

# Radiative decays of radially excited mesons $\pi^{0'}$ , $\rho^{0'}$ , and $\omega'$ in the Nambu–Jona-Lasinio model

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Radiative decays  $\pi^0(\pi^{0'}) \rightarrow \gamma + \gamma$ ,  $\pi^{0'} \rightarrow \rho^0(\omega) + \gamma$ ,  $\rho^{0'}(\omega') \rightarrow \pi^0 + \gamma$ , and  $\rho^{0'}(\omega') \rightarrow \pi^{0'} + \gamma$  are considered in the framework of the  $SU(2) \times SU(2)$  Nambu–Jona-Lasinio (NJL) model. Radially excited mesons are described with the help of a simple polynomial form factor. In spite of mixing of the ground and excited meson states in this model, the decay widths of  $\pi^0 \rightarrow \gamma + \gamma$  and  $\rho^0(\omega) \rightarrow \pi^0 + \gamma$  are found to be in good agreement with experimental data, as in the standard NJL model. Our predictions for decay widths of radially excited mesons can be verified in future experiments.

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*Introduction.* In Ref. [1], a version of the Nambu–Jona-Lasinio (NJL) model with a polynomial form factor was developed. This model allows one to describe both the ground and the first radially excited meson states. All the low-energy chiral theorems are valid in it. Further applications of the model gave a description of the mass spectrum of the scalar, pseudoscalar, and vector meson nonets and of the corresponding first radially excited states that is in satisfactory agreement with the experimental data. The main strong decays of these mesons were also described [2–6]. In this Brief Report, we describe radiative decays with participation of radially excited mesons in the framework of the same model. The amplitudes of processes considered here can further be used for predictions of production rates of radially excited mesons at electron-positron colliders in processes like  $l^+l^- \rightarrow \gamma^* \rightarrow PV$ . In particular, experimental studies of all these processes can be performed at high-luminosity modern electron-positron accelerators with center-of-mass energy of about several GeV [e.g., BEPC-II (Beijing), VEPP-2000 (Novosibirsk), DAΦNE (Frascati)].

The mass spectra and properties of light mesons, including their radially excited states, were also considered in the literature with other approaches, such as nonrelativistic [7,8] relativized quark models [9,10], the Dyson-Schwinger and Bethe-Salpeter equations [11–14], the Tamm-Dancoff method [15,16], the QCD sum rules [17,18], lattice QCD calculations [19], anti-de Sitter-space (AdS)/QCD models [20,21], etc. Theoretical studies of radiative decays involve detailed treatment of meson structure. This potentially leads to differences in predictions for decay rates, so that future experiments can discriminate among specific models.

*The Lagrangian.* Recall the main elements of the version of the NJL model given in Refs. [1,2]. The  $SU(2) \times SU(2)$  NJL model Lagrangian with a form factor in four-fermion interactions can be written in the form [3]

$$\mathcal{L}(\bar{q}, q) = \int d^4x \bar{q}(x) (i \partial_\mu \gamma^\mu - m^0 + e Q A_\mu \gamma^\mu) q(x) + \int d^4x \sum_{i=1}^N \sum_{a=1}^3 \left[ \frac{G_1}{2} (j_{\sigma,i}(x) j_{\sigma,i}(x) + j_{\pi,i}^a(x) j_{\pi,i}^a(x)) \right]$$

$$\begin{aligned} & - \frac{G_2}{2} (j_{\rho,i}^a(x) j_{\rho,i}^a(x) + j_{a_1,i}^a(x) j_{a_1,i}^a(x)) \Big], \\ j_{\sigma,i}(x) &= \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{\sigma,i}(x; x_1, x_2) q(x_2), \\ j_{\pi,i}^a(x) &= \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{\pi,i}^a(x; x_1, x_2) q(x_2), \\ j_{\rho,i}^{a,\mu}(x) &= \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{\rho,i}^{a,\mu}(x; x_1, x_2) q(x_2), \\ j_{a_1,i}^{a,\mu}(x) &= \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{a_1,i}^{a,\mu}(x; x_1, x_2) q(x_2), \end{aligned} \quad (1)$$

where  $q(\bar{q})$  is the doublet of light quarks,  $m^0$  is the current quark mass,  $e$  is the electron charge,  $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3})$  is the quark charge matrix,  $A^\mu$  is the electromagnetic field,  $G_1$  and  $G_2$  are the four-quark interaction constants in the (pseudo)scalar and (axial) vector cases, respectively.

Here we consider only the ground ( $i = 1$ ) and the first excited meson states ( $i = 2$ ). The form factors (in the momentum space) are chosen as a simple polynomial type [1–3]:

$$\begin{aligned} F_{\sigma,2} &= f_\pi(k^\perp{}^2), & F_{\pi,2}^a &= i\gamma_5 \tau^a f_\pi(k^\perp{}^2), \\ F_{\rho,2}^{a,\mu} &= \gamma^\mu \tau^a f_\rho(k^\perp{}^2), & F_{a_1,2}^{a,\mu} &= \gamma_5 \gamma^\mu \tau^a f_\rho(k^\perp{}^2), \\ f_{\pi,\rho}(k^\perp{}^2) &= c_{\pi,\rho} f(k^\perp{}^2), & k^\perp &= k - \frac{kp}{p^2} p, \\ f(k^\perp{}^2) &= (1 - d|k^\perp{}^2|) \Theta(\Lambda^2 - |k^\perp{}^2|), \end{aligned} \quad (2)$$

where  $k$  and  $p$  are the quark and meson four-momenta, respectively. In the rest frame of the external meson  $k^\perp = \{0, \vec{k}\}$  and  $k^\perp{}^2 = -\vec{k}^2$ . The parameter  $c_{\pi,\rho}$  defines only the meson masses and can be omitted in the description of meson interactions. The cutoff parameter is taken to be  $\Lambda = 1.03$  GeV.

In our model, the slope parameter  $d$  is chosen from the condition so that the excited scalar meson states do not influence the value of the quark condensate, i.e., do not change the constituent quark mass (see Refs. [1,2,22]). This condition

can be written in the form

$$I_1^f = -i \frac{N_c}{(2\pi)^4} \int \frac{d^4 k f(k^{\perp 2})}{m^2 - k^2} = 0, \quad (3)$$

where  $N_c = 3$  is the number of colors. This gives  $d \approx 1.78 \text{ GeV}^{-2}$ . This means that the quark tadpole connected with excited scalar mesons vanishes. Therefore, the quark condensate acquires only a contribution from the quark tadpole connected with the ground scalar state [23].

After bosonization of the  $SU(2) \times SU(2)$  chiral-symmetric four-quark Lagrangian and renormalization of the meson fields, we get the following form of the quark-meson interaction (we show only the terms relevant to the present study):

$$\begin{aligned} \mathcal{L}^{\text{int}} = & \bar{q}(k) \{ e Q \gamma_\mu A^\mu(p) + \tau^3 \gamma_5 [g_{\pi_1} \pi_1(p) \\ & + g_{\pi_2} \pi_2(p) f(k^{\perp 2})] + \frac{1}{2} \gamma_\mu [g_{\rho_1} (\rho_1^\mu(p) \tau^3 + \omega_1^\mu(p)) \\ & + g_{\rho_2} f(k^{\perp 2}) (\rho_2^\mu(p) \tau^3 + \omega_2^\mu(p))] \} q(k') + \dots \end{aligned} \quad (4)$$

Here  $\pi_i$ ,  $\rho_i$ , and  $\omega_i$  are nonphysical pseudoscalar and vector meson fields. The coupling constants have the form [1,2]

$$\begin{aligned} g_{\pi_1} &= \left[ 4I_2 \left( 1 - \frac{6m_u^2}{M_{a_1}^2} \right) \right]^{-1/2}, \quad g_{\pi_2} = [4I_2^{f^2}]^{-1/2}, \\ g_{\rho_1} &= \left[ \frac{2}{3} I_2 \right]^{-1/2}, \quad g_{\rho_2} = \left[ \frac{2}{3} I_2^{f^2} \right]^{-1/2}, \end{aligned} \quad (5)$$

where

$$I_2 = -i N_c \int \frac{d^4 k}{(2\pi)^4} \frac{\Theta(\Lambda^2 - \vec{k}^2)}{(m_u^2 - k^2)^2}, \quad (6)$$

$$I_m^{f^n} = -i N_c \int \frac{d^4 k}{(2\pi)^4} \frac{(f(k^{\perp 2}))^n}{(m_u^2 - k^2)^m}, \quad n, m = 1, 2.$$

Note that in  $g_{\pi_1}$  we take into account  $\pi$ - $a_1$  transitions [23], where  $M_{a_1} = 1.23 \text{ GeV}$  is the  $a_1$  meson mass. In the constant  $g_{\pi_2}$  these transitions can be neglected; see Refs. [1,2].

The free part of the Lagrangian for the pion fields contains nondiagonal kinetic terms:

$$\mathcal{L}_\pi^{\text{free}} = \frac{p^2}{2} (\pi_1^2 + 2\Gamma_\pi \pi_1 \pi_2 + \pi_2^2) - \frac{M_{\pi_1}^2}{2} \pi_1^2 - \frac{M_{\pi_2}^2}{2} \pi_2^2.$$

The mass terms have a diagonal form because of condition (3), and

$$\begin{aligned} \Gamma_\pi &= \frac{I_2^f}{\sqrt{I_2 I_2^{f^2}}}, \quad M_{\pi_1}^2 = g_{\pi_1}^2 \left[ \frac{1}{G_1} - 8I_1 \right], \\ M_{\pi_2}^2 &= g_{\pi_2}^2 \left[ \frac{1}{G_1 c_\pi} - 8I_1^{f^2} \right], \end{aligned}$$

where  $G_1 = 3.47 \text{ GeV}^{-2}$  is the interaction constant of scalar and pseudoscalar quark currents in the initial NJL Lagrangian (1).

The following transformation allows us to get a diagonal form of the free meson Lagrangian [2,3]:

$$\begin{aligned} \pi^0 &= \pi_1 \cos(\alpha - \alpha_0) - \pi_2 \cos(\alpha + \alpha_0), \\ \pi^{0'} &= \pi_1 \sin(\alpha - \alpha_0) - \pi_2 \sin(\alpha + \alpha_0), \end{aligned} \quad (7)$$

where

$$\begin{aligned} \sin \alpha_0 &= \sqrt{\frac{1 + \Gamma_\pi}{2}}, \\ \tan(2\alpha - \pi) &= \sqrt{\frac{1}{\Gamma_\pi^2} - 1} \left[ \frac{M_{\pi_1}^2 - M_{\pi_2}^2}{M_{\pi_1}^2 + M_{\pi_2}^2} \right]. \end{aligned} \quad (8)$$

We obtained the values  $\alpha_0 = 59.06^\circ$  and  $\alpha = 59.38^\circ$  for the angles. The free pion Lagrangian takes the standard form

$$\mathcal{L}_\pi^{\text{free}} = \frac{p^2}{2} (\pi^{0^2} + \pi^{0'^2}) - \frac{M_\pi^2}{2} \pi^{0^2} - \frac{M_{\pi'}^2}{2} \pi^{0'^2}. \quad (9)$$

For  $c_\pi = 1.36$  the values  $M_\pi \approx 134.8 \text{ MeV}$  and  $M_{\pi'} \approx 1308 \text{ MeV}$  were obtained, in agreement with the experimental values  $134.9766 \pm 0.0006 \text{ MeV}$  and  $1300 \pm 100 \text{ MeV}$ , respectively [24].

As a result, the interaction Lagrangian for physical pion fields with quarks takes the form

$$\begin{aligned} \mathcal{L}_\pi^{\text{int}} = & \bar{q}(k) \tau^3 \gamma_5 \\ & \times \left\{ \left[ g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{\perp 2}) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] \pi^0(p) \right. \\ & \left. - \left[ g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{\perp 2}) \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] \pi^{0'}(p) \right\} \\ & \times q(k'). \end{aligned} \quad (10)$$

An analogous procedure for the vector mesons leads to [2,3]

$$\begin{aligned} \mathcal{L}_{\rho,\omega}^{\text{int}} = & \bar{q}(k) \frac{\gamma_\mu}{2} \left\{ \left[ g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^{\perp 2}) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] \right. \\ & \times (\tau^3 \rho_\mu^0(p) + \omega_\mu(p)) - \left[ g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right. \\ & \left. \left. + g_{\rho_2} f(k^{\perp 2}) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right] (\tau^3 \rho_\mu^{0'}(p) + \omega'_\mu(p)) \right\} q(k'), \end{aligned}$$

where the angles  $\beta_0 = 61.53^\circ$  and  $\beta = 76.78^\circ$  are defined analogously to Eq. (8), using

$$M_{\rho_1}^2 = \frac{3}{8G_2 I_2}, \quad M_{\rho_2}^2 = \frac{3}{8c_\rho G_2 I_2^{f^2}}. \quad (11)$$

For  $c_\rho = 1.15$  and  $G_2 = 13.1 \text{ GeV}^{-2}$  we get  $M_\rho = M_\omega \approx 783 \text{ MeV}$  and  $M_{\rho'} = M_{\omega'} = 1450 \text{ MeV}$ . The corresponding experimental values are  $M_\rho = 775.49 \pm 0.34 \text{ MeV}$ ,  $M_\omega = 782.65 \pm 0.12 \text{ MeV}$ ,  $M_{\rho'} = 1465 \pm 25 \text{ MeV}$ , and  $M_{\omega'} = 1400\text{--}1450 \text{ MeV}$  [24].

*Results for radiative decays.* Let us consider now the standard two-photon decay described by the triangle quark diagram of the anomaly type. The decay amplitude has the

TABLE I. Ground-state meson radiative decay widths.

Decay	$\pi^0 \rightarrow \gamma\gamma$	$\rho^0 \rightarrow \pi^0\gamma$	$\omega \rightarrow \pi^0\gamma$
Theory	7.7 eV	77 keV	710 keV
Experiment	$7.5 \pm 1.1$ eV	$88 \pm 12$ keV	$700 \pm 30$ keV

form

$$\begin{aligned}
A_{\mu\nu}^{\pi^0 \rightarrow \gamma\gamma} &= 8m_u \varepsilon_{\mu\nu\gamma\sigma} q_1^\gamma q_2^\sigma e^2 (Q_u^2 - Q_d^2) \frac{(-i)N_C}{(2\pi)^4} \\
&\times \int d^4k \left\{ g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{\perp 2}) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right\} \\
&\times \frac{1}{(k^2 - m_u^2 + i0)((k - q_1)^2 - m_u^2 + i0)} \\
&\times \frac{1}{((k + q_2)^2 - m_u^2 + i0)},
\end{aligned}$$

where  $Q_{u,d}$  are the  $u$  and  $d$  quark charges and  $q_{1,2}$  are the photon momenta. The amplitude contains two types of one-loop integrals: with and without form factor in the quark-pion vertex. The expression for the  $\pi^{0'} \rightarrow \gamma\gamma$  amplitude only differs from the above expression by the coupling constant of pion and quarks [see Eq. (10)] and by the mass of the decaying particle. In the calculation we take into account only the real part of the loop integrals. This ansatz corresponds to the naïve confinement definition [25], which was used in some of our recent works [26–28].

Consider now two-particle decay modes of pseudoscalar and vector mesons with a single photon. The amplitude of  $\rho^0 \rightarrow \pi^0\gamma$  takes the form

$$\begin{aligned}
A_{\mu\nu}^{\rho^0 \rightarrow \pi^0\gamma} &= 4m_u \varepsilon_{\mu\nu\gamma\sigma} q_1^\gamma q_2^\sigma 2e(Q_u + Q_d) \frac{(-i)N_C}{(2\pi)^4} \\
&\times \int \frac{d^4k}{(k^2 - m_u^2 + i0)((k - q_1)^2 - m_u^2 + i0)} \\
&\times \frac{1}{((k + q_2)^2 - m_u^2 + i0)} \\
&\times \left\{ \frac{g_{\rho_1}}{2} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \frac{g_{\rho_2}}{2} f(k^{\perp 2}) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right\} \\
&\times \left( g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{\perp 2}) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right),
\end{aligned}$$

where  $q_1$  and  $q_2$  are the vector meson and photon momenta, respectively.

First, we recalculate the width of radiative decays of the ground meson states; see Table I. The results are in satisfactory agreement with the experimental data [24]. Note that the similar strong decays of radially excited mesons  $\rho' \rightarrow \omega\pi$  and  $\omega' \rightarrow \rho\pi$ , considered within the same nonlocal model, were found previously in Ref. [3] to be also in reasonable

TABLE II. Radiative decay widths of  $\pi^{0'}$  and  $\rho^{0'}$ .

Decay	$\pi^{0'} \rightarrow \gamma\gamma$	$\pi^{0'} \rightarrow \rho^0\gamma$	$\rho^{0'} \rightarrow \pi^0\gamma$	$\rho^{0'} \rightarrow \pi^{0'}\gamma$
Theory	3.2 keV	1.8 keV	450 keV	24 keV

TABLE III. Decay widths of radiative processes with  $\omega$  and  $\omega'$  mesons.

Decay	$\pi^{0'} \rightarrow \omega\gamma$	$\omega' \rightarrow \pi^0\gamma$	$\omega' \rightarrow \pi^{0'}\gamma$
Theory	17 keV	3.7 MeV	93 keV

agreement with observations [24,29]:

$$\begin{aligned}
\Gamma_{\rho' \rightarrow \omega\pi}^{\text{theor.}} &\approx 75 \text{ MeV}, & \Gamma_{\rho' \rightarrow \omega\pi}^{\text{exper.}} &= 65.1 \pm 12.6 \text{ MeV}, \\
\Gamma_{\omega' \rightarrow \rho\pi}^{\text{theor.}} &\approx 225 \text{ MeV}, & \Gamma_{\omega' \rightarrow \rho\pi}^{\text{exper.}} &= 174 \pm 60 \text{ MeV}.
\end{aligned} \tag{12}$$

We recall that the decay widths of the ground states was obtained within the local NJL model in 1986 [23], in good agreement with the experimental data. In the considered nonlocal version of the NJL model the radially excited meson states are mixed with the ground states. However, this mixing does not lead to the distortion of the description of the ground meson state interactions with each other, obtained in the local model.

Tables II and III contain our results of theoretical calculations of radiative decay widths of the radially excited states of  $\pi^0$ ,  $\rho^0$ , and  $\omega$  mesons calculated in the framework of the NJL model. For the calculations of the phase space we used the present experimental values for the excited meson masses. The widths of these decay channels are not yet measured experimentally. So we made predictions that could be relevant to future experiments. Note also that the decay widths  $\omega'(\rho^{0'}) \rightarrow \pi^{0'}\gamma$  are very sensitive to the meson masses, which have, at the present time, large experimental uncertainties [24].

Table IV shows a considerable discrepancy between our results and the ones obtained earlier within a nonrelativistic quark model [8]. However, we have several arguments in favor of our approach. First, within our model we obtained a theoretical description of the ground meson state decays in good agreement with experimental data. On the other hand, the strong decay widths of  $\omega' \rightarrow \rho^0\pi^0$  and  $\rho^{0'} \rightarrow \omega\pi^0$  computed within the same model in Ref. [3] also show a satisfactory agreement with the experiment; see Eq. (12). It is worth noting that all these decays in our model have the same mechanism: they are described by triangular quark loop diagrams of the anomaly type. So we hope that the predictions for radially excited meson decay widths will also be in a reasonable agreement with the future experimental data. Let us emphasize that in the present study to describe the radiative decays we did not use any additional parameters with respect to earlier works [2,3]. Moreover, the  ${}^3P_0$  model applied in Ref. [8] has

TABLE IV. Comparison with results of Ref. [8].

Decay	$\omega \rightarrow \pi^0\gamma$	$\rho^{0'} \rightarrow \pi^0\gamma$	$\rho^{0'} \rightarrow \pi^{0'}\gamma$	$\omega' \rightarrow \pi^0\gamma$	$\omega' \rightarrow \pi^{0'}\gamma$
This work	710 keV	450 MeV	24 keV	3.7 MeV	93 keV
Ref. [8]	520 keV	61 keV	5.9 keV	510 keV	29 keV

serious problems in the description of excited meson states, as discussed in that paper.

With our model, using the interaction amplitudes obtained here, one can compute the production probabilities of the ground and excited meson states at lepton colliders. There, processes like  $e^+e^-\gamma^* \rightarrow PV$  can be studied, where  $P$  and  $V$  are the ground and excited pseudoscalar and vector meson states, respectively. In these processes, it is necessary to take into account the transitions  $\gamma^* \rightarrow$

$V(V')$ . The Primakoff effect and the two-photon production mechanism of radially excited mesons can be considered as well. We plan to perform similar calculations to describe radiative decay channels of radially excited states of  $\eta$ ,  $\eta'$ , and  $\phi$  mesons in the framework of the  $U(3) \times U(3)$  NJL model.

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