One-quasiparticle states in odd-Z heavy nuclei

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(Received 22 April 2010; revised manuscript received 16 September 2010; published 10 November 2010)

The isotopic dependencies of one-quasiparticle states in Es and Md are treated. In ^{253,255}Lr, the energies of the lowest one-quasiproton states are calculated. The one-quasiparticle isomer states are revealed in the nuclei of an α-decay chain starting from ^{269}Rg. The α decays from some isomer states are predicted. The population of isomer states in the complete fusion reactions is discussed.

DOI: 10.1103/PhysRevC.82.054304 PACS number(s): 21.10.Hw, 21.10.Pc, 21.60.−n, 27.90.+b

I. INTRODUCTION

The spectroscopic study of low-lying one-quasiparticle states in the odd-mass actinides and transactinides has been performed for a long time and remains up to date [1–7] because of the problem of unambiguous identification of new superheavy nuclei [8] and unknown isotopes of heavy nuclei. The investigation of transfermium elements is a step toward a better knowledge of the heaviest elements, their single-particle structure, location of the shell gaps and magic numbers, and decay modes. In recent years the set of experimental data has been considerably expanded due to the development of devices such as SHIP at GSI, GREAT at JYFL, BEST at GANIL, and GABRIELA at JINR (Dubna) combining α, electron, and γ spectroscopy. Low-lying states have been identified in Es, Md, Lr, Db, and Bh isotopes. ^{255}Lr is the heaviest odd-Z nucleus for which such spectroscopic information as single- and multiparticle excitations and rotational bands is now available. The systematic experimental study of the single-particle states in the Es isotopes with mass numbers from 243 to 251 has been performed.

The similarities in the α-decay pattern as well as in excitation energies and ordering of low-lying Nilsson levels have been established for many isotopes of Es [4]. In even-Z nuclei such similarities are well known for the isotones [1]. The detailed study of the structures of one-quasiparticle states is important for the unambiguous definition of the half-lives of γ transitions from the isomer states. Detailed decay spectroscopy has been performed for the K isomers of several nuclei with $Z \geqslant 99$. The γ decay of these isomers populates rotational levels. Isomers decaying by α emission are of particular interest. The α decay from the isomer state can occur into the isomer state of the daughter nucleus, which can decay again by α emission. As is known, in odd-mass heavy nuclei the α decays from the ground state preferably occur into the same one-quasiparticle state of the daughter nucleus.

New experimental results challenge modern theories attempting to reproduce the properties and structures of the heaviest nuclei. The comparison of calculated and experimental one-quasiparticle states allows us to test the parameters used in the theory and to assign the quantum numbers to the experimental levels important in the analysis of γ transitions and α decays. Existing semimicroscopical approaches [9–13] based on the Nilsson-Strutinsky method supply the basis for the intensive calculations of the properties of low-lying states of heavy nuclei. These approaches use the parametrization of nuclear shape and the single-particle Hamiltonian and thus are not self-consistent. However, they provide a powerful tool for systematic calculations and predictions which are important for the planned experiments.

Low-lying states in odd-mass nuclei are essentially determined by the unpaired nucleon. The study of single-particle states in odd-mass heavy nuclei and the comparison with available experimental data is one of the goals of the present paper. Our purpose is to predict the isomer states and possibilities of α decays from them in the region of deformed heaviest nuclei. In comparison with available experimental data we want to verify the method of our calculations.

II. MICROSCOPIC-MACROSCOPIC METHOD

In the microscopic-macroscopic approach [11,12] the macroscopic E_{mac} and microscopic E_{mic} parts of the potential energy of the nucleus are calculated with the certain shape parametrization. The ground state of the nucleus corresponds to the global minimum of the potential-energy surface. In the present paper we suggest the shape parametrization adopted for the two-center shell model (TCSM) [13] and use it for finding the single-particle levels at the ground state of the nucleus. The mirror symmetric shape parametrization used in this model effectively includes all even multipolarities. The TCSM has been intensively exploited in the reaction and fission theory. Here, we verify and apply the TCSM for describing the nuclear structure near the ground state. Calculating the quadrupole, hexadecapole, and hexacontatetrapole moments, one can find the relationship between the deformation parameters used in the TCSM and the parameters β_2, β_4, and β_6 used in the models [11,12]. The main advantage of the TCSM shape parametrization is that one can easily trace the evolution from the ground state to the separate fission fragments with a small number of collective coordinates. The small number of shape parameters λ and β considerably simplifies finding the global potential minimum corresponding to the ground state. The value of λ characterizes the length of the nucleus along the symmetry axis z in the units of the diameter $2R_0$ of the spherical nucleus. The ratio of the semiaxes a and b of the purely ellipsoidal parts of the nuclear surface defines $\beta = a/b$.

The single-particle Hamiltonian, which is used here for calculating the single-particle energies and $E_{\rm mic}$, is as follows:

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(\rho,z) + V_{ls} + V_{l^2}, \tag{1}$$

where the single-particle potential for the nucleus near the ground state is parametrized as

$$V(\rho,z) = \begin{cases} \frac{1}{2}m\omega_z^2(z-z_1)^2 + \frac{1}{2}m\omega_\rho^2\rho^2, & z < z_1, \\ \frac{1}{2}m\omega_\rho^2\rho^2, & z_1 < z < z_2, \\ \frac{1}{2}m\omega_z^2(z-z_2)^2 + \frac{1}{2}m\omega_\rho^2\rho^2, & z > z_2. \end{cases} \tag{2}$$

Here, m is the nucleon mass, $\omega_\rho/\omega_z = a/b = \beta$, $z_2 - z_1 = 2R_0\lambda - 2a$, and $\omega_\rho = \beta\omega_0 R_0/a$ with $\hbar\omega_0 = 41$ MeV $A^{-1/3}$. The value of a is related to λ and β via the volume conservation $3a^2R_0\lambda - a^3 = 2R_0^3\beta^2$. The spin-orbit term,

$$V_{ls} = -\frac{2\hbar\kappa}{m\omega_0'}(\nabla V \times \mathbf{p})\mathbf{s}, \tag{3}$$

and the l^2 term,

$$V_{l^2} = -\kappa\mu\hbar\omega_0'\mathbf{l}^2 + \kappa\mu\hbar\omega_0'\frac{\mathcal{N}(\mathcal{N}+3)}{2}\delta_{if}, \tag{4}$$

contain the parameters κ and μ discussed in Ref. [14] and the diagonal operator δ_{if}. Here, $\hbar\omega_0' = 41$ MeV $A'^{-1/3}$, where $A' = Aab^2/R_0^3$. The value of $\mathcal{N}$ in Eq. (4) corresponds to the principal quantum number of the spherical oscillator.

The ground state of the nucleus results from the calculation of the potential-energy surface as a function of deformation parameters [14]. The contribution of an odd nucleon, occupying a single-particle state $|\mu\rangle$ with energy e_μ, to the energy of a nucleus is described by the one-quasiparticle energy $\sqrt{(e_\mu - e_F)^2 + \Delta^2}$. Here, the Fermi energy e_F and the pairing-energy gap parameter Δ are calculated with the BCS approximation. A pairing interaction of the monopole type with strength parameters $G_{n,p} = (19.2 \mp 7.4\frac{N-Z}{A})A^{-1}$ MeV [10] for neutrons (minus sign) and protons (plus sign) is used. The values of Δ obtained in our calculations differ from those in Refs. [11,12] within 0.1 MeV. For example, while for a proton in ^{265}Mt we get $\Delta = 0.53$ MeV, the results in Refs. [11] and [12] are $\Delta = 0.5$ and 0.553 MeV, respectively. Problems with the BCS approximation could occur when there is a large gap in the single-particle spectrum near the Fermi level. In this case the Fermi energy seems to be close to the energy of the last occupied level and an inaccuracy of the definition of Δ does not affect the order of low-lying one-quasiparticle levels with $e_\mu < e_F$. The levels with $e_\mu > e_F$ would not be the lowest ones in which we are interested.

The microscopic corrections, quadrupole parameters of deformation [15] calculated with the TCSM, are close to those obtained with the microscopic-macroscopic approaches in Refs. [11,12]. For the nuclei of the α-decay chain of ^{269}Rg, the parameters β_2, β_4, and β_6 extracted from the ground-state nuclear shapes in the TCSM are compared in Fig. 1 with those obtained in Refs. [11,12,16]. As is seen, the nuclear shapes produced by the TCSM result in the hexadecapole deformation parameters which are rather different from those in Refs. [11,12] and close to those obtained with the self-consistent calculations [16]. The TCSM shapes result in the other sign of β_6 as compared with Refs. [11,12]. The difference in β_4 and β_6 may cause the difference in the order of some single-particle levels as compared with Refs. [11,12]. Therefore the experimental definition of the order of low-lying quasiparticle states seems to be important for verifying the shell-model calculations.

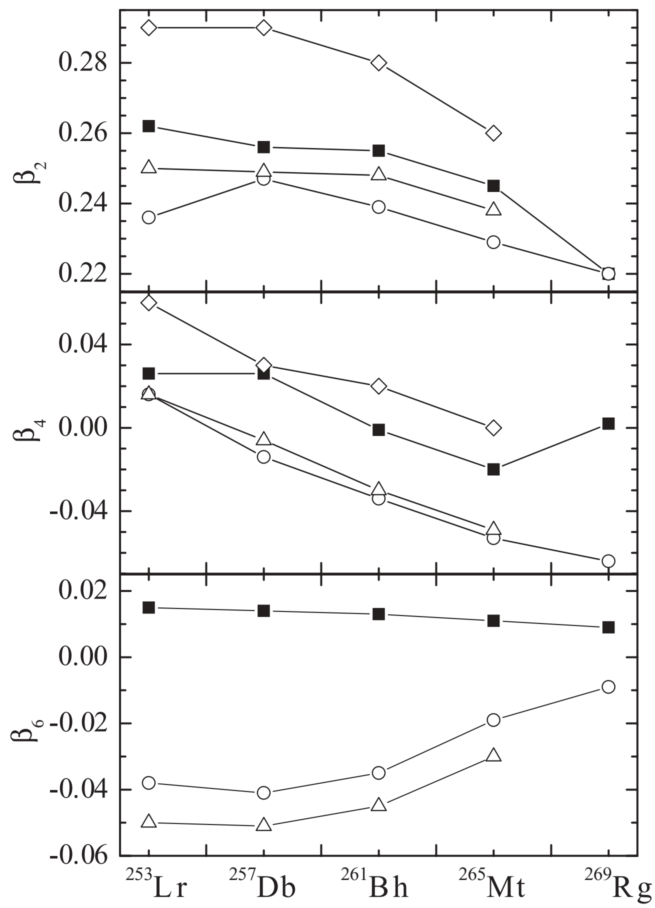

FIG. 1. For the nuclei of α decay chain of ^{269}Rg, the deformation parameters β_2, β_4, and β_6 related to the ground states in the TCSM (closed squares) are compared with those obtained in Ref. [11] (open triangles), Ref. [12] (open circles), and Ref. [16] (open diamonds).

The example of potential-energy surface calculated with the TCSM for ^{249}Md is presented in Fig. 2. The global minimum on this surface corresponds to the ground state in which the one-quasiproton states are treated. The image of the potential-energy surface in the coordinate space of β_2 and β_4 is presented as well.

As for the macroscopic-microscopic approach using the global fit of the parameters to describe the nuclear binding energies, the agreement of both the ground-state spin and parity for spherical nuclei is about 90% and about 40% for well-deformed nuclei [17]. In the Nilsson-Strutinsky approach used in the present paper, the dependence of the parameters of the **ls** and l^2 terms on A and $N - Z$ are modified [14] for the correct description of the ground-state spins and parities of known odd actinides. As found, this modification

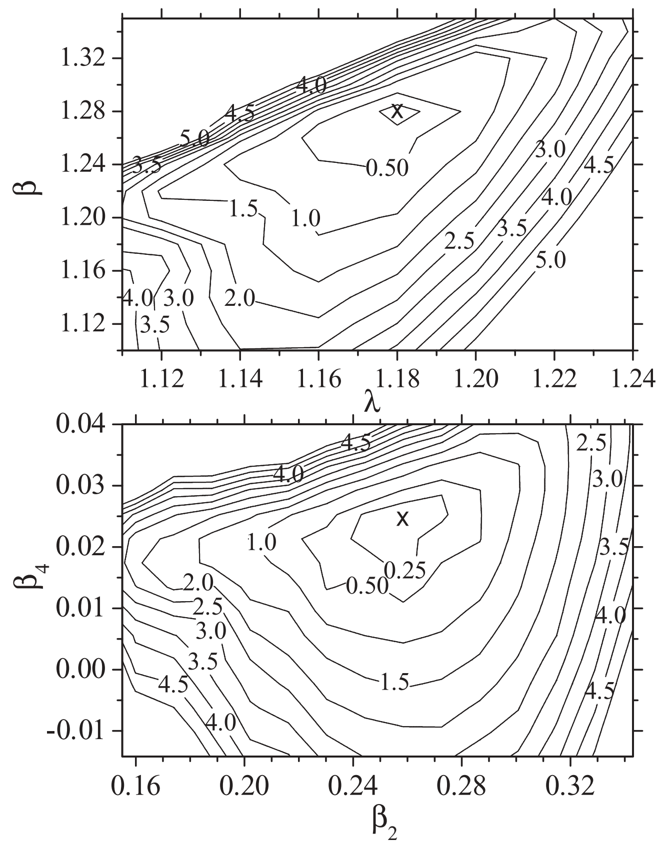

FIG. 2. Potential-energy surface of ^{249}Md calculated with the TCSM (upper part) and the image of this surface into the coordinate space of β_2 and β_4 (lower part). The potential energy is with respect to the ground state marked by the cross. The explanation of the TCSM coordinates λ and β is given in Refs. [13,14].

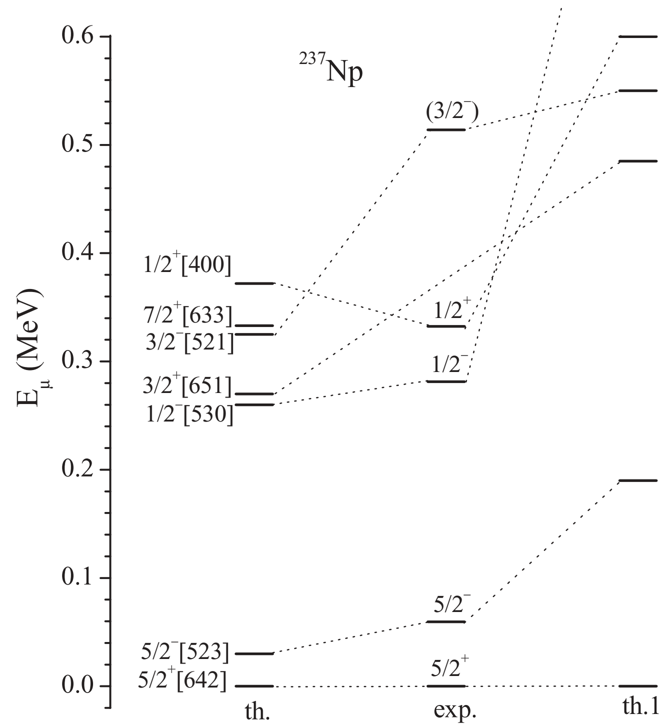

FIG. 3. Comparison between the calculated (th. and th.1) and experimental (exp.) [20] one-quasiproton spectra for the ^{237}Np nucleus. The results (th.1) are taken from Ref. [11].

weakly influences the potential-energy surface for the nuclei treated. The order of calculated single-particle levels seems to be close to the one in Ref. [9] for the same values of quadrupole and hexadecapole moments. Although we did not fit the parameters of the model for describing precisely the nuclear binding energies $B(Z, A) = E_{\rm mac}({\rm g.s.}) + E_{\rm mic}({\rm g.s.})$ in whole regions of the nuclear chart, the calculated $Q_\alpha(Z, A) = B(Z, A) + 28.296 - B(Z-2, A-4)$ values for the ground-state to ground-state α decay differ within 0.5 MeV from the experimental data, which is comparable with the accuracy of other approaches. The value of $E_{\rm mac}({\rm g.s.})$ is calculated with the same expression as in Refs. [18,19]. For ^{255,257}Md, we get the mass excesses 84.9 and 89.44 MeV, respectively, which are in good agreement with the experimental data [20] of 84.843 and 88.996 MeV, respectively. For ^{253,255}L, the calculated mass excesses are 88.64 and 89 MeV, respectively. The model in Ref. [12] results in 84.72 and 89.2 MeV and 87.89 and 89.3 MeV for ^{255,257}Md and ^{253,255}L, respectively.

To demonstrate the quality of our calculations, the energies of one-quasiproton states were calculated for ^{237}Np. The Nilsson (asymptotic) quantum numbers $[\mathcal{N} n_z \Lambda]$ are assigned to each state. One can see in Fig. 3 that the discrepancy in energy between the calculated and experimental values [20] of quasiparticle energies does not exceed 300 keV, which is quite satisfactory. The one-quasiproton spectra of ^{237}Np is described better with our approach than with the approach based on the Woods-Saxon potential and conventional multipole parametrization of nuclear shape [11]. As will be shown, the $7/2^-[514]$ state is ascribed by us to the ground states of the isotopes of Md, while in Ref. [11] the $1/2^-[521]$ state is predicted. The spins and parities of the ground states as well as the energies of some levels, which are different in our approach than in Ref. [11], can be experimentally verified.

In order to estimate the α-decay half-lives T_α, one can use the expression recently suggested in Ref. [21] and the calculated value of Q_α for the α decay treated. If the α decay is accompanied by the structure changes (transition to the one-quasiparticle state with the same K but with different other quantum numbers) the obtained T_α is increased by one to two orders of magnitude [9]. If an α particle would carry the angular momentum l, this α decay would be hindered by a factor of about 4^l [22]. This hindrance, which is consistent with the systematics in Ref. [23], is larger than that resulting from the simple addition of the centrifugal part to the one-dimensional potential barrier because the recoil effect (in which the daughter nucleus obtains or loses some angular momentum to supply the conservation of the total angular momentum) is taken into account. Note that the estimated values of T_α are consistent with the predictions of Refs. [18,19].

Using the Weisskopf estimate and the selection rules for the asymptotic quantum numbers [9], one can calculate the half-lives T_γ for γ transitions between the one-quasiparticle

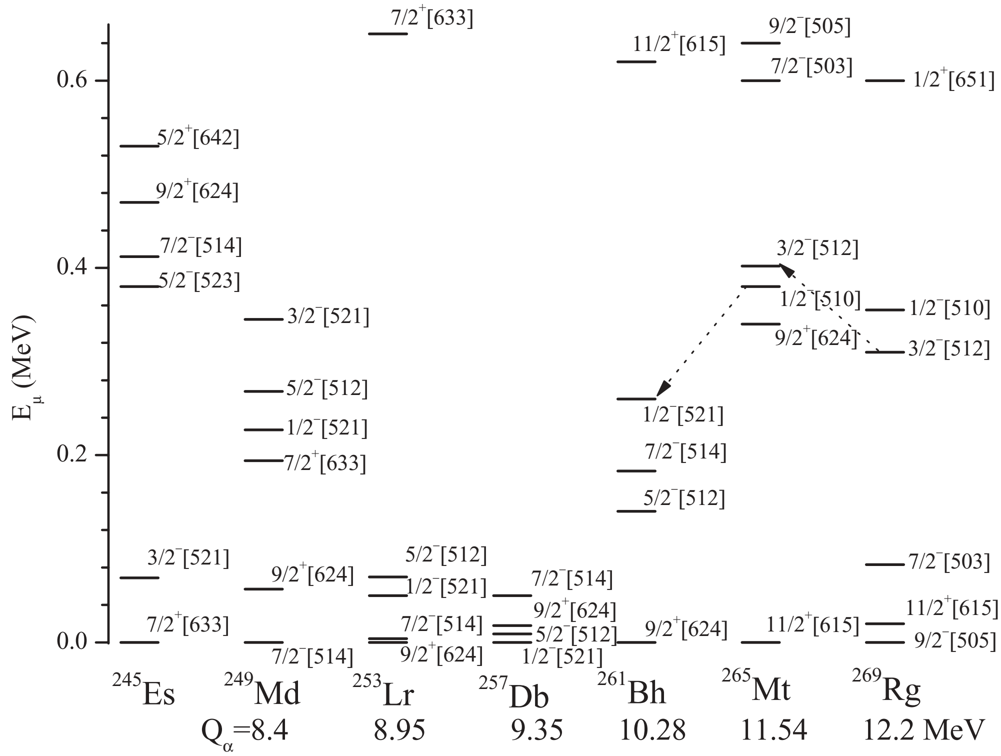

FIG. 4. Calculated energies of low-lying one-quasiproton states in the indicated nuclei of the α-decay chain of ^{269}Rg. The calculated values of Q_α are for the ground-state to ground-state α decay. Possible α decays from the isomer states of ^{269}Rg and ^{265}Mt are traced by arrows.

states. For example, in ^{249}Cf the calculated T_γ for the $M2$ transition between $5/2^+[622]$ and $9/2^-[734]$ one-quasineutron states is 7.5 μs while the experimental value is 45 μs. The calculation becomes more complicated when the transitions between one-quasiparticle states with $\Delta K > 2$ are treated. In ^{249}Cm the calculated $T_\gamma = 500$ s for direct $M3$ transition between the $7/2^+[613]$ and $1/2^+[620]$ one-quasineutron states is much larger than the experimental value of 23 μs for the isomer $7/2^+[613]$. Since the low-lying states of ^{249}Cm contain a noticeable phonon-quasiparticle admixture [24], the K-forbiddenness may be removed by the collective enhancement of the $B(E2)$ value. Then, for the $E2$ transition from the $7/2^+[613]$ state to the $3/2^+$ level of the rotational band built upon the $1/2^+[620]$ ground state we obtain $T_\gamma = 7.7$ μs, which is already close to the experimental value.

Usually, the one-quasiparticle isomers of interest exist at energy $E_{\text{is}} = 0.1$–0.4 MeV with respect to the ground state. The change of the excitation energies of the compound nucleus by these values of E_{is} does not significantly influence the survival probability. After the compound nucleus is cooled by the neutron emission until reaching the energy $E^* < 8$ MeV (i.e., less than the neutron separation energy) one can calculate the probability of population of the one-quasiparticle isomer state as $p_{\text{is}} = \exp(-E_{\text{is}}/T)/[1+\exp(-E_{\text{is}}/T)]$ (where $T \approx 0.6$ MeV is the thermodynamics temperature corresponding to E^* in the nuclei considered) by assuming the existence of only one low-lying isomer in the nucleus. As is found, $p_{\text{is}} \geqslant 0.35$ in the fusion-evaporation reactions treated here (i.e., the population of the isomer states) is quite probable.

III. RESULTS AND DISCUSSION

A. α-decay chain starting from ^{269}Rg

The low-lying one-quasiproton states in the nuclei of a possible α-decay chain starting from ^{269}Rg are shown in Fig. 4. The calculated values of Q_α for the ground-state to ground-state α decay are presented as well. In spite of the large value of Q_α for the ground-state to ground-state α decay, the nucleus ^{269}Rg lives long enough to be detected. The α decay of the ^{269}Rg ground state is preferable to the $9/2^-[505]$ state in ^{265}Mt with $Q_\alpha = 11.56$ MeV and $T_\alpha = 59.5$ μs. In ^{265}Rg the $1/2^-[510]$ excited state, if it is populated, decays within 20 ns into the $3/2^-[512]$ state by the $M1$ transition. The $E2$ transition between the $3/2^-[512]$ and $7/2^-[503]$ states mainly occurs from a 0.82% admixture of the $3/2^-[501]$ component to the $3/2^-$ state and needs $T_\gamma = 2$ μs. The α decay from the $3/2^-[512]$ state of ^{269}Rg to the same state in ^{265}Mt with $Q_\alpha = 12.11$ MeV needs $T_\alpha = 4.2$ μs, which is rather comparable with the value of T_γ estimated before. So, one can observe the isomer $3/2^-[512]$ state decaying by α emission. In the ^{63}Cu + ^{207}Pb $\rightarrow$ ^{269}Rg + $1n$ reaction the population of possible $3/2^-[512]$ isomer state can occur with about half the cross section of that of the population of the ground state. The calculated evaporation residue cross section $\sigma_{\text{ER}}^{\text{th}}$ for this reaction is 0.7 pb.

There is a prompt γ transition from the $9/2^-[505]$ state to the $11/2^+[615]$ ground state in ^{265}Mt. The α decay from the ^{265}Mt ground state occurs into the same state in ^{261}Bh with $Q_\alpha = 10.92$ MeV and $T_\alpha = 412$ μs. In ^{265}Mt, the $1/2^-[510]$ state seems to be the long-living isomer and decays by

α emission with $Q_\alpha = 11.66$ MeV and $T_\alpha = 160$ μs to the $K^\pi = 1/2^-$ isomer state in ^{261}Bh, which contains 1.4% of the $1/2^-[510]$ component. The population of the $1/2^-[510]$ isomer state in the ^{59}Co + ^{207}Pb $\rightarrow$ ^{265}Mt + $1n$ ($\sigma_{ER}^{th} = 4$ pb) reaction is estimated as 0.56 of the population of the ground state.

Because of the structure of the $K^\pi = 1/2^-$ isomer state in ^{261}Bh, the $E2$ transition from this state to the $5/2^-[512]$ state occurs with $T_\gamma = 16.4$ μs, which is considerably faster than the α decay expected in $T_\alpha = 790$ μs. The $5/2^-[512]$ state decays by fast $M2$ transition to the ground state. Indeed, no evidence for an isomer state in ^{261}Bh decaying by α emission was experimentally found in Ref. [5]. Therefore the α decay of ^{261}Bh is expected only from the ground state with $Q_\alpha = 10.26$ MeV and $T_\alpha = 3.82$ ms.

In ^{257}Db, the $1/2^-[521]$ ground state and the $5/2^-[512]$ and $9/2^+[624]$ excited states are close in energy (Fig. 4). There is a gap of about 0.7 MeV which separates the $7/2^-[514]$ and $7/2^+[633]$ levels. This gap in the one-quasiproton spectrum is probably related to the specific deformation parameters required by the closed neutron subshell $N = 152$. The strongly forbidden $E2$ transition from $5/2^-[512]$ to $1/2^-[521]$ results in T_γ larger than 1 s. Therefore one can expect the α decays of ^{257}Db from the $1/2^-$, $5/2^-$, and $9/2^+$ states with $Q_\alpha = 9.28$, 9.26, and 9.37 MeV and $T_\alpha = 0.29$, 0.34, and 0.19 s, respectively. The estimated times T_α and T_γ for the $5/2^-$ and $9/2^+$ states are comparable. So, two low-lying one-quasiproton states, $5/2^-$ and $9/2^+$, can be treated as long-living isomer states. They are populated with almost the same cross sections (0.6 nb) as the ground state in the ^{51}V + ^{207}Pb $\rightarrow$ ^{257}Db + $1n$ reaction. The values of Q_α in the proposed scheme of α decay of ^{257}Db (Fig. 5) are consistent with the experimental values within the accuracy of our calculations. However, there is a difference in the level ordering between our results and the approximate assignments in Ref. [2]. For example, the $7/2^-[514]$ level was assigned to the ground state of ^{253}Lr that is supported by the calculations in Refs. [11,25]. The model in Ref. [12] suggests that the $9/2^+[624]$ level is assigned to the ground state.

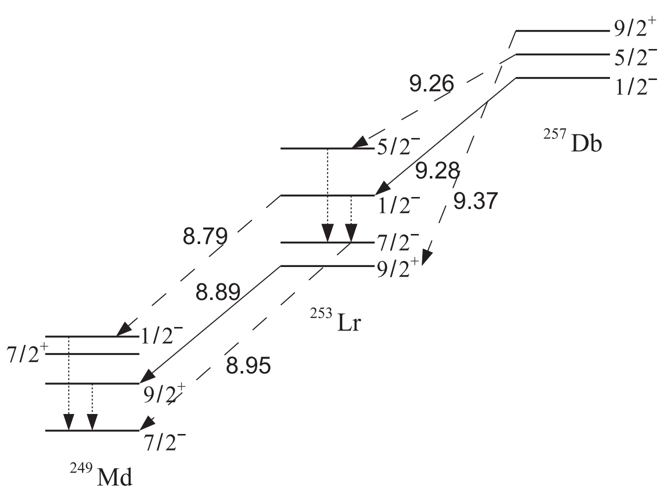

FIG. 5. Proposed decay scheme for ^{257}Db based on the calculated one-quasiproton spectra in Fig. 4. The dashed arrows indicate the α decays which can compete with the γ transitions.

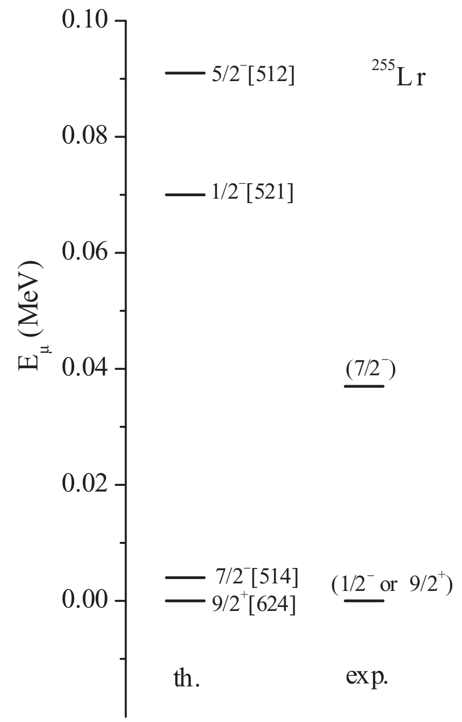

FIG. 6. Calculated (th.) energies of low-lying one-quasiproton states of ^{255}Lr. The available experimental states (exp.) [3] are depicted. The tentative assignments of these states are from Refs. [3,7].

For ^{253}Lr, our calculations predict that the $9/2^+[624]$ ground state is nearly degenerate with the $7/2^-[514]$ state, since the $1/2^-[521]$ and $5/2^-[512]$ levels lie below an excitation energy of 80 keV. There are no calculated levels in the interval from 0.1 to 0.6 MeV (see Fig. 4). For the α decay from the ground state of ^{253}Lr we obtain $Q_\alpha = 8.89$ MeV and $T_\alpha = 0.95$ s while the experimental values for the ^{253}Lr(2) state are $Q_\alpha = 8.918 \pm 0.020$ MeV and $T_\alpha = 0.44$ s [5]. If the energy difference between the $9/2^+[624]$ and $7/2^-[514]$ configurations is less than 0.02 keV (an accuracy within which

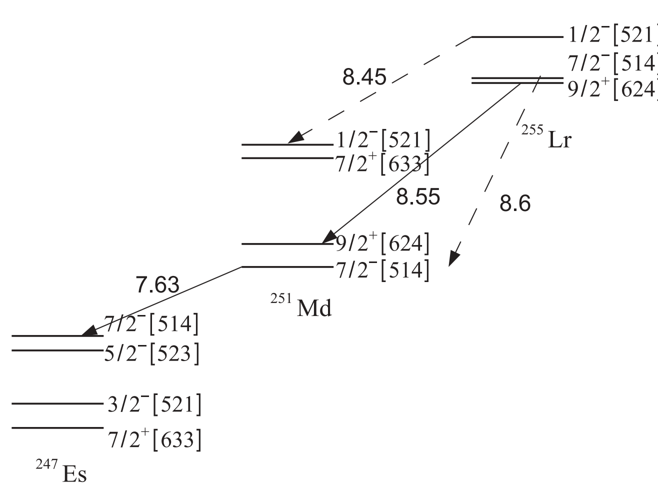

FIG. 7. Proposed decay scheme for ^{255}Lr based on the calculated one-quasiproton spectra in Fig. 6.

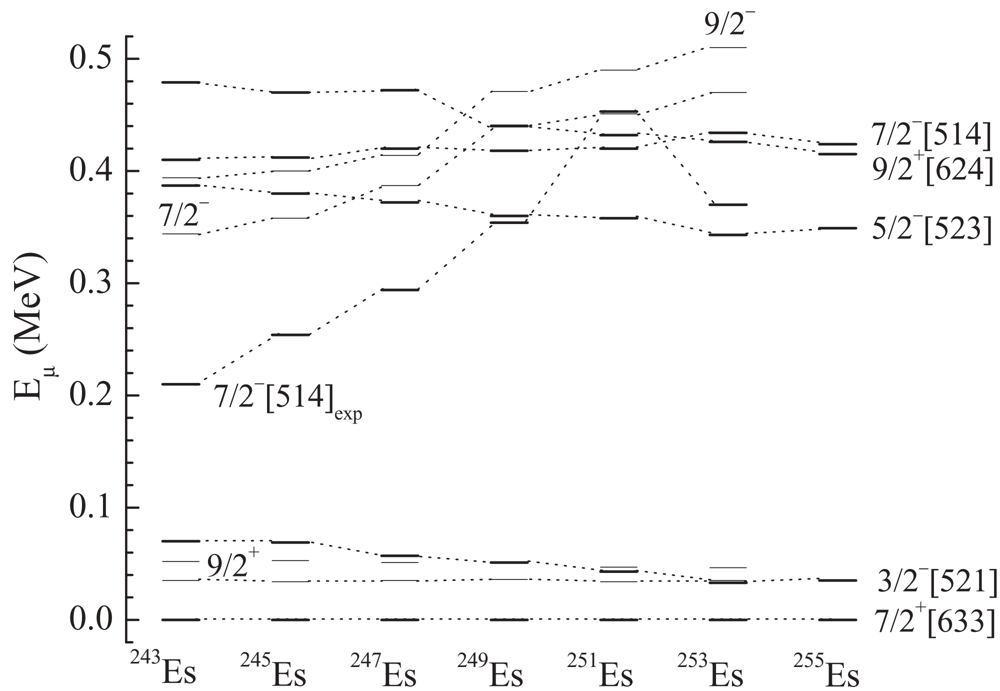

FIG. 8. Calculated energies of low-lying one-quasiproton states in the indicated isotopes of Es. The $9/2^+$, $7/2^-$, and $9/2^-$ members of the rotational band built on the $3/2^-[521]$ state are marked by thin lines. The experimental energies of the $7/2^-[514]$ states and $9/2^+$ rotational states (the thin lines which are not traced by a dashed line) [4] are shown.

one cannot calculate), the $E1$ transition from the $7/2^-[514]$ level would need more than 1 s. In this case the $7/2^-[514]$ configuration is expected to be an isomer. The α decay from this state would be observable with $Q_\alpha = 8.95$ MeV and $T_\alpha = 0.63$ s. In ^{253}Lr, the α decay from the $1/2^-[521]$ isomer state to the same state in ^{249}Md can occur with $Q_\alpha = 8.79$ MeV and $T_\alpha = 1.9$ s, which is perhaps too long to exclude the decay from the $1/2^-[521]$ state into the ground state via internal transitions. This isomer state can be related to the experimental isomer state ^{253}Lr(1) ($Q_\alpha = 8.850 \pm 0.020$ MeV and $T_\alpha = 0.7$ s [5]). The calculated population of the $1/2^-[521]$ isomer state with respect to the ground state is 0.52, which is in good agreement with the experimental intensity ratio $I[^{253}\mathrm{Lr}(1)]/I[^{253}\mathrm{Lr}(2)] = 0.41 \pm 0.11$ obtained in the reaction ^{209}Bi(^{48}Ca,4n)^{253}Lr and 0.50 ± 0.04 obtained for the production of ^{253}Lr in the α decay of ^{257}Db.

The α decay from the possible $1/2^-[521]$ isomer state in ^{249}Md would need $T_\alpha = 213$ s, which is too long for delaying the γ transition from this state to the $7/2^-[514]$ ground state. In the experimental study [5] the indications for an α-decay branch were not found either. Thus in ^{249}Md the α decay can be observed only from the ground state with $Q_\alpha = 7.99$ MeV and $T_\alpha = 130$ s. The experiments [4] provide $Q_\alpha = 8.16$ MeV, which corresponds to $T_\alpha = 35$ s in our calculations.

B. Spectrum of ^{255}Lr

In Fig. 6 the one-quasiproton spectrum of ^{255}Lr is squeezed near the ground state like the one-quasiproton spectrum of ^{253}Lr. The level order is the same as that in ^{253}Lr. There are no calculated levels in the interval from 0.1 to 0.5 MeV. The experimentally found excited state at 0.037 MeV [3] was tentatively assigned to the $7/2^-[514]$ level. As seen in Fig. 6, it can also be related to the $1/2^-[521]$ level according to our calculations. While in Refs. [3,26] the $1/2^-[521]$ level was assigned to the ground state, in Ref. [7] the lowest rotational band was found to be built on the $9/2^+[624]$ state. As in Ref. [12], we obtain that the $9/2^+[624]$ level corresponds to the ground state. The $7/2^-[514]$ and $9/2^+[624]$ states are too close in energy (about a 3-keV difference in our calculations) to be reliably distinguished. Because these states are almost degenerate and the $E1$ transition between them is strongly suppressed, the $7/2^-[514]$ configuration is expected to be the isomer. Note that in the literature the isomer character usually suggests a low-spin (high-spin) isomer state above a high-spin (low-lying) ground state (i.e., $\Delta K \geqslant 3$). The classical example of that is the $1/2^-[521]$ isomer state in ^{255}Lr. The schematic picture of favorable α decays of ^{255}Lr and ^{251}Md is presented in Fig. 7. The values of Q_α listed in Fig. 7 are in satisfactory agreement with the experimental data [3]. In contrast to ^{255}Lr, the energy difference between the $7/2^-[514]$ and $9/2^+[624]$ states in ^{251}Md becomes about 50 keV and the $7/2^-[514]$ configuration cannot be the isomer one. The α decay of ^{251}Md from the isomer state $1/2^-[521]$ needs more time than the γ transitions to lower levels. One can expect that its α-decay branch is very small compared with the γ-transition branch.

The two observed γ decays with 243 and 293 keV in ^{247}Es [3] can be explained as follows: The $M1$ transition $7/2^-[514] \to 5/2^-[523]$ (about 50 keV) occurs first, and then the transition into the $3/2^-[521]$ state (the 293-keV γ ray) or the transition into the $5/2^-$ level of the rotational band built

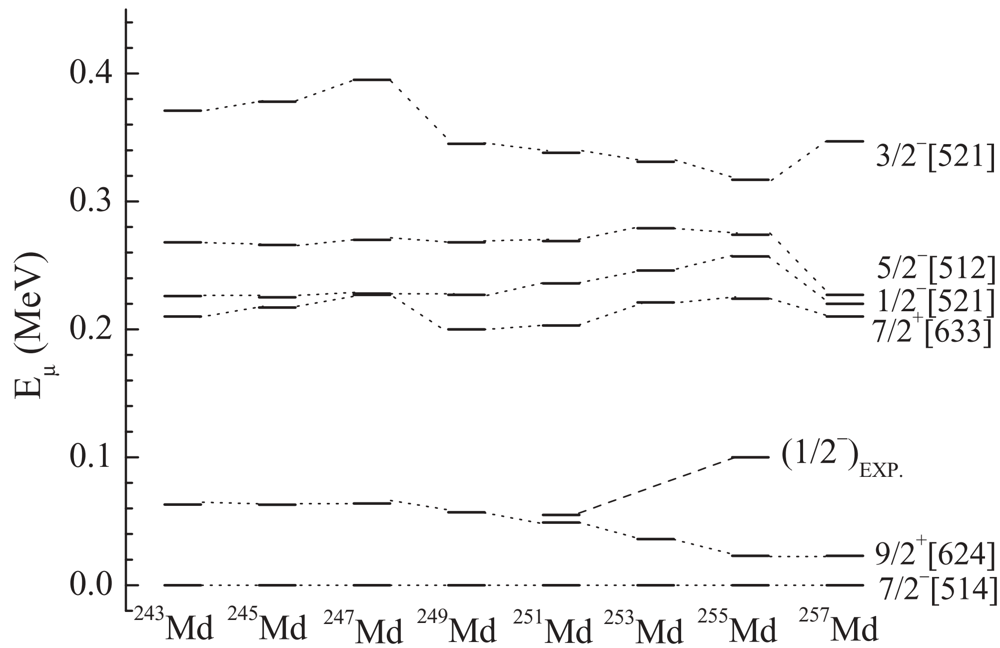

FIG. 9. Calculated energies of low-lying one-quasiproton states in the indicated isotopes of Md. The experimental first-known excited states of ^{251,255}Md with tentative assignments are shown [20]. In ^{255}Md the energy of the first excited state is approximately known.

on the $3/2^-[521]$ state (the 243-keV γ ray). One can expect the $M1$ transition $5/2^- \rightarrow 3/2^-[521]$ with an energy of about 50 keV between the members of the rotational band. Indeed, the 243-keV γ ray was detected in coincidence with the highly converted transition at 50 keV [3]. In addition, the calculation leads to the intensity ratio $T(M1, 293)/T(M1, 243) = 4.1$. Thus the experimental value of 4.4 ± 2.0 [3] supports the $M1$ assignment for both the 243- and 293-keV transitions.

C. Spectra of isotopes of Es and Md

The calculated low-lying one-quasiproton states of isotopes of Es and Md are presented in Figs. 8 and 9. The available experimental data are shown as well. The ground state of odd

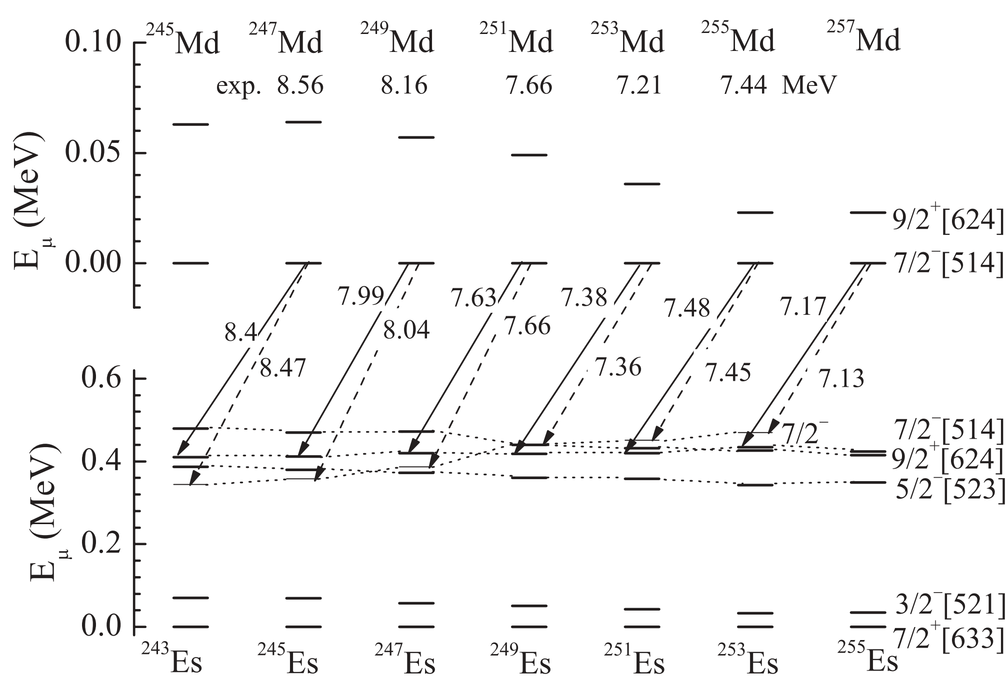

FIG. 10. Proposed α decays from the ground states of indicated isotopes of Md. The α decays to the $7/2^-$ levels of rotational bands built on the $3/2^-[521]$ states are shown by dashed arrows. The calculated values of Q_α, which are near the corresponding arrows, can be compared with the experimental data [4] presented.

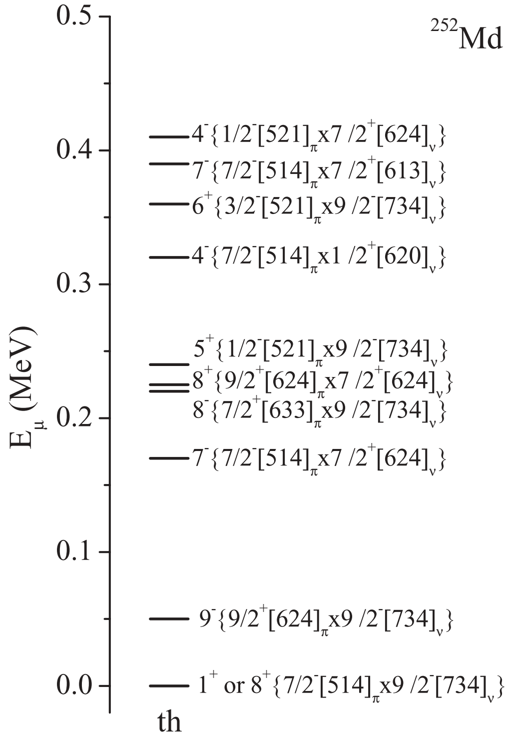

FIG. 11. Calculated energies of low-lying two-quasiparticle states with $K \geqslant 4$ in ^{252}Md. The structure of each state is indicated.

isotopes of Md is $7/2^-[514]$, which is in good agreement with experiments [1,4,27]. Note that in Refs. [11,25] and Ref. [12] the $1/2^-[521]$ and $7/2^-[514]$ levels are ascribed, respectively, to the ground states of the Md isotopes considered. For the isotopes of Es, we assigned $7/2^+[633]$ to the ground state. From the experimental side the ground states of ^{249,253,255}Es and ^{243,245,247,251}Es have been assigned to $7/2^+[633]$ and to $7/2^+[633]$ or $3/2^-[521]$, respectively. In the isotopes of Es the $M2$ transitions between the $3/2^-[521]$ and $7/2^+[633]$ states would occur with $T_\gamma \approx 0.4$ ms (i.e., $3/2^-$ states can be treated as the isomer state). As is the case for the isotonic dependence in even-Z nuclei, the isotopic dependence of the energy of a one-quasiparticle state with certain K^π is rather smooth. However, the isotopic dependence of the energy of the experimental $7/2^-[514]$ state is steeper. For several isotopes of Es, the energies of $9/2^+$, $7/2^-$, and $9/2^-$ members of the rotational band built on the $7/2^+[633]$ state were calculated with the formalism of Ref. [28]. Note the good description of the available experimental rotational $9/2^+$ states in Fig. 8. The isotopic dependence of the energy of the $7/2^-$ rotational state is steeper than that of the energy of the $7/2^-[514]$ one-quasiproton state.

In Fig. 10 the values of Q_α for the α decays from the ground states of Md isotopes are compared with the available experimental data [4]. Within the accuracy of our calculation the calculated results are in good agreement with experiment. The values of Q_α corresponding to α decays into the $7/2^-$ rotational states are presented in Fig. 10 as well. In ^{243,245,247}Es the $7/2^-$ rotational states are below the $7/2^-[514]$ states.

For the odd-odd nucleus ^{252}Md, the calculated two-quasiparticle spectra are presented in Fig. 11. We show the states with $K \geqslant 4$. Each of these states has a doublet with the smaller value of K. As is seen, there are no isomer states at $E_\mu \geqslant 50$ keV in this nucleus because the $E1$ transitions to the ground state are expected to be very fast.

IV. SUMMARY

In conclusion, the modified TCSM used here seems to be suitable for describing the single-particle structure of odd-Z heaviest nuclei and to predict the isomer configurations and the isotopic trends of one-quasiproton states. Based on our calculations the one-quasiproton spectra for ^{243}Md, ^{245}Md, ^{247}Md, ^{249}Md, ^{251}Md, ^{253}Md, ^{255}Md, ^{257}Md, ^{243}Es, ^{245}Es, ^{247}Es, ^{249}Es, ^{251}Es, ^{253}Es, ^{255}Es, ^{253,255}Lr, ^{257}Db, ^{261}Bh, ^{265}Mt, and ^{269}Rg are proposed. As is the case for the isotonic dependence in even-Z nuclei, the isotopic dependence of the energy of one-quasiparticle states in Md and Es is rather smooth. In ^{257}Db and ^{253,255}Lr the spectra of one-quasiparticle states are squeezed near the ground states. The calculations predict four closely spaced one-quasiproton states below an excitation energy of 80 keV. There are no calculated levels in the interval from 0.1 to 0.5–0.7 MeV. This gap is probably related to the deformation parameters resulting from the closed neutron subshell $N = 152$. In ^{253,255}Lr, the ground state would be $9/2^+[624]$ and the first excited state $7/2^-[514]$ is very close to it in energy. Because of this quasidegeneracy, the $7/2^-[514]$ configuration could be expected as the isomer one. In ^{269}Rg and ^{265}Mt the α decays from the isomer one-quasiproton states $3/2^-[512]$ and $1/2^-[510]$, respectively, seem to be possible for observing in future experiments. The α-decay chain ^{269}Rg $\rightarrow$ ^{265}Mt $\rightarrow$ ^{261}Bh over the isomer states is predicted. These isomers have not been discussed in the literature so far. The α decays of isotopes of Md from the $1/2^-$ isomer states are unlikely.

ACKNOWLEDGMENTS

We thank F. P. Hessberger, S. Hofmann, J. Khuyagbaatar, and A. Sobiczewski for fruitful discussions. This work was supported in part by Deutsche Forschungsgemeinschaft and the Russian Foundation for Basic Research. The IN2P3-JINR and Polish-JINR Cooperation Programmes are gratefully acknowledged.

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