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Model-independent analysis of $p + p \to pp(^1S_0) + \gamma$

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The pp hard bremsstrahlung reaction, $p + p \to pp(^1S_0) + \gamma$, in which the two protons in the final state are in the $^1S_0$ state, is investigated theoretically. Here, the most general spin structure of the NN hard bremsstrahlung reaction, consistent with symmetry principles, is derived from a partial-wave expansion of this amplitude. Based on this spin structure, it is shown that there are only four independent spin matrix elements in this reaction, which is a direct consequence of reflection symmetry in the reaction plane. It is also shown that it requires at least eight independent observables to determine them uniquely. The present method provides the coefficients multiplying each spin operator in terms of the partial-wave or, equivalently, multipole amplitudes. Some observables are expressed explicitly in terms of these multipoles and a partial-wave analysis is performed. The results should be useful in the analyses of the experimental data on the $p + p \to pp(^1S_0) + \gamma$ reaction being taken, in particular, at the COSY accelerator facility, as well as in providing some theoretical guidance to the future experiments in this area.

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I. INTRODUCTION

The nucleon-nucleon (NN) bremsstrahlung and the related photoabsorption reactions are among the most fundamental processes in hadron physics and, as such, they have been used as tools to probe the hadron and/or quark dynamics in a variety of strongly interacting systems. As an elementary process, the photodisintegration reaction $\gamma + d \to p + n$ has been studied widely and for many years to probe the properties of the NN interaction at short distances. The NN bremsstrahlung reaction, on the other hand, has been investigated intensively in the past, mainly, to learn about the off-shell properties of the NN interaction. Unfortunately, however, in field theories off-shell properties are unmeasurable quantities. Recently, in the absence of free bound diproton, hard bremsstrahlung in $p + p \to pp(^1S_0) + \gamma$ has been measured by the COSY-ANKE Collaboration [1,2] as an alternative to the $\gamma + pp(^1S_0) \to p + p$ process; the latter is complementary to the deuteron photodisintegration process. Here, the hardness of the bremsstrahlung is due to the fact that the invariant mass of the two protons in the final state is constrained experimentally to be less than 3 MeV above its minimum value of twice the proton mass. In this kinematic regime, the two protons in the final state are practically confined to the $^1S_0$ state and most of the available energy is carried by the bremsstrahlung. Therefore, this kinematic regime is maximally away from the soft-photon limit. The proton-proton (pp) hard bremsstrahlung reaction has been also measured at CELSIUS-Uppsala [3]. In spite of extensive studies of the NN bremsstrahlung reaction in the past, no dedicated experiments of pp hard bremsstrahlung with the diproton in the final state were available until the measurements of Refs. [1–3]. Also, theoretical investigation of the $p + p \to pp(^1S_0) + \gamma$ reaction is virtually nonexistent so far.

In the present work, we derive the most general spin structure of the reaction amplitude for the NN bremsstrahlung process with the final two nucleons in the $^1S_0$ state, $N + N \to NN(^1S_0) + \gamma$, following closely the method of Ref. [4]. We will show that there are four independent spin amplitudes in this reaction and that it requires at least eight independent observables to determine them. Usually, the structure of a transition amplitude is derived based solely on symmetry principles. The present method also provides explicit formulas for the coefficients multiplying each spin structure in terms of the partial-wave or, equivalently, multipole amplitudes. This should be particularly useful in model-independent analyses based on the partial-wave expansion of the reaction amplitude. Indeed, we shall show that some of the spin observables involving the spins of the two nucleons in the initial state, together with the unpolarized cross sections, can determine the low angular momentum multipoles and/or their combinations. These observables can be measured at the currently existent facilities such as COSY-ANKE at Jülich.

The present paper is organized as follows. In Sec. II the most general spin structure for the $N + N \to NN(^1S_0) + \gamma$ reaction is derived. In Sec. III, the coefficients multiplying each of the structures of the bremsstrahlung amplitude, which are expressed in terms of the partial wave matrix elements, are related to spin observables. In Sec. IV, we establish the relationship between the partial wave and multipole amplitudes. Section V is dedicated to a partial-wave analysis of the $p + p \to pp(^1S_0) + \gamma$ reaction. Finally, Sec. VI contains the summary. Some of the details of the derivations are given in Appendices A and B.

II. SPIN STRUCTURE OF THE $p + p \to pp(^1S_0) + \gamma$ AMPLITUDE

We start by making a partial-wave expansion of the $N + N \to N + N + \gamma$ reaction amplitude. For completeness, we

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allow the photon to be virtual. We have

$$\begin{aligned}\langle 1m_s; S'M_{S'}|\hat{M}(\vec{k},\vec{p}';\vec{p})|SM_S\rangle &= \sum i^{L-L'-l}(SM_SLM_L|JM_J)Y^*_{LM_L}(\hat{p})\\ &\times (S'M_{S'}L'M_{L'}|J'M_{J'})Y_{L'M_{L'}}(\hat{p}')\\ &\times (1m_slm_l|jm_j)(jm_jJ'M_{J'}|JM_J)\\ &\times Y_{lm_l}(\hat{k})\tilde{M}^{J'S'JS}_{lL'L}(k,p';p), \end{aligned} \quad (1)$$

where S, L, J stand for the total spin, total orbital angular momentum, and the total angular momentum, respectively, of the initial NN state. M_S, M_L, and M_J denote the corresponding projection quantum numbers. The primed quantities stand for the quantum numbers of the final NN state. $|1m_s\rangle$ stands for the spin state of the emitted photon; l and j denote the orbital and total angular momentum of the emitted photon, respectively, relative to the center of mass (c.m.) of the final NN system. m_l and m_j denote the corresponding projections, respectively. The summation runs over all the quantum numbers not specified in the left-hand side (l.h.s.) of Eq. (1). $\vec{p}$ and $\vec{p}'$ denote the relative momenta of the two nucleons in the initial and final states, respectively. $\vec{k}$ denotes the momentum of the emitted photon with respect to the center of mass of the two nucleons in the final state.

Equation (1) can be inverted to solve for the partial-wave matrix element $\tilde{M}^{S'J'SJ}_{lL'L}(k,p';p)$; we obtain

$$\begin{aligned}\tilde{M}^{J'S'JS}_{lL'L}(k,p';p) &= \sum i^{L'+l-L}(SM_SL0|JM_J)\\ &\times (S'M_{S'}L'M_{L'}|J'M_{J'})\\ &\times (1m_slm_l|jm_j)(jm_jJ'M_{J'}|JM_J)\\ &\times \frac{8\pi^2}{2J+1}\sqrt{\frac{2L+1}{4\pi}}\int d\Omega_{p'}Y^*_{L'M_{L'}}(\hat{p}')\\ &\times \int_{-1}^{+1} d(\cos\theta_k)Y^*_{lm_l}(\theta_k,0)\\ &\times \langle 1m_s; S'M_{S'}|\hat{M}(\vec{k},\vec{p}';\vec{p})|SM_S\rangle, \end{aligned} \quad (2)$$

where, without loss of generality, the z axis is chosen along $\vec{p}$, and $\vec{p}$ and $\vec{k}$ define the xz plane (the reaction plane); $\cos\theta_k = \hat{k}\cdot\hat{p}$. The summation is over all the quantum numbers not specified in the l.h.s. of the equality.

Following the method of Ref. [4], the most general spin structure of the transition operator can be extracted from Eq. (1) as

$$\begin{aligned}\hat{M}(\vec{k},\vec{p}';\vec{p}) &= \sum |1m_s; S'M_{S'}\rangle\\ &\times \langle 1m_s; S'M_{S'}|\hat{M}(\vec{k},\vec{p}';\vec{p})|SM_S\rangle\langle SM_S|, \end{aligned} \quad (3)$$

where the summation is over all the spin quantum numbers.

Inserting Eq. (1) into Eq. (3) and recoupling gives

$$\begin{aligned}\hat{M}(\vec{k},\vec{p}';\vec{p}) &= \sum i^{L-L'-l}(-)^{j+J'}[jJ'][J]^2\\ &\times \tilde{M}^{J'S'JS}_{lL'L}(k,p';p)\\ &\times \sum_{\alpha\beta\gamma}(-)^{\gamma}[\gamma\beta]\begin{Bmatrix} S' & L' & J' \\ 1 & l & j \\ \gamma & \beta & J \end{Bmatrix}\begin{Bmatrix} \gamma & \beta & J \\ L & S & \alpha \end{Bmatrix}\\ &\times [B_S\otimes[A_{S'}\otimes\epsilon\,]^{\gamma}]^{\alpha}\cdot[Y_L(\hat{p})\otimes[Y_{L'}(\hat{p}')\\ &\otimes Y_l(\hat{k})]^{\beta}]^{\alpha}, \end{aligned} \quad (4)$$

where we have introduced the notations $\epsilon_{1m_s}\equiv|1m_s\rangle$, $A_{S'M_{S'}}\equiv|S'M_{S'}\rangle$, and $B_{SM_S}\equiv(-)^{S-M_S}\langle S-M_S|$, in addition to

$$[j]\equiv\sqrt{2j+1},\qquad [j_1\ldots j_n]\equiv[j_1]\ldots[j_n]. \quad (5)$$

In the following we restrict ourselves to NN bremsstrahlung with the final NN subsystem in the $^1S_0$ state ($S'=L'=J'=0$). Equation (4) then reduces to

$$\begin{aligned}\hat{M}(\vec{k},\vec{p}';\vec{p}) &= \frac{1}{\sqrt{4\pi}}\sum i^{L-l}(-)^{-J+1}[J]^2M^{JS}_{lL}(k,p';p)\\ &\times\sum_{\alpha}\begin{Bmatrix} S & L & J \\ l & 1 & \alpha \end{Bmatrix}\\ &\times [B_S\otimes[A_{S'=0}\otimes\epsilon]^1]^{\alpha}\cdot[Y_L(\hat{p})\otimes Y_l(\hat{k})]^{\alpha}, \end{aligned} \quad (6)$$

where $\tilde{M}^{JS}_{lL}\equiv\tilde{M}^{00JS}_{l0L}$.

We now expand $[B_S\otimes[A_0\otimes\epsilon]^1]^{\alpha}$, for each tensor rank α, in terms of the complete set of available spin operators in the problem, i.e., the polarization of the photon $\vec{\epsilon}$, and the Pauli spin matrices $\vec{\sigma}_1$ and $\vec{\sigma}_2$ corresponding to the two nucleons together with the corresponding spin identity matrices. Note that the final NN subsystem is in the $^1S_0$ state ($S'=0$). There are, then, four cases to be considered:

$$\begin{aligned}&S=0,\alpha=1:\ [B_S\otimes[A_0\otimes\epsilon]^1]^1=\vec{\epsilon}P_{S=0},\\ &S=1,\alpha=0:\ [B_S\otimes[A_0\otimes\epsilon]^1]^0=\frac{1}{\sqrt{3}}\vec{\epsilon}\cdot\vec{T}_{S=1},\\ &S=1,\alpha=1:\ [B_S\otimes[A_0\otimes\epsilon]^1]^1=\frac{i}{\sqrt{2}}(\vec{\epsilon}\times\vec{T}_{S=1}),\\ &S=1,\alpha=2:\ [B_S\otimes[A_0\otimes\epsilon]^1]^2=-[\vec{\epsilon}\otimes\vec{T}_{S=1}]^2, \end{aligned} \quad (7)$$

where P_S stands for the spin projection operator onto the (initial) spin singlet and triplet state, respectively, as $S=0$ or 1. $\vec{T}_S$ is the spin-flip operator from the initial NN spin state S to the final NN spin state $S'=S\pm1$. In terms of the Pauli spin matrices, they are given by

$$\begin{aligned}&P_S\equiv\tfrac{1}{4}[2S+1-(-)^S\vec{\sigma}_1\cdot\vec{\sigma}_2],\\ &\vec{T}_S\equiv\tfrac{1}{2}(\vec{\sigma}_1-\vec{\sigma}_2)P_S=\tfrac{1}{4}[(\vec{\sigma}_1-\vec{\sigma}_2)+(-)^Si(\vec{\sigma}_1\times\vec{\sigma}_2)]. \end{aligned} \quad (8)$$

In Eq. (7), the numerical factors on the right-hand side (r.h.s.) of the equalities are such that the spin matrix elements of the l.h.s equal the corresponding ones of the r.h.s. For this purpose, we have made use of the formulas

$$\begin{aligned}&[\vec{A}\otimes\vec{B}]^1=\frac{i}{\sqrt{2}}(\vec{A}\times\vec{B}),\\ &\vec{A}\cdot\vec{T}_{S=1}|SM_S\rangle=\delta_{S,1}A_{M_S}|00\rangle \end{aligned} \quad (9)$$

for arbitrary vectors $\vec{A}$ and $\vec{B}$.

With the quantization axis $\hat{z}$ chosen along $\hat{p}$, $[Y_L(\hat{p}) \otimes Y_l(\hat{k})]^\alpha$ in Eq. (6) can be expressed as

$$[Y_L(\hat{p}) \otimes Y_l(\hat{k})]^0 = 0,$$
$$[Y_L(\hat{p}) \otimes Y_l(\hat{k})]^1 = \frac{[Ll]}{4\pi}(1-\delta_{L0})\left[\sqrt{\frac{2}{l(l+1)}}(L0l1|11) \times P_l^1(\hat{k}\cdot\hat{p})\hat{n}_1 + (L0l0|10)P_l(\hat{k}\cdot\hat{p})\hat{p}\right] + \frac{\sqrt{3}}{4\pi}\delta_{L0}\delta_{l1}\hat{k},$$
$$[Y_L(\hat{p}) \otimes Y_l(\hat{k})]^2 = a_{lL}[\hat{p}\otimes\hat{n}_2]^2 + b_{lL}[\hat{k}\otimes\hat{n}_2]^2, \tag{10}$$

where

$$\hat{n}_1 \equiv [(\hat{p}\times\hat{k})\times\hat{p}]/|\hat{p}\times\hat{k}|,$$
$$\hat{n}_2 \equiv (\hat{p}\times\hat{k})/|\hat{p}\times\hat{k}|. \tag{11}$$

Note that total parity conservation demands that $(-)^{L+l} = -1$. Also, in the second equality in Eq. (10), we have explicitly isolated the $L = 0$ state contribution for further convenience. The coefficients a_{lL} and b_{lL} are calculated explicitly in Appendix A.

Inserting Eqs. (7) and (10) into Eq. (6) and using the identity

$$3[\vec{\epsilon}\otimes\vec{T}_S]^2\cdot[\hat{a}\otimes\hat{b}]^2 = \tfrac{3}{2}[\vec{\epsilon}\cdot\hat{a}\vec{T}_S\cdot\hat{b} + \vec{\epsilon}\cdot\hat{b}\vec{T}_S\cdot\hat{a}] - (\hat{a}\cdot\hat{b})\vec{\epsilon}\cdot\vec{T}_S, \tag{12}$$

with $\hat{a}$ and $\hat{b}$ denoting arbitrary unit vectors, we have

$$\begin{aligned}\hat{M}(\vec{k},\vec{p}\,';\vec{p}) = {}& [F_1\vec{\epsilon}\cdot\hat{k} + F_2\vec{\epsilon}\cdot\hat{p} + F_3\vec{\epsilon}\cdot\hat{n}_1]P_{S=0} \\ & + [iF_4(\vec{\epsilon}\times\hat{k}) + iF_5(\vec{\epsilon}\times\hat{p}) + iF_6(\vec{\epsilon}\times\hat{n}_1) \\ & + F_7(\vec{\epsilon}\cdot\hat{p}\hat{n}_2 + \vec{\epsilon}\cdot\hat{n}_2\hat{p}) \\ & + F_8(\vec{\epsilon}\cdot\hat{k}\hat{n}_2 + \vec{\epsilon}\cdot\hat{n}_2\hat{k})]\cdot\vec{T}_{S=1},\end{aligned} \tag{13}$$

where the coefficients F_i ($i = 1,\ldots,8$) are given in Eq. (14) below. The above result is the most general spin structure of the NN bremsstrahlung amplitude consistent with symmetry principles when the final NN subsystem is in the $^1S_0$ state. The first three terms are the central NN spin singlet transitions. The fourth, fifth, and sixth terms are tensors of rank 1 describing the NN spin triplet → singlet transitions. The last two terms involving the coefficients F_7 and F_8 correspond to the tensor interaction of rank 2. Apart from the first three terms, all the other terms are spin-flip transitions.

Here we note that the gauge freedom always allows one to make the choice for the photon polarization $\epsilon_\mu = \epsilon'_\mu - \eta k_\mu$ for any value of the gauge parameter η. For example, with the choice $\eta = \epsilon'_0/k_0$, which leads to $\epsilon_0 = 0$, the photon polarization has no scalar component and the reaction amplitude contains a virtual photon with transverse and longitudinal components. Equation (13) corresponds to this choice. Alternatively, one could make the choice $\eta = (\vec{\epsilon}\,'\cdot\hat{k})/|\vec{k}|$ for the gauge parameter, leading to $\vec{\epsilon}\cdot\hat{k} = 0$ but $\epsilon_0 \neq 0$, i.e., the virtual photon has no longitudinal component and the reaction amplitude would correspond to a virtual photon with transverse and scalar components. Of course, amplitudes with different gauge choices are equivalent to each other by gauge invariance.

The coefficients multiplying the spin operators in Eq. (13) contain the dynamics of the reaction process and are given by

$$\begin{aligned}
F_1 &= \frac{1}{(4\pi)^{\frac{3}{2}}} i\delta_{L0}\delta_{l1}\delta_{J0}\tilde{M}^{00}_{10}(k,p';p),\\
F_2 &= \frac{1}{(4\pi)^{\frac{3}{2}}}\frac{1}{\sqrt{3}}\sum i^{L-l}(-)^l[L]^2[l](L0l0|10)(1-\delta_{L0}) \times \tilde{M}^{L0}_{lL}(k,p';p)P_l(\hat{k}\cdot\hat{p}),\\
F_3 &= \frac{1}{(4\pi)^{\frac{3}{2}}}\frac{1}{\sqrt{3}}\sum i^{L-l}(-)^l[L]^2[l]\sqrt{\frac{2}{l(l+1)}}(L0l1|11) \times (1-\delta_{L0})\tilde{M}^{L0}_{lL}(k,p';p)P^1_l(\hat{k}\cdot\hat{p}),\\
F_4 &= \frac{1}{(4\pi)^{\frac{3}{2}}}\sqrt{\frac{3}{2}}\delta_{L0}\delta_{l1}\delta_{J1}\tilde{M}^{11}_{10}(k,p';p),\\
F_5 &= \frac{1}{(4\pi)^{\frac{3}{2}}}\frac{1}{\sqrt{2}}\sum i^{L-l}(-)^{-J}[J]^2[Ll](L0l0|10) \times (1-\delta_{L0})\begin{Bmatrix}1 & L & J\\ l & 1 & 1\end{Bmatrix}\tilde{M}^{J1}_{lL}(k,p';p)P_l(\hat{k}\cdot\hat{p}),\\
F_6 &= \frac{1}{(4\pi)^{\frac{3}{2}}}\frac{1}{\sqrt{2}}\sum i^{L-l}(-)^{-J}[J]^2[Ll]\sqrt{\frac{2}{l(l+1)}} \times (L0l1|11)(1-\delta_{L0})\begin{Bmatrix}1 & L & J\\ l & 1 & 1\end{Bmatrix} \times \tilde{M}^{J1}_{lL}(k,p';p)P^1_l(\hat{k}\cdot\hat{p}),\\
F_7 &= \frac{1}{2\sqrt{4\pi}}\sum i^{L-l}(-)^{-J}[J]^2\begin{Bmatrix}1 & L & J\\ l & 1 & 2\end{Bmatrix} \times \tilde{M}^{J1}_{lL}(k,p';p)a_{lL},\\
F_8 &= \frac{1}{2\sqrt{4\pi}}\sum i^{L-l}(-)^{-J}[J]^2\begin{Bmatrix}1 & L & J\\ l & 1 & 2\end{Bmatrix} \times \tilde{M}^{J1}_{lL}(k,p';p)b_{lL}.
\end{aligned} \tag{14}$$

The summations are over all the quantum numbers appearing explicitly in the r.h.s. of the equalities. Note that they are restricted by total parity conservation, $(-)^{L+l} = -1$, and the Pauli principle. In the case of pp bremsstrahlung, the latter leads to $(-)^{L+S} = 1$. These imply that the pp spin triplet → singlet transitions involve only the even photon orbital angular momenta, while the singlet → singlet transitions involve only the odd orbital angular momenta.

Note that, in Eq. (13), the terms proportional to $\vec{\epsilon}\cdot\vec{k}$ contribute only for virtual photons, such as in dilepton productions. Also, the coefficient F_4 is absent in pp bremsstrahlung due to the Pauli principle.

Although the primary focus of the present work is on the hard bremsstrahlung production in pp collisions, all the results derived in this and the following two sections apply to pn collisions as well.

III. OBSERVABLES

In this section, we will relate the coefficients F_i in Eq. (13) to the corresponding spin matrix elements and then relate these matrix elements to some observables. In what follows, we consider the real photon and take the two independent (linear) photon polarizations to be parallel and perpendicular to the reaction plane,[1]

$$\vec{\epsilon}_\parallel \equiv \cos\theta_k \hat{x} - \sin\theta_k \hat{z}, \qquad \vec{\epsilon}_\perp \equiv \hat{y}. \tag{15}$$

Then, using the shorthand notation

$$M^{\lambda'}_{SM_S} \equiv \langle 1\lambda'; 00|\hat{M}(\vec{k}, \vec{p}'; \vec{p})|SM_S\rangle, \tag{16}$$

the spin matrix elements of the amplitude given by Eq. (13), with a specific photon polarization state $\vec{\epsilon}_{\lambda'}$, are given by

$$\begin{aligned} M^\parallel_{00} &= -F_2 \sin\theta_k + F_3 \cos\theta_k, \\ M^\perp_{00} &= 0, \end{aligned} \tag{17}$$

for the transition of NN spin singlet → singlet state, and

$$\begin{aligned} M^\parallel_{10} &= 0, \\ M^\perp_{10} &= -i(F_4 \sin\theta_k + F_6) + F_7 + F_8 \cos\theta_k, \\ M^\parallel_{1\pm1} &= \frac{-1}{\sqrt{2}}[F_4 + F_5 \cos\theta_k + (F_6 - iF_7)\sin\theta_k], \\ M^\perp_{1\pm1} &= \mp\frac{1}{\sqrt{2}}[i(F_4 \cos\theta_k + F_5) + F_8 \sin\theta_k], \end{aligned} \tag{18}$$

for the triplet → singlet transition. Here, we made use of Eq. (9). It should be emphasized that Eqs. (17) and (18) reveal that there are only four independent spin matrix elements in this reaction. This is a direct consequence of reflection symmetry in the reaction plane [6–8]. The results obtained in the remainder of this section and in Sec. V, are consequences of this property of the spin matrix elements.

In this work, we consider the unpolarized cross section, $d\sigma/d\Omega$, and the spin observables involving the spins of the initial state nucleons only, namely, the analyzing powers, A_i, and the spin correlation coefficients, A_{ij}. They are defined as

$$\begin{aligned} \frac{d\sigma}{d\Omega} &\equiv \frac{1}{4}\mathrm{Tr}[\hat{M}\hat{M}^\dagger], \\ \frac{d\sigma}{d\Omega}A_i &\equiv \frac{1}{4}\mathrm{Tr}[\hat{M}\sigma_i(1)\hat{M}^\dagger], \\ \frac{d\sigma}{d\Omega}A_{ij} &\equiv \frac{1}{4}\mathrm{Tr}[\hat{M}\sigma_i(1)\sigma_j(2)\hat{M}^\dagger]. \end{aligned} \tag{19}$$

Following Appendix D of Ref. [4], the above observables are readily expressed in terms of the spin matrix elements $M^{\lambda'}_{SM_S}$. We have for unpolarized cross section and (diagonal) spin correlation coefficients

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{4}\left(|M^\parallel_{00}|^2 + 2|M^\parallel_{11}|^2 + |M^\perp_{10}|^2 + 2|M^\perp_{11}|^2\right), \\ \frac{d\sigma}{d\Omega}A_{xx} &= \frac{1}{4}\left(-|M^\parallel_{00}|^2 + 2|M^\parallel_{11}|^2 + |M^\perp_{10}|^2 - 2|M^\perp_{11}|^2\right), \\ \frac{d\sigma}{d\Omega}A_{yy} &= \frac{1}{4}\left(-|M^\parallel_{00}|^2 - 2|M^\parallel_{11}|^2 + |M^\perp_{10}|^2 + 2|M^\perp_{11}|^2\right), \\ \frac{d\sigma}{d\Omega}A_{zz} &= \frac{1}{4}\left(-|M^\parallel_{00}|^2 + 2|M^\parallel_{11}|^2 - |M^\perp_{10}|^2 + 2|M^\perp_{11}|^2\right), \end{aligned} \tag{20}$$

[1]Note that for a circularly polarized photon, $\vec{\epsilon}_{\lambda'} \to \vec{\epsilon}^{\,*}_{\lambda'}$, for the photon is being emitted in this reaction instead of being absorbed.

which can be inverted to yield

$$\begin{aligned} |M^\parallel_{00}|^2 &= \frac{d\sigma}{d\Omega}(1 - A_{xx} - A_{yy} - A_{zz}), \\ |M^\perp_{10}|^2 &= \frac{d\sigma}{d\Omega}(1 + A_{xx} + A_{yy} - A_{zz}), \\ 2|M^\parallel_{11}|^2 &= \frac{d\sigma}{d\Omega}(1 + A_{xx} - A_{yy} + A_{zz}), \\ 2|M^\perp_{11}|^2 &= \frac{d\sigma}{d\Omega}(1 - A_{xx} + A_{yy} + A_{zz}). \end{aligned} \tag{21}$$

Therefore, the measurement of the cross section and the spin correlation coefficients can determine the magnitude of the individual spin matrix elements.

The off-diagonal spin correlation coefficients vanish identically as a consequence of reflection symmetry, except for $A_{xz}(= A_{zx})$ which is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega}A_{xz} &= \frac{d\sigma}{d\Omega}A_{zx} \\ &= -\frac{1}{\sqrt{2}}\mathrm{Re}[M^\parallel_{00}M^{\parallel\,*}_{11} - M^\perp_{10}M^{\perp *}_{11}]. \end{aligned} \tag{22}$$

The analyzing power reads

$$\frac{d\sigma}{d\Omega}A_y = \frac{1}{\sqrt{2}}\mathrm{Im}[M^\parallel_{00}M^{\parallel\,*}_{11} - M^\perp_{10}M^{\perp *}_{11}]. \tag{23}$$

$A_x = A_z = 0$ by symmetry.

Equations (22) and (23) show that the spin correlation coefficient A_{xz}, together with the analyzing power A_y, determines the relative phase of the combination of the spin matrix elements involving the two independent photon polarization states, $\vec{\epsilon}_\perp$ and $\vec{\epsilon}_\parallel$. Note that both the real and imaginary parts are required to determine the relative phase uniquely. It is also immediate from Eqs. (22) and (23) that one obvious possibility to disentangle the combination of those two terms with different photon polarization is to measure A_{xz} and A_y together with the polarization of the emitted photon. This means that, in total, we need at least eight independent observables to determine these spin amplitudes uniquely, apart from the irrelevant overall phase.[2] Evidently, a complete determination of the hard bremsstrahlung amplitude poses an enormous experimental challenge.

It is also interesting to note another consequence of reflection symmetry that relates the double polarization observable with the single polarization observable; namely, [8]

$$A_{yy} = -\Sigma, \tag{24}$$

where Σ stands for the outgoing photon asymmetry given by

$$\frac{d\sigma}{d\Omega}\Sigma \equiv \frac{d\sigma^\parallel}{d\Omega} - \frac{d\sigma^\perp}{d\Omega}, \tag{25}$$

with $\sigma^{\lambda'}$ denoting the cross section with a given photon polarization $\vec{\epsilon}_{\lambda'}$. Equation (24) can also be verified simply by an inspection of the result for A_{yy} in Eq. (20) together with the definition of Σ given in the above equation.

[2]A general discussion on the closely related issue of "complete experiment" in pseudoscalar meson photoproduction has been given by Chiang and Tabakin [9].

Some combinations of the spin matrix elements can be also extracted from the measurements of the transverse spin correlation coefficients A_{xx} and A_{yy} in conjunction with the unpolarized cross section. The coefficient A_{zz}, which involves the longitudinal polarization of the beam and target nucleon spins, are experimentally more challenging to be measured than the transverse coefficients. We have, from Eq. (20),

$$\begin{aligned} 2\frac{d\sigma}{d\Omega}(A_{xx}+A_{yy}) &= |M^{\perp}_{10}|^2 - |M^{\parallel}_{00}|^2,\\ 2\frac{d\sigma}{d\Omega}(A_{xx}-A_{yy}) &= 2|M^{\parallel}_{11}|^2 - 2|M^{\perp}_{11}|^2,\\ 2\frac{d\sigma}{d\Omega}(1+A_{xx}) &= 2|M^{\parallel}_{11}|^2 + |M^{\perp}_{10}|^2,\\ 2\frac{d\sigma}{d\Omega}(1+A_{yy}) &= 2|M^{\perp}_{11}|^2 + |M^{\perp}_{10}|^2. \end{aligned} \tag{26}$$

IV. MULTIPOLE AMPLITUDES

The bremsstrahlung amplitude can also be decomposed in terms of more familiar multipole amplitudes instead of the partial wave expansion given by Eq. (14). The multipole decomposition of the inverse process, $\gamma + pp(^1S_0) \to p + p$, has been given elsewhere [5]. In this section we will establish a relationship between the multipole and partial-wave amplitudes introduced in the previous section. We do this in the overall c.m. frame of the system with the coordinate system as chosen in Sec. II [see below Eq. (2)].

The matrix element for the transition $p + p \to pp(^1S_0) + \gamma$ in the helicity basis is then related to that of the plane-wave basis by [10]

$$\begin{aligned} &\langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle \\ &\quad = \sum_{m_s} D^{1^*}_{m_s\lambda'}(R)\langle 1m_s|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|SM_S\rangle, \end{aligned} \tag{27}$$

where $D^1_{m_s\lambda'}(R)$ stands for the spin rotation matrix which rotates our coordinate frame by an angle θ_k about the y axis ($\alpha=\gamma=0, \beta=\theta_k$) such that the new z' axis is along the photon momentum $\vec{k}$. Note that, since $\vec{p}$ is along the positive z axis, the initial plane-wave state coincides with the helicity state, i.e., $\lambda = M_S$. In the above equation and in what follows, we have suppressed the reference to the spin ($S' = M_{S'} = 0$) of the two nucleons in the final state.

Now, the partial wave decomposition of the plane-wave matrix element given by Eq. (1) for the case of the final two nucleons in the $^1S_0$ state reduces to

$$\begin{aligned} &\langle 1m_s|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|SM_S\rangle \\ &\quad = \frac{1}{4\pi}\sum [L](lm_l 1m_s|jM_S) \\ &\qquad \times (L0SM_S|jM_S)Y_{lm_l}(\hat{k})M^{jS}_{lL}(k,p';p), \end{aligned} \tag{28}$$

where the summation runs over all the quantum numbers not appearing on the l.h.s of the equality. In the above equation, we have redefined the partial wave matrix elements to incorporate a phase, i.e.,

$$M^{jS}_{lL}(k,p';p) \equiv i^{L-l}(-)^S \tilde{M}^{jS}_{lL}(k,p';p) \tag{29}$$

for further convenience. Note also the change in the angular momentum coupling scheme in Eq. (28) compared to Eq. (1). The reason for this change is to have a close comparison of the results of this section with those in Ref. [5].

Inserting Eq. (28) into Eq. (27), and using the relations [10]

$$\begin{aligned} Y_{l\mu}(\hat{k}) &= \frac{[l]}{\sqrt{4\pi}}D^{l^*}_{\mu 0}(R),\\ (l01\lambda'|j\lambda')D^{j^*}_{\lambda\lambda'}(R) &= \sum_{\mu\mu'}(l\mu 1\mu'|j\lambda)D^{l^*}_{\mu 0}(R)D^{1^*}_{\mu'\lambda'}(R), \end{aligned} \tag{30}$$

yields

$$\begin{aligned} \langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle &= \sum_{jL}\frac{[L]}{(4\pi)^{\frac{3}{2}}}(L0S\lambda|j\lambda)d^j_{\lambda\lambda'}(\theta_k)\\ &\quad\times\sum_l [l](l01\lambda'|j\lambda')M^{jS}_{lL}(k,p';p). \end{aligned} \tag{31}$$

Note that, in our case, $D^{j^*}_{\lambda\lambda'}(R) = d^{j^*}_{\lambda\lambda'}(\theta_k) = d^j_{\lambda\lambda'}(\theta_k)$.

Now, the electric (E) and magnetic (M) multipoles are classified according to $(-)^{j+l+1} = +1$ and $(-)^{j+l+1} = -1$, respectively. In addition, they are transverse amplitudes, i.e., for $\lambda' = \pm 1$ only. The longitudinal multipole amplitude (L) corresponds to $\lambda' = 0$. Then, due to the properties of the Clebsh-Gordon coefficient $(l01\lambda'|j\lambda')$,

$$\begin{aligned} (l01\lambda'|j\lambda') &= (-)^{l+1+j-2\lambda'}(l01-\lambda'|j-\lambda'),\\ (l010|j0) &= 0 \quad \text{if} \quad l+1+j = \text{odd}, \end{aligned} \tag{32}$$

one sees that Eq. (31) can be rewritten as

$$\begin{aligned} &\langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle \\ &\quad = \sum_{jL}\frac{[L]}{(4\pi)^{\frac{3}{2}}}(L0S\lambda|j\lambda)d^j_{\lambda\lambda'}(\theta_k)\big[\delta_{\lambda' 0}L_{jSL}(k,p';p)\\ &\qquad + |\lambda'|E_{jSL}(k,p';p) + \lambda' M_{jSL}(k,p';p)\big], \end{aligned} \tag{33}$$

with

$$\begin{aligned} L_{jSL} &\equiv \sum_l [l](l010|j0)M^{jS}_{lL},\\ E_{jSL} &\equiv \sum_l\left(\frac{1+(-)^{l+1+j}}{2}\right)[l](l011|j1)M^{jS}_{lL},\\ M_{jSL} &\equiv \sum_l\left(\frac{1-(-)^{l+1+j}}{2}\right)[l](l011|j1)M^{jS}_{lL}, \end{aligned} \tag{34}$$

where the momentum arguments of the amplitudes have been omitted. The above equations relate the multipole amplitudes to the partial wave amplitudes.

For a real (transverse) photon no longitudinal multipoles exist. Therefore, Eq. (33) reduces to

$$\begin{aligned} &\langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle \\ &\quad = \sum_{jL}\frac{[L]}{(4\pi)^{\frac{3}{2}}}(L0S\lambda|j\lambda)d^j_{\lambda\lambda'}(\theta_k)\\ &\qquad \times\big[E_{jSL}(k,p';p) + \lambda' M_{jSL}(k,p';p)\big], \end{aligned} \tag{35}$$

which agrees with Eq. (2) of Ref. [5] apart from an overall normalization factor.

Equation (33) can be readily inverted to solve for the multipole amplitudes. We have

$$L_{jSL} = [L]\sum_{\lambda}(L0S\lambda|j\lambda)A^{jS}_{0,\lambda},$$
$$E_{jSL} = [L]\sum_{\lambda}(L0S\lambda|j\lambda)\frac{1}{2}\left[A^{jS}_{+1,\lambda} + A^{jS}_{-1,\lambda}\right], \quad (36)$$
$$M_{jSL} = [L]\sum_{\lambda}(L0S\lambda|j\lambda)\frac{1}{2}\left[A^{jS}_{+1,\lambda} - A^{jS}_{-1,\lambda}\right],$$

where

$$A^{jS}_{\lambda',\lambda} \equiv \sqrt{4\pi}\int d\Omega_k\, d^j_{\lambda\lambda'}(\theta_k)\langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle. \quad (37)$$

Equation (27) can be inverted to obtain

$$\langle 1m_s|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|SM_S\rangle = \sum_{\lambda'} D^1_{\lambda' m_s}(R)\langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle. \quad (38)$$

Inserting the multipole expansion of the helicity amplitude, Eq. (33), into the above equation yields

$$\begin{aligned}\langle 1m_s|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|SM_S\rangle &= \frac{1}{(4\pi)}\sum [L](lm_l1m_s|jM_S)\\ &\times (L0SM_S|jM_S)Y_{lm_l}(\hat{k})\frac{[l]}{[j]^2}(l01\lambda'|j\lambda')\\ &\times \left[\delta_{\lambda'0}L_{jSL}(k,p';p)\right.\\ &\left.+|\lambda'|E_{jSL}(k,p';p)+\lambda' M_{jSL}(k,p';p)\right], \end{aligned} \quad (39)$$

where the summation runs over all the quantum numbers not appearing in the l.h.s. of the equality. Recall that $\lambda' = M_S$. In arriving at the above result, we have made used of the property [10]

$$\begin{aligned} d^{j'}_{\lambda\lambda'}d^j_{\mu\mu'} &= \sum_{kmm'}(j'\lambda j-\mu|km)(j'\lambda' j-\mu'|km')\\ &\times(-)^{\mu-\mu'}d^k_{m,m'}\\ &= \sum_{kmm'}(-)^{2j+2k+\lambda+\lambda'}\frac{[k]^2}{[j]^2}\\ &\times(k-mj'\lambda|j\mu)(k-m'j'\lambda'|j\mu')d^k_{m,m'}. \end{aligned} \quad (40)$$

Equation (39) gives a direct decomposition of the hard bremsstrahlung amplitude in the plane-wave basis in terms of the multipole amplitudes.

Finally, comparing Eqs. (28) and (39), we have

$$M^{jS}_{lL} = \frac{[l]}{[j]^2}\sum_{\lambda'}(l01\lambda'|j\lambda')\times\left[\delta_{\lambda'0}L_{jSL} + |\lambda'|E_{jSL} + \lambda' M_{jSL}\right], \quad (41)$$

which gives the partial wave amplitudes in terms of the multipole amplitudes.

V. PARTIAL-WAVE ANALYSIS OF THE $p + p \to pp(^1S_0) + \gamma$ REACTION

The $\gamma + pp(^1S_0) \to p + p$ reaction has been analyzed in terms of few multipoles with $j \leqslant 2$ in the past [5]. Here, we perform a similar analysis of the $p + p \to pp(^1S_0) + \gamma$ reaction, by extending it to include the spin observables which have not been considered in that reference. In addition, we will keep the $^3F_2 \to {}^1S_0d$ partial wave state in the analysis. This state, which contributes to the M_{213} multipole, has been ignored in Ref. [5].

Restricting the partial-wave expansion in Eq. (14) to $J(=j) \leqslant 2$, we have the following partial wave states[3]:

$$\begin{aligned} &E_{111}: {}^3P_1 \to {}^1S_0 s,\ \ L_{000}: {}^1S_0 \to {}^1S_0 p,\\ &E_{111}: {}^3P_1 \to {}^1S_0 d,\ \ E_{202}: {}^1D_2 \to {}^1S_0 p,\\ &M_{211}: {}^3P_2 \to {}^1S_0 d,\ \ E_{202}: {}^1D_2 \to {}^1S_0 f,\\ &M_{213}: {}^3F_2 \to {}^1S_0 d. \end{aligned} \quad (42)$$

In the above list we have also displayed the corresponding multipole amplitudes to which the specified partial waves contribute according to the classification given in the previous section. The $^1S_0 \to {}^1S_0p$ partial wave, which enter the L_{000} multipole, does not contribute in the present reaction because of the transversality of the (real) photon. We therefore have four independent multipoles for $j \leqslant 2$.

Now, the spin matrix elements $M^{\perp/\parallel}_{SM_S}$ introduced in Sec. III can be expressed in terms of the multipole amplitudes specified above. They are given in Appendix B. Inserting those results [Eqs. (B5) and (B6) with the overall normalization factor of $1/(4\pi)^{3/2}$ restored] into Eq. (20), we have for the unpolarized cross section

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{3}{(8\pi)^3}\left\{|E_{111}+M2^+|^2 + \left[2|E_{111}-M2^+|^2\right.\right.\\ &\left.-|E_{111}+M2^+|^2\right]\cos^2\theta_k + \left[10|E_{202}|^2 + 4(|M2^-|^2\right.\\ &\left.\left.-|M2^+|^2)\right]\times\cos^2\theta_k\sin^2\theta_k\right\}, \end{aligned} \quad (43)$$

where $M2^\pm$ is defined as

$$M2^{\pm} \equiv M_{211} \pm \left(\frac{3}{2}\right)^{\mp\frac{1}{2}} M_{213}. \quad (44)$$

Any deviation in the measured angular dependence from the above result is due to higher j states. In this connection, we note that the recent angular distribution data from CELSIUS [3]

[3]Here we adopt the same notation for the partial-wave states as used in Ref. [11], i.e., $^{2S+1}L_J \to {}^{2S'+1}L'_{J'}l$, where S, L, and J stand for the total spin, relative orbital angular momentum, and the total angular momentum of the initial pp system, respectively. The primed quantities refer to the final state. l' denotes the orbital angular momentum of the produced photon. We use the spectroscopic notation for the orbital angular momenta.

at 310 MeV proton incident energy reveals a

$$\frac{d\sigma}{d\Omega_k} = \frac{3}{8\pi}\left[a + 3b\cos^2\theta_k + c\sin^2\theta_k\cos^2\theta_k\right] \quad (45)$$

dependence with $a = (2.3 \pm 0.3)$ nb, $b = (11.9 \pm 0.5)$ nb, and $c < 2$ nb. This indicates that, at this energy, the multipoles with $j > 2$ are relatively small and may be neglected.

If we now exclude the ${}^3F_2 \to {}^1S_0 d$ transition (M_{213} multipole), as has been done in Ref. [5], Eq. (43) reduces to[4]

$$\frac{d\sigma}{d\Omega} = \frac{3}{(8\pi)^3}\{|E_{111} + M_{211}|^2 + [3|E_{111} - M_{211}|^2 - 2(|E_{111}|^2 + |M_{211}|^2)]\cos^2\theta_k + 10|E_{202}|^2\cos^2\theta_k\sin^2\theta_k\}. \quad (46)$$

Given the smallness of the coefficient c in Eq. (45), here, it is tempting to conclude that the E_{202} multipole should be very small at that energy. In fact, this is precisely the conclusion reached in the analysis of Ref. [3] based on Eq. (46). However, although this might be the case, one should be cautious in drawing such a conclusion, as can be seen from a more general formula given by Eq. (43). The conclusion of the smallness of E_{202} is warranted only if the M_{213} multipole can be neglected.

In principle, the E_{202} multipole can be determined directly if the initial state spin-singlet cross section can be extracted:

$$\frac{d({}^1\sigma)}{d\Omega} = \frac{3}{(8\pi)^3}10|E_{202}|^2\cos^2\theta_k\sin^2\theta_k, \quad (47)$$

which means measuring not only the transverse but also the longitudinal spin correlation coefficient A_{zz} (cf. the first equality in Eq. (21). See, also Eq. (D8) in Ref. [4]), a quantity that is more challenging to be measured experimentally than the transverse coefficients A_{xx} and A_{yy}.

In terms of the multipoles with $j \leqslant 2$, the analyzing power given by Eq. (23) becomes

$$\frac{d\sigma}{d\Omega}A_y = \frac{3}{4(4\pi)^3}\sqrt{10}\,\mathrm{Im}[E_{202}(E_{111} - M2^+)^*]\sin\theta_k\cos^2\theta_k + \frac{3}{2(4\pi)^3}\mathrm{Im}[M2^-(E_{111} + M2^+)^*]\sin\theta_k\cos\theta_k - \frac{3}{(4\pi)^3}\mathrm{Im}[M2^-(M2^+)^*]\sin\theta_k\cos^3\theta_k. \quad (48)$$

Neglecting again the ${}^3F_2 \to {}^1S_0 d$ partial wave state, the above equation reduces to

$$\frac{d\sigma}{d\Omega}A_y = \frac{3}{4(4\pi)^3}\sqrt{10}\,\mathrm{Im}[E_{202}(E_{111} - M_{211})^*]\sin\theta_k\cos^2\theta_k + \frac{3}{2(4\pi)^3}\mathrm{Im}[M_{211}(E_{111} + M_{211})^*]\sin\theta_k\cos\theta_k. \quad (49)$$

Comparing the results of Eqs. (48) and (49), it is clear that the angular dependence of the analyzing power (the $\sin\theta_k\cos^3\theta_k$ term) can tell us about the size of the ${}^3F_2 \to {}^1S_0 d$ matrix element, which has an immediate consequence on the size of the E_{202} multipole as has been discussed above in connection with the (unpolarized) cross section.

As for the combinations of the (transverse) spin correlation coefficients in Eq. (26), we obtain

$$\frac{d\sigma}{d\Omega}(A_{xx} + A_{yy}) = \frac{3}{4(4\pi)^3}\{4|M2^-|^2 - 10|E_{202}|^2\} \times \sin^2\theta_k\cos^2\theta_k,$$
$$\frac{d\sigma}{d\Omega}(A_{xx} - A_{yy}) = \frac{3}{4(4\pi)^3}\{4|M2^+|^2\cos^2\theta_k - |E_{111} + M2^+|^2\}\sin^2\theta_k,$$
$$\frac{d\sigma}{d\Omega}(1 + A_{xx}) = \frac{3}{4(4\pi)^3}\{|E_{111} - M2^+|^2 + 4|M2^-|^2\sin^2\theta_k\}\cos^2\theta_k,$$
$$\frac{d\sigma}{d\Omega}(1 + A_{yy}) = \frac{3}{4(4\pi)^3}\{|E_{111} + M2^+|^2\sin^2\theta_k + |E_{111} - M2^+|^2\cos^2\theta_k + 4(|M2^-|^2 + |M2^+|^2) \times \sin^2\theta_k\cos^2\theta_k\}, \quad (50)$$

which reveal that the measurements of the transverse spin correlation coefficients may be used to extract the multipole amplitudes involved. In particular, from the angular distributions in the second and third equalities, one can extract $|M2^\pm|$ and the combinations $|E_{111} \pm M2^+|$. Then, the first equality can be used to determine $|E_{202}|$. Furthermore, we have $|E_{111} + M2^+|^2 - |E_{111} - M2^+|^2 = 4\mathrm{Re}[E_{111}(M2^+)^*]$. The fourth equality may be used to check for consistency of restricting to multipoles with $j \leqslant 2$. Any deviation in the measured angular dependences from the above results is an indication of the contribution from higher multipoles.

VI. SUMMARY

In the present work, we have derived the most general spin structure of the NN bremsstrahlung amplitude where the two nucleons in the final state is restricted to the ${}^1S_0$ state. The structure is consistent with general symmetry principles. The coefficient multiplying each spin operator is expressed in terms of a linear combination of the partial wave matrix elements. This is useful in analyses based on partial wave expansion. These partial wave matrix elements have been related to more familiar multipole amplitudes. This allows to study the hard bremsstrahlung in NN collisions in terms of either the partial wave or multipole amplitudes.

Based on the general spin structure obtained, we have shown that there are only four independent spin matrix elements in the present reaction process which is a direct consequence of reflection symmetry in the reaction plane. It requires at least eight independent observables to determine uniquely these spin matrix elements. The magnitude of them can be determined from the (diagonal) spin correlation coefficients in conjunction with the unpolarized cross section [cf. Eq. (21)]. The analyzing power, together with the

[4]Recently, in Ref. [3], an expression for the cross section has been given which is at odds with the present result of Eq. (46). They have corrected, however, in a later Erratum [3] which agrees with the present result.

(off-diagonal) spin correlation coefficient, determines the relative phase of a combination [cf. Eqs. (22) and (23)] of the spin matrix elements involving two independent photon polarization states. This combination may be disentangled if one measures both the analyzing power and off-diagonal spin correlation coefficient together with the polarization of the photon. Obviously, such an experiment will be extremely challenging by any standard.

We have also performed a partial-wave analysis of the $p + p \to pp(^1S_0) + \gamma$ reaction considering the multipoles with the total angular momentum $j \leqslant 2$. It has been shown that these multipoles (or some combination of them) can be determined from the measurements of the transverse spin correlation coefficients, A_{xx} and A_{yy}, and the analyzing power, A_y, in conjunction with the unpolarized cross section. These observables can be measured at the currently existing facilities such as COSY-ANKE at Jülich.

Finally, the results of the present study should be useful for the on going experimental efforts [1–3] to investigate the hard bremsstrahlung production in pp collisions as well as in providing some theoretical guidance to the future experiments in this area.

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APPENDIX A

In this appendix we will determine the coefficients a_{lL}, b_{lL} in Eq. (10).

Taking the scalar product of the last equality in Eq. (10) with $[\hat{p} \otimes \hat{n}_2]^2$ and $[\hat{k} \otimes \hat{n}_2]^2$, respectively, we have

$$\begin{aligned} 2r &= a_{lL} + \cos\theta_k b_{lL}, \\ 2t &= \cos\theta_k a_{lL} + b_{lL}, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} r &\equiv [Y_L(\hat{p}) \otimes Y_l(\hat{k})]^2 \cdot [\hat{p} \otimes \hat{n}_2]^2, \\ t &\equiv [Y_L(\hat{p}) \otimes Y_l(\hat{k})]^2 \cdot [\hat{k} \otimes \hat{n}_2]^2. \end{aligned} \tag{A2}$$

In order to arrive at Eq. (A1), we have also made use of the results

$$\begin{aligned} [\hat{p} \otimes \hat{n}_2]^2 \cdot [\hat{p} \otimes \hat{n}_2]^2 &= \tfrac{1}{2}, \\ [\hat{p} \otimes \hat{n}_2]^2 \cdot [\hat{k} \otimes \hat{n}_2]^2 &= \tfrac{1}{2}\cos\theta_k. \end{aligned} \tag{A3}$$

Equation (A1) can be readily inverted to yield

$$\begin{aligned} a_{lL} &= \frac{2}{\sin^2\theta_k}(r - t\cos\theta_k), \\ b_{lL} &= \frac{2}{\sin^2\theta_k}(t - r\cos\theta_k). \end{aligned} \tag{A4}$$

With the quantization axis $\hat{z}$ chosen along $\hat{p}$, the quantities r and t defined in Eq. (A2) can be expressed as

$$\begin{aligned} r &= -i\frac{1}{4\pi}\frac{[Ll]}{\sqrt{l(l+1)}}(L0l1|21)P_l^1(\hat{k}\cdot\hat{p}) \\ t &= i\frac{5\sqrt{2}}{4\pi}(-)^L\frac{[L]^2}{\sqrt{L(L+1)}}(10L0|l0)(11L-1|L0) \\ &\quad \times \begin{Bmatrix} 1 & L & L \\ l & 2 & 1 \end{Bmatrix} P_L^1(\hat{k}\cdot\hat{p}). \end{aligned} \tag{A5}$$

If we consider the partial wave states with $j \leqslant 2$, the following a_{lL} and b_{lL} are required:

$$\begin{aligned} a_{21} &= 0, \\ a_{23} &= -\frac{i}{4\pi}\frac{5}{2}\sqrt{15}\sin\theta_k\cos\theta_k, \\ b_{21} &= \frac{i}{4\pi}\sqrt{15}\sin\theta_k, \\ b_{23} &= \frac{i}{4\pi}\frac{\sqrt{15}}{2}\sin\theta_k. \end{aligned} \tag{A6}$$

APPENDIX B

Here, we give the spin matrix elements in terms of the multipole amplitudes with $j \leqslant 2$. Using the shorthand notation $M_{S\lambda}^{\lambda'} \equiv \langle 1\lambda'|\hat{M}(\vec{k},\vec{p}\,';\vec{p})|S\lambda\rangle$, and leaving out the common factor of $1/(4\pi)^{3/2}$ for the moment, we have, from Eq. (35),

$$\begin{aligned} M_{00}^{\pm1} &= d_{0,\pm1}^2(\theta_k)\sqrt{5}E_{202} = \pm\sqrt{\frac{3}{2}}\sqrt{5}E_{202}\sin\theta_k\cos\theta_k, \\ M_{10}^{\pm1} &= d_{0,\pm1}^1(\theta_k)(1010|10)\sqrt{3}E_{111} \\ &\quad \pm d_{0,\pm1}^2(\theta_k)\big[(1010|20)\sqrt{3}M_{211} + (3010|20)\sqrt{7}M_{213}\big] \\ &= \sqrt{\frac{3}{2}}\sqrt{2}\left[M_{211} - \sqrt{\frac{3}{2}}M_{213}\right]\sin\theta_k\cos\theta_k, \end{aligned} \tag{B1}$$

$$\begin{aligned} M_{1,1}^{\pm1} &= d_{1,\pm1}^1(\theta_k)(1011|11)\sqrt{3}E_{111} \\ &\quad \pm d_{1,\pm1}^2(\theta_k)\big[(1011|21)\sqrt{3}M_{211} + (3011|21)\sqrt{7}M_{213}\big] \\ &= -\frac{1}{2}\sqrt{\frac{3}{2}}E_{111}(1\pm\cos\theta_k) \\ &\quad + \frac{1}{2}\sqrt{\frac{3}{2}}\left[M_{211} + \sqrt{\frac{2}{3}}M_{213}\right](2\cos^2\theta_k \pm \cos\theta_k - 1), \end{aligned} \tag{B2}$$

$$\begin{aligned} M_{1,-1}^{\pm1} &= d_{1,\pm1}^1(\theta_k)(101-1|1-1)\sqrt{3}E_{111} \\ &\quad \pm d_{-1,\pm1}^2(\theta_k)\big[(101-1|2-1)\sqrt{3}M_{211} \\ &\quad + (301-1|2-1)\sqrt{7}M_{213}\big] \\ &= \frac{1}{2}\sqrt{\frac{3}{2}}E_{111}(1\mp\cos\theta_k) \\ &\quad - \frac{1}{2}\sqrt{\frac{3}{2}}\left[M_{211} + \sqrt{\frac{2}{3}}M_{213}\right](2\cos^2\theta_k \mp \cos\theta_k - 1). \end{aligned} \tag{B3}$$

The spin matrix elements $M^{\pm 1}_{S\lambda}$ given above are related to the spin matrix elements $M^{\perp/\parallel}_{SM_S}$ introduced in Sec. III by

$$
\begin{aligned}
M^{\pm 1}_{S\lambda} &= \mp \frac{1}{\sqrt{2}}\left[M^{\parallel}_{SM_S} \mp i M^{\perp}_{SM_S}\right],\\
M^{\parallel}_{SM_S} &= \frac{1}{\sqrt{2}}\left[M^{-1}_{S\lambda} - M^{+1}_{S\lambda}\right],\\
M^{\perp}_{SM_S} &= \frac{-i}{\sqrt{2}}\left[M^{-1}_{S\lambda} + M^{+1}_{S\lambda}\right],
\end{aligned} \tag{B4}
$$

with $\lambda = M_S$.

Using Eqs. (B1)–(B3), we then have

$$
\begin{aligned}
M^{\parallel}_{00} &= \sqrt{\frac{3}{2}}\sqrt{10}E_{202}\sin\theta_k\cos\theta_k,\\
M^{\perp}_{00} &= 0,
\end{aligned} \tag{B5}
$$

for the spin singlet → singlet transitions, and

$$
\begin{aligned}
M^{\parallel}_{10} &= 0,\\
M^{\perp}_{10} &= -2\sqrt{\frac{3}{2}}\left[M_{211} - \sqrt{\frac{3}{2}}M_{213}\right]\sin\theta_k\cos\theta_k,\\
M^{\parallel}_{1\pm} &= \frac{1}{\sqrt{2}}\sqrt{\frac{3}{2}}\left(E_{111} - \left[M_{211} + \sqrt{\frac{2}{3}}M_{213}\right]\right)\cos\theta_k,\\
M^{\perp}_{1\pm} &= \pm\frac{i}{\sqrt{2}}\sqrt{\frac{3}{2}}\left(E_{111} - \left[M_{211} + \sqrt{\frac{2}{3}}M_{213}\right]\cos\theta_k\right),
\end{aligned} \tag{B6}
$$

for spin triplet → singlet transition matrix elements. Note that here we see an explicit realization of the properties of the spin matrix elements $M^{\perp/\parallel}_{SM_S}$ exhibited in Eqs. (17), (18) in Sec. III.

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