Nuclear symmetry energy at subnormal densities from measured nuclear masses

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The symmetry energy coefficients for nuclei with mass number $A = 20$–250 are extracted from more than 2000 measured nuclear masses. With the semiempirical connection between the symmetry energy coefficients of finite nuclei and the nuclear symmetry energy at reference densities, we investigate the density dependence of the symmetry energy of nuclear matter at subnormal densities. The obtained results are compared with those extracted from other methods.

I. INTRODUCTION

The nuclear symmetry energy $e_{sym}(\rho)$, which represents the energy cost per nucleon to convert all the protons to neutrons in symmetric nuclear matter, has attracted lots of attention in astrophysics and nuclear physics, because it intimately relates to a wealth of astrophysical phenomena, the structure character of nuclei, and the dynamical process of nuclear reactions. The density dependence is a key point in the study of symmetry energy. Many theoretical and experimental efforts have been paid to constrain the density dependence of the symmetry energy [1–3]. Presently, some constraints on the symmetry energy at subnormal densities have been made from the double n/p ratio and isospin diffusion in intermediate energy heavy-ion collisions of isospin asymmetric nuclei [1,2,4] and from the nuclear properties such as the thickness of the neutron skin and the binding energy of finite nuclei [5–8]. The uncertainty of the symmetry energy coefficient and the density dependence of symmetry energy at subnormal densities is still large, and more study is still needed.

We try in this work to constrain the symmetry energy from more than 2000 precisely measured nuclear masses. By directly fitting the measured nuclear masses with the liquid drop mass formula, one can obtain the symmetry energy coefficients of nuclei in which both volume and surface terms are included [9–11]. It is known that the symmetry energy coefficients of finite nuclei a_{sym} are considerably smaller than that of the infinite nuclear matter due to the influence of the surface region of nuclei. A semiempirical connection between the symmetry energy of nuclear matter at reference density and the properties of finite nuclei was proposed in Ref. [12]. More recently, the relation $a_{sym}(A) = e_{sym}(\rho_A)$ between the symmetry energy coefficients of finite nuclei and the symmetry energy of nuclear matter at reference density was proposed in Ref. [5]. For ^{208}Pb, the reference density has a value of about $\rho_{208} \simeq 0.1$ fm^{-3}. The relation provides a possible way

to explore the property of nuclear matter from the property of finite nuclei. Combining this relation and the symmetry energy coefficients of finite nuclei extracted from the measured nuclear masses, we investigate the density dependence of nuclear symmetry energy at subnormal densities.

The paper is organized as follows. In Sec. II, we determine the symmetry energy coefficients for nuclei with mass number $A = 20$–250 by analyzing the measured nuclear masses [13]. In Sec. III, we constrain the nuclear symmetry energy at subnormal density with the relation $a_{sym}(A) = e_{sym}(\rho_A)$, and we compare our results with those obtained by using other approaches. The reference density as a function of nuclear mass number is also deduced. A short summary is given in Sec. IV.

II. SYMMETRY ENERGY COEFFICIENTS OF FINITE NUCLEI

In Ref. [14], the symmetry energy coefficients of finite nuclei were studied. The energy per particle $e(A, I)$ of a nucleus can be expressed as a function of mass number A and isospin asymmetry $I = (N - Z)/A$ according to the Weizsäcker nuclear energy formula:

$$e(A, I) = a_v + a_s A^{-1/3} + e_{Coul}(A, I) + a_{sym}(A)I^2$$
$$+ a_p A^{-3/2}\Delta_{np} + e_w, \quad (1)$$

with

$$\Delta_{np} = \begin{cases} 1 & \text{for even-even nuclei,} \\ 0 & \text{for odd-}A \text{ nuclei,} \\ -1 & \text{for odd-odd nuclei.} \end{cases} \quad (2)$$

Here a small correction term (i.e., the Wigner term e_w) is introduced for a better description of the systematic behavior in the symmetry energy coefficients of nuclei. The a_v, a_s, and a_p denote the coefficients of the volume, the surface, and the pairing term, respectively. Subtracting the Coulomb term and the Wigner term from the energy per particle, one obtains

$$e_m(A, I) = e(A, I) - e_{Coul}(A, I) - e_w$$
$$= e_0(A) + a_{sym}(A)I^2. \quad (3)$$

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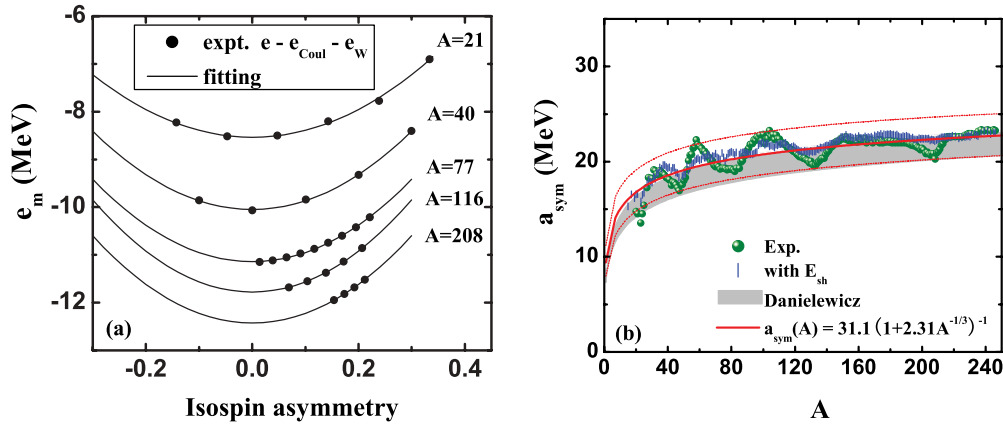†lizwux@ciae.ac.cn
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FIG. 1. (Color online) (a) Value of e_m as a function of isospin asymmetry for selected series of isobaric nuclei. The solid circles and the solid curves denote the experimental data and the results by fitting the experimental data, respectively. (b) Symmetry energy coefficients of nuclei as a function of mass number. The solid circles denote the extracted results from the measured nuclear masses. The shading denotes the results of Danielewicz *et al.* [9]. The vertical short dashes denote the results when the shell corrections of nuclei from Ref. [10] are removed from the measured energy per particle. The (red) curves denote the results by fitting the circles.

Here we assume that the a_v and a_s terms are independent of isospin asymmetry and can be rewritten as $e_0(A)$. For the Coulomb energy, we take the same form as in Ref. [14]. For the Wigner term $e_w = E_w/A$, we take the form as in Ref. [15],

$$E_w = -C_0 \exp(-W|I|/C_0), \qquad (4)$$

with two parameters $C_0 = -10\,\text{MeV}$ and $W = 42\,\text{MeV}$, which is a direct consequence of the independent-particle model. We checked that the obtained symmetry energy coefficients of finite nuclei do not change appreciably by varying the parameter W in a reasonable region of 42–47 MeV [16].

Now we can take $e_0(A)$ and $a_{\text{sym}}(A)$ as parameters and perform a two-parameter parabola fitting to the $e_m(A, I)$ for each series nuclei with the same mass number A. The experimental $e(A, I)$ data are taken from the mass table AME2003 [13]. For nuclei with even mass number, only even-even nuclei are taken into account in our calculations to consider the pairing effects. All available isobaric nuclei with mass number $A = 20$–250 are considered in the calculations. Figure 1(a) shows the values of e_m as a function of isospin asymmetry for a series of isobaric nuclei with $A = 21, 40,$ 77, 116, and 208 as examples. The solid circles and the solid curves denote the experimental data and the results with two-parameter parabola fitting, respectively. The curvature of each curve gives the corresponding symmetry energy coefficient $a_{\text{sym}}(A)$ of the nuclei with mass number A. The extracted symmetry energy coefficients of finite nuclei as a function of mass number are shown in Fig. 1(b). The solid circles denote the extracted $a_{\text{sym}}(A)$ from the measured nuclear masses. The shading denotes the results of Danielewicz and Lee [9]. The vertical short dashes denote the results when the shell corrections of nuclei [10] are removed from the measured values of energy per particle. In the region $A < 120$, the values of $a_{\text{sym}}(A)$ obtained in our approach show some oscillations and fluctuations. For heavy nuclei, our results of $a_{\text{sym}}(A)$ are comparable with those of Danielewicz and Lee. When the shell corrections are taken into account, the fluctuations

in the extracted $a_{\text{sym}}(A)$ are reduced effectively. The mass dependence of the symmetry energy coefficients of nuclei is written by Danielewicz and Lee [9] as

$$a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1}, \qquad (5)$$

where S_0 is the volume symmetry energy coefficient of the nuclei [i.e. the nuclear symmetry energy at normal density $e_{\text{sym}}(\rho_0)$] and κ is the ratio of the surface symmetry coefficient to the volume symmetry coefficient. By performing a two-parameter fitting to the $a_{\text{sym}}(A)$ obtained previously [i.e., the solid circles in Fig. 1(b)], we can obtain the values of S_0 and κ. With 95% confidence intervals of S_0 and κ, we obtain $S_0 = 31.1 \pm 1.7$ MeV and $\kappa = 2.31 \pm 0.38$, respectively. The results of fitting the circles are shown in Fig. 1(b) by the (red) curves. The obtained value of S_0 is in good agreement with the range of $S_0 = 30.2$–33.8 MeV given by the pygmy dipole resonance (PDR) data [6].

III. NUCLEAR SYMMETRY ENERGY AT SUBNORMAL DENSITY

Now let us turn to study the density dependence of nuclear symmetry energy at subnormal densities. In experiments, recent research in intermediate energy heavy-ion collisions is consistent with a dependence at subnormal densities [2,4,17]:

$$e_{\text{sym}}(\rho) = S_0(\rho/\rho_0)^\gamma. \qquad (6)$$

With the relation

$$e_{\text{sym}}(\rho_A) = a_{\text{sym}}(A) \qquad (7)$$

proposed by Centelles *et al.* [5], one can obtain the symmetry energy for nuclear matter from the symmetry energy coefficients of finite nuclei. The ρ_A is the reference density in nuclear matter to make the equation hold [5,12], which is significantly smaller than the saturation density because of the surface region of the nuclear density profile. It is known that
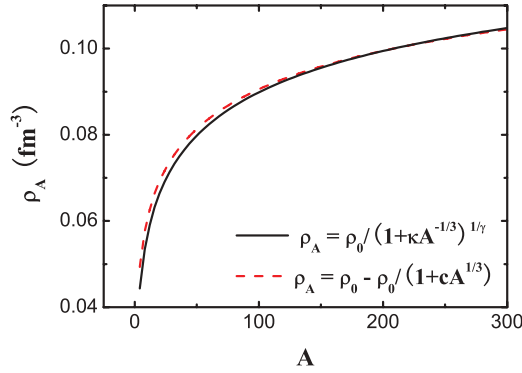
FIG. 2. (Color online) Reference density ρ_A as a function of mass number. The solid and dashed curves denote the results in this work and those from Ref. [5], respectively.

the relation of Eq. (7) is hold at $\rho_A \simeq 0.1$ fm^{-3} for ^{208}Pb from various effective interactions [5,18]. Now inserting Eqs. (5) and (6) into Eq. (7), one obtains

$$(\rho_A/\rho_0)^\gamma = (1 + \kappa A^{-1/3})^{-1}. \quad (8)$$

Applying $\rho_{208} = 0.1$ fm^{-3} [5,12,18] and the confidence interval $\kappa = 2.31 \pm 0.38$ determined by $a_{\text{sym}}(A)$ in the previous section, we obtain the following range of γ:

$$\gamma = 0.7 \pm 0.1. \quad (9)$$

With the relation of Eq. (8), one can obtain the expression of the reference density ρ_A for a nucleus with mass A; that is,

$$\rho_A = \rho_0/(1 + \kappa A^{-1/3})^{1/\gamma}. \quad (10)$$

The value of ρ_A decreases with the decrease of nuclear size because of the enhanced surface effects in light nuclei. Moreover, Eq. (10) indicates that the reference density ρ_A depends on the ratio κ and the parameter γ, which describes the stiffness of the symmetry energy. Figure 2 shows the reference densities ρ_A as a function of nuclear mass number. The black solid curve denotes the results of Eq. (10) with $\rho_0 = 0.16$ fm^{-3}, $\kappa = 2.31$, and $\gamma = 0.7$. From the figure, one can see that the reference densities for finite nuclei with $A = 20$–250 cover the densities in the range $0.42\rho_0 \leqslant \rho \leqslant 0.64\rho_0$. The results from the parametrized expression of $\rho_A = \rho_0 - \rho_0/(1 + cA^{1/3})$, proposed by Centelles *et al.* [5] with various effective interactions, are also shown in the figure with the (red) dashed curve for comparison. One can see that the reference densities obtained from the two different approaches are in good agreement with each other. The validity of the parametrized expression proposed by Centelles *et al.* has been tested in the mass region $40 \leqslant A \leqslant 208$ [5]. For the mass region $A < 40$ and $A > 208$, the two models give similar extrapolation. From Fig. 2, one sees that for heavy nuclei the reference densities change slowly with the mass number and are close to 0.1 fm^{-3}, whereas for light nuclei the reference densities fall very fast with decrease of mass. Therefore, it is better to apply relation (7) by choosing the symmetry energy coefficient of heavy nuclei to relate the symmetry energy for nuclear matter at the reference density.

Figure 3 shows the symmetry energy of nuclear matter as a function of ρ/ρ_0 obtained in this work and the comparison
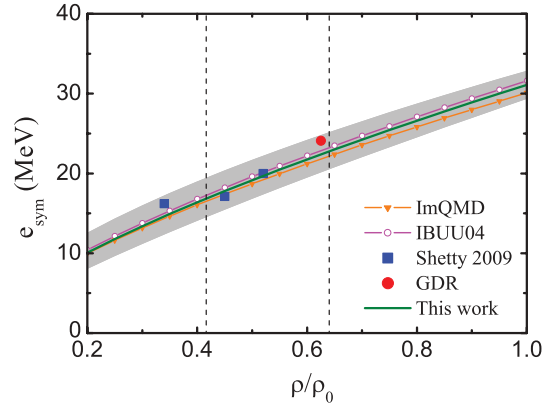

FIG. 3. (Color online) Density dependence of the nuclear symmetry energy with different methods. The two vertical dashed lines give the corresponding density region of nuclei with $A = 20$–250. The gray shading denotes the region with $S_0 = 31.1 \pm 1.7$ MeV and $\gamma = 0.7 \pm 0.1$, using the form $e_{\text{sym}}(\rho) = S_0(\rho/\rho_0)^\gamma$.

with the results obtained by other methods. The (green) solid curve corresponds to the symmetry energy calculated with $e_{\text{sym}}(\rho) = 31.1(\rho/\rho_0)^{0.7}$, in which $S_0 = 31.1$ MeV is the favorite value from $a_{\text{sym}}(A)$ and $\gamma = 0.7$ is obtained from ρ_A. The two vertical dashed lines show the corresponding density region $0.42\rho_0 \leqslant \rho \leqslant 0.64\rho_0$ of nuclei with $A = 20$–250. One should note that the results in this work for density regions ($\rho < 0.5\rho_0$ and $\rho > 0.63\rho_0$, corresponding to $A < 40$ and $A > 208$, respectively) represent an extrapolation. The (orange) dashed curve denotes the symmetry energy constrained by comparing the measurements of the isospin diffusion and the neutron-to-proton double ratio in ^{124,112}Sn $+^{124,112}$ Sn reactions with the calculations of an improved quantum-molecular dynamics model (ImQMD), where $e_{\text{sym}}(\rho) = 12.5(\rho/\rho_0)^{2/3} + 17.6(\rho/\rho_0)^\gamma$ with $\gamma = 0.7$ [2]. The dot-dashed curve denotes the symmetry energy as $e_{\text{sym}}(\rho) = 31.6(\rho/\rho_0)^\gamma$ with $\gamma = 0.69$, obtained by the comparison between National Superconducting Cyclotron Laboratory at Michigan State University (NSCL-MSU) isospin diffusion data and the IBUU04 [4]. The (blue) solid squares give the mapped $e_{\text{sym}}(\rho)$ from the correlation between temperature, excitation energy, density, and the isoscaling parameter [19,20]. The (red) solid circle gives the constrained $e_{\text{sym}}(0.1)$ from the excitation energy of giant dipole resonance (GDR) in ^{208}Pb [18]. The gray shading in Fig. 3 is bounded by $S_0 = 31.1 \pm 1.7$ MeV and $\gamma = 0.7 \pm 0.1$. One can see from Fig. 3 that the obtained nuclear symmetry energy in this work is located between the results of ImQMD and IBUU04. The gray area is consistent with the results from ImQMD, IBUU04, GDR, and the mapped data from the isoscaling parameter in Refs. [19,20] at subnormal densities.

TABLE I. Range of slope parameter L for different cases of S_0.

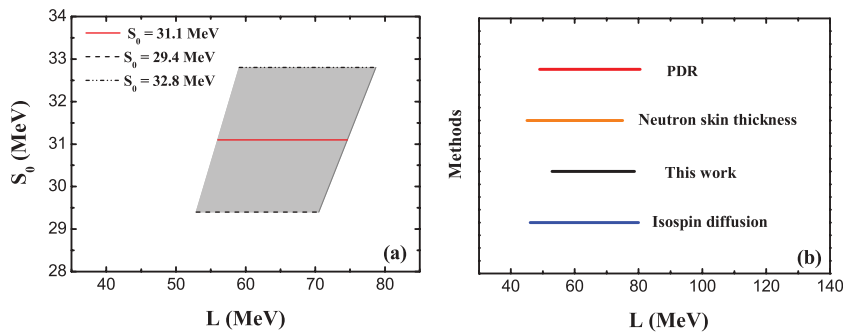| S_0 (MeV)	L_{min} (MeV)	L_{max} (MeV)	L (MeV)
29.4	52.9	70.6	61.7 ± 8.8
31.1	56	74.6	65.3 ± 9.3
32.8	59.1	78.7	68.9 ± 9.8

FIG. 4. (Color online) (a) Range of slope parameter L determined by $S_0 = 31.1 \pm 1.7$ MeV and $\gamma = 0.7 \pm 0.1$ in this work. (b) Range of slope parameter L determined from different observables. The results are taken from Ref. [21] with PDR, from Ref. [22] with neutron skin thickness, and from Ref. [2] with isospin diffusion.

On the basis of the extracted value of S_0 and the extrapolated nuclear symmetry energies at densities around the normal density, we further study the slope parameter $L = 3\rho_0 \frac{\partial e_{\text{sym}}(\rho)}{\partial \rho}|_{\rho_0}$, which is an effective quantity to characterize the density dependence of symmetry energy. From the previously obtained range $\gamma = 0.7 \pm 0.1$ and the value of S_0, the value of L can be obtained directly. The gray shading in Fig. 4(a) shows the area of L determined by $S_0 = 31.1 \pm 1.7$ MeV and $\gamma = 0.7 \pm 0.1$. The range of L for $S_0 = 31.1$ MeV is shown by the solid line. The dashed and the dot-dot-dashed lines correspond to the cases of $S_0 = 29.4$ and 32.8 MeV, respectively, which are the boundaries of the confidence interval of S_0. The values of L for the three cases are listed in Table I. From the calculations, we obtain the largest range of the slope parameter, $53 \lesssim L \lesssim 79$ MeV, for $S_0 = 31.1 \pm 1.7$ MeV. To compare with the values of L obtained from other methods, we show the range of slope parameter L determined in this work and those from other recent analyses with different observables in Fig. 4(b). The black solid line is the result from the measured nuclear masses in this work. The result from isospin diffusion is obtained from Ref. [2] with $S_0 = 30.1$ MeV and $\gamma = 0.4$–1.05. The result from PDR is taken from Ref. [21] with $L = 64.8 \pm 15.7$ MeV and $S_0 = 32.3 \pm 1.3$ MeV. The result from the neutron skin thickness is taken from Ref. [22]. The range of the values of L obtained in this work is consistent with those obtained from other analyses.

IV. SUMMARY

The symmetry energy coefficients $a_{\text{sym}}(A)$ for nuclei with mass numbers $A = 20$–250 were determined from more than 2000 precisely measured nuclear masses based on the liquid drop mass formula with the contribution of the Wigner term. By taking $a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1}$, we obtain the 95% confidence intervals of the volume symmetry energy coefficient $S_0 = 31.1 \pm 1.7$ MeV and the surface-to-volume symmetry-coefficient ratio $\kappa = 2.31 \pm 0.38$ from the determined $a_{\text{sym}}(A)$.

On the basis of the relation between the nuclear symmetry energy at reference density and the symmetry energy coefficients of nuclei $e_{\text{sym}}(\rho_A) = a_{\text{sym}}(A)$, we investigated the nuclear symmetry energy at a narrow subnormal density range, $0.42\rho_0 \leqslant \rho \leqslant 0.64\rho_0$. Applying the reference density for ^{208}Pb, $\rho_{208} \simeq 0.1$ fm^{-3}, and the symmetry parameter ratio $\kappa = 2.31 \pm 0.38$, we determined the range of γ (i.e., $\gamma = 0.7 \pm 0.1$) by inserting the nuclear symmetry energy $e_{\text{sym}}(\rho) = S_0(\rho/\rho_0)^\gamma$ and the symmetry energy coefficients of nuclei $a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{1/\gamma}$ into the relation $e_{\text{sym}}(\rho_A) = a_{\text{sym}}(A)$. The range of γ obtained in this way is independent of the nuclear symmetry energy coefficient S_0. Simultaneously, we deduced the mass dependence of the reference density ρ_A, which explicitly depends on κ and γ. Finally, the range of the slope parameter L of nuclear symmetry energy at normal density was determined to be $53 \lesssim L \lesssim 79$ MeV, based on the extracted value of S_0 and the extrapolated nuclear symmetry energies at densities around the normal density. The constraint on the nuclear symmetry energy at subnormal density provided in this work is in good agreement with the results from other recent analyses.

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