β **decay of 81Zn and migrations of states observed near the** *N* **= 50 closed shell**

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The *β* decay of the $N = 51$ nucleus ⁸¹Zn was studied by means of β -*γ* spectroscopy and from isotopically pure beams produced at the Holifield Radioactive Ion Beam Facility (HRIBF). We observe several competing *β* transitions populating 81Ga that are interpreted as allowed Gamow-Teller decays to positive parity, core excited states, and first-forbidden decays to negative parity states. The measured *β*-decay pattern suggests an assignment of $I^{\pi} = 5/2^{+}$ for the ⁸¹Zn ground state. The systematics of core excited states in $N = 50$ isotones indicate a strengthening of the $N = 40$ subshell gap near ⁷⁸Ni.

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I. INTRODUCTION

The determination of nuclear properties near expected shell closures is critical for the description of nuclear structure and astrophysical processes involving atomic nuclei. The nucleus ⁷⁸Ni, with $Z = 28$ and $N = 50$, is expected to be a doubly magic nucleus since both N , $Z = 28$ and 50 are experimentally verified magic numbers in nuclei closer to stability.

This work addresses the strength of the 78 Ni core and effects of apparent single-particle level migration for nuclei with very asymmetric neutron-to-proton ratios. These properties are essential not only for understanding nuclear structure, but also for expanding our knowledge of nucleosynthesis processes [\[1\]](#page-8-0).

The apparent migration of single-particle levels was observed in the vicinity of ⁷⁸Ni along $Z = 28$ [\[2–4\]](#page-8-0) and along $N = 50$ lines [\[5,6\]](#page-8-0). The so-called monopole migration of $Z =$ 28 Ni isotopes was interpreted as a result of the interactions between protons and neutrons from the tensor force that can only occur far from stability $[7,8]$. Such migrations change the detailed microscopic properties of the affected nuclei, such as the ground-state spin switch of copper isotopes [\[2–4\]](#page-8-0). This could also introduce a more dramatic effect on the entire region if new shell gaps develop while others are quenched.

It was suggested in the recent work by Winger *et al.* [\[6\]](#page-8-0) that the evolution of excited states observed in the study of neutron-rich Ge isotopes $[6,9]$ is supported by the calculations in the Hartree-Fock-Bogoliubov (HFB) framework. These calculations predict the migration of the *ν*g_{9/2}, *ν*s_{1/2}, and *ν*d_{5/2} levels, leading to a reduction of the $N = 50$ shell gap and formation of a new $N = 58$ shell closure. Quenching of the $N = 50$ gap may offer a possible explanation for the hitherto unexplained phenomena in semimagic nickel isotopes (i.e., systematically lower than expected energies of the 2^+ excited

states $[10]$ and large $B(E2)$ values $[11]$). However, it will be demonstrated in this work that the shell model calculations can naturally explain the spectroscopic data without the need to invoke strong shell quenching effects.

This work investigates properties of the $N = 51$ nucleus ⁸¹Zn and the $N = 50$ isotone ⁸¹Ga, which is a semimagic nucleus with only three protons above the 78Ni core. The results presented here, combined with experimental systematics in the 78 Ni region, indicate that for neutron-rich nuclei the $N = 40$ neutron shell gap grows while the $N = 50$ shell gap diminishes, effectively separating out the *ν*g_{9/2} orbital. The origins of this apparent shell migration can be traced to proton-neutron interactions.

II. SINGLE-PARTICLE STATES NEAR 78Ni

The shell structure of $N > 50$ and $Z > 28$ nuclei near ⁷⁸Ni is determined by the occupation of neutron $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $h_{11/2}$ and proton $f_{7/2}$, $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbitals. The precise information about the ordering of single-particle levels is unknown because of apparent strong migrations of singleparticle states. For protons, extensive studies of $Z = 29$ copper isotopes established a reversal of the ordering of *p*3*/*² and *f*5*/*² states beyond 75 Cu with increasing $g_{9/2}$ neutron occupation [\[2–4\]](#page-8-0). This was interpreted as an effect of tensor interactions and was since quantified by shell-model calculations [\[12,13\]](#page-8-0). Observed modification of neutron states in $N = 51$ nuclei has a less apparent origin. Previous studies [\[5,6,14\]](#page-8-0) established a systematic decrease of the energy difference between the lowest $5/2^+$ and $1/2^+$ levels, attributed to the population of single-particle $d_{5/2}$ and $s_{1/2}$ levels. The measurement at the Holifield Radioactive Ion Beam Facility (HRIBF) of the ⁸³Ge

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states via (d, p) reactions and the decay of ⁸³Ga determined the gap to be at 247 keV [\[6\]](#page-8-0), smaller than the 461.9-keV spacing observed in 85Se [\[15\]](#page-8-0). The ground-state spin of these nuclei are assigned as $I^{\pi} = 5/2^{+}$, because of occupation of the neutron $d_{5/2}$ level [\[5\]](#page-8-0). Considering the case of the copper isotopes and taking an extrapolation of the $N = 51$ systematics, one may expect such a trend to continue leading to a decrease of energy between $5/2^+$ and $1/2^+$ states at ⁸¹Zn and ultimately a reversal of the $d_{5/2}$ and $s_{1/2}$ orbital ordering. Such a scenario was suggested previously in [\[16\]](#page-8-0), which postulated a 1*/*2⁺ ground-state spin assignment for ⁸¹Zn. One of the aims of this work is to verify if such a spin reversal occurs at 81 Zn. We will show that the experimental data for ⁸¹Zn do not support the spin-reversal hypothesis and our shell-model calculations reveal that a reversal is not to be expected because of the details of the proton-neutron interactions.

Although the study of the ground-state and low-lying excited states near closed-shell nuclei provide information on the single-particle states of the valence nucleons, observation of core excited states can be used to obtain information on the size of major shell gaps [\[17\]](#page-8-0). The systematic analysis of the yrast levels in $N = 50$ isotones [\[9,18\]](#page-8-0) led to claims of the weakening of the $N = 50$ shell closure of the ⁷⁸Ni core. These investigations analyzed properties of the yrast, high-spin core excitations dominated by the *g*9*/*² orbital. However, the selection rules of Gamow-Teller (GT) *β*-decay transitions imply that they will populate proton states in the πfp shell, leaving deeply bound neutron holes in the *νfp* shell. Such core excited states were previously observed in the decay of $N = 51$ isotones of ⁸³Ge and ⁸⁵Se [\[15,19\]](#page-8-0). Our results on $N = 51$, $Z = 30$ ⁸¹Zn, and on semimagic $N = 50$, $Z = 31$ 81Ga nuclei provide experimental information closer to doubly magic 78Ni.

III. EARLIER REPORTS ON 81Zn DECAY

There have been three previously published attempts to measure the decay properties of ${}^{81}Z$ n. The half-life of 0.29(5) s and *β*-delayed neutron branching ratio $p_n = 7.5(30)\%$ were reported by Kratz *et al.* [\[20\]](#page-8-0). The spectra obtained in a study at the PARRNe facility $[16,21]$ suffered from strong activities of isobaric contaminants 81Ga, 81Ge, and 81As present in the studied samples. Two γ transitions were interpreted as following the decay of $8^{17}Zn$, at 351 and 452 keV, and coincidences between them were observed. The assignment to the decay of $81Zn$ was based on the half-life of 391(65) ms derived for the 351.1-keV line. The levels at 351.1 and 802.8 keV in 81Ga were proposed because the 351.1-keV transition appeared to have larger *γ* intensity than the 452-keV transition. A third transition at 1621.6 keV was tentatively assigned as de-exciting a level of the same energy. In this work a p_n of 7(3)% was estimated from the β -neutron daughter 80Ga activity. A later experiment by Koester *et al.* presented the *β*-gated γ spectra of $A = 81$ activities measured later at ISOLDE-CERN using resonant laser ionization [\[22\]](#page-8-0). The two *γ* transitions at 351 and 452 keV were confirmed as following the decay of ⁸¹Zn. A lower limit of the ⁸¹Zn $p_n \ge 10\%$ was suggested and was based on the presence of 80 Ga activity in

the observed spectra. A recent measurement of the *β*-delayed neutron activity using a 3 He neutron counter by Hosmer and collaborators reports a p_n of 30(13)%, considerably higher than previous estimates [\[23\]](#page-8-0). A compilation of the latest experimental results in the nuclear data evaluation suggests an average value of 0.32(5) s for the ⁸¹Zn β -decay half-life [\[24\]](#page-8-0).

There is one previous in-beam study by deAngelis and collaborators, which for ${}^{81}Ga$ reports a single previously unobserved level at 1236 keV $[25]$. The authors assign a spin and parity of 9*/*2[−] by comparison with their shell-model calculation, which such spin and parity of this level would make it difficult to observe in *β*-decay measurements.

IV. EXPERIMENTAL SETUP

A beam of 81Zn ions was produced at HRIBF at Oak Ridge National Laboratory using the isotope separation online (ISOL) technique [\[26\]](#page-8-0). The radioactive species were produced on a high-voltage (160-kV) platform by proton-induced fission of a high-temperature UC_x target, transported as a vapor to a closely coupled ion source and extracted as positive ions at 40 keV. Mass 81 ions were then selected using the first-stage mass separator dipole magnet located on the high-voltage platform, accelerated off the platform to 200 keV, and transported to the high-resolution, second-stage mass separator. This high-resolution separator consists of a pair of large-radius, double-focusing dipole magnets with a design mass-resolving power of $m/\Delta m = 20000$ for performing isobaric separation. In practice, the resolving power is limited by the beam emittance and energy spread, which vary depending on the source being used and the charge state of the beam. The resolving power required to separate ⁸¹Zn from neighboring isobar ${}^{81}Ga$ is about 1:6603 [\[27,28\]](#page-8-0), based upon measured masses.

The radioactive beam was sent to the Low-Energy Radioactive Ion Beam Spectroscopy Station (LeRIBSS) for radiation measurements. The average rate of pure ⁸¹Zn beam measured at the decay station was about 30 particles per second (pps). This was about 50% of the initial 81 Zn rate measured in the beam contaminated with ⁸¹Ga ions. One should note that the optimum rate of ${}^{81}Ga$ is about 10⁶ pps and the HRIBF high-resolution magnet was able to suppress it to below 1 pps level. The mass separated ions were implanted onto a tape in the moving tape collector (MTC) in the middle of the *β*-*γ* counting setup. The MTC transported the collected samples about 50 cm away from the measuring station within 210 ms. During the ${}^{81}Zn$ study the MTC was operated in take-away mode, with 1-s grow-in followed by 1-s decay cycle, where the beam was deflected away during the decay cycle.

The γ and β radiations were measured with four high-purity Ge clover detectors and two plastic *β* scintillators surrounding the implantation point. The photo peak efficiency for the closely packed geometry of the clover array is 34% at 81 keV and 6% at 1.33 MeV. The photo peak efficiency for transitions above 4 MeV was calibrated relative to known transitions in ⁷⁹Zn decay. As the ⁸¹Zn γ transitions are weak compared to the natural *γ* -ray background, the relative intensities of transitions below 4 MeV presented in this work were obtained from the *β*-gated spectrum. This requires a good understanding of the *β* detection efficiency. The two plastic *β* scintillators are molded to fit the 0.5-mm thin aluminum beam pipe from outside, covering nearly 4*π* solid angle. Their *β*-particle detection efficiency was established in a previous experiment at LeRIBSS [\[4\]](#page-8-0) as a function of the effective *Q* value, *Q*eff, for a given β -decay transition, ranging from 35% at $Q_{\text{eff}} = 4.5$ MeV to 60% at $Q_{\text{eff}} = 7$ MeV. The effective Q value is defined as the weighted average of the *Q* values to the different states connected by γ transitions to the parent level of the transition in question (for more details see Ref. [\[4\]](#page-8-0)).

V. DECAY SCHEME OF 81Zn

Eleven γ transitions observed in the β -gated spectrum were assigned to the decay of ${}^{81}Zn$ by measuring their half-life during the decay cycle (see Table I and Fig. [1\)](#page-3-0). We estimated the ${}^{81}Zn \beta$ -decay half-life from the growth and decay curves of the most intense transition at 351 keV, obtaining 304(13) ms. This value is close to error bar limits of the previous measurements [\[16,20\]](#page-8-0), but the improved statistics allowed us to reduce the experimental uncertainty. We do not confirm the reported 1621.6-keV transition as belonging to the ${}^{81}Zn$ decay [\[16\]](#page-8-0). We notice that this energy region is contaminated by the 1620.5(1)-keV line from ²¹²Bi decay [\[29\]](#page-8-0). The γ - γ coincidences allowed us to place all 11 transitions in the decay scheme (see Fig. [2\)](#page-4-0). The spin assignments shown in Fig. [2](#page-4-0) follow shell-model calculations of the excited states in ${}^{81}Ga$ (see next section for details). Our work confirms the observation and placement of the 351.1- and 451.7-keV transitions $[16,21,22]$ in the decay scheme. In addition to the 351.1- and 451.7-keV γ lines, we observed a crossover 802-keV transition, confirming the excited state in 81Ga at the same energy. Analysis of the intensities and detection efficiencies excluded the possibility that the 802-keV *γ* line was from *γ* -*γ* summing in the germanium crystal. According to the spin assignments shown in Fig. [2,](#page-4-0) the 802 keV would be an M1 transition strongly hindered because of nuclear structure effects, as known for the classic case of 51 V, which has three

TABLE I. Gamma transitions in 81 Ga populated in the decay of 81 Zn. All transitions have half-lives compatible with the decay of 81 Zn and the transitions observed in γ - γ coincidences are marked in the last column.

E (keV)	I_{γ (rel) $\%$	ν - ν
350.9(3)	100(4)	451, 916, 1107, 1585, 2358
451.6(4)	19(3)	351
802.2(7)	6(3)	
916.2(8)	5(3)	351
1107.6(9)	4(3)	351
1458.3(12)	4(2)	
1585.4(13)	9(5)	351
1936.3(17)	10(6)	
2358.4(20)	17(7)	351, 1585
4294.0(40)	13(4)	
4880.0(42)	19(5)	

protons above a double closed shell [\[30\]](#page-8-0). A similar hindrance effect is observed for the *γ* transitions in the $N = 50^{83}$ As [\[19\]](#page-8-0), with five protons above the 78 Ni core.

The branching ratios presented here (see Fig. [2\)](#page-4-0) only account for the intensity directly observed in this experiment. The large direct β feeding to the ground state of ⁸¹Ga, 52(7)%, is determined through the analysis of the ${}^{81}Ga$ to ${}^{81}Ge$ decays in the measured spectra and observing *γ* lines feeding the 81Ga ground state. The 81Ga decay data, including the absolute intensity of the 216-keV line, were taken from Ref. [\[31\]](#page-8-0). To obtain a meaningful value for the ${}^{81}Zn$ ground-state branching ratio it is necessary to determine the fraction of the 81Ga activity contaminating the 81Zn beam. We have studied the growth and decay curves of the ${}^{81}Ga$ 216-keV transition using the full solution of the Bateman equations [see Figs. $3(a)$ and $3(b)$]. The ⁸¹Ga production in the decay of ⁸¹Zn as well as direct contribution of the beam component were considered. The Bateman equations were fit to the data and the ⁸¹Zn and ⁸¹Ga implantation rates were left as free parameters. Our measured half-life of 81 Zn, 304(13) ms, and the half-life of 81 Ga, 1.217(5) s, from literature [\[31\]](#page-8-0) were used as constants. The best fit was found for no ${}^{81}Ga$ in the implanted beam, with an upper limit of 5% of the 81Zn implantation rate at three standard deviations from the minimum.

A fraction of the β intensity might go to unobserved states at high energy. Inspection of the β decay of $N = 51$ ⁸³Ge into $N = 50$ ⁸³As shows that 13 states are populated at energies greater than 3.5 MeV, with two of them at 4.13 and 4.43 MeV whose branching ratios are significantly higher than the others [\[19\]](#page-8-0). The total intensity observed to states above 3.5 MeV is large, 29.3%, with the two states of highest feeding taking 9.4%. Only two states above 4 MeV are observed in this work in the decay of 81Zn. Direct comparison to the decay of 83As allows us to match these two states with the two states of highest β feeding in ⁸³As. Thus, we estimate an upper limit of 20% for the unobserved *β* feeding in this experiment.

The large branching ratio and corresponding $log(f t) = 5.5$ for the ground state β feeding indicate the β transition is either allowed or first-forbidden nonunique. The lower limit of the 81Zn ground-state branching ratio, assuming up to 20% missing feeding to high-energy states, is estimated as 40(6)%. The corresponding $log(f t)$ value is about 5.6, again in the range of both allowed and first-forbidden nonunique transitions. Although most likely caused by a forbidden type transformation, a large ground-state branching ratio is not unusual in this region. An $I_\beta = 28\%$ was observed in the ν 5/2⁺ → π 3/2⁻ decay of ⁸⁵Se [\[15\]](#page-8-0). An even larger value of $I_\beta = 50\%$ led the authors to suggest that decay of $\frac{85}{3}$ As occurs between the $v5/2^+ \rightarrow \pi 5/2^-$ single-particle states [\[32\]](#page-8-0).

The neutron branching ratio, $P_n = 12(4)\%$, was determined using the known β -decay pattern of the ⁸¹Zn delayed neutron daughter 80Ga. As some transitions in 80Ge are also populated in the ${}^{81}Ga$ *β*-delayed neutron channel [\[33\]](#page-8-0), we used the 1083-keV transition in 80 Ge, with 48(9)% absolute intensity [\[31\]](#page-8-0), which was not observed in the *β*-delayed neutron channel. A value of the neutron branching ratio, $P_n = 10(3)\%$, can be estimated assuming up to 20% unobserved feeding as discussed earlier.

FIG. 1. The *γ* spectrum for 81Zn decay observed in this work. The two upper panels show the spectrum in coincidence with *β* decays. The lower panel corresponds to the singles γ spectrum in add-back mode, for increased photo-peak efficiency at high energy. The transitions assigned to the decay of ⁸¹Zn are marked by numbers, with the single escape peak (SE) and double escape peak (DE) marked where appropriate. The black triangles indicate decays of ⁸¹Ga. The stars indicate transitions from background contamination. The open triangles mark transitions in 80 Ge. The 4880-keV single escape area contains 40(10)% of intensity that may be attributed to ${}^{81}Zn$ or ${}^{81}Ga$. This would correspond to 1.3% branching ratio and because of the uncertainty it was not added to the level scheme.

A recent theoretical calculation of the *β*-decay rates including both allowed and forbidden transitions predicts a $β$ -n branching ratio of 13% for ⁸¹Zn [\[34\]](#page-8-0), close to the value obtained in this work.

Our neutron branching ratio result is an intermediate value between the $P_n(^{81}Zn)$ value of 7.5(30)% quoted earlier [\[20\]](#page-8-0), and the result of $30(13)\%$ reported in [\[23\]](#page-8-0). Similar discrepancies were found earlier for *β*-n emitters in the ⁷⁸Ni region; see Ref. [\[6\]](#page-8-0). The lower limit of 10% for this *Pn* value postulated in Ref. [\[22\]](#page-8-0) is in agreement with our measurement.

A. Nuclear structure and decay discussion

The $N = 51$ isotope ⁸¹Zn has one neutron above the $N =$ 50 closed shell. Presently, its decay marks the nearest approach to the decay of 79 Ni, a nucleus with one neutron added to

the doubly magic core. The ⁷⁹Ni \rightarrow ⁷⁹Cu decay is not yet experimentally accessible for spectroscopic studies. Within the single-particle picture allowed Gamow-Teller β decay of ⁸¹Zn proceeds via transformation of one of the core neutrons ($p_{1/2}$, $p_{3/2}$, $f_{7/2}$) into a proton on a spin-orbit partner orbital ($p_{3/2}$, $p_{1/2}, f_{5/2}$). The spin flip transformation of the valence $N = 51$ neutron in the ⁸¹Zn ground state, the $s_{1/2}$ or $d_{5/2}$ neutron, is energetically unavailable. Therefore, the states populated in the daughter decay should be at high excitation energy because of the creation of a vacancy in a strongly bound neutron state. Such a scenario was observed by Winger *et al.* in the decay of $Z = 32$, $N = 51$ isotone ⁸³Ge [\[19\]](#page-8-0). Since Gamow-Teller transformations populate highly excited states, the branching ratios are quenched by the strongly decreasing values of the Fermi integral (f) . Therefore, a competitive decay path via forbidden *β* transformations has to be considered and in

FIG. 2. Level scheme of 81Ga, populated in the *β* decay of 81Zn. The placement of *γ* transitions in this level scheme are supported by both coincidence data and half-life measurements. Relative intensities of each *γ* transition, shown above each arrow, are normalized to the 351-keV transition. Solid circles mark transitions with observed coincidences. There is weak evidence of a state at 4368 keV of 1.3% branching ratio and because of the uncertainty it was not added to the level scheme. See Fig. [1](#page-3-0) for more details.

fact was observed for nuclei in this region [\[6,19,35\]](#page-8-0). The empirical selection rules for the forbidden decays [\[36\]](#page-8-0) reveal a dependence of the log *f t* values on the spin and parity changes occurring between states and can be used to establish spins and parities in mother or daughter nuclei.

The systematics in the $N = 51$ isotones and nuclear models do not give a very clear indication for the ground-state spin of ${}^{81}Z$ n. The simple extrapolations from trends observed in this region may result in a spin of 1*/*2+, which was favored by Verney *et al.* [\[16\]](#page-8-0). The calculations of single-particle energies in the HFB framework presented in the recent work by Winger [\[6\]](#page-8-0) and the $N = 51$ systematics [\[5\]](#page-8-0) support a $5/2^+$ assignment for 81Zn ground state with crossing predicted for lighter nuclei. Both possible spin assignments of $1/2^+$ and $5/2^+$ are of positive parity, which has a significant consequence for the decay pattern. In the case of the ground-state spin of ${}^{81}Ga$ the simple extrapolations from the behavior of neutron-rich copper isotopes favors the 5*/*2[−] assignments because of the observed monopole migration of single-particle orbitals. The $I = 5/2$ assignment was recently confirmed for the ⁸¹Ga ground state by Cheal *et al.* [\[37\]](#page-8-0).

Moreover, our shell-model calculations predict the ground state of ⁸¹Ga to have $I^{\pi} = 5/2^-$. These calculations utilize the residual interaction developed by Lisetskiy *et al.* [\[12\]](#page-8-0) for the fp g orbitals. They also use a ⁵⁶Ni core and take into account systematics in the region such as the *f*5*/*2-*p*3*/*² migration. The same calculations have been tested to correctly predict the energy spacing between the 3*/*2[−] and 5*/*2[−] excited states for the odd-Z nuclei $N = 48^{79}$ Ga and $N = 50^{83}$ As.

Several decay scenarios have to be considered in the decay of ${}^{81}Zn$ from the possible single-particle configurations of ${}^{81}Zn$ and 81Ga ground states: first forbidden decays of 5*/*2⁺ to 5*/*2−, 5*/*2⁺ to 3*/*2−, and 1*/*2⁺ to 3*/*2[−] with log *ft >* 5*.*1 or first forbidden unique decay $1/2^+$ to $5/2^-$ with log $ft > 7.5$. As mentioned earlier, there is strong evidence of a 5*/*2[−] spin and parity assignment for the ground state of ${}^{81}Ga$. This knowledge enables us to use β decay to suggest the ground-state spin of 81 Zn as two clearly distinct possibilities remain. They are the first forbidden decay in the case of $5/2^+$ and first forbidden unique for $1/2^+$.

Previous study of $81Zn$ decay [\[16\]](#page-8-0) suggested that the ground-state spin of ${}^{81}Zn$ is $1/2^+$ with the dominant

FIG. 3. (Color online) (Top) Growth and decay curve of the $81Zn$ *β*-decay 351-keV transition with the fit to the Bateman equations superimposed in red. (Bottom) Growth and decay curve of the ⁸¹Ga *β*-decay 216-keV transition with the fit to the Bateman equations superimposed in red. The Bateman equations were fitted to include direct implantation rate from the beam. The best fit was obtained for zero 81Ga implantation rate, with a maximum of 5% of the 81Zn implantation rate at three standard deviations.

component of *νs*1*/*2. This assignment was based on the argument that two states with postulated spin and parity 3*/*2[−] in 81 Ga were populated. However, this argument does not take into account that the 3*/*2[−] states in 81Ga should be similarly populated through forbidden transitions in the decay of 1*/*2⁺ or $5/2^+$ states. In [\[16\]](#page-8-0) the decay of 81 Zn was observed in the presence of very strong contamination from ${}^{81}Ga$ decay, therefore the ground-state feeding could not be derived in their work. Our work is based on data obtained with an isotopically pure beam of 81Zn, which allowed us to estimate the direct *β* population of the ground state and excited states. The apparent $β$ branching ratio to the ground state is 52(7)% (see Fig. [2\)](#page-4-0) and it is at least 40% of the total taking into account up to 20% potentially missing *β* intensity from the comparison to the ⁸³Ge decay. A *β* transition from a $1/2^+$ mother state to a 5*/*2[−] daughter state is by spin and parity selection rules a first-forbidden unique transition. The lowest value of the log *f t* of first-forbidden unique transitions in the literature is above seven [\[36\]](#page-8-0), and a transition with such high log *f t* value would have negligible feeding (I_β < 0.1%) to the 5/2⁻ ground state. The apparent log $ft = 5.5$ measured here is much smaller than that of first-forbidden unique values. Given this large branching ratio and the ${}^{81}Ga$ ground-state configuration as

 $I^{\pi} = 5/2^-$ (compare [\[3,4,6,37\]](#page-8-0)) we conclude that the ground state of 81 Zn must be $5/2^+$.

There might be undetected, weak *γ* transitions depopulating high-lying states closer to the neutron separation energy and their absence in our data may in effect decrease our apparent ground-state *β* branching ratio. Only more sensitive experiments in the future can reveal these weak *γ* transitions and improve our ground-state *β* branching ratio measurements. However, the comparison with 83Ge decay mentioned earlier shows that such an effect is very unlikely to change the log *f t* value to a level that would change our spin assignment.

To further understand the ${}^{81}Zn$ ground-state spin-parity assignment we used shell-model analysis [\[38\]](#page-8-0) to study the evolution of the $1/2^+$ first excited state relative to the first $5/2^+$ state in $N = 51$ isotones. Within the shell-model description the $N = 51$ isotones have the following valence nucleons in ⁸¹Zn: one $d_{5/2}$ or $s_{1/2}$ neutron above the $N = 50$ gap and two of the $f_{5/2}p_{3/2}p_{1/2}$ protons above the $Z = 28$ gap. We performed our calculations with a closed 78 Ni core and the N3LO [\[38,39\]](#page-8-0) nucleon-nucleon interactions in the $\pi f_{5/2} p_{3/2} p_{1/2} g_{9/2}$ and $\nu d_{5/2} s_{1/2} g_{7/2} d_{3/2} h_{11/2}$ shells. Figure 4 shows the experimental energies of the first excited state in $N = 51$ isotones (red triangles) compared to the results of our shell-model calculation (black squares). The calculation shows a parabolic behavior, with lower excitation energy until $Z = 34$ (⁸⁵Se) followed by increasing excitation energy of the same trend observed experimentally. The sharp change in the excitation energy evolution occurs in our model in ⁸⁵Se (the last $\pi f_{5/2}$ nucleus), suggesting that the evolution of the first excited state is driven by the proton occupation number, which goes from an attractive proton-neutron interaction between the $\pi f_{5/2} - v s_{1/2}$ shells

FIG. 4. (Color online) The red triangles show the observed energies of the $1/2^+$ first excited state in $N = 51$ isotones [\[5,6,19\]](#page-8-0). The black squares correspond to our shell-model calculation using the full $\pi f_{5/2} p_{3/2} p_{1/2} g_{9/2}$ and $\nu d_{5/2} s_{1/2} g_{7/2} d_{3/2} h_{11/2}$ shells. The overall behavior of the calculation matches well the experimental trend except for 83 Ge and 85 Se, and predicts increased $1/2^+$ energy for *Z <* 34 isotones. To investigate the effect of configuration mixing in the proton valence shell we performed a shell-model calculation where the proton valence shell was truncated to the $f_{5/2}$ orbital. Then the shell model correctly predicts the minimum at $Z = 32$. The green points represent the $5/2^+$ ground state in the $N = 51$ isotones.

to a repulsive interaction between $\pi p_{3/2} - v s_{1/2}$ for $Z > 34$ nuclei.

However, the shell model predicts the minimum 1*/*2⁺ excitation energy to occur in 85 Se, compared to the lower excitation energy observed in 83 Ge [\[15,19\]](#page-8-0). In our calculation we see that the ground state and first excited state proton wave functions in 83 Ge and 85 Se are highly mixed, with up to 30% $\pi p_{3/2}$ component. To investigate the effect of configuration mixing we performed an auxiliary shell-model study where the proton valence space was truncated to the $f_{5/2}$ shell. Here the first excited state of 83 Ge is closer to the ground state than in the ⁸⁵Se case, the same behavior observed experimentally.

This suggests that the shell-model calculation, with the current proton-proton interaction, overestimates *fp* configuration mixing for 83 Ge and 85 Se, which results in the $1/2^+$ state to be lower in ⁸⁵Se than in ⁸³Ge. From these observations we can conclude that the overall evolution of the first excited state energy is determined from the proton-neutron interaction and thus from the proton occupation number. Therefore, our shellmodel analysis supports the 5*/*2⁺ ground-state assignment derived from the *β*-decay branching ratio study and predicts an increase of the energy of the first excited state relative to that of 83 Ge. It does not indicate a decrease as simple extrapolations would imply. This suggests that none of the $N = 51$ isotones will have a $1/2^+$ ground state and it is likely that the excitation energy of this level is lowest in 83Ge at 247 keV [\[6\]](#page-8-0).

The ${}^{81}Zn$ nucleus decays to ${}^{81}Ga$, a nucleus with three protons above $Z = 28$ and a magic number of 50 neutrons. The allowed β decay of ⁸¹Zn will connect positive parity states via Gamow-Teller transformation of the 5*/*2⁺ ground state with $3/2^{+}$ -7/2⁺ excited states in ⁸¹Ga. These decays occur via conversion of a neutron into a proton spin-orbit partner. The relevant *fp* shell nucleons are below *N* = 50 for neutrons and above $Z = 28$ for protons. Therefore, the respective particlehole configurations in ⁸¹Ga will have high excitation energy. This energy directly carries information about the size of the shell gaps in the ($N = 50$, $Z = 28$) ⁷⁸Ni core.

The core excited states were observed and discussed by Winger *et al.* [\[19\]](#page-8-0) in the context of the decay of ⁸³Ge. Most of the observed decay strength went through strong β transitions to highly excited states in ⁸³As, at about 4.2 MeV (see Fig. 4 in Ref. [\[19\]](#page-8-0)). The high *β* strength deduced for observed transitions was interpreted as a signature of Gamow-Teller transformation, while the states at low excitations were fed via forbidden β decays. We expect the decay pattern of the positive parity $81Zn$ to exhibit a similar pattern, where forbidden *β* decays mainly populate low-energy states and allowed GT transitions populate states above 3 MeV. The observed branching ratios will be comparable because forbidden decays will be enhanced by the large phase space factor (or Fermi integral), while large transition matrix elements for allowed GT transitions compensate for smaller decay energy.

We used a shell-model calculation $[12]$ to make predictions for the low-energy excited states in ${}^{81}Ga$. All states below 3.9 MeV in ${}^{81}Ga$ are of negative parity according to the shell-model calculation. These states are created by various couplings of protons in the *fp* shell. The lowest positive parity

state, a $9/2^+$ at about 4-MeV excitation energy, is because of promotion of the valence proton into the *g*9*/*² orbital and will not be populated significantly in the β decay of the $5/2^+$ state. Other positive parity states are placed around 5.5 MeV. Therefore, according to these predictions any *β* transitions from the ground state of the parent to low-lying states in the daughter must be classified as first forbidden because of the change in parity between initial and final states. Unfortunately, within this shell model it is impossible to generate positive parity states with large B(GT) because these calculations do not include core excitation of neutrons into $d_{5/2}$ or $s_{1/2}$ orbitals.

The group of states in ${}^{81}Ga$ de-exciting via the high-energy *γ* transitions at 4 MeV is populated with log *f t* values compatible with allowed Gamow-Teller transitions [\[36\]](#page-8-0). Therefore, we interpret these states as particle-hole excitations in the $\nu p_{3/2}$ $p_{1/2}$ $f_{5/2}$ single-particle states of the ⁷⁸Ni core. The distribution of the Gamow-Teller strength above 4 MeV is similar to the patterns previously seen for core excited states populated in the 83 Ge [\[19\]](#page-8-0) and 85 Se decays [\[15\]](#page-8-0).

A deeper insight into systematic effects can be achieved while inspecting the details of the populations of the core excited levels in $N = 50$ nuclei. To obtain a parameter of the strength distribution we calculated a position of its centroid (weighted average of energy distribution) for the states observed above 3 MeV in the $N = 50$ isotopes ⁸¹Ga, 83 As, and 85 Br. These centroid positions are plotted in Fig. 5. We observe that the position of the centroid is shifted upward upon approaching 78 Ni, gaining 370 keV when each proton pair is removed. This may be indicative that the *νp*3*/*2*,*1*/*² hole states, populated in the transformation of a *p* neutron into a *p* proton, become more deeply bound with respect to the available configurations in the *ds* shell with decreasing number

FIG. 5. (Color online) Distribution of the core excited states in $N = 50$ isotones. The systematics for the odd-Z isotones have been obtained in *β*-decay studies. The states populated with the largest B(GT) values are shown in black lines. The circles show the evolution of the centroid of the B(GT) distribution. The systematics for even-Z isotopes are taken from [\[9\]](#page-8-0) and shows excitations of the 6^+ states (green). The red dot is a linear extrapolation for the core excitation of 79Cu.

of protons. This interpretation assumes that the single-particle energies of proton orbitals remain unchanged in the isotonic chain. This result has to be put in perspective of previous measurements of the yrast states [\[9,18\]](#page-8-0). The experimental systematics show a decrease of the excitations of the $5⁺$ and 6⁺ states. These states were interpreted as *νg*9*/*² dominated particle-hole configurations and are also plotted in Fig. [5.](#page-6-0) For these states the opposite trend is observed, the energies of core excited states drop by about 390 keV per removed proton pair. This was interpreted by Rzaca-Urban *et al.* [\[9\]](#page-8-0) as an effect of the decrease of the $N = 50$ shell gap [i.e., decrease of the spacing between neutron $g_{9/2}$ orbital and $d_{5/2}$ (or $s_{1/2}$)]. This trend is opposite to what we have observed for the low-spin core excited states populated in *β* decays. Effectively, the systematics presented in this work indicate the $N = 50$ shell gap may shrink by 1.1 MeV between $Z = 34$ and $Z = 28$ while the $N = 40$ subshell grows by a similar amount.

While the evolution of the core excited states could be a manifestation of the changes in the shell gap, this effect could be also traced to the proton-particle and neutron-hole configuration changes along the $N = 50$ isotones caused by the residual interactions. The shell model predicts that the excitation of negative parity states for $N = 50$ nuclei will increase for the lighter nuclei. This may be a simpler explanation for the apparent increase of the energy of the core excitations. However, a similar feature of increased energies should have been observed for the yrast states seen in Refs. [\[9,18\]](#page-8-0).

B. Predictions for the decay of 79Ni

The systematics presented here allow us to predict some features of the β decay of ⁷⁹Ni. The estimated Q_{β} value for 79 Ni is about 14 MeV [\[40\]](#page-8-0), and a neutron separation energy in 79 Cu daughter is about 5.65(64) MeV [\[41\]](#page-8-0).

We estimate the position of the core-excited states in ${}^{79}Cu$ at around 5 MeV using the systematics presented in this work, a 185-keV increase of the centroid for each proton removed. This implies these are largely neutron bound states and will be de-excited with the emission of high-energy *γ* rays.

Because the ground state of 79 Ni will be $5/2^+$, the population pattern might be very similar to the one observed in the decay of ${}^{81}Zn$, with very strong ground-state population. According to shell-model calculations [\[12\]](#page-8-0) the low-energy spectrum will consist of transitions between the lowest singleparticle proton states in the fp shell with energies $E = 0$ MeV for the $5/2^-$ state, $E = 1.5$ MeV for $3/2^-$, and $E = 2.9$ MeV for the 1*/*2−. The 9*/*2⁺ *g*9*/*² orbital at 6 MeV is not likely to be directly populated. This decay can be studied most effectively with a very high efficiency array even at the expense of the energy resolution using a total absorption spectrometer. It is also interesting to look at the properties of the excited states of the 79Ni. From our shell-model calculation we predict the first excited state to have an *s*1*/*² single-particle configuration and have excitation energy of 1.3 MeV. This level might be close to the neutron separation energy or even unbound according to some mass formulas [\[40\]](#page-8-0), making it possibly the

only particle bound excited state in 79 Ni, which may have interesting astrophysical consequences [\[1,5\]](#page-8-0).

VI. CONCLUSIONS AND SUMMARY

We have produced a pure ${}^{81}Zn$ beam with about 30 pps by means of the ISOL technique at the Holifield Radioactive Ion Beam Facility and studied *β* decay at the Low Energy Radioactive Ion Beam Spectroscopy Station. The measured *β*-delayed *γ* and *β*-delayed neutron emission from ⁸¹Zn allowed us to construct the decay scheme with nine new *γ* transitions added to the two known ones $[16,22]$. The large $β$ branching ratio, around 50%, to the ground state of ⁸¹Ga deduced in this work corresponds to a ground-state transition with an approximate log $ft = 5.5$ that can only be classified as allowed or first forbidden. As the ground state of ${}^{81}Ga$ is most likely 5*/*2−, from the systematics [\[3,4,6\]](#page-8-0) and a magnetic moment measurement [\[37\]](#page-8-0), an assignment of $1/2^+$ for the ground state of ${}^{81}Zn$ [\[16\]](#page-8-0) is impossible for a transition of this high strength. Thus, we conclude that the I^{π} of the ⁸¹Zn ground state is $5/2^+$. An expected $s_{1/2}$ and $d_{5/2}$ level crossing was not observed in $81Zn$. This is in agreement with shell-model calculations, which in fact predict a rapid increase of the excitation energy of the $1/2^+$ state. A highly excited $s_{1/2}$ state may be even unbound in ⁷⁹Ni, which might be of consequence to neutron capture cross sections in the r process.

The excitation energies of the levels identified in $N = 50$ 81Ga as particle-hole excitations in the 78Ni core provide evidence for a strong neutron shell closure in contradiction to the assertions made in Refs. [\[9,18\]](#page-8-0).

The systematics of the B(GT) distributions of the core excited states suggest an increase of the gap between the *fp* and *sdg* shells. It could be explained as an effect of the development of a strong $N = 40$ shell gap near ⁷⁸Ni. This may offer an alternative explanation for the structure phenomena observed in the ⁷⁸Ni region revealed through 2^+ energies and B(E2) systematics, which are broadly assigned to "core excitation" effects. Our ⁸¹Zn measurement marks the closest approach to the decay of the single valence neutron nucleus 79 Ni, which will populate states in the single valence proton nucleus 79Cu. Spectroscopic study of the 79Ni *β* decay will be valuable because it will not only probe the single-particle excitations in $\frac{79}{2}$ Cu, but also explore the robustness of the 78 Ni shell closure. It may also provide a sensitive test for the operators responsible for the forbidden *β* transitions.

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