Transverse electron scattering response function of ³He with Δ-isobar degrees of freedom

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A calculation of the ³He transverse (e,e') inclusive response function, R_T, which includes Δ degrees of freedom, is performed using the Lorentz integral transform method. The resulting coupled equations are treated in impulse approximation, where the NNN and $NN\Delta$ channels are solved separately. As NN and NNN potentials, we use the Argonne V18 and Urbana IX models, respectively. Electromagnetic currents include the Δ-isobar currents, one-body N currents with relativistic corrections, and two-body currents consistent with the Argonne V18 potential. R_T is calculated for the breakup threshold region at momentum transfers near 900 MeV/c. Our results are similar to those of Deltuva *et al.* in that large Δ-isobar current contributions are found. However, we find that these are largely canceled by the relativistic contribution from the one-body N currents. Finally, a comparison is made between theoretical results and experimental data.

I. INTRODUCTION

It is well known [1] that subnuclear degrees of freedom play an important role in nuclear dynamics. In conventional low-energy nuclear physics, the relevant subnuclear degrees of freedom are considered to be mesons and nucleon isobars. Electron scattering affords an excellent tool for studying these degrees of freedom, which are manifested in the transverse response through meson exchange (MEC) and isobar (IC) currents. The consideration of such subnuclear currents has a long history in the physics of few-nucleon systems. For two-body systems, a review can be found in [2]. One important issue in the MEC is their consistency with the nucleon-nucleon (NN) potential being used. Such consistent MECs have been taken into account not only in deuteron electrodisintegration but also in the electrodisintegration of three-body systems [3–7]. On the other hand, IC have not received the attention in three-body systems which they have in the two-nucleon sector. Nevertheless, there exists a rather complete calculation by [5,8,9] where N and Δ degrees of freedom have been treated on an equivalent level via a coupled channel calculation with NNN and $NN\Delta$ channels. Also Δ effects have been studied [3] in ³He electrodisintegration below the three-body breakup threshold using the transition-correlation-operator method [10].

The present work incorporates the dynamics of the Δ resonance into the many-body wave function by means of the impulse approximation (IA) [11]. This method, as in the transition-correlation-operator and coupled-channel methods, avoids the static approximation by fully including the kinetic energy in the Δ propagator. For electromagnetic deuteron breakup, it has been shown that the Δ effects resulting from an IA calculation are rather similar to those resulting from a coupled-channel calculation if the energy is sufficiently below the resonance position of the Δ [12].

A calculation of R_T in our case requires an integration over continuum states of the coupled $NNN + NN\Delta$ system. As

has been demonstrated previously [13,14], the Lorentz integral transform (LIT) method is well suited for calculating inclusive quantities such as response functions. Examples of its use in calculating electron-scattering response functions of three- and four-body nuclei employing realistic nuclear forces (two- and three-body forces) include (1) nonrelativistic calculations of R_L for three-nucleon systems [15], for ⁴He [16], and inclusion of relativistic effects in ³H and ³He [17] and (2) calculations of R_T with relativistic and consistent MEC contributions for three-body nuclei [6,7,18]. There is an LIT calculation [19] of R_T for ⁴He but with semirealistic NN forces. Here the method is applied to the coupled $NNN + NN\Delta$ system.

The article is organized as follows. In Sec. II, we describe the general formalism, including the incorporation of Δ degrees of freedom in the LIT formalism. Section III specifies the input to the dynamical equations developed in Sec. II. This includes Sec. III A, detailing the potentials used in the NNN and $NN\Delta$ sectors; Sec. III B, describing the electromagnetic current operators used; and Sec. III C, outlining calculational details. Finally, our results are discussed in Sec. IV. There we compare our results for R_T to those from another calculations and to experimental data.

II. FORMALISM

In the one-photon-exchange approximation, the cross section for the process of inclusive electron scattering on a nucleus is given by

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}}\left[\frac{Q^4}{q^4}R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2}\right)R_T(q,\omega)\right],$$
(1)

where R_L and R_T are the longitudinal and transverse response functions, respectively; ω is the electron energy loss; q is

the magnitude of the electron momentum transfer; θ is the electron-scattering angle; and $Q^2 = q^2 - \omega^2$.

In the present work, we study the transverse response function,

$$R_T(q, \omega) = \overline{\sum}_{M_i} \sum\!\!\!\!\!\!\!\int df (\mathbf{j}_t^\dagger)_{if} \cdot (\mathbf{j}_t)_{fi} \delta(E_{\bar{f}} - E_i - \omega), \tag{2}$$

with Δ degrees of freedom within a nonrelativistic approach. The low-ω region is considered. In (2), the subscripts i and f label, respectively, an initial state and final state, and the matrix elements are taken between internal states, center-of-mass motion being excluded.

As mentioned in Sec. I, we employ the IA in order to take into account the Δ resonance. This approximation is used for both the ^{3}He ground state and the final state. Subsequently, we outline the various theoretical aspects required to include Δ degrees of freedom in a calculation of R_T via the LIT method.

We consider the three-nucleon system with $N(939)$ and $\Delta(1232)$ degrees of freedom, which leads to the following Hamiltonian:

$$H = T + \delta m + V = \sum_{i=1}^3 (T_i + \delta m_i) + \sum_{i<j}^3 V_{ij}, \tag{3}$$

where T_i is the kinetic energy of particle i with mass m_i, $\delta m_i = m_i - m_N$ is its mass difference with nucleon $N(939)$, and $V_{ij} = V_{ji}$ is the potential between particles i and j. By omitting the contribution from more than one Δ-isobar excitation, we construct the three-particle bound state from an NNN part and a $NN\Delta$ part, that is,

$$|\Psi_0\rangle = \left|\Psi_0^N\right\rangle + \left|\Psi_0^\Delta\right\rangle. \tag{4}$$

The wave function is determined by the Schrödinger equation:

$$(T_N + \bar{V}^{NN} - E)\left|\Psi_0^N\right\rangle = -V^{NN,N\Delta}\left|\Psi_0^\Delta\right\rangle, \tag{5}$$

$$\begin{aligned}(H_\Delta - E)\left|\Psi_0^\Delta\right\rangle &= (\delta m + T_\Delta + V^{N\Delta} - E)\left|\Psi_0^\Delta\right\rangle \\ &= -V^{N\Delta,NN}\left|\Psi_0^N\right\rangle,\end{aligned} \tag{6}$$

where $V^{N_1'N_2',N_1N_2} = \sum_{i<j}^3 V_{ij}(N_1N_2 \to N_1'N_2')$, $\bar{V}^{NN} = \bar{V}^{NN,NN}$, $V^{N\Delta} = V^{N\Delta,N\Delta}$, and $\sum_{i=1}^3 T_i$ is denoted by T_N and T_Δ for the NNN and $NN\Delta$ channels, respectively. One should note that $\bar{V}^{NN}$ is different from the usual sum of realistic NN potentials, V^{NN}, because the latter already contain implicit effects due to the Δ (e.g., in meson-theoretical NN potentials realized via part of the σ-meson exchange).

Here we do not search for a direct solution of the coupled channel problem represented by Eqs. (5) and (6). Instead we use the IA, where one computes Ψ_0^N and Ψ_0^Δ separately. More specifically, one first determines the NNN part by solving

$$(H_N - E)\left|\Psi_0^N\right\rangle = 0, \tag{7}$$

with

$$H_N = T_N + V^{NN} + V^{NNN}, \tag{8}$$

where $V^{NNN} = \sum_{i<j<k}^3 V_{ijk}$ is a three-nucleon force. In the IA, one then uses the solution $|\Psi_0^N\rangle$ in order to calculate $|\Psi_0^\Delta\rangle$ through (6).

Treatment of the continuum in the LIT technique requires the calculation of a localized Lorentz state $|\tilde{\Psi}\rangle$. This Lorentz state $|\tilde{\Psi}\rangle$ also has NNN and $NN\Delta$ parts written as

$$|\tilde{\Psi}\rangle = |\tilde{\Psi}^N\rangle + |\tilde{\Psi}^\Delta\rangle. \tag{9}$$

These fulfill the coupled equations

$$\begin{aligned}(T_N + \bar{V}^{NN} - E_0 - \sigma)|\tilde{\Psi}^N\rangle &\\ = -V^{NN,N\Delta}|\tilde{\Psi}^\Delta\rangle + O_{NN}|\Psi_0^N\rangle + O_{N\Delta}|\Psi_0^\Delta\rangle,\end{aligned} \tag{10}$$

$$\begin{aligned}(\delta m + T_\Delta + V^{N\Delta} - E_0 - \sigma)|\tilde{\Psi}^\Delta\rangle &\\ = -V^{N\Delta,NN}|\tilde{\Psi}^N\rangle + O_{\Delta N}|\Psi_0^N\rangle + O_{\Delta\Delta}|\Psi_0^\Delta\rangle,\end{aligned} \tag{11}$$

where E_0 is the three-body ground-state energy, the complex $\sigma = \sigma_R + i\sigma_I$ is the argument of the LIT in the transformed space, and the $O_{N_1N_2}$ denote the various diagonal ($N_1 = N_2$) and transition ($N_1 \neq N_2$) electromagnetic current operators. One first solves for the NNN part using H_N of Eq. (8):

$$\begin{aligned}(H_N - E_0 - \sigma)|\tilde{\Psi}^N\rangle &\\ = -V^{NN,N\Delta}(H_\Delta - E_0 - \sigma)^{-1}\big(O_{\Delta N}|\Psi_0^N\rangle + O_{\Delta\Delta}|\Psi_0^\Delta\rangle\big) &\\ + O_{NN}|\Psi_0^N\rangle + O_{N\Delta}|\Psi_0^\Delta\rangle.\end{aligned} \tag{12}$$

The preceding equation is derived by solving (11) formally for $|\tilde{\Psi}^\Delta\rangle$, inserting the solution in (10), and dropping the term $V^{NN,N\Delta}(H_\Delta - E_0 - \sigma)^{-1}V^{N\Delta,NN}$, since, as mentioned, Δ effects to the nuclear interaction are already contained in the realistic H_N. With $|\tilde{\Psi}^N\rangle$ thus obtained, one then calculates $|\tilde{\Psi}^\Delta\rangle$ in a second step through (11). Given the solutions $|\tilde{\Psi}^N\rangle$ and $|\tilde{\Psi}^\Delta\rangle$, the LIT is obtained from the norm of the Lorentz state as

$$\langle\tilde{\Psi}|\tilde{\Psi}\rangle = \langle\tilde{\Psi}^N|\tilde{\Psi}^N\rangle + \langle\tilde{\Psi}^\Delta|\tilde{\Psi}^\Delta\rangle. \tag{13}$$

These two terms correspond to different contributions. It can be shown that the piece $\langle\tilde{\Psi}^N|\tilde{\Psi}^N\rangle$ describes contributions due to final states with nucleons only. In this case, the Δ degrees of freedom only contribute as virtual intermediate states. On the contrary, the term $\langle\tilde{\Psi}^\Delta|\tilde{\Psi}^\Delta\rangle$ describes contributions from final states containing a real Δ. The contribution to R_T from this term vanishes below the threshold for Δ production. Since the present study is for energies below that threshold, this term will not contribute here to R_T. Such a real Δ has to decay into a nucleon and a pion eventually, and thus the contribution $\langle\tilde{\Psi}^\Delta|\tilde{\Psi}^\Delta\rangle$ corresponds to resonant pion production.

III. INPUT TO DYNAMICAL EQUATIONS

A. Potentials

In the pure nucleonic sector, we use the Argonne V18 (AV18) NN potential [20] and the Urbana IX (UIX) NNN potential [21], while for the pure $NN\Delta$ sector, we take $V^{N\Delta} = 0$, as in the IA calculation of [12]. The NNN and $NN\Delta$ sectors are coupled via the $V^{NN,N\Delta}$ and $V^{N\Delta,NN}$ potentials. We use the same form for this potential as described in [12], except that here we use the short-range cutoff given in the AV18 [20] potential. In detail, we take $V(NN \to N\Delta)$

between particles 1 and 2 to have the form

$$V_{12} = a_{NN'\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \left[V_0(m_\pi) + \frac{2a_{NN'\rho}}{a_{NN'\pi}} V_0(m_\rho) \right] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right.$$
$$\left. + \left[V_2(m_\pi) - \frac{a_{NN'\rho}}{a_{NN'\pi}} V_2(m_\rho) \right] S_{12} \right\}, \tag{14}$$

with

$$V_0(m) = (mr)^{-1} e^{-mr} \left(1 - e^{-cr^2} \right), \tag{15}$$

$$V_2(m) = \left[1 + \frac{3}{mr} + \frac{3}{(mr)^2} \right] \frac{e^{-mr}}{mr} \left(1 - e^{-cr^2} \right)^2, \tag{16}$$

where $\boldsymbol{\sigma}_i$ ($\boldsymbol{\tau}_i$) are regarded as transition operators for spin (isospin) of particle i, the coupling constants $a_{NN'\pi}$ and $a_{NN'\rho}$ are taken from [12], $c = 2.1$ fm^{-2} is the same value as in the AV18 potential, and $r = |\mathbf{r}_1 - \mathbf{r}_2|$ is proportional to the Jacobi vector $\boldsymbol{\xi}_1$ [see Eq. (22)].

B. Electromagnetic currents

In Eqs. (10) and (11), the driving terms are the transverse electromagnetic currents acting on the ground state. The term O_{NN} represents the purely nucleonic one- and two-body currents. For these, the same one-body and two-body currents as employed in [18] are used. There the one-body currents included relativistic corrections to order M^{-3}, and the two-body currents were consistent π- and ρ-MEC currents constructed using the method of Arenhövel and Schwamb [22]. The other terms, $O_{N\Delta}$, $O_{\Delta N}$, and $O_{\Delta\Delta}$, are transition and diagonal one-body Δ-isobar currents. The currents involving the Δ resonance are given in Fig. 1. For one-body Δ-isobar currents, we use the forms

$$\mathbf{j}_{\Delta N} = \sum_{k=1}^{3} e^{i\mathbf{q}\cdot\mathbf{r}_k'} \frac{i\left(\sigma_k^{\Delta N} \times \mathbf{q}\right)}{2m_\Delta} G_{M1}^{\Delta N} \tau_{zk}^{\Delta N}, \tag{17}$$

$$\mathbf{j}_{\Delta\Delta} = \sum_{k=1}^{3} e^{i\mathbf{q}\cdot\mathbf{r}_k'} \left[\frac{2\mathbf{p}_k' + \kappa\mathbf{q}}{2m_\Delta} G_{E0}^{\Delta} + \frac{i(\sigma_k^{\Delta} \times \mathbf{q})}{2m_\Delta} G_{M1}^{\Delta} \right]$$
$$\times \frac{1 + \tau_{zk}^{\Delta}}{2}, \tag{18}$$

where $\mathbf{r}_k' = \mathbf{r}_k - \mathbf{R}_{cm}$ and $\mathbf{p}_k' = \mathbf{p}_k - \mathbf{P}_{cm} A_k / A$ are the relative coordinate and momentum operators of the kth particle, while $\mathbf{R}_{cm}$ and $\mathbf{P}_{cm}$ are the center-of-mass coordinate and initial total momentum variable of the system. With the assumption that $\mathbf{P}_{cm}$ is directed along $\mathbf{q}$, the term κ in (18) has the value $\kappa = 1 + 2P_{cm} A_k / (Aq)$. However, since here we are dealing with transverse currents, this term does not contribute. The form factors for the preceding currents are the same as in [5,9]

(a)   (b)
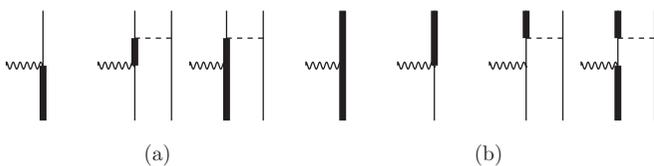
FIG. 1. Operators involving the Δ resonance in the (a) NNN and (b) $NN\Delta$ channels.

and take the form

$$G_{M1}^{\Delta N}(Q^2) = \frac{m_\Delta}{m_N} \frac{4.59}{\left(1 + Q^2/\Lambda_{\Delta N1}^2\right)^2 \left(1 + Q^2/\Lambda_{\Delta N2}^2\right)^{1/2}},$$

$$G_{E0}^{\Delta}(Q^2) = G_E^p(Q_2),$$

$$G_{M1}^{\Delta}(Q^2) = \frac{m_\Delta}{3m_N} \frac{(4.35/2.0)}{\left(1 + Q^2/\Lambda_{\Delta N1}^2\right)^2}, \tag{19}$$

with $\Lambda_{\Delta N1} = 840$ MeV and $\Lambda_{\Delta N2} = 1200$ MeV. As in our previous NNN calculations [6,7,18], we use the approximations

$$G_{M1}^{\Delta N}(Q^2) \approx \bar{\mu}_p^{\Delta N}\left(Q_{av}^2\right) G_E^p(Q^2), \quad \bar{\mu}_p^{\Delta N}\left(Q_{av}^2\right) = \frac{G_{M1}^{\Delta N}\left(Q_{av}^2\right)}{G_E^p\left(Q_{av}^2\right)},$$

$$G_{M1}^{\Delta}(Q^2) \approx \bar{\mu}_p^{\Delta}\left(Q_{av}^2\right) G_E^p(Q^2), \quad \bar{\mu}_p^{\Delta}\left(Q_{av}^2\right) = \frac{G_{M1}^{\Delta}\left(Q_{av}^2\right)}{G_E^p\left(Q_{av}^2\right)}.$$

C. Calculational details

In order to solve Eqs. (7), (11), and (12) for the ground state and Lorentz vectors, we expand the bound and Lorentz states on a complete antisymmetric basis. The reason for antisymmetrizing $NN\Delta$ states is that they couple to purely antisymmetric nucleonic states through symmetric operators. Thus the excitation of an antisymmetric NNN state to a $NN\Delta$ state occurs via an operator symmetric with respect to nucleons. Such an operator is a sum of operators which replace a nucleon with a Δ, which therefore leads to an antisymmetric $NN\Delta$ state. For the NNN part, we take the same correlated hyperspherical basis as in our previous three-body calculations, without considering Δ degrees of freedom (see, e.g., [14]). For the part with one Δ excitation, we use the following hyperspherical basis:

$$|\varphi_k\rangle = \frac{1 + P}{\sqrt{3}} \frac{1 - (12)}{\sqrt{2(1 + B^2)}}$$
$$\times |\{h_{NK(Ll)\mathcal{L}} \otimes [(s_1 s_2)Ss_3]\mathcal{S}\}_{\mathcal{J}\mathcal{M}_{\mathcal{J}}}, [(t_1 t_2)Tt_3]\mathcal{T}\mathcal{M}_{\mathcal{T}}\rangle, \tag{20}$$

where N is the order of the hyperradial function, K is the hyperspherical angular momentum, and L and l are the orbital angular momentum of the pair and of the spectator, respectively, coupled to the total orbital angular momentum $\mathcal{L}$. Individual spin (isospin) quantum numbers of the three particles are denoted by s_i (t_i), $i = 1, 2, 3$, while the pair spin (isospin) is denoted by S (T), the total spin (isospin) is denoted by $\mathcal{S}$ ($\mathcal{T}$), and $\mathcal{M}_{\mathcal{T}}$ stands for the projection of the total isospin. Quantum numbers $\mathcal{J}$ and $\mathcal{M}_{\mathcal{J}}$ denote the total angular momentum and its projection, and $(\cdots \otimes \cdots)_{\mathcal{J}\mathcal{M}_{\mathcal{J}}}$ denotes $(\mathcal{LS})\mathcal{J}$ coupling. The index k denotes collectively $\{N, K, L, l, \mathcal{L}, S, \mathcal{S}, \mathcal{J}, \mathcal{M}_{\mathcal{J}}, T, \mathcal{T}, \mathcal{M}_{\mathcal{T}}\}$ and B. We define the quantity B to be 0 if the pair of the three-particle system contains one Δ, and therefore $(s_1, s_2, s_3) = (t_1, t_2, t_3) = (1/2, 3/2, 1/2)$, and to be 1 if the pair contains no Δ, and therefore $(s_1, s_2, s_3) = (t_1, t_2, t_3) = (1/2, 1/2, 3/2)$. Note that if particles 1 and 2 are nucleons, we always assume that $L + S + T = $ odd so that the NN pair is already antisymmetric. The spatial basis functions in coordinate representation

are products of hyperradial functions and hyperspherical harmonics:

$$\langle \boldsymbol{\xi}_1 \boldsymbol{\xi}_2 | h_{NK(Ll)\mathcal{L}\mathcal{M}_{\mathcal{L}}} \rangle = R_N(\rho) Y^{Ll}_{K\mathcal{L}\mathcal{M}_{\mathcal{L}}}(x, x_1, \varphi_1, x_2, \varphi_2). \quad (21)$$

The coordinates ρ and x are defined in terms of the Jacobi vectors,

$$\boldsymbol{\xi}_1 = \sqrt{\frac{A_1 A_2}{A_1 + A_2}}(\mathbf{r}_2 - \mathbf{r}_1),$$

$$\boldsymbol{\xi}_2 = \sqrt{\frac{(A_1 + A_2)A_3}{A_1 + A_2 + A_3}}\left(\mathbf{r}_3 - \frac{A_1\mathbf{r}_1 + A_2\mathbf{r}_2}{A_1 + A_2}\right), \quad (22)$$

as $\rho = (\xi_1^2 + \xi_2^2)^{1/2}$, $x = (\xi_2^2 - \xi_1^2)/\rho$. The coordinates $x_i \equiv \cos\theta_i$ and φ_i are spherical coordinates of the unit vectors in the directions of $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, respectively. We use the notation $A_i \equiv m_i/m_N$. Note that particle permutations entering the antisymmetrization operator interchange not only particle position vectors $\mathbf{r}_i$ but also their mass numbers A_i. One has $\rho^2 = \xi_1^2 + \xi_2^2 = A_1 r_1^2 + A_2 r_2^2 + A_3 r_3^2 - A R^2$, where $\mathbf{R}$ is the center-of-mass position. Thus ρ remains invariant under particle permutations.

The operator $[1 - (12)]/[2(1 + B^2)]^{1/2}$ makes the (12) pair explicitly antisymmetric. Note that it gives unity if the pair contains no Δ but rather is an antisymmetric NN pair. Finally, the operator $(1 + P)/\sqrt{3}$, where $P \equiv (123) + (132)$, makes the three-particle states with antisymmetric (12) pair totally antisymmetric. It turns out to be convenient to keep both $B = 0$ and $B = 1$ in (20) (thereby resulting in an overcomplete basis) and to finally select out numerically the linearly independent states. This enables one to select out those states which give negligible contributions to the results. Application of the operator $(1 + P)/\sqrt{3}$ in (20) results in the more practical form

$$|\varphi_k\rangle = \sum_{B'S'T'} \frac{1 - (12)}{\sqrt{2(1 + B'^2)}}$$
$$\times |\{F_{j\mathcal{L}} \otimes [(s_1's_2')S's_3']\mathcal{S}\}_{\mathcal{J}\mathcal{M}_{\mathcal{J}}}, [(t_1't_2')T't_3']T\mathcal{M}_T\rangle, \quad (23)$$

where j denotes collectively $\{B'S'T', B\mathcal{S}\mathcal{S}\mathcal{T}\mathcal{T}NK(Ll)\}$ and

$$|F_{j\mathcal{L}\mathcal{M}_{\mathcal{L}}}\rangle = \frac{1}{\sqrt{3}}\big(\delta_{B',B}\delta_{S',\mathcal{S}}\delta_{T',\mathcal{T}}|h_{NK(Ll)\mathcal{L}\mathcal{M}_{\mathcal{L}}}\rangle$$
$$+ g^c_{B'S'T',B\mathcal{S}\mathcal{S}\mathcal{T}\mathcal{T}}|h^c_{NK(Ll)\mathcal{L}\mathcal{M}_{\mathcal{L}}}\rangle$$
$$+ g^d_{B'S'T',B\mathcal{S}\mathcal{S}\mathcal{T}\mathcal{T}}|h^d_{NK(Ll)\mathcal{L}\mathcal{M}_{\mathcal{L}}}\rangle\big), \quad (24)$$

with $(Ll)\mathcal{L}$ coupling for the total orbital angular momentum $\mathcal{L}$ and its projection $\mathcal{M}_{\mathcal{L}}$. As mentioned earlier, components with $B' = 1$ in Eq. (23) represent configurations $NN\Delta$ in which particles 1 and 2 are nucleons. Those components with $B' = 0$ represent $N\Delta N$ configurations in which particle 1 is a nucleon and particle 2 is a Δ. More details of the spin-isospin factors g^c and g^d, and the spatial functions h^c and h^d, are given in Appendix A. Techniques employed in calculating the kinetic energy and the $NN \to N\Delta$ or $N\Delta \to NN$ potential are given in Appendixes B and C, respectively.

As in [6], all currents are expressed in terms of multipole expansions. Explicit expressions for the multipoles of the one-body current (containing relativistic corrections) are given

in [18]. The multipoles for the π- and ρ-MEC are found in [6], with modifications because of the implementation of consistent MECs for the AV18 potential listed in [7]. Finally, the multipoles required here for the one-body currents relating to the Δ are listed in Appendix D. With these multipoles, one can then decompose the LIT of the response function according to its multipole content as

$$\tilde{R}_T(\sigma) = \frac{4\pi}{2\mathcal{J}_0 + 1} \sum_{\lambda=\text{el,mag}} \sum_{\mathcal{J}j}(2\mathcal{J} + 1)(\tilde{R}_T)^{j\lambda}_{\mathcal{J}}, \quad (25)$$

where

$$(\tilde{R}_T)^{j\lambda}_{\mathcal{J}} = \langle \tilde{\Psi}^{j\lambda}_{\mathcal{J}\mathcal{M}} | \tilde{\Psi}^{j\lambda}_{\mathcal{J}\mathcal{M}} \rangle = \langle \tilde{\Psi}^{Nj\lambda}_{\mathcal{J}\mathcal{M}} | \tilde{\Psi}^{Nj\lambda}_{\mathcal{J}\mathcal{M}} \rangle + \langle \tilde{\Psi}^{\Delta j\lambda}_{\mathcal{J}\mathcal{M}} | \tilde{\Psi}^{\Delta j\lambda}_{\mathcal{J}\mathcal{M}} \rangle. \quad (26)$$

Here $\mathcal{J}_0$ is the initial state total angular momentum and $\mathcal{J}$ and $\mathcal{M}$ are the final state total angular momentum and its projection. The $|\tilde{\Psi}^{j\lambda}_{\mathcal{J}}\rangle$ are the solutions of (11) and (12), where the following replacement is made on the right-hand side (rhs) of these equations:

$$O|\Psi_0\rangle \to |q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle = \big[T^{\lambda}_j \otimes |\Psi_0(\mathcal{J}_0)\rangle\big]_{\mathcal{J}\mathcal{M}}. \quad (27)$$

In Eq. (26), $\mathcal{M}$ is arbitrary. In Eq. (27), above Ψ_0 is either Ψ_0^N or Ψ_0^{Δ}, while O represents the various electromagnetic current operators on the rhs of (10) and (11). By projecting the rhs of (27) on the basis states (23), one obtains

$$\langle \{F_{j'\mathcal{L}'} \otimes [(s_1's_2')S's_3']\mathcal{S}'\}_{\mathcal{J}'\mathcal{M}'} | q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle$$
$$= \delta_{\mathcal{J}',\mathcal{J}}\delta_{\mathcal{M}',\mathcal{M}} \frac{\langle \{F_{j'\mathcal{L}'} \otimes [(s_1's_2')S's_3']\mathcal{S}'\}_{\mathcal{J}'} \| T^{\lambda}_j \| \Psi_0(\mathcal{J}_0)\rangle}{\hat{\mathcal{J}}'}. \quad (28)$$

We use the Lanczos method to calculate the response function, as described in [23]. The response function is thus calculated using

$$\langle \tilde{\Psi}^{Nj\lambda}_{\mathcal{J}\mathcal{M}} | \tilde{\Psi}^{Nj\lambda}_{\mathcal{J}\mathcal{M}} \rangle = -\frac{1}{\sigma_I}\text{Im}\langle Q^{j\lambda}_{\mathcal{J}\mathcal{M}}|\frac{1}{E_0 + \sigma - H_N}|Q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle$$
$$= -\sum_{m,n}\frac{1}{\sigma_I}\text{Im}\langle Q^{j\lambda}_{\mathcal{J}\mathcal{M}}|\phi_m\rangle\langle\phi_m|$$
$$\times \frac{1}{E_0 + \sigma - H_N}|\phi_n\rangle\langle\phi_n|Q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle, \quad (29)$$

where Q corresponds to the rhs of (12) and $|\phi_n\rangle$ is the set of orthogonal Lanczos vectors. As starting vector $|\phi_0\rangle$, we choose the rhs of (12) at one particular value of σ. The expression $\langle\phi_m|(E_0 + \sigma - H_N)^{-1}|\phi_n\rangle$ can be calculated using Eq. (71) of [23]. A difference from previous LIT applications appears in the potential term on the rhs of the LIT equation, namely, the $N\Delta$ transition potential $V^{NN,N\Delta}$ in (12). The contribution of this term to $\langle\phi_n|Q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle$ is given by

$$-\sum_{l'k'lkm}\langle\phi_n|\varphi_{l'}\rangle N^{-1}_{l'k'}\langle\varphi_{k'}|V^{NN,N\Delta}|\varphi_l\rangle$$
$$\times N^{-1}_{lk}\langle\varphi_k|\phi_m\rangle\langle\phi_m|(H_{\Delta} - E_0 - \sigma)^{-1}|q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle, \quad (30)$$

with

$$|q^{j\lambda}_{\mathcal{J}\mathcal{M}}\rangle = \big[\mathcal{T}^{\lambda}_{\Delta N,j} \otimes |\Psi_0^N(\mathcal{J}_0)\rangle\big]_{\mathcal{J}\mathcal{M}} + \big[\mathcal{T}^{\lambda}_{\Delta\Delta,j} \otimes |\Psi_0^{\Delta}(\mathcal{J}_0)\rangle\big]_{\mathcal{J}\mathcal{M}} \quad (31)$$

and the norm matrix $N = (\langle\varphi_k|\varphi_l\rangle)$. Also, the second term in Eq. (26), $\langle\tilde{\Psi}^{\Delta j\lambda}_{\mathcal{J}M}|\tilde{\Psi}^{\Delta j\lambda}_{\mathcal{J}M}\rangle$, can be calculated in a similar way with the Lanczos method.

Because, in this article, we are working at low energies near the ^{3}He breakup threshold, only the lowest multipole transitions contribute. We found sufficient accuracy by restricting the maximum value of $\mathcal{J}$ to 5/2. The LIT is computed with $\sigma_I = 5$ MeV. The LIT inversion [24] is made with our standard inversion method [14,25]. As discussed in [6], we subtract from the LIT of the M1 transition the elastic contribution and invert the remaining inelastic piece separately from the other multipole contributions.

IV. RESULTS

In the present work, we have used the LIT method to calculate Δ-IC effects on the transverse electron scattering response function $R_T(q,\omega)$ for $q \approx 900$ MeV/c and ω up to 20 MeV above the breakup threshold. This is an application of the LIT method to include Δ degrees of freedom in the calculation of inclusive (e,e') response functions. The importance of Δ effects at these kinematics has previously been shown by [5]. As NN and NNN interactions, we used the AV18 and the UIX potentials, respectively. Following the IA calculation of [12], we do not consider a diagonal $N\Delta$ interaction, that is, $V_{N\Delta} = 0$. For the ^{3}He ground state, our interaction model leads to a Δ probability of 1.14%, which compares to 1.44% obtained by [5], who used a CDBonn + Δ coupled channel potential model [26]. In addition to the Δ-ICs $\mathbf{j}_{N\Delta}$, $\mathbf{j}_{\Delta N}$, and $\mathbf{j}_{\Delta\Delta}$, the purely nucleonic currents include the nonrelativistic one-body current with relativistic corrections up to order M^{-3} [18] and an MEC consistent with the AV18 potential [7]. Concerning the relativistic corrections, we leave out the ω-dependent relativistic piece in the present work as its contribution is negligible in the threshold region we consider. For the neutron magnetic and the proton form factors, we take the dipole fit, while the neutron electric form factor is taken from [27].

Figure 2 displays our R_T results for several calculational options. The dominant transition multipolarity contributing at these near-threshold energies is M1. One sees that relativistic effects reduce the M1 transition strength considerably. If, in addition, MECs are also taken into account, then the M1 contribution drops markedly, leading to a rather different low-energy behavior of R_T. Inclusion of Δ-IC restores some of this lost strength and demonstrates, as anticipated, that the Δ effect is quite large.

Now we turn to a comparison of our results with those of [5]. For the comparison, one should keep in mind that there are differences between the two calculations. Thus, in [5], (1) relativistic currents have not been considered; (2) the Coulomb force is neglected in the final state interaction; (3) their nucleonic potential model, the CDBonn [28], is different from ours and does not reproduce the ^{3}He binding energy; and (4) the full coupled channel calculation, CDBonn + Δ [26], is a more consistent treatment of Δ degrees of freedom than the IA but leads to a slight underestimation of the ^{3}He binding energy. There is another point which makes the comparison a
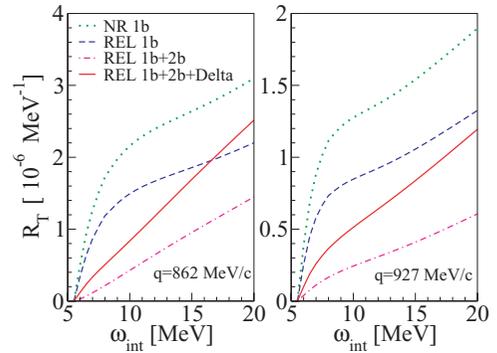
FIG. 2. (Color online) Theoretical results for R_T of ^{3}He as a function of internal excitation energy ω_{int} at $q = 862$ and 927 MeV/c with various current operators: nonrelativistic nucleon one-body (dotted lines), with additional relativistic corrections (dashed lines), plus additional MEC (dash-dotted lines) and further addition of Δ-IC (solid lines).

bit more difficult, namely, the R_T of [5] is not calculated for a constant momentum transfer; in fact, q is slightly decreasing with growing energy. Therefore, in Fig. 3, we prefer to display the results for each q in two panels. One sees that despite the various differences mentioned earlier, the Δ effects in both calculations are very similar. However, one also notes that relativistic corrections lead to an opposite effect which is of the same size at $q = 862$ MeV/c but somewhat weaker at $q = 927$ MeV/c. By comparison of results for R_T at about $q = 500$ MeV/c from [7] against those of [5], one finds again an at least partial cancellation of relativistic and Δ contributions close to the breakup threshold. The stronger increase of R_T in the calculation of [5] at higher energies, seen in Fig. 3, partly originates from the nonconstant momentum transfers used in [5], as mentioned earlier (see also the discussion of Fig. 4).
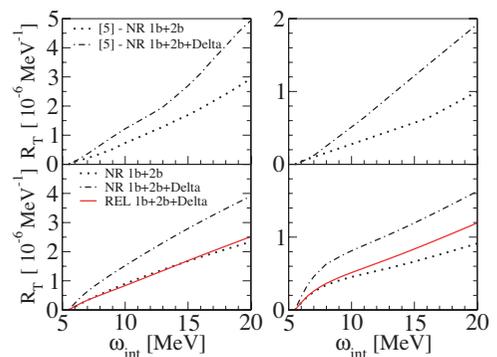
FIG. 3. (Color online) R_T of ^{3}He. Lower panels ($q = 862$ MeV/c, left; $q = 927$ MeV/c, right) show the theoretical result from the present work with nonrelativistic nucleon one-body current and MEC (dotted line) and additional Δ-IC (dash-dotted lines) and further addition of relativistic corrections for nucleon one-body current (solid lines). Upper panels (q at threshold as in lower panels, but slightly varying with ω_{int}; see text) show the theoretical results from Deltuva *et al.* [5] with nonrelativistic nucleon one-body current and MEC (dotted lines) and additional Δ contributions (dash-dotted lines).
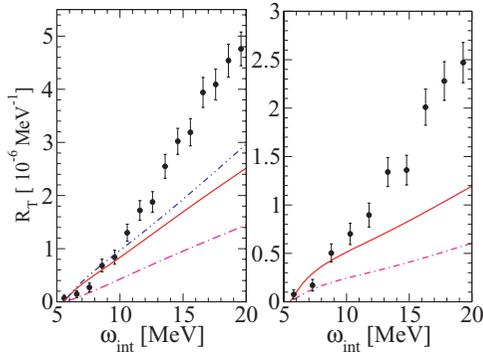
FIG. 4. (Color online) R_T of ^{3}He for same kinematics and the same dash-dotted and solid curves as in Fig. 2; in addition, the result with all current contributions at $q = 850$ MeV/c (dash-double-dotted curve). Experimental data with slightly varying q values from [29].

Finally, in Fig. 4, we compare the results of our calculation with experimental data [29]. Again, as in the calculation of [5], the momentum transfer is only quasiconstant. Therefore, in the left panel, we show R_T including all current contributions for the two extreme q values, that is, $q = 862$ and 850 MeV/c, and in addition, for $q = 862$ MeV/c, the result where only Δ-IC are left out. The lower q corresponds to the data at about $\omega_{int} = 20$ MeV, while the higher q corresponds to the threshold energy. In the right panel of the figure, we only show the results for $q = 927$ MeV/c that correspond to the data close to threshold. The Δ-IC contribution is seen to be essential for obtaining a good agreement between theory and experiment below 10 MeV. However, at higher energies, the increase because of Δ-IC is not sufficient to describe the data, even if one considers the slight shift of q with higher energies represented by the dotted curve. The present case is rather similar to deuteron electrodisintegration at higher momentum transfer, where at low excitation energy, the leading M1 transition also has a minimum. For the deuteron case, it is known (see, e.g., [30]) that various theoretical ingredients, for example, the potential model dependence, can lead to rather large variations of the theoretical result. Thus our present study cannot give a final answer concerning the comparison of theory and experiment.

We summarize our work as follows. We have illustrated how Δ degrees of freedom are integrated into the LIT formalism for a calculation of the inelastic inclusive transverse (e,e') response function R_T of ^{3}He. The resulting coupled equations for the Lorentz states of the NNN and $NN\Delta$ channels contain, as opposed to the corresponding coupled Schrödinger equation, source terms with electromagnetic operators acting on the nuclear ground state. The Δ degrees of freedom are present in three different forms: (1) in the potentials $V^{N_1'N_2',N_1N_2}$, (2) in the Δ propagator, and (3) in the current operators $\mathbf{j}_{N_1'N_1}$. The coupled channel equation is solved in impulse approximation, where the NNN and $NN\Delta$ channels are treated separately. First, the NNN part is solved using a realistic nuclear interaction with NN and NNN potentials. The result thus obtained is then used for the solution of the $NN\Delta$ channel. The former gives a contribution to the electrodisintegration of a purely nucleonic final state, whereas the latter leads to a contribution to the pion production channel. In the present work, we have studied Δ effects in R_T of ^{3}He close to the breakup threshold at a momentum transfer of about 900 MeV/c. The response function is affected by sizable MEC contributions and, as in a previous full coupled-channel calculation [5], we find a considerable increase of R_T because of Δ degrees of freedom. Unlike the calculation of [5], we here take into account relativistic corrections to the nonrelativistic one-body current operator. At the kinematics considered here, these relativistic corrections nearly cancel the Δ-IC contribution. This cancellation in fact leads to good agreement of our theoretical R_T with experimental data at very low energy transfer, while the experimental R_T is underestimated at somewhat higher energies.

APPENDIX A: DETAILS OF g^c, h^c, g^d, h^d

The spin-isospin factors are

$$g^c_{B'S'T',B\mathcal{S}\mathcal{T}} = \begin{cases} \frac{1}{\sqrt{2}} f_{B'S'T',B\mathcal{S}\mathcal{T}} & \text{if} \quad B' = 1 \quad \text{and} \quad B = 0, \\ -(-)^{S+T} f_{B'S'T',B\mathcal{S}\mathcal{T}}|_{s_1 \leftrightarrow s_2, t_1 \leftrightarrow t_2} & \text{if} \quad B' = 0 \quad \text{and} \quad B = 0, \\ 0 & \text{otherwise}, \end{cases} \tag{A1}$$

$$g^d_{B'S'T',B\mathcal{S}\mathcal{T}} = \begin{cases} -(-1)^{S'+T'} \frac{1}{\sqrt{2}} f_{B'S'T',B\mathcal{S}\mathcal{T}}|_{s_1' \leftrightarrow s_2', t_1' \leftrightarrow t_2'} & \text{if} \quad B' = 1 \quad \text{and} \quad B = 0, \\ -(-)^{S'+T'} \sqrt{2} f_{B'S'T',B\mathcal{S}\mathcal{T}}|_{s_1' \leftrightarrow s_2', t_1' \leftrightarrow t_2'} & \text{if} \quad B' = 0 \quad \text{and} \quad B = 1, \\ 0 & \text{otherwise}, \end{cases} \tag{A2}$$

$$f_{B'S'T',B\mathcal{S}\mathcal{T}} = (-)^{S+T} \hat{S}'\hat{S}\hat{T}'\hat{T} \begin{Bmatrix} s_1' & s_2' & S' \\ s_3' & \mathcal{S} & S \end{Bmatrix} \begin{Bmatrix} t_1' & t_2' & T' \\ t_3' & \mathcal{T} & T \end{Bmatrix} \delta_{s_1's_3}\delta_{s_2's_1}\delta_{s_3's_2}. \tag{A3}$$

The spatial functions in coordinate representation are

$$\langle \boldsymbol{\xi}_1 \boldsymbol{\xi}_2 | h^c_{NK(Ll)\mathcal{L}\mathcal{M}_\mathcal{L}} \rangle$$
$$= (-)^L \langle \boldsymbol{\xi}'_1(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) \boldsymbol{\xi}'_2(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) | h_{NK(Ll)\mathcal{L}\mathcal{M}_\mathcal{L}} \rangle, \qquad (A4)$$

$$\langle \boldsymbol{\xi}_1 \boldsymbol{\xi}_2 | h^d_{NK(Ll)\mathcal{L}\mathcal{M}_\mathcal{L}} \rangle$$
$$= \langle \boldsymbol{\xi}'_1(-\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) \boldsymbol{\xi}'_2(-\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) | h_{NK(Ll)\mathcal{L}\mathcal{M}_\mathcal{L}} \rangle, \qquad (A5)$$

where $\boldsymbol{\xi}'_1(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$ and $\boldsymbol{\xi}'_2(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$ are connected to $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ through the cycle operator (123) by $\langle \mathbf{r}'_1 \mathbf{r}'_2 \mathbf{r}'_3 s'_1 s'_2 s'_3 t'_1 t'_2 t'_3 | = \langle \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_1 s_2 s_3 s_1 t_2 t_3 t_1 |$, and using (22), we have

$$\boldsymbol{\xi}'_1(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) = -\sqrt{\frac{A_3 A_1}{(A_2 + A_3)(A_1 + A_2)}}\, \boldsymbol{\xi}_1$$
$$+ \sqrt{\frac{A_2(A_1 + A_2 + A_3)}{(A_2 + A_3)(A_1 + A_2)}}\, \boldsymbol{\xi}_2,$$
$$\qquad (A6)$$
$$\boldsymbol{\xi}'_2(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) = -\sqrt{\frac{(A_1 + A_2 + A_3)A_2}{(A_2 + A_3)(A_1 + A_2)}}\, \boldsymbol{\xi}_1$$
$$- \sqrt{\frac{A_1 A_3}{(A_2 + A_3)(A_1 + A_2)}}\, \boldsymbol{\xi}_2.$$

APPENDIX B: KINETIC ENERGY CALCULATIONAL DETAILS

For a basis of antisymmetric states, the kinetic energy can be written as

$$T = \frac{3}{2m_N} \frac{A_1 + A_2}{A_1 + A_2 + A_3} \boldsymbol{\pi}_1^2, \qquad (B1)$$

where $\boldsymbol{\pi}_1 = -i\partial/\partial \boldsymbol{\xi}_1$ is the Jacobi momentum conjugate to $\boldsymbol{\xi}_1$. Noting that for calculating the matrix elements of $\boldsymbol{\pi}_1^2$ between the basis states (23), one may drop $(1 - (12))/[2(1 + B''^2)]^{1/2}$, we get

$$\langle \varphi_{k'} | T | \varphi_k \rangle = \frac{3}{2m_N} \delta_{\mathcal{L}',\mathcal{L}} \delta_{\mathcal{S}',\mathcal{S}} \delta_{\mathcal{J}',\mathcal{J}} \delta_{\mathcal{M}_\mathcal{J}',\mathcal{M}_\mathcal{J}} \delta_{\mathcal{T}',\mathcal{T}} \delta_{\mathcal{M}_\mathcal{T}',\mathcal{M}_\mathcal{T}}$$
$$\times \sum_{B''S''T''} \frac{A''_1 + A''_2}{A''_1 + A''_2 + A''_3}$$
$$\times \langle F_{B''S''T'', B'S'S\mathcal{T}'\mathcal{T}N'K'(L'l')\mathcal{L}\mathcal{M}_\mathcal{L}}$$
$$\times |\boldsymbol{\pi}_1^2| F_{B''S''T'', BS\mathcal{S}\mathcal{T}\mathcal{T}NK(Ll)\mathcal{L}\mathcal{M}_\mathcal{L}} \rangle. \qquad (B2)$$

For the calculation of the spatial matrix elements, we use the technique as described in [31] to get

$$\langle F_{j'\mathcal{L}'\mathcal{M}_{\mathcal{L}'}} | \boldsymbol{\pi}_1^2 | F_{j\mathcal{L}\mathcal{M}_\mathcal{L}} \rangle$$
$$= \frac{\delta_{\mathcal{L}',\mathcal{L}} \delta_{\mathcal{M}_{\mathcal{L}'},\mathcal{M}_\mathcal{L}} 8\pi^2}{2\mathcal{L}+1} \int d\tau_{\text{int}} \left\{ \sum_{\mathcal{M}''} \left[\frac{\partial}{\partial \xi_1} F_{j'\mathcal{L}\mathcal{M}''} \right] \right.$$
$$\left. \times \left[\frac{\partial}{\partial \xi_1} F_{j\mathcal{L}\mathcal{M}''} \right] + \xi_1^{-2} \sum_{\mathcal{M}'',\mu=\pm} l_\mu F_{j'\mathcal{L}\mathcal{M}''} l_\mu F_{j\mathcal{L}\mathcal{M}''} \right\},$$
$$\qquad (B3)$$

where $d\tau_{\text{int}} = \xi_1^2 \xi_2^2 d\xi_1 d\xi_2 dt$, $t = \hat{\boldsymbol{\xi}}_1 \cdot \hat{\boldsymbol{\xi}}_2$. The underlines mean that the space points take the value

$$\xi_{1x} = 0, \quad \xi_{1y} = 0, \quad \xi_{1z} = \xi_1, \quad \xi_{2x} = \xi_2 \sqrt{1 - t^2},$$
$$\xi_{2y} = 0, \quad \xi_{2z} = \xi_2 t; \qquad (B4)$$

the orbital angular momentum is given by $\mathbf{l} = \boldsymbol{\xi}_1 \times \boldsymbol{\pi}_1$; and

$$l_{\pm 1} F_{j\mathcal{L}\mathcal{M}} = -\frac{\xi_1}{\sqrt{2}} [(\partial/\partial \xi_{1x}) \pm i(\partial/\partial \xi_{1y})] F_{j\mathcal{L}\mathcal{M}}, \qquad (B5)$$

with the derivatives taken at the space point of (B4).

APPENDIX C: CALCULATIONAL DETAILS FOR $V(NN \to N\Delta)$ POTENTIAL

In calculating matrix elements of the transition potentials between antisymmetric basis states, one may omit the factor $[1 - (12)]/\sqrt{2(1 + B'^2)}$ from Eq. (23) using the substitutions

$$\frac{1 - (12)}{\sqrt{2(1 + B'^2)}} V^{N\Delta,NN} \to 3 \cdot \sqrt{2} V_{12}(NN \to N\Delta),$$
$$V^{NN,N\Delta} \frac{1 - (12)}{\sqrt{2(1 + B^2)}} \to 3 \cdot \sqrt{2} V_{12}(N\Delta \to NN).$$

Each basis state in (23) is the sum of several terms of the form

$$\frac{1 - (12)}{\sqrt{2(1 + B'^2)}} |\{F_{j\mathcal{L}} \otimes [(s'_1 s'_2)S' s'_3]\mathcal{S}\}_{\mathcal{J}\mathcal{M}_\mathcal{J}}, [(t'_1 t'_2)T' t'_3]\mathcal{T}\mathcal{M}_\mathcal{T}\rangle,$$

and this is also true for the basis of the NNN part, but with $B = B' = 1$, $(s_1, s_2, s_3) = (t_1, t_2, t_3) = (1/2, 1/2, 1/2)$, and $[1 - (12)]/[2(1 + B'^2)]^{1/2} = 1$. Therefore, for the matrix elements of the operator V_{12}, we need

$$\langle \{F_{j'\mathcal{L}'} \otimes [(s'_1 s'_2)S' s'_3]\mathcal{S}'\}_{\mathcal{J}'\mathcal{M}_{\mathcal{J}'}}$$
$$\times |V_0(m_B)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2| \{F_{j\mathcal{L}} \otimes [(s_1 s_2)Ss_3]\mathcal{S}\}_{\mathcal{J}\mathcal{M}_\mathcal{J}} \rangle$$
$$= -\delta_{\mathcal{L}',\mathcal{L}} \delta_{\mathcal{S}',\mathcal{S}} \delta_{s'_3,s_3} \delta_{S',\mathcal{S}} \delta_{\mathcal{J}',\mathcal{J}} \delta_{\mathcal{M}_{\mathcal{J}'},\mathcal{M}_\mathcal{J}} \frac{1}{\hat{\mathcal{L}}} \langle F_{j'\mathcal{L}} || V_0(m_B) || F_{j\mathcal{L}} \rangle$$
$$\times (-1)^{1 + S' + s_1 + s'_2} \begin{Bmatrix} s'_1 & s_1 & 1 \\ s_2 & s'_2 & S' \end{Bmatrix} \langle s'_1 ||\sigma_1|| s_1 \rangle \langle s'_2 ||\sigma_2|| s_2 \rangle.$$
$$\qquad (C1)$$

In the spatial matrix elements entering here, we note that the NNN component and the $NN\Delta$ component of the wave function are given in terms of the Jacobi vectors of the same form (22), but with different mass numbers. To perform the integration, one needs to express one set of the Jacobi vectors in terms of the other via

$$\bar{\boldsymbol{\xi}}_1(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) = \sqrt{\frac{A'_2 A'_1(A_1 + A_2)}{(A'_1 + A'_2)A_1 A_2}}\, \boldsymbol{\xi}_1,$$
$$\bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) = \frac{(A'_1 A_2 - A'_2 A_1)}{\sqrt{A'(A'_1 + A'_2)A_1 A_2(A_1 + A_2)}}\, \boldsymbol{\xi}_1$$
$$+ \sqrt{\frac{(A'_1 + A'_2)A'_3 A}{A'(A_1 + A_2)A_3}}\, \boldsymbol{\xi}_2, \qquad (C2)$$

where $A' = A'_1 + A'_2 + A'_3$, $A = A_1 + A_2 + A_3$. The integration is done as

$$\langle F_{j'\mathcal{L}}||V_0(m_B)||F_{j\mathcal{L}}\rangle$$
$$= 8\pi^2(2\mathcal{L}+1)^{-1/2} \int d\tau_{\text{int}} \sum_{\mathcal{M}} \underline{F_{j'\mathcal{L}\mathcal{M}}}(\bar{\xi}_1, \bar{\xi}_2)$$
$$\times V_0(m_B)\underline{F_{j\mathcal{L}\mathcal{M}}}(\xi_1, \xi_2). \tag{C3}$$

In addition, we need [using $\mathcal{N}_M^{(2)} = 4(\pi/5)^{1/2}Y_{2M}(\mathbf{n})$]

$$\langle\{F_{j'\mathcal{L}'} \otimes [(s'_1 s'_2)S's'_3]\mathcal{S}'\}_{\mathcal{J}'\mathcal{M}_{\mathcal{J}'}}$$
$$\times |V_2(m_B)S_{12}|\{F_{j\mathcal{L}} \otimes [(s_1 s_2)Ss_3]\mathcal{S}\}_{\mathcal{J}\mathcal{M}_{\mathcal{J}}}\rangle$$
$$= \delta_{\mathcal{J}',\mathcal{J}}\delta_{\mathcal{M}_{\mathcal{J}'},\mathcal{M}_{\mathcal{J}}}(-1)^{\mathcal{L}+\mathcal{S}'+\mathcal{J}'}3/2 \begin{Bmatrix} \mathcal{L}' & \mathcal{L} & 2 \\ \mathcal{S} & \mathcal{S}' & \mathcal{J}' \end{Bmatrix} \langle F_{j'\mathcal{L}'}|$$
$$\times |V_2(m_B)\mathcal{N}^{(2)}||F_{j\mathcal{L}}\rangle\langle[(s'_1 s'_2)S's'_3]\mathcal{S}'||\Sigma^{(2)}||[(s_1 s_2)Ss_3]\mathcal{S}\rangle, \tag{C4}$$

$$\langle F_{j'\mathcal{L}'}||V_2(m_B)\mathcal{N}^{(2)}||F_{j\mathcal{L}}\rangle$$
$$= 16\pi^2(2\mathcal{L}'+1)^{-1/2} \int d\tau_{\text{int}} \sum_{\mathcal{M}} C^{\mathcal{L}'\mathcal{M}}_{\mathcal{L}\mathcal{M}20}\underline{F_{j'\mathcal{L}'\mathcal{M}}}(\bar{\xi}_1, \bar{\xi}_2)$$
$$\times V_2(m_B)\underline{F_{j\mathcal{L}\mathcal{M}}}(\xi_1, \xi_2), \tag{C5}$$

$$\langle[(s'_1 s'_2)S's'_3]\mathcal{S}'||\Sigma^{(2)}||[(s_1 s_2)Ss_3]\mathcal{S}\rangle$$
$$= \delta_{s'_3,s_3}(-1)^{S'+s'_3+\mathcal{S}}\sqrt{30}/3\hat{S}'\hat{S}\hat{\mathcal{S}}'\hat{\mathcal{S}} \begin{Bmatrix} \mathcal{S}' & \mathcal{S} & 2 \\ S & S' & s'_3 \end{Bmatrix} \begin{Bmatrix} s'_1 & s_1 & 1 \\ s'_2 & s_2 & 1 \\ S' & S & 2 \end{Bmatrix}$$
$$\times \langle s'_1||\sigma_1||s_1\rangle\langle s'_2||\sigma_2||s_2\rangle. $$

APPENDIX D: T_{jm}^l MILTIPOLES OF ONE-BODY CURRENTS RELATING Δ

For the magnetic multipoles, one has

$$T_{jm}^j = \sum_i [T_{jm}^{j,\text{spin}}(i) + T_{jm}^{j,\text{conv}}(i)]. \tag{D1}$$

We have

$$T_{\Delta N,jm}^{j,\text{spin}}(i) = \frac{1}{m_\Delta}\frac{q}{2}\bar{\mu}_p^{\Delta N}\tau_{zi} \left\{ \sqrt{\frac{j}{2j+1}}j_{j+1}(qr'_i) \right.$$
$$\times [Y_{j+1}(\hat{\mathbf{r}}'_i) \otimes \sigma_i]_{jm} - \sqrt{\frac{j+1}{2j+1}}j_{j-1}(qr'_i)$$
$$\left. \times [Y_{j-1}(\hat{\mathbf{r}}'_i) \otimes \sigma_i]_{jm} \right\}, \tag{D2}$$

$$T_{\Delta\Delta,jm}^{j,\text{spin}}(i) = \frac{1}{m_\Delta}\frac{q}{2}\bar{\mu}_p^\Delta \left(\frac{1}{2}+\frac{1}{2}\tau_{zi}\right) \left\{ \sqrt{\frac{j}{2j+1}}j_{j+1}(qr'_i) \right.$$
$$\times [Y_{j+1}(\hat{\mathbf{r}}'_i) \otimes \sigma_i]_{jm} - \sqrt{\frac{j+1}{2j+1}}j_{j-1}(qr'_i)$$
$$\left. \times [Y_{j-1}(\hat{\mathbf{r}}'_i) \otimes \sigma_i]_{jm} \right\}, \tag{D3}$$

$$T_{\Delta\Delta,jm}^{j,\text{conv}}(i) = \frac{1}{m_\Delta} \left(\frac{1}{2}+\frac{1}{2}\tau_{zi}\right) j_j(qr'_i)[Y_j(\hat{\mathbf{r}}'_i) \otimes \partial'_i]_{jm}. \tag{D4}$$

The quantity ∂' is defined by the relationship $-i\partial'_\mu = p'_\mu$, and

$$\partial'^{(3)}_\mu = \left[\frac{(A_1+A_2)A_3}{A}\right]^{1/2} \frac{\partial}{\partial\xi_{2,\mu}}.$$

For the electric multipoles,

$$T_{jm}^l = \sum_i [T_{jm}^{l,\text{spin}}(i) + T_{jm}^{l,\text{conv}}(i)], \tag{D5}$$

where $l = j \pm 1$. One obtains

$$T_{\Delta N,jm}^{j\pm 1,\text{spin}}(i) = -\frac{1}{m_\Delta}\frac{q}{2}\bar{\mu}_p^{\Delta N}\tau_{zi}\sqrt{\frac{j+(1\mp 1)/2}{2j+1}}j_j(qr'_i)$$
$$\times [Y_j(\hat{\mathbf{r}}'_i) \otimes \sigma_i]_{jm}, \tag{D6}$$

$$T_{\Delta\Delta,jm}^{j\pm 1,\text{spin}}(i) = -\frac{1}{m_\Delta}\frac{q}{2}\bar{\mu}_p^\Delta\left(\frac{1}{2}+\frac{1}{2}\tau_{zi}\right)\sqrt{\frac{j+(1\mp 1)/2}{2j+1}}$$
$$\times j_j(qr'_i)[Y_j(\hat{\mathbf{r}}'_i) \otimes \sigma_i]_{jm}, \tag{D7}$$

$$T_{\Delta\Delta,jm}^{j\pm 1,\text{conv}}(i) = \pm\frac{1}{m_\Delta}\left(\frac{1}{2}+\frac{1}{2}\tau_{zi}\right)\left\{ j_{j\pm 1}(qr'_i) \right.$$
$$\times [Y_{j\pm 1}(\hat{\mathbf{r}}'_i) \otimes \partial'_i]_{jm} - \kappa\frac{q}{2}\sqrt{\frac{j+(1\pm 1)/2}{2j+1}}$$
$$\left. \times j_j(qr'_i)Y_{jm}(\hat{\mathbf{r}}'_i) \right\}. \tag{D8}$$

For the combined r and spin space operator, we only need the reduced matrix element for the calculation of the response function, as seen in (28). The reduced matrix element is given by

$$\langle\{F_{j'\mathcal{L}'} \otimes [(s'_1 s'_2)S's'_3]\mathcal{S}'\}_{\mathcal{J}'}|$$
$$\times |(O_l \otimes O_s)_j||\{F_{j\mathcal{L}} \otimes [(s_1 s_2)Ss_3]\mathcal{S}\}_{\mathcal{J}}\rangle$$
$$= \hat{\mathcal{J}}'\hat{\mathcal{J}}\hat{j} \begin{Bmatrix} \mathcal{L}' & \mathcal{L} & l \\ \mathcal{S}' & \mathcal{S} & s \\ \mathcal{J}' & \mathcal{J} & j \end{Bmatrix} \langle F_{j'\mathcal{L}'}||O_l||F_{j\mathcal{L}}\rangle\langle[(s'_1 s'_2)S's'_3]\mathcal{S}'|$$
$$\times |O_s||[(s_1 s_2)Ss_3]\mathcal{S}\rangle, \tag{D9}$$

where

$$\langle F_{j'\mathcal{L}'}||O_l||F_{j\mathcal{L}}\rangle = 8\pi^2(2\mathcal{L}'+1)^{-1/2} \int d\tau_{\text{int}} \sum_{\mathcal{M}m} C^{\mathcal{L}'(\mathcal{M}+m)}_{\mathcal{L}\mathcal{M}lm}$$
$$\times \underline{F_{j'\mathcal{L}'\mathcal{M}}}(\bar{\xi}_1, \bar{\xi}_2)O_l\underline{F_{j\mathcal{L}\mathcal{M}}}(\xi_1, \xi_2),$$

and O_l is a function of relative coordinate and momentum:

$$\mathbf{r}_3 - \mathbf{R}_{cm} = \left[\frac{(A_1+A_2)}{AA_3}\right]^{1/2} \xi_2,$$

$$i\left(\mathbf{p}_3 - \frac{A_3}{A}\mathbf{P}_{cm}\right) = \left[\frac{(A_1+A_2)A_3}{A}\right]^{1/2} \frac{\partial}{\partial\xi_2}.$$

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