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Spin and spin-isospin instabilities in asymmetric nuclear matter at zero and finite temperatures using Skyrme functionals

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Self-consistent mean-field methods based on phenomenological Skyrme effective interactions are known to exhibit spurious spin and spin-isospin instabilities both at zero and finite temperatures when applied to homogeneous nuclear matter at the densities encountered in neutron stars and in supernova cores. The origin of these instabilities is revisited in the framework of the nuclear energy density functional theory, and a simple prescription is proposed to remove them. The stability of several Skyrme parametrizations is reexamined.

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I. INTRODUCTION

The self-consistent mean-field method with Skyrme effective interactions has been very successful in describing the structure and the dynamics of medium-mass and heavy nuclei [1]. These interactions have been also widely applied to the description of extreme astrophysical environments such as neutron stars and supernova cores. Actually very soon after Skyrme [2] introduced his eponymous effective interaction, Cameron [3] applied it to calculate the structure of neutron stars. Assuming that neutron stars were made only of neutrons, he found that their maximum mass was significantly higher than the Chandrasekhar mass limit. His work thus brought support to the scenario of neutron star formation from the catastrophic gravitational collapse of massive stars in supernova explosions, as proposed much earlier by Baade and Zwicky [4]. The interior of neutron stars is highly neutron rich but contains also a non-negligible amount of protons, leptons, and possibly other particles. However, microscopic calculations in uniform infinite nuclear matter using bare nucleon-nucleon potentials have been usually restricted to symmetric nuclear matter (SNM) and pure neutron matter (NeuM). Even though effective interactions are phenomenological, they can provide a convenient interpolation of realistic calculations to determine the equation of state of neutron star cores. Mean-field calculations can be easily extended to finite temperatures and can thus be also used to describe the hot nuclear matter found in supernova cores and protoneutron stars. Moreover, the mean-field method allows a consistent and tractable treatment of both homogeneous matter and inhomogeneous matter (e.g., neutron star crusts [5]) with a reduced computational cost. This opens the way to a unified description of all regions of neutron stars and supernova cores [6].

Nevertheless, the application of these effective forces to nuclear matter at high densities has been limited by the occurrence of spurious instabilities [7,8]. In particular, Skyrme forces predict a spontaneous transition to a spin-polarized phase when the density exceeds a critical threshold which depends on the isospin asymmetry [9–13]. Besides, it is found that for some forces, the energy density of the spin-polarized phase decreases with increasing density. In this case, the phase transition is accompanied by a catastrophic collapse [14], which is contradicted by the existence of neutron stars (note, however, that observations alone do not exclude the possibility of a ferromagnetic core inside neutron stars; see, for instance, Refs. [15,16]). Moreover, the critical density predicted within the Skyrme formalism generally decreases with temperature due to an anomalous behavior of the entropy, which is larger in the spin-ordered phase than in the unpolarized phase [17,18]. This instability can strongly affect the neutrino propagation in hot dense nuclear matter [12,19,20] which is believed to play an important role in the supernova explosion mechanism and in the evolution of protoneutron stars [21]. However, no such spin-polarized phase transition is found by microscopic calculations using realistic nucleon-nucleon potentials. Indeed, several calculations based on different methods, such as the lowest-order constrained variational method [22–26], the Brueckner-Hartree-Fock method [27–29], the auxiliary field diffusion Monte Carlo method [30] and the Dirac-Brueckner-Hartree-Fock method [31], show that nuclear matter remains unpolarized well above the nuclear saturation density ρ_0 both at zero and finite temperatures.

The prediction of spin-ordering in nuclear matter is one of the main deficiencies of the mean-field method with effective forces. Different extensions of the standard Skyrme force have been recently proposed in order to prevent these phase transitions at zero temperature [12,13]. In this paper, the origin of the spin and spin-isospin instabilities is revisited in the more general framework of the nuclear energy density functional (EDF) theory (see, for instance, Ref. [32] for a review) and a simpler prescription is proposed to ensure stability of dense nuclear matter for any degree of spin and spin-isospin polarizations and for any temperature. The paper is organized as follows. The Skyrme functionals that we consider here are defined in Sec. II. Section III is devoted to the discussion about the stability of nuclear matter. Several Skyrme functionals are reexamined in Sec. IV.

II. SKYRME FUNCTIONALS

The nuclear EDFs that we consider here are of the form

$$E = E_{\rm kin} + E_{\rm Coul} + E_{\rm Sky}, \tag{1}$$

where $E_{\rm kin}$ is the kinetic energy, $E_{\rm Coul}$ is the Coulomb energy, and $E_{\rm Sky}$ is a functional of the local densities and currents ($q = n$, p for neutron, proton, respectively): the density ρ_q, the current density $\boldsymbol{j_q}$, the kinetic density τ_q, the spin density $\boldsymbol{s_q}$, the spin kinetic density $\boldsymbol{T_q}$, and the spin-current tensor $J_{q,\mu\nu}$ (see, for instance, Ref. [1] for precise definitions). It is convenient to introduce the isospin index $t = 0, 1$ for isoscalar and isovector quantities, respectively. Isoscalar quantities (also written without any subscript) are sums over neutrons and protons (e.g., $\rho_0 = \rho = \rho_n + \rho_p$), while isovector quantities are differences between neutrons and protons (e.g., $\rho_1 = \rho_n - \rho_p$). The Skyrme functional $E_{\rm Sky}$ is then given by

$$E_{\rm Sky} = \int {\rm d}^3\boldsymbol{r}\,\mathcal{E}_{\rm Sky}(\boldsymbol{r}), \quad \mathcal{E}_{\rm Sky} = \sum_{t=0,1}\left(\mathcal{E}_t^{\rm even} + \mathcal{E}_t^{\rm odd}\right), \tag{2a}$$

$$\mathcal{E}_t^{\rm even} = C_t^\rho \rho_t^2 + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^\tau\rho_t\tau_t + C_t^{\nabla J}\rho_t\nabla\cdot\boldsymbol{J_t} + C_t^J\sum_{\mu,\nu}J_{t,\mu\nu}J_{t,\mu\nu}, \tag{2b}$$

$$\mathcal{E}_t^{\rm odd} = C_t^s s_t^2 + C_t^{\Delta s}\boldsymbol{s_t}\cdot\Delta\boldsymbol{s_t} + C_t^T\boldsymbol{s_t}\cdot\boldsymbol{T_t} + C_t^j j_t^2 + C_t^{\nabla j}\boldsymbol{s_t}\cdot\nabla\times\boldsymbol{j_t}. \tag{2c}$$

The spin current vector is defined by $J_{t\kappa} = \sum_{\mu,\nu}\varepsilon_{\kappa\mu\nu}J_{t,\mu\nu}$, where $\varepsilon_{\kappa\mu\nu}$ is the Levi-Civita tensor. The so-called time-even part $\mathcal{E}_t^{\rm even}$ (time-odd part $\mathcal{E}_t^{\rm odd}$) contains only even (odd) densities and currents with respect to time reversal.

The coupling "constants" C_t^ρ and C_t^s generally depend on the isoscalar density $\rho = \rho_n + \rho_p$ as follows:

$$C_t^\rho = a_t^\rho + b_t^\rho\rho^\alpha, \tag{3a}$$

$$C_t^s = a_t^s + b_t^s\rho^\alpha. \tag{3b}$$

Moreover, local gauge invariance [33,34] imposes the following relations:

$$C_t^j = -C_t^\tau, \quad C_t^J = -C_t^T, \quad C_t^{\nabla j} = C_t^{\nabla J}. \tag{4}$$

Historically the type of functionals given by Eqs. (2a)–(2c) were obtained from the Hartree-Fock approximation using effective zero-range interactions of the Skyrme type [1,6]

$$\begin{aligned} v_{i,j} &= t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}_{ij}) \\ &+ \frac{1}{2}t_1(1+x_1P_\sigma)\frac{1}{\hbar^2}\left[p_{ij}^2\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})p_{ij}^2\right] \\ &+ t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{p}_{ij} \\ &+ \frac{1}{6}t_3(1+x_3P_\sigma)\rho(\boldsymbol{r})^\alpha\delta(\boldsymbol{r}_{ij}) \\ &+ \frac{\rm i}{\hbar^2}W_0(\hat{\boldsymbol{\sigma}}_{\boldsymbol{i}} + \hat{\boldsymbol{\sigma}}_{\boldsymbol{j}})\cdot\boldsymbol{p}_{ij}\times\delta(\boldsymbol{r}_{ij})\boldsymbol{p}_{ij}, \end{aligned} \tag{5}$$

where $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\boldsymbol{r} = (\boldsymbol{r}_i + \boldsymbol{r}_j)/2$, $\boldsymbol{p}_{ij} = -{\rm i}\hbar(\nabla_i - \nabla_j)/2$ is the relative momentum, and P_σ is the two-body spin-exchange operator. The relations between the coupling constants in Eqs. (2b) and (2c) and the parameters of the effective force in Eq. (5) can be found, for instance, in Appendix A of Ref. [1].

Kutschera and Wójcik [14] pointed out that for some Skyrme forces not only is the ground state of NeuM polarized, but also the energy density of polarized NeuM decreases with increasing density. However, such a catastrophic ferromagnetic collapse is ruled out by neutron star observations. The origin of this singular behavior can be traced back to the parameters t_2 and x_2 of the Skyrme force. In particular, the authors of Ref. [14] found that in order to prevent a ferromagnetic collapse of NeuM, the parameters of Skyrme forces must satisfy the following inequality:

$$t_2(1+x_2) \geqslant 0. \tag{6}$$

This constraint was taken into account to construct the Saclay-Lyon Skyrme parametrizations [35], which were fitted with the parameter $x_2 = -1$. These forces, which were specifically developed for astrophysics, have been widely used in neutron star studies. However, it has been found that these forces predict various transitions to spin-ordered phases in nuclear matter [10,12,13,17,18] even though Eq. (6) was enforced. Actually, this is a general feature of standard Skyrme forces [7,8]. We will now reexamine this issue in the framework of the nuclear EDF theory.

III. STABILITY OF NUCLEAR MATTER

Let us consider the case of static uniform (possibly polarized) infinite isospin asymmetric nuclear matter. The Skyrme energy density, Eq. (2a), thus reduces to

$$\mathcal{E}_{\rm Sky} = \sum_{t=0,1}\left(C_t^\rho\rho_t^2 + C_t^\tau\rho_t\tau_t + C_t^s s_t^2 + C_t^T\boldsymbol{s_t}\cdot\boldsymbol{T_t}\right). \tag{7}$$

Let us choose the spin-quantization axis so that the only nonvanishing components of the spin density $\boldsymbol{s_q}$ and the spin kinetic density $\boldsymbol{T_q}$ are along the z axis. For brevity we will simply write s_q and T_q instead of s_{qz} and T_{qz}. In the following we will neglect the anisotropies induced by the polarization of matter [36]. Introducing the density $\rho_{q\sigma}$ of nucleons with spins $\sigma = \uparrow, \downarrow$ and the kinetic density of polarized nucleons defined by

$$\tau_{q\sigma} = \tfrac{3}{5}(6\pi^2)^{2/3}\rho_{q\sigma}^{5/3}, \tag{8}$$

the spin density and the spin kinetic density can now be expressed as

$$s_q = \rho_{q\uparrow} - \rho_{q\downarrow}, \tag{9}$$

$$T_q = \tau_{q\uparrow} - \tau_{q\downarrow}. \tag{10}$$

In fully polarized NeuM with all spins up ($\rho = \rho_{n\uparrow}$), Eq. (7) reduces to

$$\mathcal{E}_{\rm NeuM}^{\rm pol} = \left[\frac{\hbar^2}{2M_n} + \left(C_0^\tau + C_1^\tau + C_0^T + C_1^T\right)\rho\right]\tau_{n\uparrow} + \left(C_0^\rho + C_1^\rho + C_0^s + C_1^s\right)\rho^2. \tag{11}$$

If the energy density is calculated from a Skyrme force in the Hartree-Fock approximation, we find

$$C_0^\rho + C_1^\rho + C_0^s + C_1^s = 0, \tag{12}$$

$$C_0^\tau + C_1^\tau + C_0^T + C_1^T = \tfrac{1}{2}t_2(1+x_2), \tag{13}$$

so that Eq. (11) reduces to

$$\mathcal{E}_{\rm NeuM}^{\rm pol} = \left[\frac{\hbar^2}{2M_n} + \frac{1}{2}t_2(1+x_2)\rho\right]\tau_{n\uparrow}. \tag{14}$$

Eq. (12) is a consequence of the Pauli exclusion principle and the zero range of the Skyrme interaction. Actually as will be shown elsewhere, Eq. (12) must still be satisfied for nuclear EDFs that are not obtained from effective forces in order to prevent self-interactions. The constraint of Kutschera and Wójcik [14], Eq. (6), can thus be more generally written as

$$C_0^\tau + C_1^\tau + C_0^T + C_1^T \geqslant 0. \tag{15}$$

If this inequality is violated, $\mathcal{E}_{\rm NeuM}^{\rm pol}$ decreases with increasing density, thus leading to a ferromagnetic collapse.

It is instructive to rewrite Eq. (11) as

$$\mathcal{E}_{\rm NeuM}^{\rm pol} = \frac{\hbar^2}{2M^*_{n\uparrow}}\tau_{n\uparrow}, \tag{16}$$

where we have introduced the effective mass of a nucleon in a spin state σ defined by

$$\frac{\hbar^2}{2M^*_{q\sigma}} = \frac{\partial \mathcal{E}}{\partial \tau_{q\sigma}} = \frac{\hbar^2}{2M^*_q} \pm \left[s\left(C_0^T - C_1^T\right) + 2s_q C_1^T\right], \tag{17}$$

with $+(-)$ for spin up (spin down), and M^*_q is the usual effective mass given by

$$\frac{\hbar^2}{2M^*_q} = \frac{\partial \mathcal{E}}{\partial \tau_q} = \frac{\hbar^2}{2M_q} + \left[\left(C_0^\tau - C_1^\tau\right)\rho + 2\rho_q C_1^\tau\right]. \tag{18}$$

It can be easily seen that in fully polarized NeuM, the effective mass reduces to

$$\begin{aligned}\frac{\hbar^2}{2M^*_{n\uparrow}} &= \frac{\hbar^2}{2M_n} + \left(C_0^\tau + C_1^\tau + C_0^T + C_1^T\right)\rho \\ &= \frac{\hbar^2}{2M^*_n} + \left(C_0^T + C_1^T\right)\rho = \frac{\hbar^2}{2M_n} + t_2(1+x_2)\rho,\end{aligned} \tag{19}$$

so that Eq. (16) coincides with Eq. (14). Setting $x_2 = -1$ as in the Saclay-Lyon Skyrme forces [35] therefore implies that the effective mass of polarized neutrons is equal to the bare mass.

We have seen that the constraint of Ref. [14] is equivalent to the requirement that the effective mass of polarized neutrons remains always positive. However, the ground state of NeuM (and more generally that of isospin asymmetric nuclear matter) could still be polarized as shown below.

A. Landau stability criterion

The stability of unpolarized homogeneous nuclear matter with respect to spin and spin-isospin polarizations has been generally addressed using the Landau Fermi-liquid theory (see, e.g., Ref. [37]). In this theory, the elementary excitations of the liquid at low temperatures are described in terms of quasiparticles which are in one-to-one correspondence with single-particle states of the noninteracting Fermi gas. Any *small* change $\delta\tilde{n}(k)$ in the distribution function of quasiparticles with wave vector $\boldsymbol{k}$ leads to a change $\delta\mathcal{E}$ in the energy density, which can be expressed (up to second order) as

$$\begin{aligned}\delta\mathcal{E} &= \int \frac{d^3k}{(2\pi)^3}\varepsilon(\boldsymbol{k})\delta\tilde{n}(\boldsymbol{k}) \\ &\quad + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} v(\boldsymbol{k},\boldsymbol{k}')\delta\tilde{n}(\boldsymbol{k})\delta\tilde{n}(\boldsymbol{k}'),\end{aligned} \tag{20}$$

where $\varepsilon(\boldsymbol{k})$ is the energy of a quasiparticle with wave vector $\boldsymbol{k}$ and $v(\boldsymbol{k},\boldsymbol{k}')$ is the residual interaction between quasiparticles with wave vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$.

In pure NeuM, the residual interaction (neglecting tensor interaction) can be expressed as

$$v^{\rm NeuM}(\boldsymbol{k},\boldsymbol{k}') = \frac{1}{N}[F^{\rm NeuM}(\boldsymbol{k},\boldsymbol{k}') + G^{\rm NeuM}(\boldsymbol{k},\boldsymbol{k}')\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}'], \tag{21}$$

where N is the density of states at the Fermi surface given by

$$N = \frac{M^*_n k_F}{\hbar^2\pi^2}, \tag{22}$$

with $k_F = (3\pi^2\rho)^{1/3}$. We have also introduced the Pauli matrices $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\sigma}}'$ in order to take into account the spin of the quasiparticles. Small perturbations involve only quasiparticles at the Fermi surface, i.e., with $k = k' = k_F$. We can thus expand each term in the residual interaction in Legendre polynomials $P_\ell(\cos\theta)$, where θ is the angle between $\boldsymbol{k}$ and $\boldsymbol{k}'$. For instance,

$$F^{\rm NeuM}(\boldsymbol{k},\boldsymbol{k}') = \sum_{\ell=0}^{+\infty} F_\ell^{\rm NeuM} P_\ell(\cos\theta), \tag{23}$$

where $F_\ell^{\rm NeuM}$ are dimensionless Landau parameters. Similarly, we can define Landau parameters $G_\ell^{\rm NeuM}$. For the Skyrme functional the only nonzero Landau parameters are of order $\ell = 0$ and $\ell = 1$. The stability of the initial state is ensured if any change in the energy per particle $e \equiv \mathcal{E}/\rho$ is positive. This condition leads to Landau's criterion

$$F_\ell^{\rm NeuM} > -(2\ell+1), \tag{24a}$$

$$G_\ell^{\rm NeuM} > -(2\ell+1). \tag{24b}$$

In particular, the condition on $G_0^{\rm NeuM}$ guarantees that NeuM is stable against small fluctuations of the (isoscalar) spin polarization $I_\sigma = s_0/\rho = (\rho_\uparrow - \rho_\downarrow)$. This can be seen by expanding the energy per particle up to second order in I_σ

$$e(I_\sigma) \simeq e(0) + \frac{1}{2}\frac{\partial^2 e}{\partial I_\sigma^2}\bigg|_{I_\sigma=0} I_\sigma^2, \tag{25}$$

with

$$\frac{\partial^2 e}{\partial I_\sigma^2}\bigg|_{I_\sigma=0} = \frac{\hbar^2 k_F^2}{3M^*_n}\left(1 + G_0^{\rm NeuM}\right). \tag{26}$$

The first-order term vanishes because of the requirement that the unpolarized phase be an equilibrium state.

Using the Skyrme functional, we find

$$G_0^{\rm NeuM} = 2N\left[C_0^s + C_1^s + k_F^2\left(C_0^T + C_1^T\right)\right]. \tag{27}$$

Now if the Skyrme functional is fitted to a realistic equation of state of NeuM [13], we find that $C_0^\rho + C_1^\rho \leqslant 0$, which according to Eq. (12) implies that

$$C_0^s + C_1^s \geqslant 0. \tag{28}$$

Ferromagnetic instabilities are therefore mainly due to the coupling constants C_t^T. In order to fulfill the Landau's stability condition $G_0^{\rm NeuM} > -1$ at any density, we must have[1]

$$C_0^T + C_1^T \geqslant 0. \tag{29}$$

The absence of a ferromagnetic transition in NeuM does not generally forbid the occurrence of spin-ordered phases in asymmetric nuclear matter. Let us consider in particular SNM. The most general form of the residual interaction (neglecting tensor interaction) can be expressed as

$$v^{\rm SNM}(\boldsymbol{k},\boldsymbol{k}') = \frac{1}{N_0}[F(\boldsymbol{k},\boldsymbol{k}') + F'(\boldsymbol{k},\boldsymbol{k}')\hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{\tau}}' + G(\boldsymbol{k},\boldsymbol{k}') \times \hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}' + G'(\boldsymbol{k},\boldsymbol{k}')\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}'\hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{\tau}}'], \tag{30}$$

where N_0 is the density of states at the Fermi surface given by

$$N_0 = \frac{2M_s^* k_{F0}}{\hbar^2\pi^2}, \tag{31}$$

with $k_{F0} = (3\pi^2\rho/2)^{1/3}$, and M_s^* is the isoscalar effective mass defined by

$$\frac{M}{M_s^*} = 1 + \frac{2M}{\hbar^2}C_0^\tau\rho, \quad \frac{2}{M} = \frac{1}{M_n} + \frac{1}{M_p}. \tag{32}$$

We have also introduced the Pauli matrices $\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\tau}}'$ in order to take into account the isospin of the quasiparticles. As before, we can define dimensionless Landau parameters F_ℓ, F'_ℓ, G_ℓ, and G'_ℓ. The Landau stability conditions are in this case given by

$$F_\ell > -(2\ell+1), \tag{33a}$$
$$F'_\ell > -(2\ell+1), \tag{33b}$$
$$G_\ell > -(2\ell+1), \tag{33c}$$
$$G'_\ell > -(2\ell+1). \tag{33d}$$

The Landau parameters F_0 and F'_0 are related to the usual compression modulus

$$K_v = \frac{3\hbar^2 k_{F0}^2}{M_s^*}(1+F_0), \tag{34}$$

and symmetry energy

$$J = \frac{\hbar^2 k_{F0}^2}{6M_s^*}(1+F'_0), \tag{35}$$

respectively. The conditions on G_0 and G'_0 ensure that SNM is stable against small fluctuations of isoscalar and isovector spin densities, respectively. These Landau parameters can be expressed in terms of the spin asymmetry coefficient, defined by

$$a_\sigma \equiv \frac{1}{2}\frac{\partial^2 e}{\partial I_\sigma^2}\bigg|_{I_\sigma=0} = \frac{\hbar^2 k_{F0}^2}{6M_s^*}(1+G_0), \tag{36}$$

and the spin-isospin asymmetry coefficient, defined by

$$a_{\sigma\tau} \equiv \frac{1}{2}\frac{\partial^2 e}{\partial I_{\sigma\tau}^2}\bigg|_{I_{\sigma\tau}=0} = \frac{\hbar^2 k_{F0}^2}{6M_s^*}(1+G'_0), \tag{37}$$

where $I_{\sigma\tau} \equiv s_1/\rho = (\rho_{n\uparrow} - \rho_{n\downarrow} - \rho_{p\uparrow} + \rho_{p\downarrow})/\rho$. Using the Skyrme functional, the Landau parameters G_0 and G'_0 are given by

$$G_0 = 2N_0\big[C_0^s + C_0^T k_{F0}^2\big], \tag{38}$$
$$G'_0 = 2N_0\big[C_1^s + C_1^T k_{F0}^2\big]. \tag{39}$$

The stability of SNM at any density thus requires

$$C_t^T \geqslant 0. \tag{40}$$

These two conditions entail Eq. (29). Note that Landau's stability conditions allow one of the coefficients C_t^s to be negative provided their sum remains positive.

Landau's stability conditions, Eqs. (24b), (33c), and (33d), guarantee that the unpolarized state is *locally* stable (metastable) against *small* fluctuations of the spin and spin-isospin polarizations. But this criterion does not necessarily imply that the unpolarized state is the ground state, i.e., the state with the lowest energy. In particular, the ground state could still be polarized with finite values of I_σ and $I_{\sigma\tau}$. Moreover, we have only considered so far the two limiting cases of SNM and NeuM. However, the outer core of neutron stars is formed of isospin asymmetric nuclear matter whose composition varies with depth. We thus need a more general stability criterion.

B. General stability criterion

Asymmetric nuclear matter is stable with respect to *any* degree of spin and spin-isospin polarizations whenever the energy density $\mathcal{E}^{\rm pol}$ of the polarized state is larger than the energy density $\mathcal{E}^{\rm unpol}$ of the unpolarized state (for a given density ρ). Using Eqs. (7), (17), and (18), we find

$$\mathcal{E}^{\rm pol} = \sum_{q,\sigma}\frac{\hbar^2}{2M_{q\sigma}^*}\tau_{q\sigma} + C_0^s s^2 + C_1^s(s_n - s_p)^2 + C_0^\rho\rho^2 + C_1^\rho(\rho_n-\rho_p)^2, \tag{41}$$

which for unpolarized matter (i.e., $s_q = 0$, $T_q = 0$) yields

$$\mathcal{E}^{\rm unpol} = \sum_q \frac{\hbar^2}{2M_q^*}\tau_q + C_0^\rho\rho^2 + C_1^\rho(\rho_n-\rho_p)^2, \tag{42}$$

with

$$\tau_q = \tfrac{3}{5}(3\pi^2)^{2/3}\rho_q^{5/3}. \tag{43}$$

The difference can thus be expressed as

$$\mathcal{E}^{\rm pol} - \mathcal{E}^{\rm unpol} = \sum_q \frac{\hbar^2}{2M_q^*}\big(\tau_q^{\rm pol} - \tau_q\big) + C_0^s s^2 + C_1^s(s_n-s_p)^2 + C_0^T sT + C_1^T(s_n-s_p)(T_n-T_p), \tag{44}$$

[1]This inequality is not strictly required if the coefficients C_t^s are allowed to depend on the density according to Eqs. (3a) and (3b) and the term in $(b_0^s + b_1^s)\rho^\alpha$ dominates at high density. However, for modern Skyrme parametrizations, such a situation does not arise because $3\alpha < 2$.

where $\tau_q^{\rm pol} = \tau_{q\uparrow} + \tau_{q\downarrow}$ is the nucleon kinetic density in the polarized phase. The absolute stability of the unpolarized phase can be ensured by requiring each term be separately positive so that $\mathcal{E}^{\rm pol} > \mathcal{E}^{\rm unpol}$. Now the first term in Eq. (44) is always positive, since mechanical stability requires $M_q^* \geqslant 0$ and the Pauli exclusion principle implies that $\tau_q^{\rm pol} > \tau_q$. Let us also remark that $(s_n - s_p)(T_n - T_p) \geqslant 0$ because $\tau_{q\sigma}$ increases monotonically with $\rho_{q\sigma}$. The following constraints

$$C_t^s \geqslant 0, \tag{45a}$$

and

$$C_t^T \geqslant 0, \tag{45b}$$

therefore guarantee the absence of any spin-ordered phase transitions in asymmetric nuclear matter. It is readily seen from Eqs. (27), (38), and (39) that these inequalities enforce Landau stability conditions, Eq. (24b) in NeuM and Eqs. (33c) and (33d) in SNM. Since Eqs. (45a) and (45b) ensure the stability of asymmetric nuclear matter, they obviously prevent a ferromagnetic collapse of NeuM, as can be seen from Eq. (15) remembering that $C_0^\tau + C_1^\tau \geqslant 0$ as a consequence of $M_n^* \geqslant 0$.

C. Anomalous behavior of the entropy

We have seen that the stability of nuclear matter requires that $C_t^T \geqslant 0$. However, these coefficients cannot take arbitrary values. From Eq. (4), large positive values of C_t^T translate into large negative values of C_t^J which, in certain circumstances, can lead to instabilities in finite nuclei whose consequence is a major rearrangement of the single-particle spectrum [38]. We will now show that these coupling constants can be further constrained by requiring the stability of nuclear matter with respect to any degree of spin and spin-isospin polarizations at nonzero temperatures.

It was shown in Refs. [17,18] that not only do Skyrme forces predict a ferromagnetic transition in NeuM above a certain critical density, but worse this density decreases with increasing temperature due to an anomalous behavior of the entropy. This argument can be easily transposed to asymmetric nuclear matter as follows. At low temperatures (compared to nucleon Fermi energies), the difference between the entropy density $\mathcal{S}^{\rm pol}$ of the polarized state and the entropy density $\mathcal{S}^{\rm unpol}$ of the unpolarized state is approximately given by

$$\mathcal{S}^{\rm pol} - \mathcal{S}^{\rm unpol} = \sum_{q,\sigma} \frac{\pi^2 T M_q^* \rho_q}{2\hbar^2 k_{Fq}^2}\left[\frac{M_{q\sigma}^*}{M_q^*}\left(\frac{2\rho_{q\sigma}}{\rho_q}\right)^{1/3} - 1\right]. \tag{46}$$

Now because the polarized phase is more ordered than the unpolarized phase, its entropy according to Boltzmann's definition should thus be lower, i.e., $\mathcal{S}^{\rm pol} < \mathcal{S}^{\rm unpol}$ as found in realistic calculations [23–25,29]. Since this should be true for any isospin asymmetry, we find from Eq. (46)

$$\sum_\sigma \frac{M_{q\sigma}^*}{M_q^*}\left(\frac{\rho_{q\sigma}}{\rho_q}\right)^{1/3} < 2^{2/3}. \tag{47}$$

This condition reduces to that of Ref. [17] in the limiting case of fully polarized NeuM. Equation (47) can be equivalently expressed as $(q' \neq q)$

$$\frac{(1+I_{\sigma q})^{1/3}}{1 + \Xi I_{\sigma q} - \Upsilon I_{\sigma q'}} + \frac{(1-I_{\sigma q})^{1/3}}{1 - \Xi I_{\sigma q} + \Upsilon I_{\sigma q'}} < 2, \tag{48}$$

with $I_{\sigma q} = (\rho_{q\uparrow} - \rho_{q\downarrow})/\rho_q$,

$$\Xi = \left(C_0^T + C_1^T\right)\rho_q \frac{2M_q^*}{\hbar^2}, \tag{49}$$

$$\Upsilon = \left(C_0^T - C_1^T\right)\rho_{q'} \frac{2M_q^*}{\hbar^2}. \tag{50}$$

We have found numerically that the inequalities (48) can be satisfied for any degree of spin and spin-isospin polarizations, i.e., $0 < |I_{\sigma q}|, |I_{\sigma q'}| \leqslant 1$, provided

$$\Xi_1 \leqslant \Xi \leqslant \Xi_2, \tag{51a}$$

$$\Upsilon = 0, \tag{51b}$$

with $\Xi_1 \simeq -0.21$ and $\Xi_2 \simeq 0.54$. We have also found solutions of Eq. (48) for $|\Upsilon| > \Upsilon_c(\Xi) > 0$. But it can be seen from Eq. (50) that such solutions cannot exist for all densities and must therefore be excluded. Inserting Eq. (49) into Eq. (51a) using Eq. (18) yields

$$\rho_q\left[\left(C_0^T + C_1^T\right) - \Xi_2\left(C_0^\tau + C_1^\tau\right)\right] - \Xi_2\rho_{q'}\left(C_0^\tau - C_1^\tau\right) \leqslant \Xi_2 \frac{\hbar^2}{2M_q}, \tag{52a}$$

$$\rho_q\left[\left(C_0^T + C_1^T\right) - \Xi_1\left(C_0^\tau + C_1^\tau\right)\right] - \Xi_1\rho_{q'}\left(C_0^\tau - C_1^\tau\right) \geqslant \Xi_1 \frac{\hbar^2}{2M_q}. \tag{52b}$$

The terms in $\rho_{q'}$ always satisfy the above inequalities. This is a consequence of the positivity of M_q^* for any density and isospin asymmetry which requires that $C_0^\tau + C_1^\tau \geqslant 0$ and $C_0^\tau - C_1^\tau \geqslant 0$, as can be seen from Eq. (18). The conditions (52a) and (52b) can be ensured for any density ρ_q by imposing that the associated terms be, respectively, negative and positive, leading to

$$\Xi_1\left(C_0^\tau + C_1^\tau\right) \leqslant C_0^T + C_1^T \leqslant \Xi_2\left(C_0^\tau + C_1^\tau\right). \tag{53}$$

On the other hand, Eq. (51b) implies

$$C_0^T = C_1^T. \tag{54}$$

Combining these inequalities with Eqs. (45b), we arrive at the following restrictions:

$$C_0^T = C_1^T, \quad 0 \leqslant C_t^T \leqslant \tfrac{1}{2}\Xi_2\left(C_0^\tau + C_1^\tau\right). \tag{55}$$

Equations (55) guarantee that asymmetric nuclear matter remains unpolarized at finite temperature T, since the free energy density of the polarized phase, defined by $\mathcal{F}^{\rm pol} = \mathcal{E}^{\rm pol} - T\mathcal{S}^{\rm pol}$, is always higher than the free energy density $\mathcal{F}^{\rm unpol} = \mathcal{E}^{\rm unpol} - T\mathcal{S}^{\rm unpol}$ of the unpolarized phase.

IV. STABILITY OF SKYRME FORCES REVISITED

Conventional Skyrme forces have been shown to predict various spin and spin-isospin instabilities in nuclear matter

[7,9,10,13,14,17,18]. We have seen in the previous section that for a nuclear functional given by Eqs. (2a)–(2c), the stability of asymmetric nuclear matter at any temperature can be ensured by imposing Eqs. (45a) and (55) [the constraint proposed in Ref. [14], Eq. (6), and more generally Eq. (15), prevents a collapse of polarized NeuM, but does not forbid a ferromagnetic transition]. While the coefficients C_t^s are generally positive (at least for not too high densities), standard Skyrme forces yield negative values of at least one of the couplings constants C_t^T. The origin of the instabilities can therefore be traced back to the time-odd terms $\boldsymbol{s_t} \cdot \boldsymbol{T_t}$, which are related to the time-even terms $\sum_{\mu,\nu} J_{t,\mu\nu} J_{t,\mu\nu}$ due to gauge invariance (4). Since the seminal work of Vautherin and Brink [39], it is commonly taken for granted that the spin-current tensor (which is usually approximated by the spin-current vector $\boldsymbol{J_q}$) is small in nuclei, and most Skyrme parametrizations therefore neglect them. We have tested this assumption by computing the HFB energies with and without the J^2 and J_q^2 terms (denoted, respectively, by $E_{\rm HFB}$ and $E^0_{\rm HFB}$) for all even-even nuclei with $Z, N > 8$ and $Z < 110$ lying between the proton and neutron drip lines. (Note that when the J^2 and J_q^2 terms are included, the associated time-odd terms in C_t^T play a role in the exact treatment of the masses of odd nuclei, but not in the equal-filling approximation [40], which we adopt here, as in all our previous papers.) The differences $\Delta M \equiv E_{\rm HFB} - E^0_{\rm HFB}$ are shown in Fig. 1 for the Skyrme parametrization BSk17 [41,42], which was originally fitted with the J^2 and J_q^2 terms, and for SkI2 [43] which was not. The impact of the J^2 and J_q^2 terms is quite large, reaching about 20 MeV for the heaviest nuclei. The impact of dropping or including the J^2 and J_q^2 terms is logically found to be correlated to the amplitude of the $C_t^T = -C_t^J$ coupling constants, especially C_0^T. For instance, in the case of the SLy4 [35] interaction ($C_0^T = -17.21$ MeV fm^5), the HFB energy is affected by no more than 5 MeV, while for SkO [44] ($C_0^T = -220.54$ MeV fm^5) values up to 30 MeV can be reached. Adding or removing the J^2 and J_q^2 terms *a posteriori* without refitting all the parameters of the force can thus lead to significant errors. However, in all previous studies of spin and spin-isospin instabilities in nuclear matter [7–14,17,18], the time-odd terms $\boldsymbol{s_t} \cdot \boldsymbol{T_t}$ were taken into account, whereas the Skyrme forces were generally fitted without the J^2 and J_q^2 terms. This treatment not only violates gauge symmetry but also introduces inconsistencies in the residual interaction hence in the Landau parameters (see the discussion in Sec. III of Ref. [45] and also in Sec. 5D of Ref. [38]).

We have therefore reexamined the stability of several Skyrme parametrizations for which the J^2 and J_q^2 terms were not included in the fit: SGII [46], SLy4 [35], SkI1-SkI5 [43], SkO [44], and LNS [47]. The parametrization SGII [46] was constructed in order to improve the Landau parameters G_0 and G_0' and the description of Gamow-Teller resonances in nuclei. The Skyrme Saclay-Lyon forces and especially the parametrization SLy4 [35] have been widely used not only in nuclear physics but also in neutron star studies, because these forces were constrained to reproduce a realistic neutron-matter equation of state. The SkI [43] forces were all constrained (except for SkI1) to reproduce the isotopic

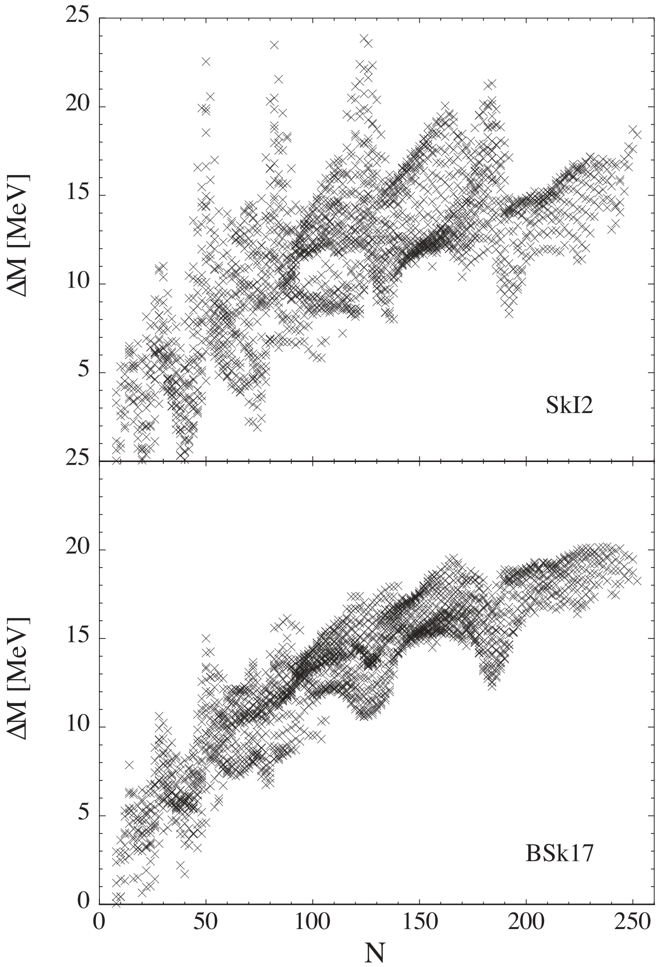

FIG. 1. Differences between the HFB energies estimated with and without the J^2 terms for two Skyrme forces SkI2 (upper panel) and BSk17 (lower panel) for all even-even nuclei with $Z, N > 8$ and $Z < 110$ lying between the proton and neutron drip lines.

shifts of the root-mean-square charge radii of neutron-rich Pb and Ca nuclei. Forces SkI3 and SkI4 were constructed with nonstandard spin-orbit couplings. For the parametrization SkI5, the ^{16}O ground-state data were excluded from the fit. We have also included the parametrization SkO [44] from the same group. The parametrization LNS [47] was fitted to Brueckner calculations. The Landau parameters in SNM and in NeuM calculated at saturation density ρ_0, with and without the terms in C_t^T, are shown in Table I. For comparison we have also indicated the predictions from Brueckner-Hartree-Fock calculations in SNM [48] and from realistic calculations based on the renormalization group approach in NeuM [49]. As can be seen in Table I, setting $C_t^T = 0$ in Eqs. (27), (33c), and (33d) tends to reduce the discrepancies between the different Skyrme functionals and generally leads to a better agreement with realistic calculations, especially for G_0'. In particular, the new values of G_0' lie closely inside the empirical range of 1.0 ± 0.1 deduced in Ref. [50] from the analysis of Gamow-Teller resonances and magnetic-dipole modes in finite nuclei. The improvement is quite spectacular for the parametrization SkI1. In the case of LNS, setting $C_0^T = 0$ actually deteriorates

TABLE I. Landau parameters G_0 and G_0' in symmetric nuclear matter and $G_0^{\rm NeuM}$ in neutron matter (at saturation density) for selected Skyrme forces which were fitted without the J^2 and J_q^2 terms. Values in parentheses were obtained by setting $C_t^T = 0$. The last line shows the Landau parameters predicted by microscopic calculations using realistic interactions: Ref. [48] for symmetric nuclear matter and Ref. [49] for neutron matter.

	G_0	G_0'	$G_0^{\rm NeuM}$
SGII	0.01(0.62)	0.51(0.93)	−0.07(1.19)
SLy4	1.11(1.39)	−0.13(0.90)	0.11(1.27)
SkI1	−8.74(1.09)	3.17(0.90)	−5.57(1.10)
SkI2	−1.18(1.35)	0.77(0.90)	−1.08(1.24)
SkI3	0.57(1.90)	0.20(0.85)	−0.19(1.35)
SkI4	−2.81(1.77)	1.38(0.88)	−2.03(1.40)
SkI5	0.28(1.79)	0.30(0.85)	−0.31(1.30)
SkO	−4.08(0.48)	1.61(0.98)	−3.17(0.97)
LNS	0.83(0.32)	0.14(0.92)	0.59(0.91)
Realistic	0.83	1.22	0.77

the value of the Landau parameter G_0, since the latter was directly fitted to the value obtained from realistic calculations. Table II shows the critical densities of the spin-ordered phase transitions according to Landau's stability criterion. It can be seen that dropping the terms $\boldsymbol{s_t} \cdot \boldsymbol{T_t}$ eliminates the instabilities in almost all Skyrme forces. This prescription is also consistent with Eqs. (55) and therefore prevents an anomalous behavior of the entropy, thus ensuring the stability of nuclear matter for any temperatures.

Moreover, setting $C_t^T = 0$ is the only prescription which guarantees the Landau stability conditions of Eqs. (24b), (33c), and (33d) at any density both for $\ell = 0$ and $\ell = 1$. Indeed the Landau parameters G_1, G_1' in SNM and $G_1^{\rm NeuM}$ in NeuM are given by

$$G_1 = -2N_0 C_0^T k_{F0}^2, \tag{56}$$

$$G_1' = -2N_0 C_1^T k_{F0}^2, \tag{57}$$

$$G_1^{\rm NeuM} = -2N k_F^2 \left(C_0^T + C_1^T\right). \tag{58}$$

TABLE II. Critical densities above which nuclear matter becomes unstable according to Landau's criterion for selected Skyrme forces which were fitted without the J^2 and J_q^2 terms. The first two columns are for symmetric nuclear matter, while the last column is for pure neutron matter. The densities indicated in parentheses were obtained by setting $C_t^T = 0$.

	$\rho_c(G_0)$ (fm^{-3})	$\rho_c(G_0')$ (fm^{-3})	$\rho_c(G_0^{\rm NeuM})$ (fm^{-3})
SGII	0.44(∞)	0.80(∞)	0.26(2.07)
SLy4	∞ (∞)	0.33(∞)	0.59(∞)
SkI1	0.04(0.71)	∞ (∞)	0.05(∞)
SkI2	0.14(∞)	∞ (∞)	0.15(∞)
SkI3	0.91(∞)	0.92(∞)	0.37(∞)
SkI4	0.07(∞)	∞ (∞)	0.09(∞)
SkI5	0.43(∞)	1.36(∞)	0.28(∞)
SkO	0.07(0.52)	∞(2.32)	0.09(0.67)
LNS	∞ (∞)	0.43(∞)	0.62(1.38)

TABLE III. Landau sum rules given by Eqs. (59a) and (59b) for selected Skyrme forces which were fitted without the J^2 and J_q^2 terms. Values in parentheses were obtained by setting $C_t^T = 0$.

	S_1	S_2
SGII	0.97(0.61)	1.13(−0.51)
SLy4	−0.31(−0.65)	1.52(0.85)
SkI1	−6.71(−0.59)	−89.2(0.86)
SkI2	6.87(−0.71)	−20.7(0.98)
SkI3	−1.46(−2.33)	2.14(1.84)
SkI4	1.01(−1.23)	−11.3(1.32)
SkI5	−1.47(−2.28)	2.17(1.77)
SkO	3.21(1.07)	−13.7(0.87)
LNS	0.49(0.63)	3.53(−0.04)

Requiring $G_1 \geqslant -3$, $G_1' \geqslant -3$, and $G_1^{\rm NeuM} \geqslant -3$ for any density thus leads to $C_t^T \leqslant 0$. Combining these inequalities with Eqs. (40) yields $C_t^T = 0$. Adopting these particular values tends to be supported by the following basic sum rules of Landau Fermi-liquid theory [51]:

$$S_1 = \sum_\ell \frac{F_\ell}{1 + F_\ell/(2\ell+1)} + \frac{F_\ell'}{1 + F_\ell'/(2\ell+1)} + \frac{G_\ell}{1 + G_\ell/(2\ell+1)} + \frac{G_\ell'}{1 + G_\ell'/(2\ell+1)} = 0, \tag{59a}$$

and

$$S_2 = \sum_\ell \frac{F_\ell}{1 + F_\ell/(2\ell+1)} - 3\frac{F_\ell'}{1 + F_\ell'/(2\ell+1)} - 3\frac{G_\ell}{1 + G_\ell/(2\ell+1)} + 9\frac{G_\ell'}{1 + G_\ell'/(2\ell+1)} = 0. \tag{59b}$$

Even though Skyrme forces generally violate these sum rules, the prescription $C_t^T = 0$ significantly improves the second sum rule, as can be seen in Table III. It is quite remarkable that dropping the terms $\boldsymbol{s_t} \cdot \boldsymbol{T_t}$ not only removes all kinds of instabilities in nuclear matter but also improves the internal consistency of the nuclear functional. Nevertheless with this prescription, the Landau parameters G_1, G_1', and $G_1^{\rm NeuM}$ all vanish, leading to unrealistic effective masses in polarized matter. Indeed, according to Eqs. (17) $M^*_{q\uparrow} = M^*_{q\downarrow} = M^*_q$ which obviously holds in the limit of vanishing spin polarizations but is otherwise contradicted by realistic calculations [23–25,29,31]. In particular, these calculations indicate that in polarized NeuM, $M_{n\uparrow} > M_{n\downarrow}$ whenever $\rho_\uparrow > \rho_\downarrow$. Imposing the less stringent stability conditions (55) leads to a splitting of effective masses but with a wrong sign. This deficiency calls for further extensions of existing Skyrme functionals.

In the discussion above, we have implicitly adopted the point of view of the nuclear EDF theory [32] that the different terms appearing in Eqs. (2b) and (2c) can be *a priori* considered as independent from each other (apart from the requirements of gauge invariance). It is therefore perfectly legitimate to set $C_t^J = -C_t^T \equiv 0$. However, in the framework of effective forces, the coupling constants are uniquely determined by the parameters of the force. In particular, the coefficients C_t^s and

C_t^T are now given by

$$C_0^s = -\frac{1}{4}t_0\left(\frac{1}{2}-x_0\right) - \frac{1}{24}t_3\left(\frac{1}{2}-x_3\right)\rho^\alpha, \quad (60a)$$

$$C_1^s = -\frac{1}{8}t_0 - \frac{1}{48}t_3\rho^\alpha, \quad (60b)$$

$$C_0^T = -\frac{1}{8}\left[t_1\left(\frac{1}{2}-x_1\right) - t_2\left(\frac{1}{2}+x_2\right)\right], \quad (60c)$$

$$C_1^T = -\frac{1}{16}(t_1 - t_2). \quad (60d)$$

We have therefore studied the stability of the few Skyrme parametrizations which were fitted with the J^2 and J_q^2 terms: SkP [52], SLy5 [35], SkO′ [44], SkX [53], and BSk17 [41,42]. The parametrization SkP, which was specifically designed to be used both in the particle-hole channel and in the particle-particle channel, is still used nowadays. The forces SLy5 and SkO′ were fitted following the same protocol as SLy4 and SkO, respectively, but they include the contribution of the J^2 and J_q^2 terms. The force SkX [53] was constructed in an attempt to improve the description of single-particle energies. BSk17 is the force underlying our nuclear mass model HFB-17, based on the Hartree-Fock-Bogoliubov method [41,42]. With this model, we were able to fit with an rms deviation of 0.581 MeV the 2149 measured masses of nuclei with N and $Z \geqslant 8$ given in the 2003 Atomic Mass Evaluation [54], while at the same time constraining the underlying Skyrme force to fit properties of SNM and NeuM, as determined by many-body calculations using realistic potentials. The values of the Landau parameters in SNM and in NeuM are shown in Table IV, and the critical densities for the onset of instabilities are shown in Table V. For those few Skyrme forces which include the J^2 and J_q^2 terms, nuclear matter is therefore unstable because of the tight correlations between the different coupling constants in the energy density.

In order to illustrate the impact of the J^2 and J_q^2 terms and their time-odd counterparts on the stability of nuclear matter, we have plotted in Fig. 2 the difference between the energy per particle in fully polarized NeuM and in unpolarized NeuM for the parametrizations SLy4 and BSk17. Both have been fitted to a realistic equation of state of NeuM, but BSk17 includes the J^2 and J_q^2 terms, while SLy4 does not. Removing all instabilities requires that we impose $C_t^s \geqslant 0$ and $C_t^T = 0$. Since the first term in t_0 of the Skyrme force is generally associated with the long-range attractive part of the nucleon-nucleon interaction while the density-dependent term in t_3 is related to the strongly repulsive short-range part, the coupling constant C_0^s can be made positive for any density by choosing $x_0 < 1/2$ and $x_3 > 0$. With $t_0 < 0$ and $t_3 > 0$, the coefficient C_1^s will be positive, at least for not too high densities. Spin- and spin-isospin instabilities thus generally arise mainly from the coupling constants C_0^T and C_1^T, which in turn are generated by the momentum-dependent terms in t_1 and t_2. Using Eqs. (60c) and (60d), the conditions $C_t^T = 0$ entail $t_1 = t_2$ and $x_1 = -x_2$. Imposing these constraints would leave no degree of freedom for adjusting surface properties of nuclei, which also depend on the momentum-dependent t_1 and t_2 terms through the coupling constants $C_t^{\Delta\rho}$. This would also have an impact on the coupling constants C_t^τ which determine the nucleon effective masses, Eq. (18). There is little doubt that such a force would yield poor results when applied to nuclei. Since thermal effects on the spin polarization are rather small for temperatures found

TABLE IV. Landau parameters G_0 and G_0' in symmetric nuclear matter and $G_0^{\rm NeuM}$ in neutron matter (at saturation density) for selected Skyrme forces which were fitted with the J^2 and J_q^2 terms. The last line shows the Landau parameters predicted by microscopic calculations using realistic interactions: Ref. [48] for symmetric nuclear matter and Ref. [49] for neutron matter.

	G_0	G_0'	$G_0^{\rm NeuM}$
SkO′	−1.62	0.79	−1.43
SLy5	1.09	−0.16	0.09
SkP	−0.23	0.06	−0.61
SkX	−0.63	0.51	−0.50
BSk17	−0.69	0.50	−0.88
BSk17st	−0.68	0.50	0.47
BSk18	−0.33	0.46	−0.57
Realistic	0.83	1.22	0.77

TABLE V. Critical densities above which nuclear matter becomes unstable according to Landau's criterion for selected Skyrme forces which were fitted with the J^2 and J_q^2 terms. The first two columns are for symmetric nuclear matter, while the last column is for pure neutron matter.

	$\rho_c(G_0)$ (fm^{-3})	$\rho_c(G_0')$ (fm^{-3})	$\rho_c(G_0^{\rm NeuM})$ (fm^{-3})
SkO′	0.12	0.97	0.14
SLy5	∞	0.33	0.57
SkP	0.74	0.30	0.19
SkX	0.22	0.40	0.19
BSk17	0.21	0.68	0.17
BSk17st	∞	∞	∞
BSk18	∞	0.62	∞

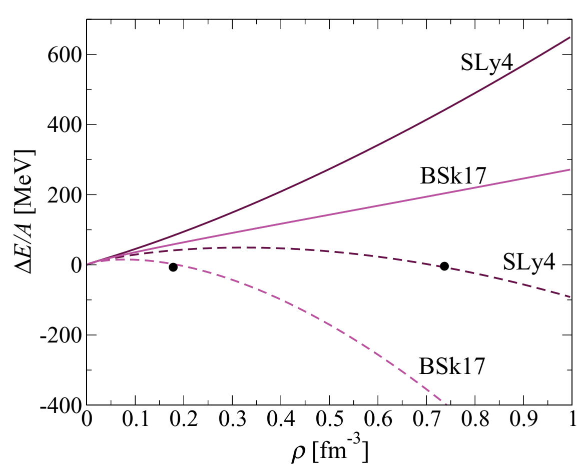

FIG. 2. (Color online) Difference between the energy per particle in fully polarized neutron matter and in unpolarized neutron matter for two Skyrme forces SLy4 and BSk17, with (dashed line) and without (solid line) the J^2 and J_q^2 terms and their time-odd part. The black dots indicate the densities at which the difference vanishes.

in protoneutron stars and supernova cores [17], one may be tempted to require the stability of cold nuclear matter only. But even in this case, it was shown in Refs. [7,8] that it is not possible to avoid spurious transitions to spin-ordered phases in nuclear matter above 2–3 times saturation density, and at the same time giving reasonable properties of SNM. We have found that the critical densities above which instabilities occur are even lower when more nuclear data are included in the fit of the effective interaction. In particular, conventional Skyrme forces fitted to essentially all experimental nuclear mass data predict a ferromagnetic transition in NeuM at a density slightly above saturation density [13] (see also Table V).

The stability of cold nuclear matter can only be restored by including additional components in the Skyrme interaction, thereby inducing new terms in the energy density. Two different extensions have been recently proposed. Margueron and Sagawa [12] considered extended Skyrme forces with two new t_3 like terms depending on the nucleon spin densities $\boldsymbol{s_q}$ of the form

$$\tfrac{1}{6}t_3^s\left(1+x_3^s P_\sigma\right)s(\boldsymbol{r})^2\delta(\boldsymbol{r}_{ij})+\tfrac{1}{6}t_3^{st}\left(1+x_3^{st} P_\sigma\right)s_1(\boldsymbol{r})^2\delta(\boldsymbol{r}_{ij}). \tag{61}$$

In the energy density, Eqs. (2b) and (2c), these new terms modify the coefficients C_t^s. The additional parameters were adjusted so as to ensure the Landau stability conditions $G_0 > -1$, $G_0' > -1$, and $G_0^{\rm NeuM} > -1$. The nuclear mass model HFB-17 [41,42] was thus refitted with these new terms [55]. With this extended Skyrme force called BSk17st, it was possible to maintain the quality of the HFB-17 mass model, and at the same time the Landau parameters were adjusted so as to remove the spin and spin-isospin instabilities present in the original force BSk17. Unfortunately, instabilities were still found for finite spin and spin-isospin polarizations [55]. The reason is that terms of the form given by Eq. (61) do not change the coefficients C_t^T, and consequently, Eq. (44) is not guaranteed to remain positive for any spin and spin-isospin polarizations. Moreover, as noted in Ref. [55], the contributions of Eq. (61) to the energy density cancel in fully polarized NeuM so that BSk17st still predicts a ferromagnetic collapse of NeuM, as BSk17 does. The extension of Ref. [12] does not affect the coefficients C_t^T hence also the effective masses of spin-up and spin-down nucleons are not affected, as can be seen from Eq. (17). This means that if the original Skyrme force violates the constraint (47), this will still be the case for the extended version of this force.

Alternatively, instabilities can be avoided by introducing into the force, density-dependent generalizations of the usual t_1 and t_2 terms of the form [13]

$$\frac{1}{2}t_4(1+x_4P_\sigma)\frac{1}{\hbar^2}\left[p_{ij}^2\rho(\boldsymbol{r})^\beta\delta(\boldsymbol{r}_{ij})+\delta(\boldsymbol{r}_{ij})\rho(\boldsymbol{r})^\beta p_{ij}^2\right] + t_5(1+x_5P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}\cdot\rho(\boldsymbol{r})^\gamma\delta(\boldsymbol{r}_{ij})\boldsymbol{p}_{ij}. \tag{62}$$

These new terms modify the coefficients C_t^τ, C_t^T, $C_t^{\Delta\rho}$, and $C_t^{\Delta s}$ thus providing more flexibility to remove instabilities without deteriorating the fit to nuclear data. We have constructed a new nuclear mass model, labeled HFB-18, with such a generalized Skyrme force [13]. The parameters t_5, x_5, and γ were chosen in order to avoid a ferromagnetic collapse of neutron star matter. For simplicity, the remaining parameters in Eq. (62) were fixed by the equations

$$\beta = \gamma, \tag{63a}$$

$$t_4 = -\frac{1}{3}t_5(5+4x_5), \tag{63b}$$

$$x_4 = -\frac{4+5x_5}{5+4x_5}, \tag{63c}$$

which ensure that the contributions of the new terms to the coefficients C_t^τ vanish identically. As a result, the t_4 and t_5 terms cancel exactly in unpolarized homogeneous nuclear matter. This new model yields almost as good a mass fit as our previous model HFB-17, with the advantage that NeuM matter is now stable with respect to any degree of spin polarizations. Even though this new force still predicts an isospin instability in SNM, this does not affect the interior of neutron stars, which is now unpolarized. Moreover, we have found that this isospin instability can be easily removed if the conditions (63a)–(63c) are released, without deteriorating the quality of the mass fit [56]. However, we did not succeed in constructing a nuclear mass model that satisfies Eq. (47). As a consequence, nuclear matter could still become unstable at finite temperatures even though no phase transitions occur at zero temperature, as shown in Ref. [17].

One might be tempted to enforce the stability conditions $C_t^T = 0$ by adding a zero-range tensor force to the conventional Skyrme interaction (5) with suitable adjustments of the parameters, like the parametrization T22 of Ref. [38]. Unfortunately, a tensor force introduces new terms in the functional which also affect the stability of nuclear matter [57]. The stability of 41 different Skyrme interactions having a tensor component has been recently studied in Ref. [58]. In particular, the recent Skyrme forces from the Saclay-Lyon group [38] which include tensor forces and which were fitted following the same protocol as the older SLy family [35], still predict various spin and spin-isospin instabilities. This is notably the case for the force T22 for which $C_t^T = 0$.

V. CONCLUSION

Nuclear energy density functional theory has been traditionally restricted to very specific phenomenological semilocal functionals of the form given by Eqs. (2a)–(2c), based on effective forces [1,6]. However, the use of effective forces introduces tight correlations between different terms of the functional, which can generate various kinds of instabilities. In particular, the time-odd terms $\boldsymbol{s_t}\cdot\boldsymbol{T_t}$ induced by the momentum-dependent part of Skyrme forces (which contribute also to the coupling constants C_t^τ, $C_t^{\Delta\rho}$, and $C_t^{\Delta s}$) are responsible for spurious spin and spin-isospin instabilities in infinite homogeneous nuclear matter at densities encountered in the interior of neutron stars. In some cases, instabilities arise in symmetric nuclear matter below saturation densities and could thus also contaminate calculations in finite nuclei. (Note that the coupling constants $C_t^{\Delta\rho}$ alone were found to drive finite-size instabilities [59].) These correlations between different parts of the nuclear energy density functional hamper the development of more accurate functionals, since adding one term in the effective force can induce several new terms in the functional. Moreover, the coupling constants of the time-odd terms are generally not directly fitted to experimental data but are calculated *a posteriori* using the parameters of Skyrme force. However, there is no guarantee that the effects associated with the time-odd terms will be correctly described in this way. As shown in Refs. [7,8], it is not possible to avoid spurious transitions to spin-ordered phases in nuclear matter above 2–3 times saturation density. The critical densities above which these instabilities occur decrease when more nuclear data are included in the fit of the parameters of the Skyrme force [13]. For instance, for our nuclear mass model HFB-17 [41,42], the ground state of neutron matter becomes ferromagnetic above 0.17 fm^{-3}. These instabilities can be (at least partially) removed by suitable extensions of the Skyrme force, as proposed, for instance, in Refs. [12,13]. However an unphysical spin-ordering could still occur at finite temperatures thus spoiling the application of Skyrme forces to the hot nuclear matter found in protoneutron stars and supernova cores. Alternatively the terms $\boldsymbol{s}_t \cdot \boldsymbol{T}_t$ that are responsible for spin and spin-isospin instabilities could be canceled by suitable adjustments of an additional tensor component to the Skyrme force [38]. Unfortunately, a tensor force would also generate new terms in the energy density which still lead to instabilities [58].

On the other hand, the concept of effective forces leads to formal inconsistencies as recently discussed in Ref. [60]. Lots of efforts are now devoted to the construction of nonempirical functionals from realistic interactions directly without resorting to effective forces [32]. If one adopts the point of view that the nuclear functional is more fundamental than effective forces, the different terms appearing in Eqs. (2b) and (2c) can be treated independently (apart from the requirements of gauge invariance and cancellation of self-interactions as will be shown elsewhere). It is therefore perfectly legitimate to set $C_t^J = -C_t^T \equiv 0$. Actually the J^2 and J_q^2 terms are dropped in most Skyrme forces, not only because of simplicity but also because it seems to be favored by global fits to nuclear data and basic nuclear matter properties [61]. Moreover, the J^2 and J_q^2 terms might even lead to instabilities in the single-particle spectra of finite nuclei, as discussed, for instance, in Ref. [38]. However, in all previous studies of spin and spin-isospin instabilities in nuclear matter [7–14,17,18], the associated time-odd terms $\boldsymbol{s} \cdot \boldsymbol{T}$ and $(\boldsymbol{s_n} - \boldsymbol{s_p}) \cdot (\boldsymbol{T_n} - \boldsymbol{T_p})$ have been included in the residual interaction, thus violating gauge symmetry. We have therefore reexamined the stability of nuclear matter by setting $C_t^T \equiv 0$ for those Skyrme parametrizations which were fitted *without* the J^2 and J_q^2 terms. We have found that this simple prescription not only improves the values of the Landau parameters G_0, G_0', and G_0^{NeuM}. But this also generally removes all kinds of instabilities in asymmetric nuclear matter both at zero and finite temperatures. Nevertheless, this prescription yields unrealistic values of the Landau parameters G_1, G_1', and G_1^{NeuM}, hence also of the effective masses $M_{q\sigma}^*$ in polarized matter. Further improvements thus require extensions of existing Skyrme functionals.

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[1] M. Bender, P.-H. Heenen, and P.-G. Reinhard, Rev. Mod. Phys. **75**, 121 (2003).
[2] T. H. R. Skyrme, Nucl. Phys. **9**, 615 (1959).
[3] A. G. W. Cameron, Astrophys. J. **130**, 884 (1959).
[4] W. Baade and F. Zwicky, Phys. Rev. **45**, 138 (1934).
[5] N. Chamel and P. Haensel, Living Rev. Relativity **11**, 10 (2008) [http://www.livingreviews.org/lrr-2008-10].
[6] J. R. Stone and P. G. Reinhard, Prog. Part. Nucl. Phys. **58**, 587 (2007).
[7] J. Margueron, J. Navarro, and N. V. Giai, Phys. Rev. C **66**, 014303 (2002).
[8] B. K. Agrawal, S. Shlomo, and V. K. Au, Phys. Rev. C **70**, 057302 (2004).
[9] A. Vidaurre, J. Navarro, and J. Bernabéu, Astron. Astrophys. **135**, 361 (1984).
[10] A. A. Isayev, Phys. Rev. C **74**, 057301 (2006).
[11] M. A. Pérez-García, Phys. Rev. C **77**, 065806 (2008).
[12] J. Margueron and H. Sagawa, J. Phys. G **36**, 125102 (2009).
[13] N. Chamel, S. Goriely, and J. M. Pearson, Phys. Rev. C **80**, 065804 (2009).
[14] M. Kutschera and W. Wójcik, Phys. Lett. B **325**, 271 (1994).
[15] P. Haensel and S. Bonazzola, Astron. Astrophys. **314**, 1017 (1996).
[16] M. Kutschera, Mon. Not. R. Astron. Soc. **307**, 784 (1999).
[17] A. Rios, A. Polls, and I. Vidana, Phys. Rev. C **71**, 055802 (2005).
[18] A. A. Isayev and J. Yang, J. Korean Astron. Soc. **43**, 161 (2010).
[19] S. Reddy, M. Prakash, J. M. Lattimer, and J. A. Pons, Phys. Rev. C **59**, 2888 (1999).
[20] J. Navarro, E. S. Hernàndez, and D. Vautherin, Phys. Rev. C **60**, 045801 (1999).
[21] M. Prakash, I. Bombaci, M. Prakash, P. J. Ellis, J. M. Lattimer, and R. Knorren, Phys. Rep. **280**, 1 (1997).
[22] G. H. Bordbar and M. Bigdeli, Phys. Rev. C **77**, 015805 (2008).
[23] G. H. Bordbar and M. Bigdeli, Phys. Rev. C **78**, 054315 (2008).
[24] M. Bigdeli, G. H. Bordbar, and Z. Rezaei, Phys. Rev. C **80**, 034310 (2009).
[25] M. Modarres and T. Pourmirjafari, Nucl. Phys. A **836**, 91 (2010).
[26] M. Bigdeli, G. H. Bordbar, and A. Poostforush, Phys. Rev. C **82**, 034309 (2010).
[27] I. Vidaña and I. Bombaci, Phys. Rev. C **66**, 045801 (2002).
[28] W. Zuo, U. Lombardo, and C. W. Shen, in *Quark-Gluon Plasma and Heavy Ion Collisions*, edited by W. M. Alberico, M. Nardi, and M. P. Lombardo (World Scientific, Singapore, 2002), p. 192.

[29] I. Bombaci, A. Polls, A. Ramos, A. Rios, and I. Vidaña, Phys. Lett. B **632**, 638 (2006).
[30] S. Fantoni, A. Sarsa, and K. E. Schmidt, Phys. Rev. Lett. **87**, 181101 (2001).
[31] F. Sammarruca and P. G. Krastev, Phys. Rev. C **75**, 034315 (2007).
[32] J. E. Drut, R. J. Furnstahl, and L. Platter, Prog. Part. Nucl. Phys. **64**, 120 (2010).
[33] Y. M. Engel, D. M. Brink, K. Goeke, S. J. Krieger, and D. Vautherin, Nucl. Phys. A **249**, 215 (1975).
[34] J. Dobaczewski and J. Dudek, Phys. Rev. C **52**, 1827 (1995); **55**, 3177(E) (1997).
[35] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A **635**, 231 (1998).
[36] J. Dąbrowski and P. Haensel, Ann. Phys. (NY) **97**, 452 (1976).
[37] G. Baym and C. Pethick, *Landau Fermi-Liquid Theory* (Wiley-VCH, New York, 2004).
[38] T. Lesinski, M. Bender, K. Bennaceur, T. Duguet, and J. Meyer, Phys. Rev. C **76**, 014312 (2007).
[39] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
[40] S. Perez-Martin and L. M. Robledo, Phys. Rev. C **78**, 014304 (2008).
[41] S. Goriely, N. Chamel, and J. M. Pearson, Phys. Rev. Lett. **102**, 152503 (2009).
[42] S. Goriely, N. Chamel, and J. M. Pearson, Eur. Phys. J. A **42**, 547 (2009).
[43] P.-G. Reinhard and H. Flocard, Nucl. Phys. A **584**, 467 (1995).
[44] P.-G. Reinhard, D. J. Dean, W. Nazarewicz, J. Dobaczewski, J. A. Maruhn, and M. R. Strayer, Phys. Rev. C **60**, 014316 (1999).
[45] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002).
[46] N. Van Giai and H. Sagawa, Phys. Lett. B **106**, 379 (1981).
[47] L. G. Cao, U. Lombardo, C. W. Shen, and N. V. Giai, Phys. Rev. C **73**, 014313 (2006).
[48] W. Zuo, C. Shen, and U. Lombardo, Phys. Rev. C **67**, 037301 (2003).
[49] A. Schwenk, B. Friman, and G. E. Brown, Nucl. Phys. A **713**, 191 (2003).
[50] I. N. Borzov, S. V. Tolokonnikov, and S. A. Fayans, Sov. J. Nucl. Phys. **40**, 732 (1984).
[51] B. L. Friman and A. K. Dhar, Phys. Lett. B **85**, 1 (1979).
[52] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A **422**, 103 (1984).
[53] B. A. Brown, Phys. Rev. C **58**, 220 (1998).
[54] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[55] J. Margueron, S. Goriely, M. Grasso, G. Colo, and H. Sagawa, J. Phys. G **36**, 125103 (2009).
[56] S. Goriely, N. Chamel, and J. M. Pearson, Phys. Rev. C **82**, 035804 (2010).
[57] S.-O. Bäckman, O. Sjöberg, and A. D. Jackson, Nucl. Phys. A **321**, 10 (1979).
[58] Li-Gang Cao, G. Colò, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010).
[59] M. Kortelainen and T. Lesinski, J. Phys. G **37**, 064039 (2010).
[60] J. Erler, P. Klüpfel, and P. G. Reinhard, J. Phys. G **37**, 064001 (2010).
[61] P. Klüpfel, P.-G. Reinhard, T. J. Bürvenich, and J. A. Maruhn, Phys. Rev. C **79**, 034310 (2009).