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Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors

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Future exact many-body theory will allow us to calculate nuclear reactions based on the adopted NN and many-body nuclear potentials. But NN potentials are not observable and there are an infinite number of the phase-equivalent NN potentials related via finite-range unitary transformations. We show that asymptotic normalization coefficients, which are the amplitudes of the asymptotic tails of the overlap functions, are invariant under finite-range unitary transformations but spectroscopic factors are not. We prove also that the exact amplitudes for the (d,p), (d,pn), and $(e,e'p)$ reactions determining the asymptotic behavior of the exact scattering wave functions in the corresponding channels, in contrast to spectroscopic factors, are invariant under finite-range unitary transformations. Moreover, the exact reaction amplitudes are not parametrized in terms of the spectroscopic factors and nuclear reactions in the exact approach cannot provide a tool to determine spectroscopic factors which are not observable.

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The beginning of the 21st century has opened up a new chapter in nuclear physics fueled by a new generation of the radioactive beam facilities allowing us to approach the nuclei away from the valley of stability. It should lead to new discoveries in nuclear structure, nuclear reaction fields, and nuclear astrophysics which is closely related to cosmology. The important part of the new programs, as has been for the previous 50 years, is the determination of spectroscopic factors (SFs) from nuclear reactions, which play an important role in nuclear physics, nuclear astrophysics, and applied physics. Spectroscopic factors (SFs) were introduced by the shell model formalism and are typically related to the shell occupancy. Since the dawn of nuclear physics, direct reactions have been the main tool for extracting SFs in nuclear laboratories worldwide, which were compared with predictions of the independent-particle shell model (see [1] and references therein) to test the validity of many-body theories. The idea of extracting SFs from nuclear reactions is based on the drastic approximations to exact reaction amplitudes leading to the DWBA. This approach has been used for more than 50 years. A new era of nuclear physics calls for new eventually exact methods of treating nuclear reactions. The question is what should we expect when an exact reaction theory based on *ab initio* calculations started from NN and many-body potentials will be delivered?

First, solution of the nuclear many-body problem requires nuclear potentials describing the interaction of two, three, and more nucleons which are the main input into many-body calculations. The next crucial step is to deliver a theory allowing us to calculate nuclear reactions based on the adopted NN and many-body nuclear potentials which is more difficult than a treatment of many-body bound states. But even identifying two- and many-body nuclear potentials turns out to be extremely difficult and represents a yet unsolved problem even for light nuclei and low-energy reactions because the NN potential is not observable. As it has been underscored in [2,3], there are an infinite number of phase equivalent potentials related via the short-range unitary transformations. These unitary transformations can soften or even remove the repulsive core without changing the tail of the potential. Short-range unitary transformations can be applied to the wave functions affecting their short-range behavior. Intuitively one may guess that the asymptotic behavior of the wave functions will not be affected by short-range unitary operators. Then the elastic scattering and reaction amplitudes being the amplitudes of the asymptotic terms of the many-body wave functions should be invariant under short-range unitary transformations. From the other side the wave functions are normalized and the unitary transformations affecting their short-range behavior can affect the asymptotic amplitudes. It is the goal of this Rapid Communication to prove that the exact direct reaction amplitudes and asymptotic normalization coefficients (ANCs) are invariant under finite-range unitary transformations while the SFs are not. We conclude that the exact reaction amplitudes (direct transfer, breakup, and electron-induced disintegration) cannot be used as a tool to determine SFs. When sooner or later an exact many-body theory will be available, the reaction amplitudes can be determined from the matrix elements or from the asymptotic behavior of the exact wave functions in the corresponding asymptotic regions while the SFs can be obtained from the overlap functions. However, these SFs depend on the adopted nuclear potentials while the reactions amplitudes do not. Thus delivering the exact reaction theory would mean the separation of the nuclear reactions and SFs.

There are different unitary transformations used in literature. Among them is the unitary correlation operator method (UCOM) (see [4] and references therein), Lee-Suzuki unitary transformation [5] used in the no-core-shell model [6], the unitary model operator approach (UMOA) (see [7] and references therein), and similarity renormalization group [8]. The unitary transformations aim to take into account effectively the short-range repulsive core in NN interaction

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potentials. For example, in the UCOM [4] the unitary transformation shifts away nucleons depleting the wave function at small nucleon-nucleon distances and making the NN potential softer. The correlated wave function in the UCOM is obtained from an uncorrelated state (linear combination of Slater determinants) using unitary correlation operators. The proof which is presented here is valid for general short-range unitary transformation requiring only the "cluster decomposition property" [4]. The potential is assumed to be energy independent, local or nonlocal.

Unitary transformation. Let us consider a system of nucleons with the wave function satisfying the Shrödinger equation $H\,\Psi = E\,\Psi$, where $H = T + V$, $V = \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots +$ is the total interaction potential, which is the sum of two- and many-body potentials, $T = \sum_i T_i - T_{\rm c.m.} = T_{\rm rel}$, T_i is the kinetic energy operator of the nucleon i, $T_{\rm rel}$ is the relative kinetic energy operator of the system, and $T_{\rm c.m.}$ is the kinetic energy operator of the center of mass of the system. A unitary transformation of the wave function

$$\Psi = U\,\tilde{\Psi} \tag{1}$$

conserving the norm transforms the matrix element $\langle\Psi|H|\Psi\rangle = \langle\tilde{\Psi}|\tilde{H}|\tilde{\Psi}\rangle$. Here, $\tilde{H} = U^{-1}\,H\,U$, the unitary operator $U = e^{i\,G}$, $\tilde{V} = \tilde{H} - T$. Even if the initial potential V contains only NN potentials the transformed $\tilde{V}$ contains not only NN potentials but also three- and many-body potentials [4]. $\tilde{\Psi}$ satisfies the Shrödinger equation

$$(\tilde{V} + T_{\rm rel})\,\tilde{\Psi} = E\,\tilde{\Psi}. \tag{2}$$

Elastic scattering. There are infinite number of the NN potentials related via finite-range unitary transformations, which modify the short-range part of the potential leaving the tail intact [2]. First we demonstrate that the NN scattering amplitude is invariant under finite-range unitary transformations of the wave functions and potentials, i.e., finite-range unitary transformations generate phase equivalent potentials. Let us consider two wave functions related by the finite-range unitary transformation $\Psi = U\,\tilde{\Psi}$. Now we take into account that the asymptotic behavior of the scattering wave function at $r \to \infty$ is given by

$$\Psi \overset{r\to\infty}{\approx} e^{i\,\mathbf{k}\cdot\mathbf{r}} - \frac{\mu}{2\,\pi}\,f\,\frac{e^{i\,k\,r}}{r}, \tag{3}$$

where f is the NN elastic scattering amplitude, μ is the NN reduced mass. Since at distances larger then the range of the unitary operator (at distances larger than the correlation radius the generator $G \to 0$) $\Psi = \tilde{\Psi}$, the scattering amplitude $f = \tilde{f}$, i.e., it is invariant under finite-range unitary transformations. Thus different potentials related via the finite-range transformation are phase-equivalent and indistinguishable.

Next let us consider the $n + A$ elastic scattering amplitude. For simplicity we disregard the spins of the particles. The asymptotic behavior of the $n + A$ scattering wave function at $r_{nA} \to \infty$ is given by

$$\Psi \overset{r_{nA}\to\infty}{\approx} \Psi^{(0)}_{nA} - \sum_j \frac{\mu_{nA_j}}{2\pi}\,f_{nA_j}\,\frac{e^{i\,k_{nA_j}\,r_{nA}}}{r_{nA}}\,\varphi_{A_j} + \Delta\Psi. \tag{4}$$

Here, $\Psi^{(0)}_{nA}$ is the incident wave in the channel $n + A$, μ_{nA} is the reduced mass of n and A, f_{nA_j} are the elastic ($j = 0$) and inelastic ($j \geqslant 1$) $n + A$ scattering amplitudes, $\mathbf{k}_{nA}$ and $\mathbf{r}_{nA}$ are the relative $n + A$ momentum and radius-vector connecting n and the center-of-mass of A, φ_{A_j} is the internal wave function of nucleus A in the bound state j. $\Delta\Psi$ is the contribution from the outgoing waves in the rearrangement and breakup channels which are of the next order compared to the elastic and inelastic outgoing waves. Thus the leading asymptotic term of the projection of the wave function on the initial two-body channel $n + A$, where $A \equiv A_0$, at $r_{nA} \to \infty$ is

$$\langle\varphi_A|\Psi\rangle \overset{r_{nA}\to\infty}{\approx} e^{i\,\mathbf{k}_{nA}\cdot\mathbf{r}_{nA}} - \frac{\mu_{nA}}{2\pi}\,f_{nA}\,\frac{e^{i\,k_{nA}\,r_{nA}}}{r_{nA}}. \tag{5}$$

Correspondingly for the wave function $\tilde{\Psi}$ we get

$$\langle\tilde{\varphi}_A|\tilde{\Psi}\rangle \overset{r_{nA}\to\infty}{\approx} e^{i\,\mathbf{k}_{nA}\cdot\mathbf{r}_{nA}} - \frac{\mu_{nA}}{2\pi}\,\tilde{f}_{nA}\,\frac{e^{i\,k_{nA}\,r_{nA}}}{r_{nA}}. \tag{6}$$

We recall now the cluster property of the unitary transformation

$$U = U_{nA}\,U_A, \tag{7}$$

where U_{nA} is the unitary transformation, which takes into account correlations between n and the nucleons of nucleus A and U_A is the unitary transformation for the nucleons of nucleus A. At $r_{nA} \to \infty$ the finite-range U_{nA} can be replaced by the unit operator. Then from Eqs. (5) and (1) taking into account that $\varphi_A = U_A\,\tilde{\varphi}_A$ we get

$$\begin{aligned}\langle\varphi_A|\Psi\rangle &= U_{nA}\,\langle\tilde{\varphi}_A|\tilde{\Psi}\rangle \overset{r_{nA}\to\infty}{=} \langle\tilde{\varphi}_A|\tilde{\Psi}\rangle \\ &= e^{i\,\mathbf{k}_{nA}\cdot\mathbf{r}_{nA}} - \frac{\mu_{nA}}{2\pi}\,f_{nA}\,\frac{e^{i\,k_{nA}\,r_{nA}}}{r_{nA}} + O\left(\frac{1}{r^2_{nA}}\right).\end{aligned} \tag{8}$$

Comparing Eqs. (6) and (8) we conclude that $f_{nA} = \tilde{f}_{nA}$, i.e., the $n + A$ scattering amplitude is invariant under the finite-range unitary transformation. Similarly one can prove that the scattering amplitude of two nuclei is also invariant under the finite-range unitary transformations.

It is interesting that the invariance of the scattering amplitude under finite-range unitary transformations can be proved in a different way using the surface-integral reaction theory (see [9] and references therein). To do it we rewrite the conventional expression for the $n + A$ elastic scattering amplitude by splitting the volume integral into two parts:

$$f_{nA} = \left\langle\psi^{(0)}_{nA}\,\varphi_A\middle|V_{nA}|\Psi^{(+)}_i\right\rangle = f_{nA}|_{r_{nA}\leqslant R} + f_{nA}|_{r_{nA}>R}, \tag{9}$$

where $\psi^{(0)}_{nA}$ is the plane wave describing the relative motion of the free n and A in the final state. The first (second) term in Eq. (9) is the volume integral, in which $r_{nA} \leqslant R$ ($r_{nA} > R$). We choose the radius R significantly larger than the radius of the unitary transformation, so that at $r_{nA} > R$ the unitary operator $U_{nA} = \mathbf{1}$ where $\mathbf{1}$ is the unit operator. Introducing the finite-range unitary transformation $\Psi^{(+)}_i = U\,\tilde{\Psi}^{(+)}_i$ and $\varphi_A = U_A\,\tilde{\varphi}_A$ we immediately conclude that the second term $f_{nA}|_{r_{nA}>R}$ is not sensitive to the finite-range unitary transformation. Using the

Green's theorem

$$L = \langle f(\mathbf{r})|\overleftarrow{T} - \overrightarrow{T}|g(\mathbf{r})\rangle = -\frac{1}{2\mu_{nA}^2}\lim_{r\to\infty} r^2 \int d\hat{\mathbf{r}}\left[g(\mathbf{r})\frac{\partial}{\partial r}f^*(\mathbf{r}) - f^*(\mathbf{r})\frac{\partial}{\partial r}g(\mathbf{r})\right] \tag{10}$$

we can transform the internal volume integral into the surface one over the coordinate $\mathbf{r}_{nA}$:

$$\begin{aligned} f_{nA}|_{r_{nA}\leqslant R} &= \left\langle \psi_{nA}^{(0)}\varphi_A\right| - V_A + V_{nA} + V_A|\Psi_i^{(+)}\rangle|_{r_{nA}\leqslant R} \\ &= \left\langle \psi_{nA}^{(0)}\varphi_A\right| - E + \overleftarrow{T} + E - \overrightarrow{T}|\Psi_i^{(+)}\rangle|_{r_{nA}\leqslant R} \\ &= \left\langle \psi_{nA}^{(0)}\varphi_A\right|\overleftarrow{T} - \overrightarrow{T}|\Psi_i^{(+)}\rangle|_{r_{nA}\leqslant R} \\ &= \left\langle \psi_{nA}^{(0)}\varphi_A\right|\overleftarrow{T}_{nA} - \overrightarrow{T}_{nA}|\Psi_i^{(+)}\rangle|_{r_{nA}\leqslant R} = f_{nA}^{(S)}|_{r_{nA}=R}. \end{aligned} \tag{11}$$

We took into account that $T = T_A + T_{nA}$ and that the operator T_A is Hermitian because of the presence in the bra-state of the bound-state wave function φ_A, i.e., using the integration by parts the operator $\overleftarrow{T}_A$ can be transformed into $\overrightarrow{T}_A$ and $\langle|\overleftarrow{T}_A - \overrightarrow{T}_A|\rangle = \langle|\overrightarrow{T}_A - \overrightarrow{T}_A|\rangle = 0$. Since the radius R of the sphere is taken large enough the surface integral is invariant under finite-range unitary transformations. Thus the scattering amplitude can be written as

$$f_{nA} = f_{nA}^{(S)}|_{r_{nA}=R} + f_{nA}|_{r_{nA}>R}, \tag{12}$$

i.e., it is invariant under finite-range unitary transformations because the matrix elements are taken in the region where $U_{nA} = \mathbf{1}$. This method of the proof is universal and can be used for rearrangement and breakup nuclear reactions.

Reaction amplitudes. The scattering wave function $\Psi_i^{(+)}$ describing the collision of two nuclei a and A asymptotically behaves as

$$\Psi_i^{(+)} = \Psi_i^{(0)} - \sum_\alpha \frac{\mu_\alpha}{2\pi} M_\alpha\, u^{(+)}(r_\alpha)\,\Phi_\alpha + \Delta\Psi, \tag{13}$$

where the sum over α contains the elastic, inelastic, and rearrangement channels, M_α is the reaction amplitude leading to the final channel α, $\Psi_i^{(0)}$ is the incident wave in the entry channel $a + A$, Φ_α is the product of the bound state wave functions of the fragments in the exit channel α, $u^{(+)}(r_\alpha)$ is the outgoing wave in the two-fragment channel α, and $\Delta\Psi$ is the contribution from the breakup channels (more than two fragments in the exit channel). Let us select a specific rearrangement channel $\alpha = y + B$ and take a projection of $\Psi_i^{(+)}$ on the exit two-fragment channel $y + B$. The asymptotic of this projection at $r_{yB} \to \infty$ behaves as

$$\langle\varphi_B\,\varphi_y|\Psi_i^{(+)}\rangle = U_{yB}\,\langle\tilde{\varphi}_B\,\tilde{\varphi}_y|\tilde{\Psi}_i^{(+)}\rangle \overset{r_{yB}\to\infty}{=} \langle\tilde{\varphi}_B\,\tilde{\varphi}_y|\tilde{\Psi}_i^{(+)}\rangle = M_{yB}\,u^{(+)}(r_{yB}). \tag{14}$$

Hence, $M_\alpha = \tilde{M}_\alpha$, i.e., the rearrangement reaction amplitudes are also invariant under finite-range unitary transformations. Evidently that this result is also true for breakup amplitudes with three or more fragments in the exit channel. In this case we need to consider the asymptotic behavior of Ψ in the asymptotic region where all fragments are well separated. The invariance of the reaction amplitudes can be also be shown directly from the conventional volume matrix element using the surface-integral formulation of the reaction theory [9].

$(e,e'p)$ reaction amplitude. As a particular case of the breakup reaction we consider the electron induced photodisintegration $(e, e'\,p)$ which is believed to be a powerful tool to determine the SFs [10]. To prove the invariance of the $(e, e'\,p)$ amplitude under unitary transformations we use the surface-integral formulation of the theory rather than the asymptotic behavior of the wave function. The $B(e, e'\,p)A$ amplitude in the prior form is given by

$$\begin{aligned} M_{ee'p} &= \langle\Psi_f^{(-)}|V_{eB} - V_{eB}^{(\mathrm{opt})}|\,\varphi_B\,\chi_i^{(+)}\rangle \\ &= \langle\Psi_f^{(-)}| - \overleftarrow{T} + \overrightarrow{T}|\,\varphi_B\,\chi_i^{(+)}\rangle, \end{aligned} \tag{15}$$

where $\chi_i^{(+)}$ is the electron distorted wave in the initial channel generated by the Coulomb optical potential $V_{eB}^{(\mathrm{opt})}$. Note that this expression is valid even if the three-body potentials $V_{e\,p\,j}$ are included in addition to the two-body potential $V_{e\,p}$. Introducing the finite-range unitary transformations we get

$$\begin{aligned} M_{ee'p} &= \left\langle\tilde{\Psi}_f^{(-)}\,U_{p\,A}^{-1}U_A^{-1}\right| - \overleftarrow{T} + \overrightarrow{T}|U_{p\,A}\,U_A\,\tilde{\varphi}_B\chi_i^{(+)}\rangle \\ &= \langle\tilde{\Psi}_f^{(-)}| - \overleftarrow{T}_{e\,B} + \overrightarrow{T}_{e\,B}|\tilde{\varphi}_B\chi_i^{(+)}\rangle, \end{aligned} \tag{16}$$

where $T = T_{e\,B} + T_{p\,A} + T_A$. To obtain the last equation we took into account that operators T_{pA} and T_A are hermitian because in the initial channel B is in the bound state. Thus the exact $(e, e'\,p)$ amplitude is invariant under finite-range unitary transformations.

Radiative capture processes. Let us consider $E1$ direct radiative capture amplitude $A(n,\gamma)B$. After applying the Siegert's theorem it can be written as

$$M = i\,k_\gamma\,\langle\varphi_B|\mathbf{D}|\Psi_i^{(+)}\rangle, \tag{17}$$

where $\Psi_i^{(+)}$ and φ_B are the exact initial scattering and final bound-state wave function of nucleus B. $\mathbf{D}$ is the dipole operator given by the sum of the nucleon operators

$$\mathbf{D} = \sum_i e_i\,(\mathbf{r}_i - \mathbf{R}), \tag{18}$$

e_i and $\mathbf{r}_i$ are the charge and the radius of the ith nucleon, k_γ is the momentum of the emitted photon, R is the coordinate of the center of mass of nucleus B. Assume that nucleus B consists of Z protons and $N = B - Z$ neutrons. Then the dipole operator can be written as

$$\mathbf{D} = \frac{e}{B}\sum_{i=1}^{Z}\sum_{j=Z+1}^{B}\mathbf{r}_{ij}, \tag{19}$$

where e is the proton charge. Applying now the unitary transformation of the wave functions we get

$$\begin{aligned} M &= i\,k_\gamma\,\langle\tilde{\varphi}_B|U_B^{-1}\,\mathbf{D}\,U_B|\tilde{\Psi}_i^{(+)}\rangle \\ &= \frac{e}{B}\sum_{i=1}^{Z}\sum_{j=Z+1}^{B}\langle\tilde{\varphi}_B|U_{ij}^{-1}\,\mathbf{r}_{ij}\,U_{ij}|\tilde{\Psi}_i^{(+)}\rangle. \end{aligned} \tag{20}$$

The dipole operator is not invariant under unitary transformations. It is evident if we consider, for example, the unitary correlation operator [4] which shifts away two nucleons increasing the distances between them, i.e., this transformations

will modify each $\mathbf{r}_{ij}$ and, hence, the dipole operator. Hence the direct radiative capture amplitude is also not invariant under unitary transformations. It is also clear from the fact that the radiative capture amplitude does not determine the asymptotic behavior of the exact wave function. However, the modification of the radiative capture amplitude can be quite small if the range of the unitary correlator is small.

Effect of unitary transformations on asymptotic normalization coefficients and spectroscopic factors. Let us consider now the effect of finite-range unitary transformations on the ANCs and SFs. The overlap function of the bound state wave functions of nuclei $B = (A\,n)$ and A is given by (for simplicity we neglect the spins of the particles)

$$I_A^B(\mathbf{r}_{nA}) = (A+1)^{1/2}\,\langle\varphi_A|\varphi_B\rangle. \tag{21}$$

The integration in the matrix element $\langle\varphi_A|\varphi_B\rangle$ is carried over all the internal coordinates of daughter nucleus A and $(A+1)^{1/2}$ is the antisymmetrization factor due to identical nucleons. Asymptotic behavior of its radial part is given by

$$I_A^B(r_{nA}) \overset{r_{nA}\to\infty}{\approx} C_{An}^B\,\frac{e^{-\kappa\,r_{nA}}}{r_{nA}}. \tag{22}$$

Here, C_{An}^B is the ANC for the virtual decay $B \to A + n$ and κ is the neutron bound state wave number. Introducing the unitary transformation (1) and taking into account that $U_B = U_{nA}\,U_A$ we get from Eq. (21)

$$\begin{aligned} I_A^B(\mathbf{r}_{nA}) &= (A+1)^{1/2}\,\langle\varphi_A|\varphi_B\rangle \\ &= (A+1)^{1/2}\,\langle\tilde{\varphi}_A|U_A^{-1}\,U_B|\tilde{\varphi}_B\rangle \\ &= (A+1)^{1/2}\,\langle\tilde{\varphi}_A|U_{nA}|\tilde{\varphi}_B\rangle. \end{aligned} \tag{23}$$

Thus the overlap function is not invariant under unitary transformations. However, the tail of the overlap function (at $r_{nA} \to \infty$) remains intact under the unitary transformations:

$$\begin{aligned} I_A^B(\mathbf{r}_{nA}) &= (A+1)^{1/2}\,\langle\varphi_A|\varphi_B\rangle \\ &= (A+1)^{1/2}\,\langle\tilde{\varphi}_A|U_{nA}|\tilde{\varphi}_B\rangle \\ &\overset{r_{nA}\to\infty}{=} (A+1)^{1/2}\,\langle\tilde{\varphi}_A|\tilde{\varphi}_B\rangle, \end{aligned} \tag{24}$$

i.e., the radial part

$$I_A^B(r_{nA}) \overset{r_{nA}\to\infty}{\approx} C_{An}^B\,\frac{e^{-\kappa\,r_{nA}}}{r_{nA}} = \tilde{C}_{An}^B\,\frac{e^{-\kappa\,r_{nA}}}{r_{nA}}. \tag{25}$$

Thus the ANC, which determines the amplitude of the projection of the bound-state wave function B on the two-body channel $A + n$, is invariant under finite-range unitary transformations. The ANC, which governs the overall normalization of the peripheral transfer reactions, can be determined from their analysis [11]. Hence, the ANC is observable. Note that if two different phase-equivalent potentials are not connected via an unitary transformation the ANCs generated by these potentials, according to the inverse scattering problem theorem, *a priori*, can differ. The measurement of the ANC allows one to select a proper NN potential among different potentials which are not unitary equivalent.

The most general model-independent definition of the SF is the square of the norm of the overlap function:

$$S = \left\langle I_A^B \middle| I_A^B \right\rangle. \tag{26}$$

Thus the SF is contributed by the overlap function at small distances where the effect of unitary transformations, which take into account short-range nucleon correlations, can be significant (see [12] and references therein and [13]). Hence, the SF, in contrast to the ANC, is not invariant under finite-range unitary transformations and is nonobservable.

(d,p) *reaction amplitude and spectroscopic factor.* For many years (d,p) reactions (see [1] and references therein), electron-induced breakup reactions (see [10] and references therein), and nucleon knockout reactions (see [14] and references therein) have been the main tool for extracting the SFs, which were compared with predictions of the independent-particle shell model. However, as it has been proved, the exact reaction amplitudes are invariant under the finite-range unitary transformations while the SFs are not.

In the conventional approach the parametrization of the reaction amplitude in terms of the SF is achieved using drastic approximations of the exact reaction amplitude leading to the DWBA. To discuss it let us recall that the exact (d,p) reaction amplitude in the post form is given by

$$M_{dp} = \langle\chi_f^{(-)}\varphi_B|V_{pB} - V_{pB}^{(\mathrm{opt})}|\Psi_i^{(+)}\rangle. \tag{27}$$

Here $\Psi_i^{(+)}$ is the scattering wave function with the incident wave in the initial $d + A$ channel and outgoing waves in all the open channels, V_{pB} and $V_{pB}^{(\mathrm{opt})}$ is the interaction potential and optical potential between p and B, $\chi_f^{(-)}$is the distorted wave in the exit $p + B$ channel. Note that if the wave function $\Psi_i^{(+)}$ is exact, Eq. (27) does not depend on the choice of the optical potential.

To get the DWBA amplitude the following approximations should be done. (i) $\Psi_i^{(+)}$ is approximated by the incident wave $\varphi_d\,\varphi_A\,\chi_i^{(+)}$ in the channel $d + A$. (ii) The bound-state wave function φ_B is approximated by the single term $\varphi_B = I_A^B\,\varphi_A$. (iii) The next step is the single-particle approximation for the overlap function given by

$$I_A^B = \left[S_{An}^{(\mathrm{sp})}\right]^{1/2}\varphi_n, \tag{28}$$

where $S_{An}^{(\mathrm{sp})}$ is the SF of the configuration $B = (A\,n)$ in the single-particle approximation and φ_n is the single-particle wave function of the neutron moving in the mean-field. Since the squares of the norms of the overlap function and the radial bound-state wave function are, correspondingly, the SF and unity, the single-particle SF in Eq. (28) will equal the SF defined in Eq. (26) if the bound-state wave function and the overlap function have very similar radial behavior both in the nuclear interior and exterior. However, for $r_{nA} < R_N$, R_N is the nuclear interaction radius, where both I_A^B and φ_n have most of their probability, the radial dependence of the overlap function and single-particle wave function, *a priori*, are different because the overlap function is a many-particle object affected by the short-range correlations [13], whereas the single-particle wave function is a solution of the single-particle Schrödinger equation. Thus, in general $S_{An}^{(\mathrm{sp})}$ does not coincide with the microscopically calculated SF. Nonetheless, for $r_{nA} > R_N$, the radial dependencies of I_A^B and φ_n are the same, and they differ only by their overall normalizations. The asymptotic behavior of the radial overlap function is given

by Eq. (22) and the asymptotic normalization of the radial bound-state wave function is defined as

$$\varphi_n(r_{nA}) \overset{r_{nA}\to\infty}{\approx} b_{An}\,\frac{e^{-\kappa\, r_{nA}}}{r_{nA}}, \tag{29}$$

where b_{An} is the single-particle ANC. By the proper choice of $S_{An}^{(\mathrm{sp})}$, one can make Eq. (28) exact for $r > R_N$. Then, from Eqs. (22) and (29) we get the relationship $S_{An}^{(\mathrm{sp})} = C_{An}^2/b_{An}^2$ connecting the single-particle SF, the ANC, and the single-particle ANC b_{An}. While the ANC is an experimentally measurable quantity, the single-particle ANC b_{An} is not. Hence, the single-particle SF is model dependent. Its model dependence comes through the single-particle ANC b_{An}, which is a function of the geometric parameters, radius r_0 and diffuseness a, of the Woods-Saxon potential conventionally used as a single-particle potential. Furthermore, unlike the SF as defined in Eq. (26), the single-particle SF is actually a property of the peripheral part of the nucleon overlap function, because its definition comes from the peripheral behavior of the overlap function and the single-particle wave function.

In contrast to the (d,p) reactions the $(e,e'p)$ reactions can probe nuclear interior (large transfer momentum) and that is why these reactions are believed to be the best tool to determine the SFs [10]. However, at small internucleon distances probed in the $(e,e'p)$ reactions the short-range correlations are important. These correlations affect the overlap function at small distances and, hence, the SF but leave intact the exact $(e,e'p)$ amplitude. Note that earlier in [15] using the effective field theory it was shown that the occupation numbers cannot be unambiguously determined from $(e,e'p)$ reactions.

Thus in the exact approach nuclear reactions cannot be used as a tool to determine the SFs, which are not observable. We proved that the exact reaction amplitudes, which are asymptotic amplitudes of the exact scattering wave functions in the corresponding asymptotic regions, are invariant under finite-range unitary transformations while SFs are not. It means that an exact reaction theory cannot be used as a tool to determine SFs. We call it separation of the exact reaction theory and SFs.

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