and $N = 21 N_{\rm part}$, where $N_{\rm part}$ is the number of participants, we obtained $\gamma \cdot N_{\rm part} \approx -0.005$, which is a factor of 3–5 (very peripheral-midcentral) smaller than the experimental data. Thus, we may conclude that the TMC alone cannot explain the data. Also, there is no charge dependence as opposed to experimental data. However, it is a significant source of background that eventually must be quantified and taken into account if one really wants to extract the possible CME from the present (and future) data.

(iii) We have demonstrated that finite-acceptance issues (i.e., the facts that particles are measured in a relatively narrow pseudorapidity bin and neutral particles are not detected) do not suppress the effect of TMC on γ.

(iv) We studied the dependence of γ vs $p_+ = (p_{1,t} + p_{2,t})/2$ and $p_- = |p_{1,t} - p_{2,t}|$. We found that γ increases with increasing p_+, $\gamma \propto (\frac{p_+}{\langle p_t\rangle})^{\alpha_+}$ with $\alpha_+ = 2$ to 3. The dependence on p_- is much weaker, $\gamma \propto (\frac{p_-}{\langle p_t\rangle})^{\alpha_-}$, with $\alpha_- \approx 1$. This behavior is qualitatively similar to what is observed in the data. We also investigated the dependence of γ on $|\eta_1-\eta_2|$ and found no pseudorapidity dependence in contrast to what is observed in the data.

(v) Finally we calculated $\delta = \langle\cos(\phi_1-\phi_2)\rangle$ and found that, in the STAR acceptance, $\delta \approx -1/N$. We found this contribution to be a factor of 1.5–4 (midcentral-very peripheral) *larger* than the experimental data, which indicates again that the TMC effect is a significant source of background.

(vi) The present calculation is based on the minimal assumption that the transverse momentum is balanced over *all* particles (in the full phase space). Thus, it is very likely that the calculated contribution to γ and δ from the TMC rather represents the lower limit. Should, as it is not reasonable to assume, the transverse momentum be balanced over a finite-rapidity interval, we predict not only a stronger effect at midrapidity, but also a rapidity dependence of the correlation functions γ and δ. Thus, a measurement of something, such as a transverse momentum balance function would be highly desirable. This problem is currently under consideration.

We end this paper with a few additional comments.

(a) While TMC alone is not sufficient to explain the data, one may combine it with other effects, such as the

TABLE I. Estimated contributions to azimuthal correlations from various effects and comparison with data. The DATA are from the STAR measurement for AuAu 200-GeV collisions at $\sim$50%–60% centrality.

$\hat{O} \times 10^3$	$\langle\cos(\phi_1+\phi_2)\rangle_{++}$	$\langle\cos(\phi_1+\phi_2)\rangle_{+-}$	$\langle\cos(\phi_1-\phi_2)\rangle_{++}$	$\langle\cos(\phi_1-\phi_2)\rangle_{+-}$
CME	$-(0.1-1)$	$+(0.01-0.1)$	$+(0.1-1)$	$-(0.01-0.1)$
LCC	~ 0	$+(0.1-1)$	~ 0	$+(1-10)$
TMC	~ -0.1	~ -0.1	~ -1	~ -1
DATA	-0.45	$+0.06$	-0.38	$+1.97$

CME or local charge conservation (LCC) [10] to get closer to the data. In Table I, we summarize the estimated contributions to the azimuthal correlations from these effects together with the STAR data. All numbers quoted are for AuAu 200-GeV collisions at $\sim$50%–60% centrality (which corresponds to $N_{\text{part}} \approx 50$ [34]).

As a precaution, all numbers (except the STAR data) bear considerable uncertainty. The numbers from the CME for observables $\langle\cos(\phi_1+\phi_2)\rangle_{++,+-}$ are extracted from Ref. [4]. The numbers for $\langle\cos(\phi_1-\phi_2)\rangle_{++,+-}$ were obtained by using the relation $\langle\cos(\phi_1-\phi_2)\rangle_{++} = -\langle\cos(\phi_1+\phi_2)\rangle_{++}$ and analogously for $(+,-)$, which hold in the case of a pure CME. The numbers from the LCC are inferred from Ref. [10]: The authors showed the difference $\langle\cos(\phi_1+\phi_2)\rangle_{+-} - \frac{1}{2}[\langle\cos(\phi_1+\phi_2)\rangle_{++} + \langle\cos(\phi_1+\phi_2)\rangle_{--}]$. From LCC, one should expect $\langle\cos(\phi_1+\phi_2)\rangle_{++,--} \sim 0$, so the $\langle\cos(\phi_1+\phi_2)\rangle_{+-}$ could be inferred. Furthermore, the value for $\langle\cos(\phi_1-\phi_2)\rangle_{+-}$ is estimated by $\langle\cos(\phi_1-\phi_2)\rangle_{+-} \sim \langle\cos(\phi_1+\phi_2)\rangle_{+-}/v_2$. Finally, as the authors pointed out, their results represent an upper limit of the magnitude for LCC because they exactly enforce local charge neutrality. Our results for TMC are as given in Secs. I–IV (with the expected uncertainty within a few tens of percent). The STAR data are from Ref. [7]. Although these are rough estimates, one still can make a few observations: First, no single effect shows a pattern for all observables that are in accord with the data; second, different correlators appear to be dominated by different effects, in particular—both CME and TMC provide important contributions to $\langle\cos(\phi_1+\phi_2)\rangle_{++}$, while TMC seems necessary to explain the $\langle\cos(\phi_1-\phi_2)\rangle_{++}$ and LCC seems necessary to explain the observed value of $\langle\cos(\phi_1-\phi_2)\rangle_{+-}$. The situation for $\langle\cos(\phi_1+\phi_2)\rangle_{+-}$, on the other hand, is more complicated, and none of the effects discussed here seems to dominate. Clearly, additional measurements will be required to disentangle this situation.

(b) As far as comparisons of the data with models are concerned, such as the ones presented by the STAR Collaboration [7], one has to ensure that the model satisfies at least two criteria. First, the model has to conserve transverse momentum not only on average, but also event by event. Second, the model has to reproduce the measured magnitude of the elliptic flow v_2, as essentially, all trivial contributions to $\langle\cos(\phi_1+\phi_2)\rangle_{++,+-}$ scale with v_2. For example, UrQMD is known to underestimate the measured v_2 [35]. This may partly be the reason that it also underestimates the measured data for $\langle\cos(\phi_1+\phi_2)\rangle_{++}$ as reported in Ref. [7].

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