Single universal curve for cluster radioactivities and α decay

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One single line of universal (UNIV) curve for α decay and cluster radioactivities is obtained by plotting the sum of the decimal logarithm of the half-life and cluster preformation probability versus the decimal logarithm of the penetrability of external barrier. This fission-like theory is compared to the universal decay law (UDL) derived using α-like R-matrix theory. The experimental data on heavy cluster decay in three groups of even-even, even-odd, and odd-even parent nuclei are reproduced with comparable accuracy by both types of universal curves, UNIV and UDL.

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I. INTRODUCTION

The field of heavy-particle radioactivity (PACS number 23.70.+j) was developed during the last 30 years. Four years after the theoretical prediction [1], Rose and Jones [2] discovered experimentally ^{14}C radioactivity of ^{223}Ra, which happens to have the highest branching ratio with respect to α decay. Both fission-like and α-like models have been proposed that allow an explanation of the experimental data (see the chapters in Refs. [3–5] and references therein). Recently a new measurement of ^{14}C decay of ^{223}Ac was reported [6]. It was one of the possible candidates for future experiments mentioned in the systematics [7] showing that the strong shell effect ($N_d = 126$, $Z_d = 82$) present in order to lead to shorter half lives was not entirely exploited. One may add to the review of the main experimental data on ^{14}C, ^{20}O, ^{23}F, $^{22,24-26}$Ne, ^{28,30}Mg, and ^{32,34}S decay modes of heavy nuclei with $Z = 87 - 96$ [8]. As can be seen by comparing the table of this review paper with half-life predictions within the analytical superasymmetric model [9–12], the agreement is rather good. Nevertheless, there are many recently published theoretical approaches on this topic, e.g., Refs. [13–21].

Several simple effective relationships for half lives are available with parameters obtained by fitting the experimental data. Among them one should mention the universal (UNIV) curves [22–25] derived by extending a fission theory to larger-mass asymmetry. They are based on the quantum mechanical tunneling process relationship [26,27] of the disintegration constant λ, valid in both fission-like or α-like theories

$$\lambda = \ln 2/T = \nu S P_s, \tag{1}$$

where T is the half life and ν, S, P_s are three model-dependent quantities: ν is the frequency of assaults on the barrier per second, S is the preformation probability of the cluster at the nuclear surface (equal to the penetrability of the internal part of the barrier in a fission theory [22,23]), and P_s is the quantum penetrability of the external potential barrier.

The famous Viola-Seaborg formula [28], very frequently used for α decay, has been also extended to cluster radioactivity by Ren *et al.* [29].

A new universal decay law (UDL) for α-decay and cluster-decay modes was introduced [17,30] starting from α-like (extension to the heavier cluster of α-decay theory) R-matrix theory. Moreover, this UDL was presented in an interesting way, which makes it possible to represent on the same plot with a single straight line the logarithm of the half lives minus some quantity versus one of the two parameters (χ' or ρ') that depend on the atomic and mass numbers of the daughter and emitted particles as well as the Q value.

In the present paper we present a slightly modified way of plotting our universal curves in order to have both α decay and cluster radioactivities on the same line. We shall also compare the results of the UNIV formula with those obtained by UDL in three groups of cluster emitters: even-even, even-odd, and odd-even parent nuclei. Up to now no odd-odd cluster emitter has been experimentally determined.

II. NEW UNIVERSAL PLOT

We study a spontaneous process of charged particle decay in which a *parent* nucleus A, Z in its ground state is split into two fragments A_d, Z_d and A_e, Z_e,

$$AZ \rightarrow \;{}^{A_d}Z_d + {}^{A_e}Z_e, \tag{2}$$

in a way that conserves the hadron numbers $A = A_d + A_e$, $Z = Z_d + Z_e$. The heavy fragment A_d, Z_d is called the *daughter particle* and the light one the *emitted particle*. The partial decay half life T of the parent nucleus is related to the disintegration constant λ of the exponential decay law in time by Eq. (1).

Very frequently the penetrability is calculated by using the one-dimensional Wentzel–Kramers–Brillouin approximation:

$$P = \exp(-K); \quad K = \frac{2}{\hbar}\int_{R_a}^{R_b} \sqrt{2B(R)E(R)}\, dR, \tag{3}$$

where B is the nuclear inertia, approximated by the reduced mass $\mu = mA_1A_2/A$, m is the nucleon mass, E is the potential energy from which the Q value has been subtracted out, R_a

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and R_b are the classical turning points, and K is the action integral. The zero-point vibration energy is $E_v = h\nu/2$, in which h is the Plank constant. The outer potential barrier is of a Coulomb nature, $E(R) = e^2 Z_1 Z_2/R - Q$, with the electron charge given by its square $e^2 = 1.43998$ MeV·fm. The other numerical constants are given by $(1/2)h \ln 2 = 1.4333 \times 10^{-21}$ MeV·s and $2\sqrt{2m}/\hbar = 0.43921$ MeV$^{-1/2}$·fm$^{-1/2}$.

By using the decimal logarithm we have

$$\log_{10} T(s) = -\log_{10} P - \log_{10} S + [\log_{10}(\ln 2) - \log_{10} \nu]. \tag{4}$$

In order to derive the universal formula we assume that $\nu =$ constant and that S depends only on the mass number of the emitted particle A_e [23,26]. A microscopic calculation of the preformation probability [31] of many clusters from ^{8}Be to ^{46}Ar had shown indeed that it is dependent only upon the size of the cluster. The corresponding numerical values [23] have been obtained by fit with experimental data for α decay: $S_\alpha = 0.0143153$, $\nu = 10^{22.01}$ s^{-1}. We denote the additive constant for an even-even nucleus

$$c_{\mathrm{ee}} = [-\log_{10} \nu + \log_{10}(\ln 2)] = -22.16917 \tag{5}$$

and the decimal logarithm of the preformation factor

$$\log_{10} S = -0.598(A_e - 1). \tag{6}$$

In order to allow for a good fit when the experimental data are changed, we shall replace c_{ee} by $c_{\mathrm{ee}} + h_{\mathrm{ee}}$, $c_{\mathrm{eo}} = c_{\mathrm{ee}} + h_{\mathrm{eo}}$, $c_{\mathrm{oe}} = c_{\mathrm{ee}} + h_{\mathrm{oe}}$, and $c_{\mathrm{oo}} = c_{\mathrm{ee}} + h_{\mathrm{oo}}$, for even-even, even-odd, odd-even, and odd-odd nuclei, where h_{eo}, h_{oe}, h_{oo} are the mean values of the hindrance factors in these groups of nuclides. In a doubly logarithmic scale Eq. (4) represents a straight line.

Now, if we would like to plot a single universal curve for all charged-particle decay modes (α decay and cluster radioactivities) we can take the function $y = \log_{10} T + \log_{10} S = f(\log_{10} P_s)$ instead of the previous $y_1 = \log_{10} T = F(\log_{10} P_s)$:

$$\log_{10} T(s) + \log_{10} S = -\log_{10} P + c_{\mathrm{ee}} + h_{\mathrm{UNIV}}, \tag{7}$$

where $h_{\mathrm{UNIV}} = h_{\mathrm{ee}}$, $h_{\mathrm{UNIV}} = h_{\mathrm{eo}}$, $h_{\mathrm{UNIV}} = h_{\mathrm{oe}}$, and $h_{\mathrm{UNIV}} = h_{\mathrm{oo}}$ for even-even, even-odd, odd-even, and odd-odd parent nuclei. Such a plot is shown in Fig. 1. As seen in this figure the experimental data on α decay of even-even emitters and cluster radioactivities of even-even, even-odd, and odd-even nuclei could be described by a single universal curve by plotting $\log_{10} T(s) + \log_{10} S = f(-\log_{10} P_s)$ instead of the previous $\log_{10} T(s) = g(-\log_{10} P_s)$.

The penetrability of an external Coulomb barrier, having as the first turning point the separation distance at the touching configuration $R_a = R_t = R_d + R_e$ and the second one defined by $e^2 Z_d Z_e/R_b = Q$, may be found analytically as

$$-\log_{10} P_s = 0.22873(\mu_A Z_d Z_e R_b)^{1/2} \times [\arccos \sqrt{r} - \sqrt{r(1-r)}], \tag{8}$$

where $r = R_t/R_b$, $R_t = 1.2249(A_d^{1/3} + A_e^{1/3})$, and $R_b = 1.43998 Z_d Z_e/Q$. We use the liquid-drop-model [32] radius constant $r_0 = 1.2249$ fm and the mass tables [33] to calculate the released energy Q.

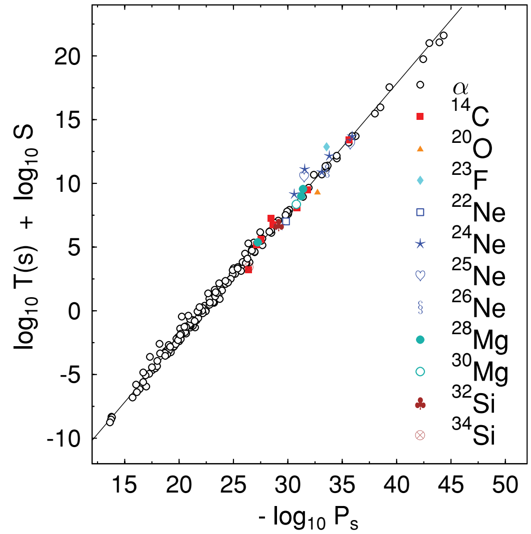

FIG. 1. (Color online) One single universal curve (straight line) for α decay and cluster radioactivities. The points correspond to 163 even-even α emitters and 27 cluster radioactivities data experimentally determined (8 ^{14}C, 6 ^{24}Ne, 3 ^{28}Mg, 2 ^{25}Ne, and only one for each of the ^{20}O, ^{23}F, ^{22}Ne, ^{26}Ne, ^{30}Mg, ^{32}Si, and ^{34}Si radioactivities). The overall standard deviation obtained by taking one value of $h_{\mathrm{UNIV}} = 0.040$ was $\sigma = 0.428$.

We can determine the hindrance factors by fitting the experimental data in order to obtain the minimum values of the standard root-mean-square deviation of $\log_{10} T$ values:

$$\sigma = \left\{ \sum_{i=1}^{n} [\log_{10}(T_i/T_{\mathrm{exp}})]^2/(n-1) \right\}^{1/2}. \tag{9}$$

The overall standard deviation obtained by taking one value of $h_{\mathrm{UNIV}} = 0.040$ for even-even α emitters and all types (even-even, even-odd, and odd-even) of cluster emitters was $\sigma = 0.428$.

For cluster radioactivities of 16 e-e, 6 e-o, and 5 o-e parents one obtains by fit $\sigma_{\mathrm{UNIV}} = 0.560$ when $c_{\mathrm{ee}} = -0.382$, $\sigma_{\mathrm{UNIV}} = 0.848$ when $c_{\mathrm{eo}} = 0.616$, and $\sigma_{\mathrm{UNIV}} = 0.665$ for $c_{\mathrm{oe}} = 0.579$ (see Table I). There is no successful experiment on cluster radioactivity of an odd-odd parent nucleus.

III. COMPARISON WITH UDL

Equation (12) from Ref. [30] may be written as

$$\log_{10} T(s) - b\rho(s) = a\chi + c_{\mathrm{UDL}}, \tag{10}$$

TABLE I. Standard deviations for universal curves of cluster radioactivities.

n	Parent	σ_{UNIV}	h_{UNIV}	σ_{UDL}	c_{UDL}
16	e-e	0.560	−0.382	0.574	−26.417
6	e-o	0.848	0.616	1.137	−25.470
5	o-e	0.665	0.579	0.426	−25.196

where

$$\chi = Z_e Z_d \sqrt{A_r/Q}, \quad A_r = A_e A_d / A, \tag{11}$$

$$\rho = \sqrt{A_r Z_e Z_d \left(A_e^{1/3} + A_d^{1/3}\right)}, \tag{12}$$

with $a = 0.3671$, $b = -0.3296$, $c_{\rm UDL} = -26.2681$ determined by fit with 11 selected experimental data of cluster decay modes of even-even parent nuclei. We use χ and ρ instead of the original χ' and ρ'. A standard error of $\sigma_{\rm UDL} = 0.608$ was obtained in Ref. [30]. When we replace the input data with a slightly different $T_{\rm exp}$ value for ^{14}C radioactivity of ^{226}Ra, ^{22}Ne emission from ^{230}Th, and ^{28}Mg radioactivity of ^{234}U taken from Ref. [8] and change $c_{\rm UDL}$ to $c_{\rm UDL} = -26.3530$ we obtain $\sigma_{\rm UDL} = 0.573$.

When we select only the 11 e-e parents that have been used in Ref. [30] but with experimental half lives from our table, we get $\sigma_{\rm UNIV} = 0.456$ with $c_{\rm ee} = -0.429$ and $\sigma_{\rm UDL} = 0.573$ with $c_{\rm UDL} = -26.3530$.

For 16 cluster radioactivities of e-e parent nuclei experimentally determined [8], we obtain $\sigma_{\rm UNIV} = 0.560$ when $h_{\rm ee} = -0.382$ and $\sigma_{\rm UDL} = 0.574$ with $c_{\rm UDL} = -26.417$ (see Fig. 2). For 6 e-o parent nuclei we obtain $\sigma_{\rm UNIV} = 0.848$ when $h_{\rm eo} = 0.616$. For 5 o-e parent nuclei we obtain $\sigma_{\rm UNIV} = 0.665$ when $h_{\rm oe} = 0.579$.

For 27 cluster radioactivities of e-e, e-o, and o-e parent nuclei experimentally determined [6,8] we got $\sigma_{\rm UNIV} = 0.792$ when $h_{\rm ee} = h_{\rm eo} = h_{\rm oe} = 0.019$ and $\sigma_{\rm UDL} = 0.882$ with $c_{\rm UDL} = -25.966$ (see Fig. 3). Despite the fact that the standard deviations of two theoretical approaches are comparable

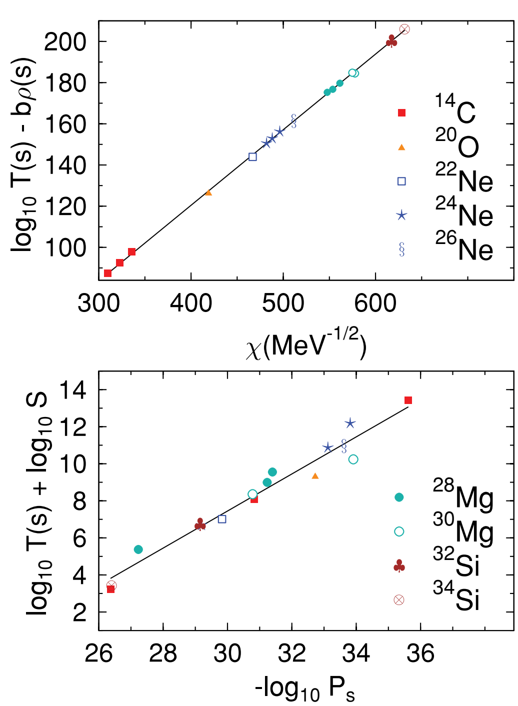

FIG. 2. (Color online) A single universal curve for cluster decay calculated with Eq. (12) of Ref. [30] (top) with standard deviation $\sigma_{\rm UDL} = 0.574$ obtained with $c_{\rm UDL} = -26.417$, and with our universal curve (bottom) $\sigma_{\rm UNIV} = 0.560$, $h_{\rm ee} = -0.382$, for the cluster-decay modes of e-e parent nuclei experimentally determined [8].

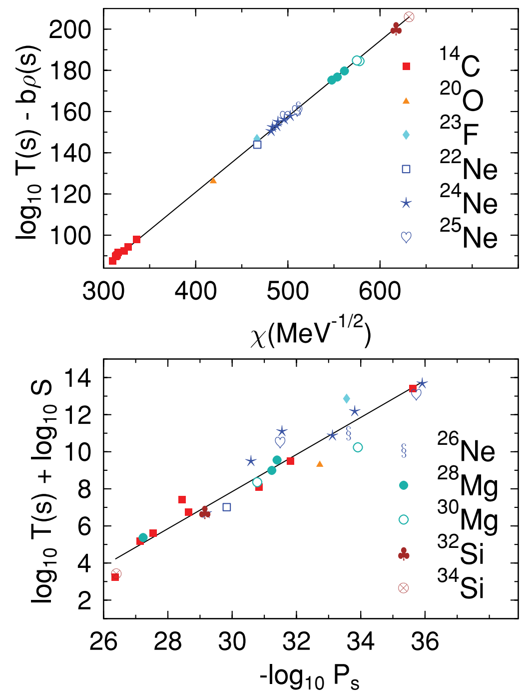

FIG. 3. (Color online) A single universal curve for cluster-decay calculated with Eq. (12) of Ref. [30] (top), with standard deviation $\sigma_{\rm UDL} = 0.882$ obtained with $c_{\rm UDL} = -25.966$, and with our universal curve (bottom) $\sigma_{\rm UNIV} = 0.792$, $h_{\rm UNIV} = 0.019$, for the cluster decay modes of e-e, e-o, and o-e parent nuclei experimentally determined [6,8].

(0.815 and 0.723, respectively), the points in the bottom panel seem to be more scattered than those in the top panel. This effect is due to the fact that for UNIV $\log_{10} T(s)$ represents an important part of the quantity $\log_{10} T(s) + \log_{10} S$, while for UDL $\log_{10} T(s)$ is just a small part of the total $\log_{10} T(s) - b\rho(s)$. As a numerical example we give the results for ^{26}Ne radioactivity of ^{234}U: for UNIV $\log_{10} T(s) = 25.88$, $\log_{10} S = -14.95$, $\log_{10} T(s) + \log_{10} S = 10.93$ at $-\log_{10} P_s = 33.61$, while for UDL $\log_{10} T(s) - b\rho(s) = 161.15$ at $\chi = 511.20$.

We can get smaller values of the standard deviations if we look for the optimum values of constants in three different groups of nuclei, as shown in Table I.

A remarkable fact concerning the agreement between the experimental data and theoretical results should be stressed. It

TABLE II. Large deviations of theoretical half lives, $T_{\rm UNIV}$ and $T_{\rm UDL}$, from the experimental ones, $T_{\rm exp}$, when $h_{\rm UNIV} = 0.019$ and $c_{\rm UDL} = -25.966$.

Parent	Emitted cluster	$\log_{10} T_{\rm UNIV}/T_{\rm exp}$	$\log_{10} T_{\rm UDL}/T_{\rm exp}$
^{223}Ra	^{14}C	−1.12	−1.69
^{228}Th	^{20}O	1.22	1.14
^{231}Pa	^{23}F	−1.45	−1.42
^{230}U	^{22}Ne	0.67	1.47
^{233}U	^{24}Ne	−1.69	−1.27
^{233}U	^{25}Ne	−1.15	−1.03
^{236}U	^{30}Mg	1.52	1.66

is connected with the observation [22,23,30] that the universal curves are not able to reproduce the shell effects present in the preformation probability. Few examples of large deviations of theoretical half lives, T_{UNIV} and T_{UDL}, from the experimental ones, T_{exp}, are given in Table II. There are similar deviations of both models for the examples given in Table II. A part of this deviation may also be due to the imprecision of measurement and in the case of ^{223}Ra to the fine structure of ^{14}C radioactivity [34].

In conclusion, one can obtain one single universal curve for α decay and cluster radioactivities by plotting $\log_{10} T + \log_{10} S$ versus $-\log_{10} P_s$. The experimental data on heavy cluster decay in three groups of even-even, even-odd, and odd-even parent nuclei are reproduced with comparable accuracy by both types of universal curves, UNIV and UDL, derived using fission-like and α-like theories.

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[1] A. Săndulescu, D. N. Poenaru, and W. Greiner, Sov. J. Part. Nucl. **11**, 528 (1980).
[2] H. J. Rose and G. A. Jones, Nature **307**, 245 (1984).
[3] D. N. Poenaru and W. Greiner, in *Nuclear Decay Modes* (Institute of Physics Publishing, Bristol, 1996), Chap. 6, pp. 275–336.
[4] D. N. Poenaru and W. Greiner, in *Handbook of Nuclear Properties* (Clarendon Press, Oxford, 1996), Chap. 5, pp. 131–182.
[5] D. N. Poenaru and W. Greiner, in *Clusters in Nuclei 1*, Lecture Notes in Physics, Vol. 818, edited by C. Beck (Springer, Berlin, 2010), Chap. 1, pp. 1–56.
[6] A. Guglielmetti *et al.*, J. Phys.: Conf. Ser. **111**, 012050 (2008).
[7] D. N. Poenaru, Y. Nagame, R. A. Gherghescu, and W. Greiner, Phys. Rev. C **65**, 054308 (2002). **66** 049902(E) (2002).
[8] R. Bonetti and A. Guglielmetti, Rom. Rep. Phys. **59**, 301 (2007).
[9] D. N. Poenaru, M. Ivaşcu, A. Săndulescu, and W. Greiner, J. Phys. G: Nucl. Phys. **10**, L183 (1984).
[10] D. N. Poenaru, M. Ivaşcu, A. Săndulescu, and W. Greiner, Phys. Rev. C **32**, 572 (1985).
[11] D. N. Poenaru, W. Greiner, K. Depta, M. Ivaşcu, D. Mazilu, and A. Săndulescu, Atomic Data Nucl. Data Tables **34**, 423 (1986).
[12] D. N. Poenaru, D. Schnabel, W. Greiner, D. Mazilu, and R. Gherghescu, Atomic Data Nucl. Data Tables **48**, 231 (1991).
[13] W. Greiner, Rom. Rep. Phys. **59**, 193 (2007).
[14] D. Ni and Z. Ren, Phys. Rev. C **82**, 024311 (2010).
[15] K. P. Santhosh, R. K. Biju, and S. Sahadevan, Nucl. Phys. A **838**, 38 (2010).
[16] B. B. Singh, S. K. Patra, and R. K. Gupta, Phys. Rev. C **82**, 014607 (2010).
[17] C. Qi, F. R. Xu, R. J. Liotta, and R. Wyss, Phys. Rev. Lett. **103**, 072501 (2009).
[18] D. S. Delion, Phys. Rev. C **80**, 024310 (2009).
[19] J. M. Dong, H. F. Zhang, J. Q. Li, and W. Scheid, Eur. Phys. J. A **41**, 197 (2009).
[20] S. Kumar, R. Rani, and R. Kumar, J. Phys. G: Nucl. Part. Phys. **36**, 105104 (2009).
[21] D. Ni, Z. Ren, T. Dong, and C. Xu, Phys. Rev. C **78**, 044310 (2008).
[22] D. N. Poenaru and W. Greiner, J. Phys. G: Nucl. Part. Phys. **17**, S443 (1991).
[23] D. N. Poenaru and W. Greiner, Phys. Scr. **44**, 427 (1991).
[24] D. N. Poenaru, I. H. Plonski, and W. Greiner, Phys. Rev. C **74**, 014312 (2006).
[25] D. N. Poenaru, I. H. Plonski, R. A. Gherghescu, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **32**, 1223 (2006).
[26] R. Blendowske and H. Walliser, Phys. Rev. Lett. **61**, 1930 (1988).
[27] R. Blendowske, T. Fliessbach, and H. Walliser, in *Nuclear Decay Modes* (IOP Publishing, Bristol, 1996), Chap. 7, pp. 337–349.
[28] V. E. Viola Jr. and G. T. Seaborg, J. Inorg. Nucl. Chem. **28**, 741 (1966).
[29] Z. Ren, C. Xu, and Z. Wang, Phys. Rev. C **70**, 034304 (2004).
[30] C. Qi, F. R. Xu, R. J. Liotta, R. Wyss, M. Y. Zhang, C. Asawatangtrakuldee, and D. Hu, Phys. Rev. C **80**, 044326 (2009).
[31] M. Iriondo, D. Jerrestam, and R. J. Liotta, Nucl. Phys. A **454**, 252 (1986).
[32] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **81**, 1 (1966).
[33] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[34] L. Brillard, A. G. Elayi, E. Hourani, M. Hussonnois, J. F. Le Du, L. H. Rosier, and L. Stab, C. R. Acad. Sci. Paris **309**, 1105 (1989).