Isospin dependence of capture cross sections: The ^{36}S+^{208} Pb reaction

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The capture-fission cross section for the ^{36}S+^{208}Pb reaction was measured for seven center-of-mass energies ranging from 147.5 to 210.2 MeV. A comparison of the deduced interaction barriers from "distribution of barriers" measurements and simple $1/E_{\text{c.m.}}$ plots for 13 well-characterized systems shows the validity of the latter approach for deducing interaction barriers, especially for reaction systems involving radioactive beams where the former measurements are not currently feasible. Application of the $1/E_{\text{c.m.}}$ plot technique for the ^{36}S+^{208}Pb reaction gives an interaction barrier height of 140.4 ± 1.4 MeV. This value as well as the deduced interaction barriers for all known studies of capture cross sections with radioactive beams are in good agreement with recent predictions of an improved isospin-dependent quantum molecular dynamics model and a modified version of capture cross-section systematics by Swiatecki *et al.* The deduced barriers for these n-rich systems are lower than one would expect from semiempirical systematics based upon the Bass potential. In addition to the barrier lowering, there is an enhanced subbarrier cross section in these n-rich systems not predicted by the Bass potential systematics. These enhanced subbarrier cross sections may be important in the synthesis of the heaviest nuclei.

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I. INTRODUCTION

The synthesis and study of the heaviest elements is one of the forefront areas of nuclear science. Most of these studies have involved complete fusion reactions where one can represent the cross section for producing a heavy reaction product, σ_{EVR}, by the equation,

$$\sigma_{\text{EVR}}(E_{\text{c.m.}}) = \sum_{J=0}^{J_{\max}} \sigma_{\text{CN}}(E_{\text{c.m.}}, J) W_{\text{sur}}(E_{\text{c.m.}}, J), \qquad (1)$$

where σ_{CN} is the complete fusion cross section and W_{sur} is the survival probability of the completely fused system. The complete fusion cross section can be written as

$$\sigma_{\text{CN}}(E_{\text{c.m.}}) = \sum_{J=0}^{J_{\max}} \sigma_{\text{capture}}(E_{\text{c.m.}}, J) P_{\text{CN}}(E_{\text{c.m.}}, J), \qquad (2)$$

where $\sigma_{\text{capture}}(E_{\text{c.m.}}, J)$ is the "capture" cross section at center-of-mass (c.m.) energy $E_{\text{c.m.}}$ and spin J and P_{CN} is the probability that the projectile-target system will evolve inside the fission saddle point to form a completely fused system rather than reseparating (quasifission). Occasionally this equation is written in its spin-independent form,

$$\sigma_{\text{EVR}}(E_{\text{c.m.}}) = \sigma_{\text{capture}}(E_{\text{c.m.}}) P_{\text{CN}}(E_{\text{c.m.}}) W_{\text{sur}}(E_{\text{c.m.}}). \qquad (3)$$

This is not, in general, correct as the formation of heavy nuclei in fusion reactions can involve significant angular momenta [1].

The "capture" cross sections are the focus of this paper. "Capture" corresponds to overcoming the interaction barrier forming a composite system. If the configuration at the interaction barrier, the contact configuration, evolves inside the fission saddle point, fusion occurs. For lighter systems, $P_{\text{CN}} = 1$ and $\sigma_{\text{capture}} = \sigma_{\text{fusion}}$. For heavy systems $\sigma_{\text{capture}} = \sigma_{\text{fusion}} + \sigma_{\text{quasifission}}$. Formally, $\sigma_{\text{capture}}(E_{\text{c.m.}}) = (\frac{\pi\hbar^2}{2\mu E_{\text{c.m.}}}) \sum_{J=0}^{J_{\max}} (2J+1)T(E_{\text{c.m.}}, J)$ and $\sigma_{\text{fusion}}(E_{\text{c.m.}}) = (\frac{\pi\hbar^2}{2\mu E_{\text{c.m.}}}) \sum_{J=0}^{J_{\max}} (2J+1)T(E_{\text{c.m.}}, J)P_{\text{CN}}(E_{\text{c.m.}}, J)$. Semiempirical systematics of capture cross sections have been developed [2–5] that are generally thought to allow predictions of σ_{capture} within a factor of two for most complete fusion reactions used in heavy element synthesis.

Recently, a great deal of attention was devoted to the possible use of neutron-rich radioactive beams to synthesize heavy nuclei. Such efforts are motivated by the possibility of enhanced subbarrier fusion cross sections and increased survival probabilities. These possibilities have been summarized recently [6].

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To accurately predict the heavy element formation rates in neutron-rich systems, such as those resulting from the use of complete fusion reactions, one needs to know the capture cross sections for very neutron-rich systems. A first attempt at examining this question was made in Ref. [7]. After examining data on capture cross sections for all known reactions involving neutron-rich/radioactive projectiles in heavy systems, these authors concluded that the systematics of the interaction barrier heights in reactions induced by very neutron-rich projectiles supports the idea that there is a systematic decrease in barrier heights that is correlated to the relative neutron richness of the composite system and this correlation is not included in current semiempirical models of capture cross sections. That conclusion was disputed [8] for the case of the ^{38}S+^{208}Pb reaction where a previous analysis [7] had shown a large lowering of the barrier height relative to systematics.

Subsequently, these neutron-rich fusion reactions were studied using an improved isospin-dependent quantum molecular dynamics (ImIQMD) model [9]. The calculated cross sections and barrier heights agreed quantitatively with the experimental data [7,10–12]. In the model, an enhancement of the N/Z ratio in the neck region of the interacting nuclei leads to enhanced subbarrier fusion cross sections and lowered barrier heights (relative to the semiempirical systematics [2,5] of capture reactions) for neutron-rich systems.

Most of the reactions involved in these experimental and theoretical determinations of the capture cross sections in neutron-rich systems involved radioactive beams. At present, such experiments can suffer from large statistical uncertainties [7] in the measured cross sections because of low beam intensities and, in some cases, uncertainties in the beam energies from the use of degraders in beam production. We thought it would be useful to study a reaction with stable beams where the relatively high beam intensities lead to low statistical uncertainties in the measured data and where the energies of the projectiles are well known but also where a large N/Z ratio exists in the composite system. We picked the ^{36}S+^{208}Pb reaction and measured the capture-fission excitation function.

In Sec. II of this article, we describe the experimental methods used to make the measurement whereas in Sec. III we describe the analysis of the measured data and in Sec. IV we attempt to extract physically significant information from the data. In Sec. V, we discuss the implications of a new capture systematics for the production of heavy nuclei. Section VI contains our conclusions.

II. EXPERIMENTAL METHOD

The capture-fission cross section for the ^{36}S+^{208}Pb reaction was measured at the ATLAS accelerator at Argonne National Laboratory. Beams of ^{36}S with energies $E_{\text{lab}} =$ 174.4, 179.4, 184.4, 189.4, 199.4, 219.4, and 247.4 MeV impinged upon ^{208}Pb targets of 400 μg/cm^2 and 600 μg/cm^2 in thickness, respectively, backed by 40 μg/cm^2 C. The thicker target was used in the three lowest energy measurements.

Fission fragments were detected in 10 Si surface barrier detectors of area 300 mm^2 each, arranged in a plane at angles $\theta = 75, 85, 95, 110, 120, 130, 140, 150, 160$, and 170°. Each detector provided an energy and a time signal measured relative to the rf signal of the accelerator. The fission detectors were placed at a distance of 171 mm from the center of the target, each subtending a solid angle of 10.26 msr, and were energy calibrated using a ^{252}Cf source. Two Si surface barrier detectors, collimated to subtend a solid angle of 0.406 msr, were placed at a distance of 244.75 mm from the center of the target, at angles $\theta = \pm 13.2^\circ$. These detectors detected the elastically scattered particles and were used to normalize the capture-fission cross section to the Rutherford scattering cross section. At each beam energy, two sets of data were taken, one in which the target was oriented such that its normal pointed in the beam direction, and the other pointing at 45°. The former position shadowed the forward detectors ($75^\circ < \theta < 110^\circ$) whereas the latter shadowed the backward detectors. In each detector, the fission fragments were separated from elastic and quasielastic particles by their time of flight and energy. The time of flight and energy were calibrated simultaneously by an iterative procedure whose only assumption is that the average fission fragment mass is half that of the fissioning system (we assumed $A_{\text{fs}} = A_{\text{CN}}/2$ for all energies.) The laboratory energy of the fission fragments was corrected for energy losses in the target and backing material where applicable, using standard range tables [13]. Losses in detector dead layers and pulse-height defects are corrected for by the Schmitt calibration procedure [14].

III. DATA ANALYSIS

The resulting center-of-mass angular distributions were fitted using the standard theory of fission angular distributions [15],

$$W(\theta) = \sum_{I=0}^{\infty}(2I+1)T_I \times \frac{\sum_{K=-I}^{I} \frac{1}{2}(2I+1)d^I_{M=0,K}(\theta)^2 \exp\left(-K^2/K_0^2\right)}{\sum_{K=-I}^{I} \exp\left(-K^2/K_0^2\right)}, \tag{4}$$

where T_I is the transmission coefficient for the Ith partial wave and $d^I_{M,K}(\theta)$ is the separable part of the symmetric top wave function [16]. The transmission coefficients used are those given by the parabolic potential of Hill and Wheeler [17]. The standard deviation K_0 of the projection K of the total angular momentum I onto the symmetry axis was treated as a fitting parameter. The formalism assumes that the projection M of I onto the space-fixed axis is always zero. This assumption is only valid for cases where the spin of the projectile and target are zero, and no evaporation has taken place (first chance fission). After the first de-excitation prior to fission, M can no longer be considered equal to zero. The theory of angular distributions including misalignment ($M \neq 0$) was worked out by Back and Bjørnholm [18]. In this formalism, M is (similarly to K) assumed to be a Gaussian distribution centered around $M = 0$ with standard deviation M_0. Therefore, the fit parameter K_0 may not represent the standard deviation of the distribution of K onto the symmetry axis. Rather, this parameter represents an intricate convolution of the true K_0

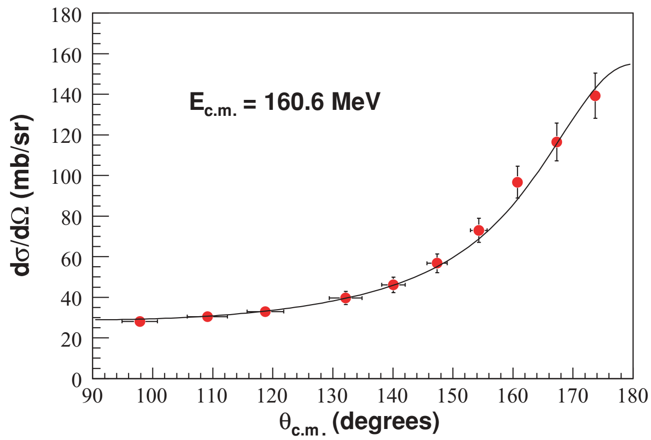

FIG. 1. (Color online) Measured fission fragment angular distribution for the $^{36}S+^{208}Pb$ reaction at $E_{c.m.} = 160.6$ MeV. Solid line is a fit with Eq. (4).

and M_0, averaged over all I. The value of the fit parameter has no direct physical meaning. It cannot, for example, be associated with the nuclear temperature T or the effective moment of inertia at the saddle $\Im_{eff}$ through the transition state model expression $K_0^2 = \Im_{eff} T/\hbar^2$, except in very special cases. Because the purpose of the fitting is to obtain a continuous function representing $d\sigma/d\Omega$ that can be integrated over the solid angle to obtain the cross section, the neglect to include a misalignment is justified, and the results should be very similar to the full treatment. K_0 was in the range of $4\hbar$–$20\hbar$.

In Fig. 1 we show the c.m. angular distribution for one of the energies measured. Errors in $d\sigma/d\Omega$ include estimated errors in $d\Omega$ and statistical errors. The former is less than 1% whereas the latter dominates the error estimation, typically ∼5%. By varying the lateral position of the beam axis and making the calculated Rutherford scattering yield coincide in both monitor detectors it was estimated that the beam could be off center by as much as 4 mm (typically <2 mm.) The error in $d\Omega$ was therefore estimated by assuming the uncertainty in the fission detector distance from the reaction spot was $\delta d = 3$ mm. The c.m. angle becomes a distribution of angles because of the varying velocity of the fission fragments. The c.m. angle is taken as the weighted average of the distribution and its error as the rms deviation. The total cross section was deduced by integration of the fitted differential cross section. The error corresponds to the integration error with a 95% confidence level band.

The measured capture-fission cross section is shown in Table I, together with the measured anisotropies, $A = W(\theta = 180^\circ)/W(\theta = 90^\circ)$ and the average angular momentum of the fissioning system $\langle I \rangle$ ($\hbar$). The errors in A correspond to the combined error of $W(\theta)$ extrapolated to 90° and 180° within a 95% confidence level band. The measured capture-fission excitation function is shown in Fig. 2.

IV. DISCUSSION

One possible representation of measured excitation functions is to plot the capture cross section as a function of $1/E_{c.m.}$.

TABLE I. Measured capture-fission cross sections, anisotropies, and deduced mean angular momenta for the $^{36}S+^{208}Pb$ reaction. The center-of-mass energy is quoted at a cross-section-weighted depth in the target.

$E_{c.m.}$ (MeV)	σ (mb)	A	$\langle I \rangle$ ($\hbar$)
147.5	192 ± 5	4.1 ± 0.8	8.2
151.7	325 ± 9	4.8 ± 0.9	14.8
156.0	400 ± 12	4.7 ± 1.3	20.8
160.6	535 ± 14	5.4 ± 0.5	25.5
169.2	789 ± 21	5.6 ± 1.1	33.0
186.3	1001 ± 27	4.6 ± 0.9	44.2
210.2	1264 ± 34	4.3 ± 0.8	56.1

The underlying assumption is that the capture cross section may be expressed as

$$\sigma_{capture} = \pi R_{int}^2 (1 - V_{int}/E_{c.m.}), \quad (5)$$

where R_{int} is the s-wave interaction barrier radius and V_{int} is the interaction barrier height. This is the classical expression for the capture cross section, and the partial wave expression for the cross section reduces to the same form in the sharp cutoff approximation. It then implies that all partial waves contributing to the capture cross section have the same barrier radius R_{int}, which is probably not true for most reactions. However, in heavy systems, this condition is largely met because the kinetic energy and angular momentum dissipation counteract each other [19,20]. Thus, the effects of varying R_{int} and V_{int} with I are largely canceled and the cross section as a whole is relatively insensitive to this I dependence. In the range of validity of this assumption, namely above the barrier, where couplings are negligible, and below energies where deep-inelastic contributions may start to appear, a plot of the capture cross section $\sigma_{capture}$ as a function of $1/E_{c.m.}$ gives V_{int} as the intercept with the $1/E_{c.m.}$ axis whereas R_{int} is determined by the slope.

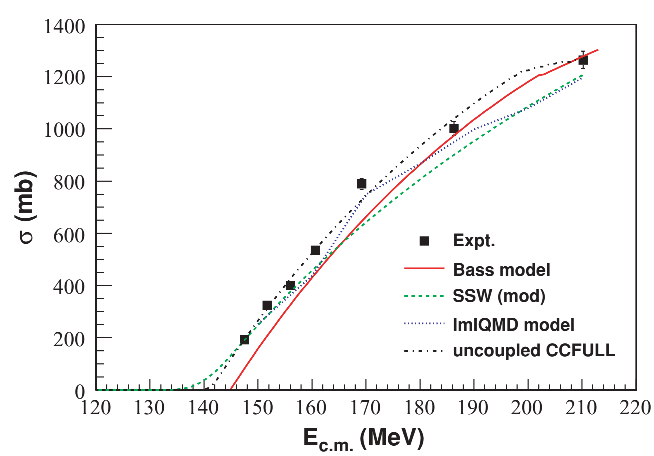

FIG. 2. (Color online) Measured capture-fission excitation function for the $^{36}S+^{208}Pb$ reaction. Shown as lines are model representations of the excitation function given by the Bass model, the Swiatecki-Siwek-Wilczynska-Wilczynski modified systematics (SSW), and the improved isospin-dependent QMD model (ImIQMD), respectively.

In previous work [7] we have used the $1/E_{c.m.}$ representation of the cross section of the ^{38}S+^{208}Pb reaction to infer the interaction barrier and radius. Ideally, one would like to infer these quantities by constructing a barrier distribution [21] for the reaction, but the low intensity of current radioactive beams, such as ^{38}S, renders it impossible, and the $1/E_{c.m.}$ representation appears as the solely accessible method. Furthermore, to infer the isospin dependence of these quantities, which would include reactions with stable beams, we would need to first establish the robustness of the $1/E_{c.m.}$ representation and its applicability in all relevant cases. To this end, we have fitted measured excitation functions with stable beams that have also been used to construct barrier distributions. The comparison between the average or most probable barrier deduced from the barrier distribution and the interaction barrier deduced by the $1/E_{c.m.}$ method should be a meaningful comparison. The systems considered are ^{16}O+^{144,148,154}Sm and ^{16}O+^{186}W of Ref. [22], ^{16}O+^{208}Pb of Ref. [23], ^{19}F+^{208}Pb of Ref. [24], ^{36}S+^{90,96}Zr of Ref. [25], ^{40}Ca+^{90,96}Zr of Ref. [26], ^{48}Ca+^{90,96}Zr of Ref. [27], and ^{34}S+^{168}Er of Ref. [28]. Capture-fission excitation functions were measured for ^{16}O+^{208}Pb, ^{19}F+^{208}Pb, and ^{34}S+^{168}Er, whereas evaporation residue excitation functions were measured for the rest. The common feature of these 13 reactions is that both the barrier distribution and the data of the excitation function are published.

In Fig. 3 we show the $1/E_{c.m.}$ representation of the cross sections of the previously mentioned reactions and the linear fits performed to infer the intercept and slope in each case. The data in the vicinity where the barrier distribution is manifested are excluded from the fit, as well as some high-energy data points where deep-inelastic scattering may be dominant. The excluded data points are shown as open symbols in the plots. In all cases, the fit resulted in reduced $\chi^2 < 1$, indicating a numerically robust fit. The result of the fits is given in Table II. Errors are given within a 95% confidence level band. To compare this data, we have estimated graphically the most probable barrier from the experimental barrier distribution, given in the third column of Table II. In some cases, the barrier distribution has a complicated structure as a result of couplings. The most probable barrier corresponds to the maximum of the distribution, not necessarily the average barrier. In the majority of cases, the $1/E_{c.m.}$ representation of the cross sections gives the most probable barrier. We conclude that this method can most likely be used with certainty to estimate V_{int} in reactions where barrier distributions have not, or cannot be measured, provided the relevant portion of the capture cross section was measured. In Table II we also give the fitted interaction radius R_{int}.

The $1/E_{c.m.}$ method was recently called into question in Ref. [8]. The authors investigated the errors associated with the method in simulated cross sections for the ^{32}S+^{208}Pb reaction with the code CCMOD. It was shown that substantial discrepancies occur. It was pointed out that the $1/E_{c.m.}$ method decouples V_{int} and R_{int} as no implicit nuclear potential is assumed. In the reasoning mentioned previously we have argued that this apparent decoupling occurs because in reactions involving heavy ions the relative kinetic energy and angular momentum dissipation effectively counteract each other [19,20]. On the other hand, coupled-channels simulation codes like CCMOD do couple these two quantities through an assumed nuclear potential, usually of a Woods-Saxon form. Hence, it may not be surprising that the $1/E_{c.m.}$ method applied to simulated cross sections under such assumptions does produce substantial discrepancies.

In the discussion that follows we intend to combine extracted interaction barrier heights in reactions involving both stable and radioactive beams. In one of the few measurements involving radioactive beams, the ^{38}S+^{208}Pb reaction [7], the capture cross section was fit with the $1/E_{c.m.}$ method. The fit performed to the data was revised by the authors of Ref. [8] with the fitting code DESCALC, which resulted in a value of $V_{int} = 134.0 \pm 6.2$ MeV, instead of the originally published value of $V_{int} = 133.3 \pm 10.4$ MeV. We have performed a new fit with the present fitting procedure and found that $V_{int} = 133.4 \pm 6.6$ MeV. The present fit is consistent with the

TABLE II. $1/E_{c.m.}$ fits to capture-fusion cross sections for measured excitation functions where barrier distributions are available. The extrapolated interaction barrier V_{int} is compared to the experimental most probable barrier and the fitted uncoupled barrier with CCFULL. Also shown is the extrapolated interaction radius R_{int} and the fitted uncoupled radius R_{int}^{CC} with CCFULL, and the fit parameters r_0 and a_0, respectively.

System	V_{int} (MeV)	V_{int}^{mp} (MeV)	V_{int}^{CC} (MeV)	R_{int} (fm)	R_{int}^{CC} (fm)	r_0 (fm)	a_0 (fm)	Ref.
^{16}O+^{144}Sm	60.3 ± 0.1	60	67.3	10.2 ± 0.1	10.4	1.18	0.25	[22]
^{16}O+^{148}Sm	59.3 ± 0.2	59	66.1	10.2 ± 0.1	10.6	1.21	0.23	[22]
^{16}O+^{154}Sm	58.9 ± 0.1	59	65.2	10.2 ± 0.1	10.5	1.08	0.48	[22]
^{16}O+^{186}W	68.1 ± 0.1	68	74.1	10.4 ± 0.1	10.7	0.97	0.77	[22]
^{16}O+^{208}Pb	73.8 ± 0.1	74	74.2	10.6 ± 0.1	11.3	0.89	1.26	[23]
^{19}F+^{208}Pb	82.4 ± 0.1	82	82.9	10.9 ± 0.1	11.6	0.97	1.07	[24]
^{36}S+^{90}Zr	77.4 ± 0.1	77	77.6	11.3 ± 0.1	11.7	1.41	0.14	[25]
^{36}S+^{96}Zr	75.4 ± 0.1	75	75.5	11.5 ± 0.1	11.6	1.20	0.54	[25]
^{40}Ca+^{90}Zr	96.1 ± 0.1	96	[a]	10.0 ± 0.1	[a]	[a]	[a]	[26]
^{40}Ca+^{96}Zr	94.0 ± 0.1	93	[a]	9.6 ± 0.1	[a]	[a]	[a]	[26]
^{48}Ca+^{90}Zr	95.0 ± 0.1	93	95.3	10.0 ± 0.1	10.4	0.87	1.34	[27]
^{48}Ca+^{96}Zr	94.2 ± 0.2	94	94.5	10.4 ± 0.1	10.7	0.91	1.21	[27]
^{34}S+^{168}Er	121.9 ± 0.1	122	122.5	10.6 ± 0.1	11.1	0.93	1.32	[28]

[a]No convergence found.

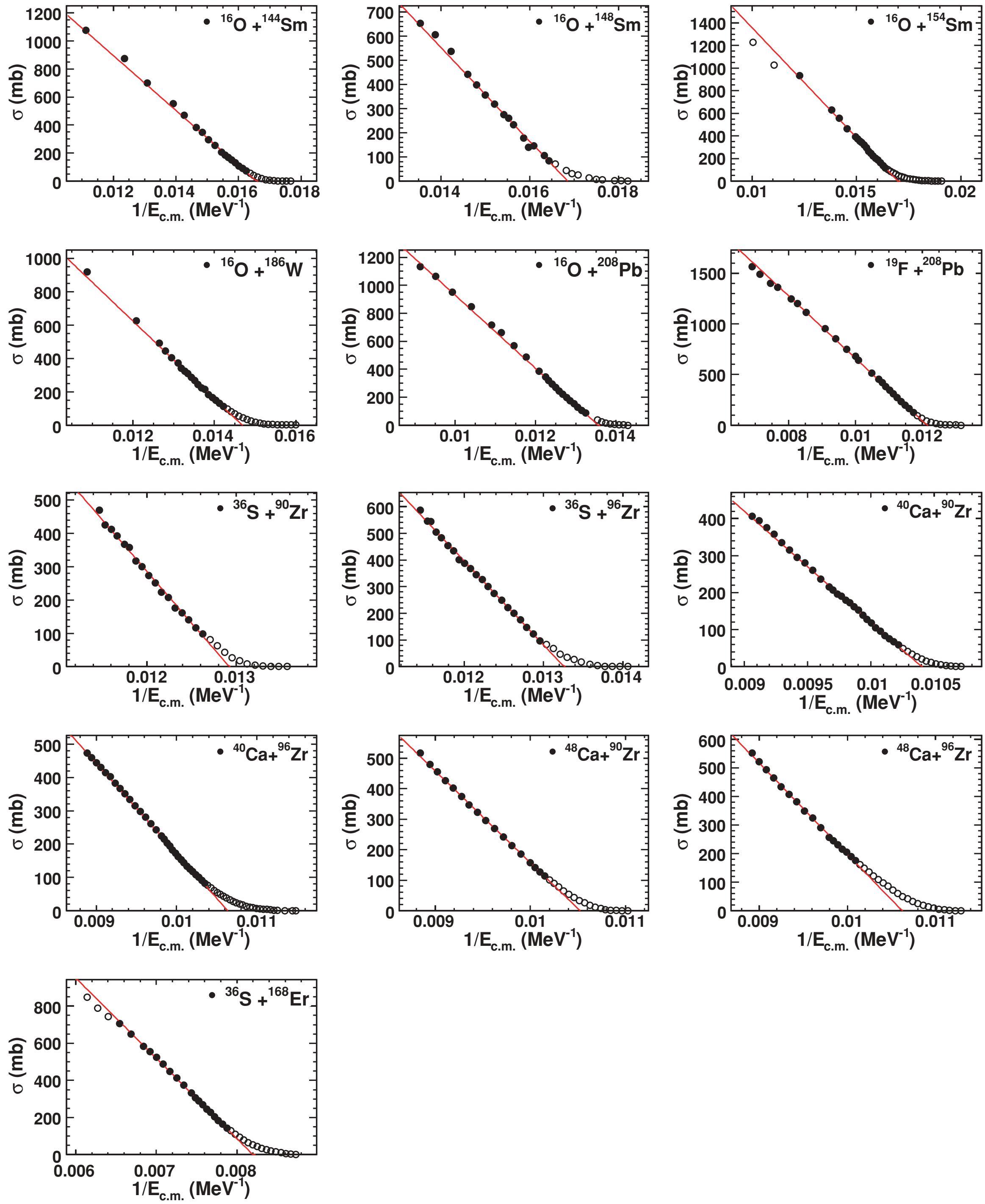

FIG. 3. (Color online) $1/E_{\text{c.m.}}$ fits to the measured data of Refs. [22–28].

fit made in Ref. [7], although the error is taken differently. To perform the fits we have used the MINUIT package within the CERN data analysis framework ROOT [29], which uses the MINGRAD algorithm. In this software package, the default fitting error estimation is made with a 95% confidence level band, which is how the error in the present fit is quoted. The fit in Ref. [7] was made with the graphical analysis software ORIGIN, which may explain the small difference in fitted value, and how errors are estimated. We are not aware of the origin of DESCALC used by Ref. [8]. We will use the value deduced by the present fit.

In Fig. 4 we show the $1/E_{\text{c.m.}}$ representation of the capture cross section for the ^{36}S+^{208}Pb reaction, giving a value of $V_{\text{int}} = 140.4 \pm 1.4$ MeV and $R_{\text{int}} = 11.5 \pm 0.2$ fm, respectively. In Table III we compare this deduced barrier height with various prescriptions and calculations. The Bass model/potential [30] significantly overestimates the barrier height, although it should be remarked that this semiempirical

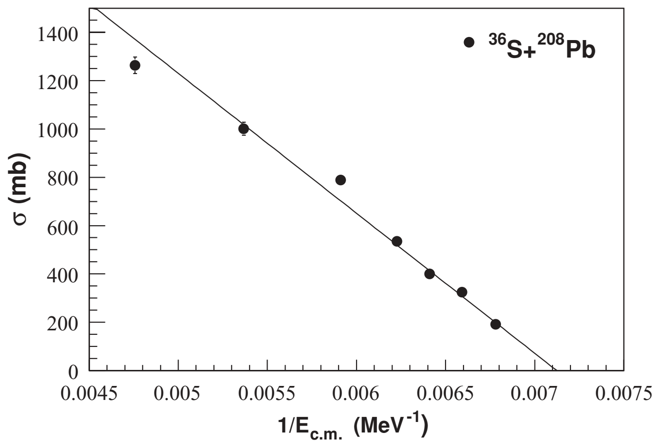

FIG. 4. $1/E_{\text{c.m.}}$ fit to the measured capture excitation function of the ^{36}S+^{208}Pb reaction.

potential is fitted to fusion rather than interaction cross sections. The original prescription of Ref. [5] would also give a barrier height that exceeds the measured height. A recent revision of that prescription [31] for neutron-rich systems gives a barrier height in agreement with the measured data. In this revision [31] the formula for the barrier B (MeV) is given as

$$B = (0.86665612z + 0.00099062z^2 - 0.000001243z^3), \quad (6)$$

where

$$z = Z_{\text{projectile}} Z_{\text{target}} \left(A_{\text{projectile}}^{1/3} + A_{\text{target}}^{1/3}\right)^{-1}, \quad (7)$$

whereas the formula for the radius (fm) is given as

$$R = 1.16 \left(A_{\text{projectile}}^{1/3} + A_{\text{target}}^{1/3}\right). \quad (8)$$

The predicted barrier height from the ImIQMD model [9] also correctly describes the measured barrier height.

When studying capture (barrier-crossing) cross sections, it is common practice to compare the measured excitation functions with predictions of coupled-channels calculations. Such calculations seldom reproduce the experimental data, unless a potential, usually of the Woods-Saxon form, with unphysical values of the parameters is used. Newton *et al.* [32] found values of the diffuseness parameter a ranging between 0.75 fm to as high as 1.5 fm were able to fit fusion excitation functions in 47 heavy-ion reactions, whereas the diffuseness parameter that reproduce elastic scattering data is close to 0.65 fm. The follow-up study in Ref. [33] intended to reconcile the nuclear potential to simultaneously reproduce elastic scattering and

TABLE III. Interaction barrier heights and radii for the ^{36}S+^{208}Pb reaction.

Source	V_{int} (MeV)	R_{int} (fm)
Expt	140.4 ± 1.4	11.5 ± 0.2
Bass	144.8	12.2
Swiatecki *et al.* (original)	142.3	10.5
Swiatecki *et al.* (model)	139.7	10.7
Bian *et al.*	138.1	
CCFULL (uncoupled)	141.2	12.1

the fusion excitation function in the ^{12}C+^{208}Pb reaction. No set of parameters was found that could reproduce both sets of experimental data. To this date, there has been no satisfactory explanation as to the meaning or correctness of the procedure put forward in Ref. [32]. This procedure was nevertheless used to deduce interaction barriers in the ^{32}S+^{208}Pb, ^{34}S+^{206}Pb, and ^{36}S+^{204}Pb reactions in Ref. [8]. We have used the CCFULL code [34] to make such deductions to be able to compare the procedure with the $1/E_{\text{c.m.}}$ method. The measured excitation function for the ^{36}S+^{208}Pb reaction was fit with the uncoupled CCFULL code by minimizing χ^2 using the Levenberg-Marquardt method with respect to the parameters V_0, the depth of the Woods-Saxon potential, r_0, the radius parameter, and a_0, the diffuseness parameter. Derivatives with respect to the three free parameters were evaluated numerically using Ridders' method. All three parameters were fit simultaneously. The goodness of the fit is poor, as evidenced by the reduced χ^2 value of 2.4. Similar reduced χ^2 values where found when fitting the previous three reactions [8]. The fit parameters obtained are $V_0 = 216.5$ MeV, $r_0 = 1.01$ fm, and $a_0 = 1.08$ fm, respectively. With these parameters the uncoupled barrier for the ^{36}S+^{208}Pb reaction deduced by CCFULL is $V_b = 141.2$ MeV, about 0.8 MeV higher than the interaction barrier deduced by the $1/E_{\text{c.m.}}$ method, but within experimental errors. The values for the uncoupled barrier and radius obtained with CCFULL can be found in Table III. In Ref. [8] several couplings where added and a modest variation of 0.1 MeV in the uncoupled barriers were found. We will not attempt here to add couplings to CCFULL, as our data is measured above the barrier where such coupling effects are unimportant.

We have attempted to apply the coupled channels fitting procedure to the 13 reactions considered earlier. Numerical convergence was found in only six of the 13 cases. By restricting $V_0 = 200$ MeV, as done in Ref. [8], we were able to obtain better results. It is interesting to note that in the cases where convergence was found in both fitting attempts, the resulting uncoupled barrier is essentially the same number, although the value of the parameters differ substantially. In column 4 of Table II we show the uncoupled barriers obtained in the latter fitting (restricted V_0.) In two cases, no numerical convergence was found. In all cases where convergence was found, the reduced $\chi^2 > 1$, indicating the fit quality is rather poor. Some interaction barriers deduced with this method are in accordance with the $1/E_{\text{c.m.}}$ method; some differ markedly. However, the parameters obtained by these fits, shown in columns 7 and 8 of Table II, are difficult to understand and reconcile with the generally accepted shape of the nuclear potential. A potential depth of 550 MeV, radius parameter of 0.5 fm, and diffuseness parameter of 3 fm, as one of the better cases yielded, or radius parameter of 1.2 fm and diffuseness parameter of 0.5 fm in the second attempt, is unrealistic and hardly meaningful. The notion that V_0, r_0, and a_0 in a coupled-channels calculation should be regarded as simple parameters [33] implies they have no physical meaning. This claim seems arbitrary, unless the quantity the parameters control, the Wood-Saxon potential, has no physical meaning. This needs further clarification before this method becomes a "standard" method of deducing interaction barriers. Meanwhile, the $1/E_{\text{c.m.}}$ method is physically justified, direct, and universally reproducible. Furthermore, it seems to deduce accurate interaction barriers, in particular, when a barrier distribution measurement is out of reach, whereas the coupled-channels fitting procedure seems erratic.

It should be mentioned that coupled-channels calculations have been an important tool in elucidating the nature and kind of couplings that produce the observed phenomenon of fusion barrier distributions. It is not our intention to discredit or discourage its use in modeling physical phenomena the model is intended for.

Equally important as the interaction barrier height $V_{\rm int}$ and the interaction radius $R_{\rm int}$, is how the cross section is predicted to vary with projectile energy. In Fig. 2 we compare the measured capture-fission excitation function for the ^{36}S+^{208}Pb reaction with various prescriptions for the excitation functions. The Bass potential/model is a poor fit to the data near the barrier, underestimating the cross section near the barrier by a factor of two although formally it refers to the fusion rather than the capture cross section. The modified potential [31] of Swiatecki *et al.* also reproduces the excitation function fairly well. The fitted coupled channels calculation done with CCFULL, as described previously, is also shown.

V. IMPLICATIONS OF THIS WORK FOR HEAVY ELEMENT PRODUCTION

In Fig. 5 we show the measured values of the interaction barriers as a function of the z parameter [see Eq. (7)] for a large number of neutron-rich systems involving stable and radioactive beams. The $1/E_{\rm c.m.}$ method was used for the reactions ^{27,29,31}Al+^{197}Au [11], ^{32,38}S+^{181}Ta [10], ^{32}S+^{208}Pb [15], ^{36}S+^{208}Pb, ^{38}S+^{208}Pb [7], ^{64}Ni+^{124}Sn [35], and ^{132}Sn+^{64}Ni [36]. The interaction barriers deduced in Ref. [8] were used for the ^{32}S+^{208}Pb, ^{34}S+^{206}Pb, and ^{36}S+^{204}Pb reactions. The best representation of the data is with the modified systematics [31] of Swiatecki *et al.* The ImIQMD model [9] also predicts the observed values. Thus it would seem the cross-section enhancements seen previously in studies [7,10–12] with radioactive beams are well described by both theoretical and semiempirical models.

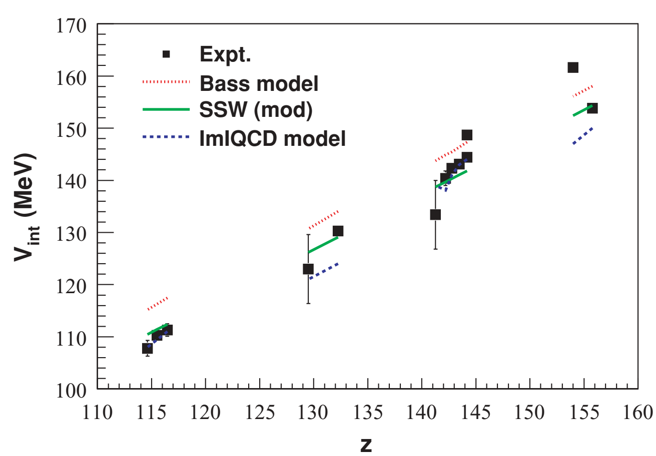

FIG. 5. (Color online) Interaction barrier plotted against the parameter $Z_1Z_2/(A_1^{1/3} + A_2^{1/3})$ for reactions involving stable and radioactive beams.

What are the implications of our measurements and the agreement between all the data on n-rich systems with the predictions of the ImIQMD and modified Swiatecki *et al.* models? One now believes one can calculate the values of the capture cross sections very well for n-rich systems likely to be involved in heavy element production using the theoretical calculations of Ref. [9] and/or the semiempirical predictions of Ref. [31].

A specific example of how these new systematics might affect the production of the heaviest nuclei is to compare some features of three different reactions leading to the production of Cf isotopes (i.e., the ^{32}S+^{208}Pb, ^{36}S+^{208}Pb, and ^{44}S+^{208}Pb reactions that lead to completely fused systems of ^{240}Cf, ^{244}Cf, and ^{252}Cf). Imagine each of these reactions takes place at the interaction barrier [31]. Of necessity, the capture cross sections are similar (i.e, 30.8, 33.1, and 33.9 mb). The excitation energies of the completely fused systems differ greatly, being 35.9, 21.8, and 23.8 MeV, respectively. The resulting survival probabilities [37] are 2×10^{-7}, 2.8×10^{-3}, and 5×10^{-2}, respectively. In short, the survival probabilities increase by $\sim 10^5$ for the most n-rich system for the same capture cross section. However, as pointed out in Ref. [7], the beam intensities expected from the planned US radioactive beam facility, FRIB, [38] are 6×10^{12}, 3×10^8, and 3×10^6 particles per second for ^{32,36,44}S, completely negating any advantages posed by the n-rich system. Nevertheless, if the stable and radioactive beam intensities are similar [39], then the use of n-rich radioactive beams would offer significant advantages.

VI. CONCLUSIONS

The conclusions and findings of this work are as follows:

(i) We have measured the capture-fission excitation function for the ^{36}S+^{208}Pb reaction deducing values of the interaction barrier height, $V_{\rm int} = 140.4 \pm 1.4$ MeV, and the interaction radius, $R_{\rm int} = 11.5 \pm 0.2$ fm.

(ii) By comparing a set of measurements of interaction barrier heights from "distribution of barriers" measurements with $1/E_{\rm c.m.}$ plots, we conclude the latter technique is a valid way to deduce $V_{\rm int}$.

(iii) By comparing our results along with deduced interaction barrier heights for all known capture cross-section measurements for intermediate mass radioactive beams, we conclude that these barrier heights are in good agreement with recent predictions of an improved isospin-dependent QMD model and a modified version of capture cross-section systematics of Swiatecki *et al.*

(iv) Although the ImIQMD model calculations and the modified capture cross-section systematics describe the shape of the excitation function for the ^{36}S+^{208}Pb reaction, coupled-channels calculations with parameters consistent with elastic scattering results do not.

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