Spectroscopic information about a hypothetical tetrahedral configuration in ^{156}Gd

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A detailed γ-ray spectroscopy of the lowest two negative-parity bands in ^{156}Gd has been performed in the framework of the TetraNuc collaboration. Relative γ-ray intensities of all transitions connecting these bands as well as the ones to the ground-state band are presented. Angular distribution analysis has been performed to determine the nature and the mixing ratio of one of the newly established transitions linking the two excited bands.

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Theoretical studies based on the nuclear mean-field approach and group theory considerations suggest [1,2] that some atomic nuclei may exhibit tetrahedral and/or octahedral symmetries. To the lowest order, tetrahedral symmetry is realized through octupole deformation $Y_{3\pm2}$ of the nuclear surface. Dudek *et al.* [3] have determined magic numbers for which tetrahedral deformation should be the easiest to observe leading to tetrahedral proton and neutron "magic" numbers $Z_t/N_t = 32, 40, 56, 64, 70, 90$, and 112, with extra gaps at $N_t = 136$ and 142. The authors of Refs. [1–3] have furthermore demonstrated that nuclei *with an exact tetrahedral symmetry* have all multipole moments $Q_{\lambda<7,\mu} = 0$ except for Q_{32}—thus, in particular, the corresponding quadrupole moments Q_2 vanish.

Because of the nonspherical shape, the presence of rotational bands is still expected, but the intraband $E2$ transitions are predicted to vanish if only a single deformation point is to contribute to the quadrupole moment. The latter idea turned out to be oversimplified, as recent works based on the symmetry-oriented formulation of the nuclear collective model indicate [4–6] (see also the following). However, from theoretical [1] and experimental points of view, ^{156}Gd seems to be an excellent candidate to test the tetrahedral symmetry. Indeed, in this nucleus a pronounced tetrahedral minimum is calculated and the corresponding candidate band, with odd spins and negative parity, has been found and, moreover, with unobserved $\Delta I = 2$ transitions at low spins.

The nucleus ^{156}Gd was produced by fusion-evaporation reaction ^{154}Sm(α, 2n) and studied [7] by using the JUROGAM γ-ray spectrometer of the Department of Physics of the University of Jyväskylä. The JUROGAM array consists of 43 anti-Compton suppressed HP-Ge detectors distributed over six rings around the beam axis. The global photopeak efficiency of the array is around 4.3% for 1.3 MeV γ rays. The bombarding energy (27 MeV) enables us to populate mainly ^{156}Gd at low and medium spin and to minimize the contaminations coming from other channels, mainly ^{155}Gd, below 8%. Self-supporting ^{154}Sm targets, 99.2% enriched, of thickness 2 mg/cm^2 were used. Correlations between the detectors were determined using the total data readout (TDR) data acquisition system [8]. The system is triggerless, all the signals being stamped by a global 100 MHz clock, and is therefore designed to reduce dead time to the minimum. The collected data were reconstructed and analyzed offline. For the sorting, a 40 ns time window has been set to build coincidences and, after Compton suppression, a total of 228×10^6 γ-γ-γ coincidence events have been used for the present analysis.

A partial level scheme, including new transitions obtained in this work, is given in Fig. 1. Transitions linking the odd-spin negative-parity band to the ground-state band in ^{156}Gd are well established; cf. Refs. [9,10]. Before our measurements, the states $I^\pi = 5^-, 7^-$, and 9^- were attributed to the odd-spin negative-parity band using only their excitation energies. The new transitions connecting even- and odd-spin

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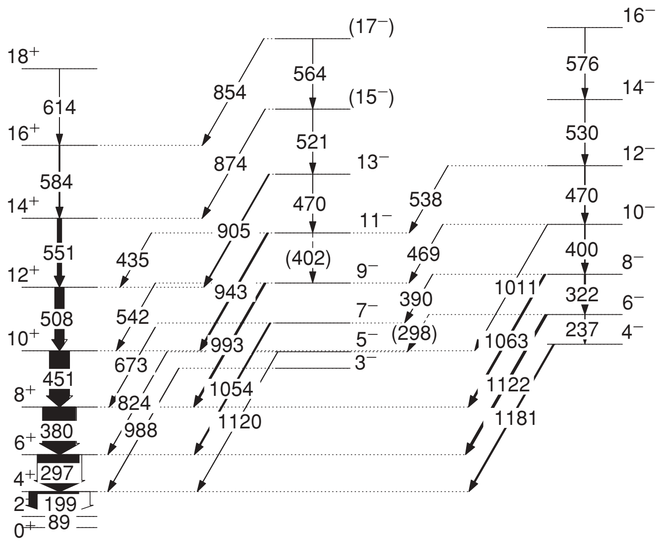

FIG. 1. Partial level scheme of ^{156}Gd, established in this work, showing the ground-state band, odd- and even-spin negative-parity bands, as well as the linking transitions.

negative-parity bands, 297.7 keV ($6^- \to 5^-$), 389.6 keV ($8^- \to 7^-$), 469.4 keV ($10^- \to 9^-$), and 538.0 keV ($12^- \to 11^-$) (cf. also Ref. [7]), suggest that the two sequences can be considered as rotational-partner bands. Intensities of all the transitions of interest have been established; see Table I. They are expressed relative to the most intense transition observed in our data set: 199.2 keV ($4^+ \to 2^+$). In this Brief Report, we also report the results of the angular-distribution analysis that allows us to deduce the multipolarity of one of the new transitions (389.6 keV). The other ones have too low intensity or are too contaminated to allow for conclusive results.

In order to obtain the multipolarity of the γ transitions, the data were sorted to deduce the angular distributions. For this purpose, six nonsymmetrized matrices have been built, placing on the first axis the γ-ray energies detected at any angle and, on the second axis, only those detected at a particular θ angle with respect to the beam direction. This allowed us to put gates on the first axis in order to clean up the spectra, which is necessary in our study (because of multiple transitions close in energy), and thus to obtain the evolution, as a function of θ, of the intensity of the γ-ray transitions in coincidence. The intensities have to be normalized to take into account the efficiency that depends of the θ angle, on the γ-ray energy, and on the sorting procedure (event reconstruction from time stamp data and gating on matrices). Because γ-ray transitions from the sources are emitted isotropically, such normalization factors could be easily obtained by producing, with the same sorting method, the equivalent matrices for the ^{152}Eu (source) run. Given a γ-ray energy, these factors were calculated using the closest ^{152}Eu transitions. For all angular distributions, 22° has been chosen arbitrarily to be the angle for which the γ-ray intensities are set to 100%.

The radiation intensity can be written down using the standard expression

$$I(\theta) = 1 + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta) \tag{1}$$

TABLE I. Relative γ-ray intensities of the transitions shown in Fig. 1. The $4^+ \to 2^+$ transition of the ground-state band has been chosen as a reference (100%). On the left-hand side, intraband transitions are listed for the ground-state (g.s.$^+$), even-spin, and odd-spin negative-parity bands, respectively. On the right-hand side, interband transitions are given.

I_i	I_f	E_γ (keV)	I_γ (%)	I_i	I_f	E_γ (keV)	I_γ (%)
		g.s.$^+$				odd spins $\pi^- \to$ g.s.$^+$	
18	16	614.0 (3)	0.27 (3)	17	16	854.9 (3)	0.07 (1)
16	14	583.6 (2)	1.96 (9)	15	14	874.4 (3)	0.54 (6)
14	12	551.3 (1)	8.7 (4)	13	12	904.8 (2)	2.0 (2)
12	10	508.1 (1)	18.6 (9)	11	10	943.8 (2)	3.2 (2)
10	8	451.0 (1)	40.0 (2)	11	12	436.4 (3)	0.20 (2)
8	6	380.3 (1)	66 (3.4)	9	8	993.1 (2)	3.6 (3)
6	4	296.4 (1)	84 (4.8)	9	10	543.0 (3)	0.43 (4)
4	2	199.2 (1)	100	7	6	1053.3 (2)	2.7 (3)
		even spins π^-		7	8	672.9 (3)	0.64 (6)
16	14	576.0 (3)	0.13 (1)	5	4	1119.9 (3)	0.53 (5)
14	12	530.1 (2)	1.4 (2)	5	6	823.4 (3)	0.59 (6)
12	10	470.3 (2)	1.4 (2)	3	4	987.8 (3)	0.73 (8)
10	8	399.8 (2)	1.5 (2)			even $\to$ odd spins π^-	
8	6	321.7 (2)	2.5 (3)	12	11	538.0 (3)	0.11 (1)
6	4	237.0 (3)	0.58 (7)	10	9	469.4 (3)	0.20 (2)
		odd spins π^-		8	7	389.6 (3)	0.51 (5)
17	15	563.6 (3)	0.05 (1)	6	5	297.7 (3)	–
15	13	521.3 (3)	0.13(2)			even spins $\pi^- \to$ g.s.$^+$	
13	11	468.3 (3)	0.32 (5)	10	10	1013.4 (3)	0.39 (4)
11	9	403.2 (3)	< 0.02	8	8	1062.6 (2)	3.6 (3)
9	7	320.0 (3)	–	6	6	1120.8 (2)	4.6 (5)
7	5	229.9 (3)	–	4	4	1080.9 (2)	2.7 (3)

for the predominantly observed dipole and quadrupole transitions. In this expression, $P_k(\cos\theta)$ denotes the Legendre polynomials. The coefficients A_2 and A_4 are determined by a least-squares fit of the experimental points and the resulting curve compared to theoretical points. The theoretical angular distributions of γ-ray transitions of mixed multipole character, $L + L'$ (generally $L' = L + 1$), emitted by an aligned initial state of spin J_i to a final state of spin J_f, can be expressed as (Krane *et al.* [11])

$$W(\theta) = 1 + \sum_{\lambda} \rho_\lambda(J_i) B_\lambda(J_f J_i L \delta) P_\lambda(\cos\theta), \tag{2}$$

where B_λ are geometrical coefficients which can be expressed in terms of the well-known F coefficients, δ being the multipole mixing ratio. The index λ takes only even values and is in practice restricted to 4. The statistical tensors $\rho_\lambda(J_i)$, which describe the initial-state orientation, are defined as

$$\sqrt{2J_i + 1} \sum_{m=-J_i}^{J_i} (-1)^{J_i - m} \langle J_i m J_i - m | \lambda 0 \rangle P(m), \tag{3}$$

where $P(m)$ gives the magnetic m-substate population distribution. As proposed by Konijn *et al.* [10], an oblate alignment of nuclei is assumed at the instant of formation. It implies a Gaussian shape for the distribution whose width σ is a measure

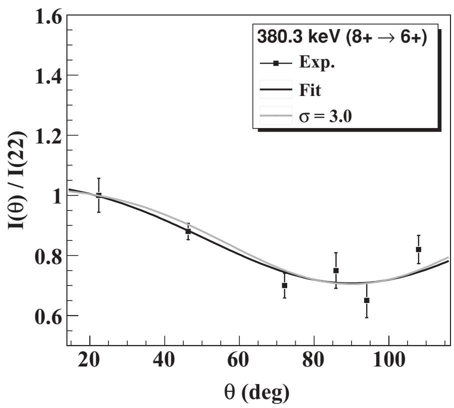

FIG. 2. Angular distributions for the 380.3 keV ($8^+ \to 6^+$, pure $\Delta I = 2$) transition. Experimental points are fitted (black curve) and well reproduced by the theoretical curve (gray) using $\sigma = 3.0$.

of the dealignment:

$$P(m) = \frac{\exp(-m^2/2\sigma^2)}{\sum_{m=-J_i}^{+J_i} \exp(-m^2/2\sigma^2)}. \tag{4}$$

The analysis procedure has been controlled using well-established pure $\Delta I = 1$ and $\Delta I = 2$ γ-ray transitions belonging to the ^{156}Gd level scheme. Known transitions are also required to determine, for a given entry spin, the value of σ before application to a new γ-ray transition. The results are displayed in Figs. 2 and 3 for the 380.3 keV ($8^+ \to 6^+$, $\Delta I = 2$) and the 943.8 keV ($11^- \to 10^+, \Delta I = 1$) transitions; the experimental points are clearly in agreement with the expected trends for such transitions. The curve (black solid line) obtained from the fit using Eq. (1) is satisfactorily matching the theoretical angular distribution (gray line) with $\sigma = 3$ and $\sigma = 3.5$, respectively. It has been checked, using several known transitions, that similar σ values are found for a given entry spin in the level scheme; this helps to establish a systematic error on this parameter. From such a procedure we

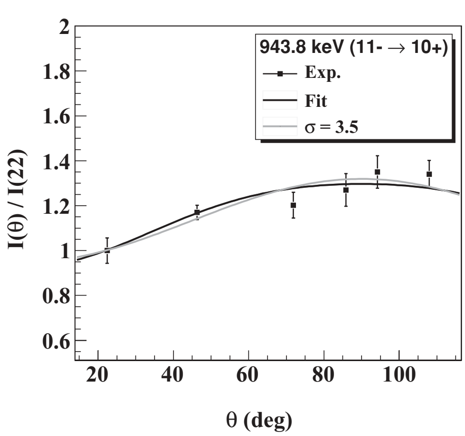

FIG. 3. Angular distributions for the 943.8 keV ($11^- \to 10^+$, pure $\Delta I = 1$) transition. Experimental points are fitted (black curve) and well reproduced by the theoretical curve (gray) using $\sigma = 3.5$.

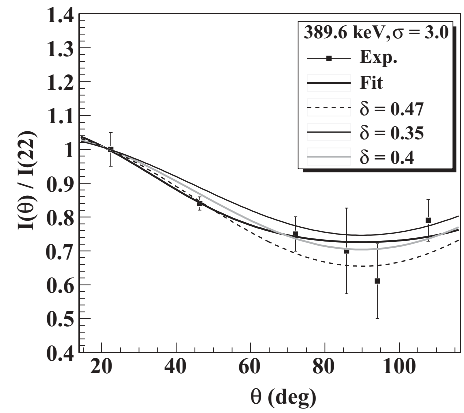

FIG. 4. Angular distributions for the 389.6 keV ($8^- \to 7^-$) transition. The fit of the experimental points is shown by the black curve. With $\sigma = 3.0$ the data are well reproduced by the theoretical curve using $\delta = 0.4$ (gray). Dot-dashed ($\delta = 0.35$) and dashed ($\delta = 0.47$) curves illustrate the sensitivity of the method to the δ parameter.

have determined that σ lies in the range (3–3.5) for the new transition under consideration.

The results concerning the 389.6 keV transition, which links the 8^- state to the 7^- state, are shown in Figs. 4 and 5 with the two extreme values of σ, 3.0 and 3.5, respectively. Because the parity of the initial and the final states are equal, one might expect this γ ray to be an $M1$ transition. This is, however, in contradiction with the experimental points, which show that the intensity is more pronounced at lower angles compared to angles around 90° with respect to the beam axis. Thus, one needs to consider a mixed $M1$-$E2$ transition with a mixing ratio δ, which can be obtained by comparing theoretical curves with the fit of the experimental points (black curves in Figs. 4 and 5). For $\sigma = 3.0$, the best matching is obtained using $\delta = 0.4$, while for $\sigma = 3.5$, the best matching is obtained using $\delta = 0.47$. In both cases, δ has been slightly modified around the

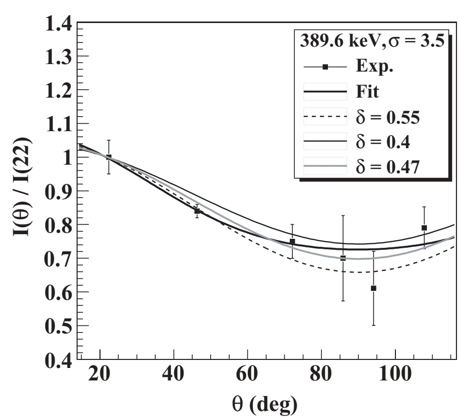

FIG. 5. Angular distributions for the 389.6 keV ($8^- \to 7^-$) transition. The fit of the experimental points is shown by the black curve. With $\sigma = 3.5$ the data are well reproduced by the theoretical curve using $\delta = 0.47$ (gray). Dot-dashed ($\delta = 0.40$) and dashed ($\delta = 0.55$) curves illustrate the sensitivity of the method to the δ parameter.

best values. The resulting curves, dashed and dot-dashed lines, are shown in Figs. 4 and 5. From this procedure we conclude that the new 389.6 keV ($8^- \rightarrow 7^-$) γ ray is a mixed $M1 + E2$ transition with a positive mixing ratio: $0.4 < \delta < 0.47$.

Using a modern γ-ray instrument, JUROGAM, and despite high statistics, we have not been able to observe the $E2$ intraband transitions of the tetrahedral-candidate band and thus to extract $B(E2)/B(E1)$ branching ratios. Recently, ultrahigh resolution γ-ray spectroscopy of ^{156}Gd has obtained an intrinsic quadrupole moment $Q_0 = 7.1$ b for the 5^- level of this band [12], in apparent disagreement with the original criterion of tetrahedral symmetry cited at the outset of this paper. However, the cited criterion is true only in the case of exact symmetry (i.e., it applies to a single static deformation point). Using the Skyrme-HFBCS approach and the generator coordinate method (GCM), Zberecki *et al.* [13] confirm that ^{156}Gd is a good candidate to search for tetrahedral effects and point to the coupling of octupole and quadrupole deformations that could obscure the signals from the tetrahedral symmetry. Góźdź *et al.* [5] and Dobrowolski *et al.* [6] reexamined the problem by using the symmetry-oriented generalization of the nuclear collective model and—while confirming the fact that various octupole components may mix—they formulate a criterion that allows distinguishing between, for example, tetrahedral- and pear-shape symmetry signals. Moreover, the aforementioned authors point out that when collective vibrations are taken into account, the equilibrium point, as a zero-measure set, contributes a null probability, whereas (especially large-amplitude) vibrations may contribute to a significant increase in the quadrupole moment, collecting contributions from a broad deformation area [4–6]. This is especially true if the potential energy surfaces are flat. The electromagnetic decay of those states is an issue of great interest, and it is receiving particular attention from various teams.

In summary, a detailed spectroscopy of the even- and odd-spin negative-parity bands in ^{156}Gd has been performed using a modern γ-ray spectrometer (JUROGAM). Our earlier work [7] has been completed in the present paper, in which all γ-ray relative intensities are given. One of the transitions (389.6 keV) linking the odd- and even-spin negative-parity bands has been characterized as an $M1 + E2$ transition with a large positive mixing coefficient δ. The characterization of the transitions linking the low spin states in the odd-spin negative-parity band, for which $E2$ transitions remain below the sensitivity of the present-day γ-ray spectrometers, with its rotational partner, together with lifetime measurements and the decay to the ground-state band, is of importance to help determine the exact structure of these states.

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