Extraction of the axial mass parameter from MiniBooNE neutrino quasielastic double differential cross-section data

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The recently published [A. A. Aguilar-Arevalo (MiniBooNE Collaboration), Phys. Rev. D **81**, 092005 (2010)] MiniBooNE charge current quasielastic double differential cross-section data are analyzed and compared with simulations based on two nuclear models: Fermi gas model and spectral function model. In both cases the axial mass parametr M_A is obtained from a fitting procedure which takes into account the multiplicative factor in the uncertainty of the flux. Bins with large (>50%) contribution from events with small momentum transfer ($q < q_{\text{cut}}$) are eliminated from the analysis. More restrictive cuts result in smaller fitted M_A values. The $q_{\text{cut}} = 500$ MeV/c leads to the value of $M_A = 1350 \pm 66$ MeV in the Fermi gas model and $M_A = 1343 \pm 60$ MeV in the spectral function model. In both cases the value of $M_A = 1030$ MeV is excluded at the confidence level greater than 5σ.

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I. INTRODUCTION

The knowledge of neutrino interactions in the ∼1-GeV energy region is important because this energy domain is characteristic for the majority of neutrino oscillations experiments performed during recent ∼5 years and also those scheduled for the near future. The list includes K2K, MiniBooNE, SciBooNE, MINOS, T2K, and NOνA.

Neutrino oscillations are an energy-dependent phenomenon and the most straightforward analysis of experimental data requires a reconstruction of the neutrino energy. The neutrino flux spectrum is typically rather wideband (despite significant improvement introduced with the idea of the off-axis beams) and the interacting neutrino energy must be estimated, based on the observation of the leptonic and/or hadronic final states. The precision of the analysis depends on the knowledge of neutrino interaction cross sections, both inclusive and exclusive in several most important channels.

In this article we discuss the charged current quasielastic reaction (CCQE)

$$\nu_\mu + n \rightarrow \mu^- + p. \tag{1}$$

This is the dominant process in the case of sub-GeV beams in MiniBooNE, SciBooNE, and T2K experiments. The theoretical description of this reaction is based on the conserved vector current (CVC) and the partially conserved axial current (PCAC) hypotheses. After a simple analysis only one unknown quantity remains, the axial form-factor $G_A(Q^2)$, for which the dipole form with an unknown parameter called the axial mass M_A [1] is typically assumed. If the deviations of $G_A(Q^2)$ from the dipole form are similar in size to those measured in the electron-nucleon scattering, it would be very difficult to observe them, and the basic assumptions described above seem to be well justified. Thus the aim of the CCQE cross-section measurements is to estimate the value of M_A. Even if the neutrinos interact with nucleons bounded in nuclei, the reported results should always refer to the parameter in the formula for free nucleon scattering. Obviously, any such measurement performed on a nuclear target depends on the model of nucleus used in the data analysis.

Measurements of M_A typically focus on investigating the shape of the Q^2 differential cross section that turns out to be sensitive enough for a precise evaluation of M_A. Such an approach has the advantage of not relying on the knowledge of the overall neutrino flux that usually carries much uncertainty. The dependence of the total cross section on M_A could also be used as a tool to fix its value provided that the overall flux is known with high precision. The limiting value of the CCQE cross section as $E_\nu \rightarrow \infty$ can be calculated in the analytical way assuming only dipole, vector, and axial form factors [2]. In the exact formula the cross section depends quadratically on M_A but with a good precision it can be considered linear in the physically relevant region. If the value of M_A is increased from 1.03 to 1.35 GeV the cross section and thus the expected number of CCQE events is raised by ∼30%.

In the past, several measurements of M_A were performed, using often the deuterium target, and until a few years ago it seemed that the results converge to a value close to 1.03 GeV. This value is consistent with the weak pion-production measurements at low Q^2 where the PCAC based computations give the value of $M_A = 1.077 \pm 0.039$ GeV [3]. On the other hand, almost all (except the NOMAD experiment) recent high statistics measurements of M_A show much larger values; see Table I.

There are several possible explanations of the discrepancy. In the simplest one, this disagreement can be treated as a result of statistical fluctuations because the effect is below the 2σ level. There are, however, several independent measurements and the question arises if nuclear effects may be the cause of the problem. Monte Carlo event generators used in the data analysis rely on the impulse approximation (IA) and the Fermi gas model (FG) and perhaps limitations of these assumptions (discussed later in the text) do not allow for a correct extraction of the M_A value.

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TABLE I. Recent M_A measurements.

Experiment	Target	Cut in Q^2 (GeV^2)	M_A (GeV)
K2K [4]	Oxygen	$Q^2 > 0.2$	1.2 ± 0.12
K2K [5]	Carbon	$Q^2 > 0.2$	1.14 ± 0.11
MINOS [6]	Iron	No cut	1.19 ± 0.17
MINOS [6]	Iron	$Q^2 > 0.2$	1.26 ± 0.17
MiniBooNE [7]	Carbon	No cut	1.35 ± 0.17
MiniBooNE [7]	Carbon	$Q^2 > 0.25$	1.27 ± 0.14
NOMAD [8]	Carbon	No cut	1.07 ± 0.07

In this article we investigate the recently released high statistics flux-averaged CCQE double differential cross-section MiniBooNE data in the muon observables: scattering angle and kinetic energy [7]. The data provide an unprecedented possibility to validate predictions from various theoretical models. The data include corrections for the detector efficiency and much effort was made to make them independent from the nuclear physics assumptions of the MC code used in the analysis. An important element of the analysis was the subtraction from the sample of QE-like events (no pion in the final state) those ones that are believed not to be QE in the primary interaction. The NUANCE [9] MC event generator based on the Fermi gas model was used in the analysis. Obviously such subtraction depends on the ingredients of the MC model. The MiniBooNE Collaboration corrected the MC prediction for this background by a phenomenological function obtained by comparing a sample of SPP-like events (a single pion is detected in the final state) to predictions of the same MC generator. The shape of the correction function is rather poorly understood but it has an obvious and presumably important impact on the extracted value of M_A. The function quantifies a lack of knowledge in describing processes like pion absorption and this affects the understanding of the QE-like and SPP-like event samples. The MiniBooNE Collaboration obtained large values of M_A not only by the investigating the shape of the event distribution in Q^2 but also as a fit to the normalized cross section, and both evaluations give similar results.

To estimate the precision of the M_A value obtained from measurements on a nucleus target good understanding of all the nuclear effects is required. In order to extract the value of the parameter for the neutrino scattering on a *free nucleon*, it is assumed that the neutrino nucleus scattering occurs on individual quasi-free nucleons (IA). This is well justified if typical values of the momentum transfer are sufficiently large ($q \geqslant 350$–400 MeV$/c$, but some authors assume even $q \geqslant 500$ MeV$/c$). In the contrary to what might be expected, in the case of neutrino QE interactions, a fraction of at least 15–20% of the total cross section comes from lower values of the momentum transfer, almost independently of the neutrino energy. For neutrino energies E_ν below 500 MeV the percentage is even higher [10]. This manifests itself as the low Q^2 (typically $Q^2 \lessapprox 0.1$ GeV^2), which is a problem reported in several neutrino experiments: The number of events in this region is smaller than expected. This is why, in the data analysis, very often appropriate cuts are imposed (see Table I). Since $q > \omega$, where ω is the energy transfer, and $Q^2 = q^2 - \omega^2$, the region of the failure of the IA is contained in the region $Q^2 < 0.1$ GeV^2. Experimental groups invented some *ad hoc* solutions to deal with the low Q^2 problem. The MiniBooNE Collaboration proposed an effective parameter κ to increase the effect of Pauli blocking [11]. The CCQE data were used to fit simultaneously M_A and κ, treated as free parameters. In the recent MiniBooNE's article [7] the best fit to κ is within 1σ consistent with $\kappa = 1$ (no modification of the Pauli blocking). It is also important that the one-parameter fit for M_A (with $\kappa = 1$) does not lead to results that differ significantly. Also the MINOS Collaboration proposed an *ad hoc* modification of the Pauli blocking [6]. On the theoretical side, from the electron-scattering data analysis it is known that the correct treatment of nucleus in the low momentum transfer region must account for collective effects (giant resonances) and computational techniques like RPA or better CRPA should be applied [12]. The impact of the limitations of the IA on the extracted value of M_A will be discussed in detail in the next section. An important result of our investigation is that cuts on the momentum transfer make the fitted value of M_A smaller, but the effect is by no means sufficiently strong to explain the discrepancy with the old deuterium measurements.

The FG model is determined by only two parameters: Fermi momentum p_F and binding energy B. It assumes that the probability that a bound nucleon has a given value of momentum is quadratic and vanishes for $p > p_F$. The removal energy is assumed to be constant and equal to B. From the electron-scattering experiments it is known that for large-enough values of the momentum transfer, in the region of the *quasielastic peak* (the terminology used in the electron-scattering community), the FG model allows for a satisfactory agreement with the data. The advantage of the model lies in its simplicity and an easy MC implementation. However, from a closer investigation of the electron-scattering data, it is known that the FG model is unable to correctly separate the longitudinal and transverse nuclear response functions.

A few, more sophisticated, approaches primarily used to describe nuclear effects in electron scattering were later applied to neutrino interactions. Many of them are described in Refs. [13–15]. In our investigation we use the *spectral function* (SF) approach [16]. In the context of neutrino interactions its use has been advocated by Omar Benhar [17]. The SF is defined as a joint probability distribution to find a nucleon with a given momentum and binding energy inside nucleus. The SF arises naturally in the calculatations of the neutrino QE cross section in the plane-wave impulse approximation (PWIA) [18], i.e., assuming that the nucleon in the final state leaves the nucleus after primary interaction with no FSI effects. The SF model gives a very good agreement with the electron-nucleus cross-section data in the quasielastic region for momentum transfers larger than ~350 MeV/c [19,20]. The available models of SF combine information from the mean-field theory (shell model) and a contribution from the short-range correlations (SRC). The shell-model orbitals are clearly seen as wide slopes in the probability distribution. The SRC part contributes to the large nucleon momentum tail. In our investigation we use the implementation of the SF formalism in the NuWro MC events generator [21]. For carbon, oxygen, and iron NuWro uses tabulized spectral functions provided by Benhar. There also exist approximate models of SF for medium-sized nuclei like calcium and argon, which have been shown to provide a good agreement with the electron-scattering data [22].

II. RESULTS

A. Definition of χ^2

The MiniBooNE double differential CCQE cross-section data are given in the form of the table [7] with 20 bins in $\cos\theta$ and 18 bins in the muon kinetic energy T_μ spanning the region between 200 to 2000 MeV. There are 360 bins altogether and the double differential cross section is nonzero in 137 of them. The single differential cross section in Q^2 is presented in the form of 17 unequal bins covering the region from 0 to 2 GeV^2.

Until now the fits to M_A were done only on the $d\sigma/dQ^2$ data. The MiniBooNE Collaboration reported the value $M_A = 1.35 \pm 0.17$ GeV and in the recent paper Butkevich [23] obtained the values 1.37 ± 0.05 and 1.36 ± 0.05 for the two theoretical models used in the analysis [in the author's nomenclature: the relativistic distorted wave impulse approximation (RDWIA) and the relativistic Fermi gas model (RFGM)]. The agreement is very good, which is an interesting result because RDWIA is a sophisticated model which includes contribution from short-range correlated nucleon pairs and corrections from the FSI effects. In the fitting procedure the impact of the overall (correlated) flux uncertainty was not taken into account.

Our analysis is the first which uses the complete set of the reported by MiniBooNE double differential cross section. On the theoretical side we compare two models: the Fermi gas and the spectral function both implemented in the NuWro MC events generator. In the case of the FG the parameters used in the simulations were: $p_F = 220\,\text{MeV}/c$ and $B = 34\,\text{MeV}$. The SF approach is parameter free. Pauli blocking is imposed in both models. In the case of SF the Fermi momentum value needed for Pauli blocking was calculated within the local density approximation. The samples of events were produced by NuWro for both the FG and SF models for the axial mass value changing in steps of 10 MeV in the 1- to 2-GeV region.

It is well known that for the same value of M_A the FG and the SF predict quite different values of the total CCQE cross section and one could expect that the fitting procedure will give rise to very different values of M_A for the two models. In a recent article [24] the conclusion is drawn that for the SF approach the best agreement with the data is obtained for $M_A = 1.6\,\text{GeV}$. The total flux averaged SF CCQE cross section at $M_A = 1.6$ GeV is approximately the same as the total flux averaged FG CCQE total cross section at $M_A = 1.3$ GeV.

Despite great effort there is still a lot of uncertainty in the knowledge of the neutrino flux [25]. MiniBooNE Collaboration estimates the overall fully correlated uncertainty as 10.7%. It is known that some other MiniBooNE measurements yield larger-than-expected cross sections [26] which are difficult to explain with standard theoretical models. On the other hand, the reported ratio CCPi+/CCQE is in reasonable agreement with many models [27]. It seems necessary also to include in the data analysis the contribution coming from the fully correlated flux uncertainty. We apply the method of D'Agostini [28] and we construct the appropriate χ^2 function:

$$\chi^2(M_A,\lambda) = \sum_{i=1}^{n} \left\{ \frac{\left(\frac{d^2\sigma}{dT_\mu d\cos\theta}\right)_j^{\text{exp}} - \lambda\left[\frac{d^2\sigma}{dT_\mu d\cos\theta}(M_A)\right]_j^{\text{th}}}{\Delta\left(\frac{d^2\sigma}{dT_\mu d\cos\theta}\right)_j} \right\}^2 + \left(\frac{\lambda^{-1}-1}{\Delta\lambda}\right)^2. \quad (2)$$

$(d^2\sigma/dT_\mu d\cos\theta)_j^{\text{exp}}$ is the measured double differential cross section in the j-th bin with the uncertainty $\Delta(d^2\sigma/dT_\mu d\cos\theta)_j$ (all the uncertainties are also provided by MiniBooNE). $(d^2\sigma/dT_\mu d\cos\theta)_j^{\text{th}}$ is the theoretical prediction from either the FG or the SF model for a fixed value of M_A. $\Delta\lambda = 0.107$ is the overall normalization uncertainty. The similar χ^2 was successfully applied in the reanalysis of the single pion production bubble chamber experiments data [29].

We also investigated the possible impact of the boundary bins in which MiniBooNE reported the vanishing cross section. For this reason we added those bins to the analysis and assumed that the uncertainty with which the null cross section is measured is equal to the average of uncertainties from all the neighboring bins. The proposed extension of the fitting procedure allows for a *punishment* of the models/parameter values which give rise to too-large predictions in the kinematical region excluded by the MiniBooNE measurements. This extension had a very small impact on the final results, shifting the best fit value of the axial mass by a few MeV only. In what follows we present the results for the χ^2 calculated on the nonzero bins only.

B. Momentum transfer cut

We propose a further refinement in the analysis. We exclude from the χ^2 expression [Eq. (2)] the bins with large contribution of small momentum transfer events. The motivation was explained in the introduction: It cannot be expected that the models based on the IA give reliable results in this kinematical region. In the article [12] it was shown that the inclusion of RPA correlations in the theoretical model improves significantly the agreement in the distribution of events in the small Q^2 region. We introduce the momentum transfer cut parameter q_{cut} and change its value in steps of

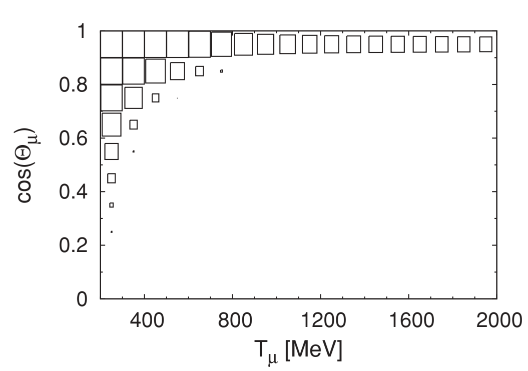

FIG. 1. Contributions of events with momentum transfer lower than $q_{\text{cut}} = 400\,\text{MeV}/c$ for the spectral function model. For each bin the contribution is proportional to the area.

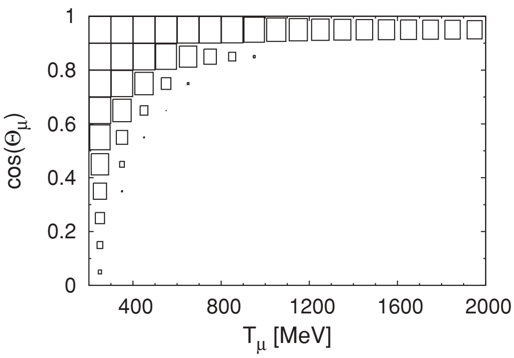

FIG. 2. Contributions of events with momentum transfer lower than $q_{\mathrm{cut}} = 500$ MeV/c for the spectral function model. For each bin the contribution is proportional to the area.

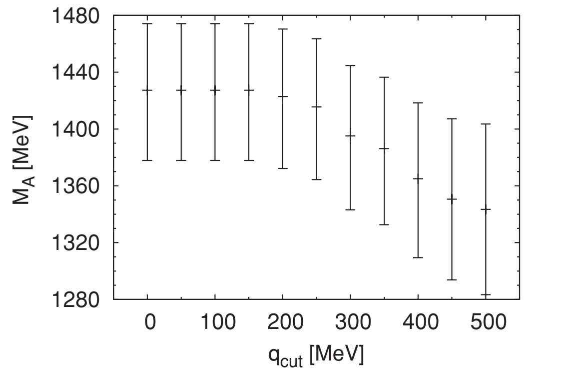

FIG. 4. Best fit values of M_A and 1σ regions for the SF model as functions of the low momentum transfer cut.

50 MeV/c. The parameter is defined in such a way that the bins for which the contribution from $q < q_{\mathrm{cut}}$ is larger than 50% are eliminated. In Figs 1 and 2 we show the contributions of events with the momentum transfer $q < q_{\mathrm{cut}} = 400$ MeV/c and $q < q_{\mathrm{cut}} = 500$ MeV/c for the SF, in every bin separately. The results for the FG are very similar and there is no need to show them independently. The bins which are excluded from the fitting procedure are shown in Fig. 3 as marked in black. For every value of q_{cut} the same bins survive for both FG and SF models. For the value $q_{\mathrm{cut}} = 500$ MeV/c there are still 108 bins taken into account, in the numerical analysis.

In a recent article Butkevich [23] concludes that for certain bins the predictions of the RDWIA and RFGM models do not agree with the data. Most of these bins are excluded from our analysis. For example, at $q_{\mathrm{cut}} = 400$ MeV/c only one bin pointed out by Butkevich is present in our analysis: $T_\mu \in (400, 500)$ MeV and $\cos\theta \in (0.7, 0.8)$.

C. Main result

Figures 4 and 5 contain our main discovery: the best fit values of M_A for various choices of q_{cut} for both SF and FG models. Contrary to what might be expected the values corresponding to the best fits for the SF model are only slightly smaller than ones for the FG model. The reason is in the interplay between M_A and λ parameters: the best fit for λ is in the case of the SF much larger. For $q_{\mathrm{cut}} = 500$ MeV/c χ^2 becomes minimal at $\lambda = 1.06$ for the FG and $\lambda = 1.23$ for the SF. The obtained best fit values for the FG and the SF are very similar: $M_A = 1350 \pm 66$ MeV for the FG and $M_A = 1343 \pm 60$ MeV for SF. The minimal values of χ^2 differ; in the case of the FG they are always smaller. For example, for $q_{\mathrm{cut}} = 500$ MeV/c the minimal values are $\chi^2_{\mathrm{min}} = 14.45$ (FG) and $\chi^2_{\mathrm{min}} = 23.2$ (SF). The goodness of fit is excellent for a wide M_A area because of very large uncertainties in the MiniBooNE data. As q_{cut} becomes larger the best fit value of M_A gets smaller, and there is less tension with the old bubble chamber measurements. The decline is noticeable but even if we take the maximal meaningful value of the cut, namely $q_{\mathrm{cut}} = 500$ MeV/c, we are still far away from the old world average $M_A = 1.03$ GeV.

In Fig. 6 we show the two-dimensional 1σ, 3σ, and 5σ regions for $q_{\mathrm{cut}} = 500$ MeV/c. Because the scale factors for both models best fits differ significantly, it is possible to show them in one figure. For the comparison we also show the old world average value of the axial mass $M_A = 1.03$ GeV. Our conclusion is that old and new measurements are incompatible. This is the most important result of our investigation.

We also checked the behavior of the best fits for M_A for even more restrictive cuts in the momentum transfer. We discovered that for $q_{\mathrm{cut}} > 550$ MeV/c the best fit values start to increase but simultaneously also the 1σ regions grow. The behavior of 1σ regions is what might be expected because as q_{cut} gets larger we loose more and more statistics and the predictions

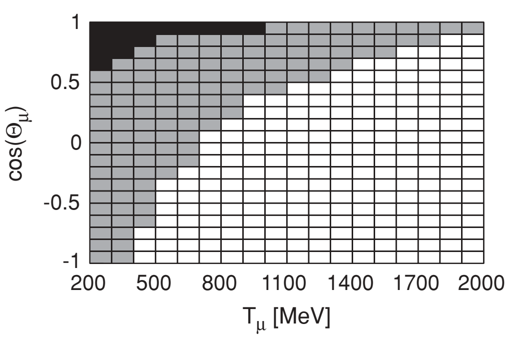

FIG. 3. Bins excluded from the fitting procedure for $q_{\mathrm{cut}} = 400$ MeV/c are shown in black. Bins with a nonzero cross section measured by MiniBooNE are shown in gray.

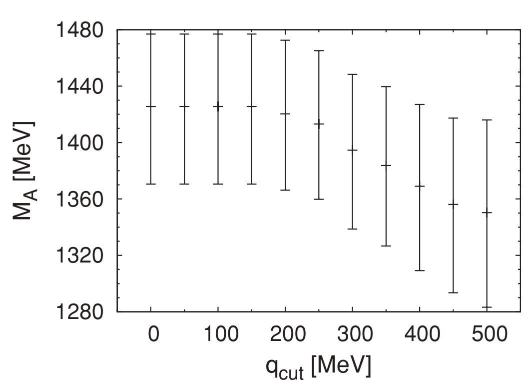

FIG. 5. Best fit values of M_A and 1σ regions for the FG model as functions of the low momentum transfer cut.

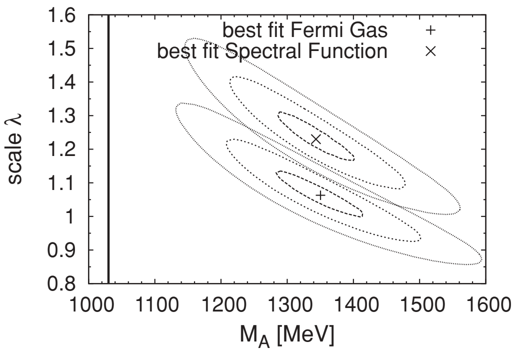

FIG. 6. Best fit values of M_A and λ together with the 1σ, 3σ, and 5σ regions for the $q_{\text{cut}} = 500$ MeV/c momentum transfer cut. The old M_A world average value is marked with a vertical line.

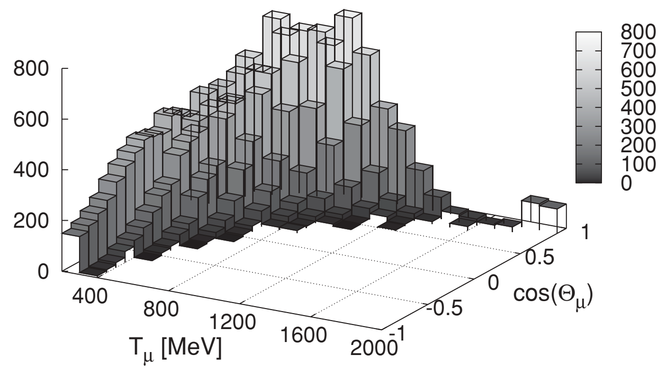

FIG. 7. The difference between the double differential cross section measured by MiniBooNE and prediction from the SF model with $M_A = 1.03$ GeV without rescaling. The units are 10^{-41} cm^2/GeV/nucleon.

become less precise. The absence of a plateau as q_{cut} increases can be caused by a deviation of the dipole form factor from the assumed dipole representation.

Finally, we note that the best fit value for the axial mass from our analysis is very close to the values obtained by MiniBooNE and Butkevich from the one-dimensional analysis of $d\sigma/dQ^2$. However, without the momentum transfer cut our results for M_A would be higher. The advantage of our analysis is that we use all the information provided by the MiniBooNE Collaboration and not only the Q^2 projection of the results.

III. CONCLUSIONS

In the comparison with the MiniBooNE CCQE double differential cross-section data we used two nuclear models: the simplest nucleus Fermi gas model, which is common in MC events generators, and the much more sophisticated spectral function model, which is well tested on the electron-scattering data in the quasielastic peak region. We eliminated from the discussion the bins dominated by events with the low momentum transfer for which the IA-based models are known to be unreliable. Our conclusion is that the new data are not compatible with the results from the old bubble chamber experiments on deuterium where the nuclear effects can be easily controlled.

It is natural to consider the possibility that the disagreement is caused by the nuclear effects which were not taken into account in the models applied so far. We know from the electron scattering that there is a need for a new dynamical mechanism in the region between quasielastic and the Δ peaks, called the DIP region. It is known that the meson exchange current (MEC) reaction in which an electron interacts with a pair of nucleons exchanging a pion adds some cross section in the DIP region, making the theoretical predictions more realistic [30]. The MEC contributes to the transverse response function where the strength is missing. In the MEC reaction two nucleons can be ejected from the nucleus. If an analogous process happened in the case of 1-GeV neutrino scattering, the event would probably be categorized as QE-like. It is unlikely that both nucleons would be detected as they typically carry insufficient kinetic energy. Clearly such events would mislead the experimentalist and contribute to the measured CCQE double differential cross section.

There are many articles devoted to the MEC in the case of electron scattering and very few evaluations of a possible significance of 2p-2h excitations in the case of neutrino scattering. According to Marteau-Martini computations [31] the contribution to the CC cross section neglected in IA models is quite large. They developed the nonrelativistic model that includes QE and Δ production primary interactions, RPA correlations, local density effects, and elementary 2p-2h excitations. The 2p-2h contribution is claimed to be a possible cause of the large CCQE cross section as measured by the MiniBooNE Collaboration. In the case of the neutrino-carbon CCQE process, after averaging over the MiniBooNE beam, the nuclear effects are expected to increase the cross section per neutron from 7.46 to 9.13 in units of 10^{-39} cm^2. This includes a reduction of the cross section due to RPA effects and an increase due to the 2p-2h contribution. In the case of the antineutrino-carbon CCQE reaction the RPA and 2p-2h effects approximately cancel each other, and the cross section is virtually unchanged (modification from 2.09 to 2.07 per proton in the same units).

The verification of the Marteau-Martini model predictions can come from the comparison of their 2p-2h contribution with the full MiniBooNE data. Because such results are not yet available, we compared the true data with the SF predictions for $M_A = 1.03$ GeV and $\lambda = 1$. The difference, shown in Fig. 7, can be treated as the contribution from a new dynamical mechanism going beyond the IA. A theoretical model able to explain the obtained distribution would solve the MiniBooNE's axial mass puzzle.

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