Parity-violating asymmetry in the ^{3}He$(\vec{n}, p)^3$H reaction

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The longitudinal asymmetry induced by parity-violating (PV) components in the nucleon-nucleon potential is studied in the charge-exchange reaction ^{3}He$(\vec{n}, p)^3$H at vanishing incident neutron energies. An expression for the PV observable is derived in terms of T-matrix elements for transitions from the $^{2S+1}L_J = {}^1S_0$ and $^3S_1$ states in the incoming n-^{3}He channel to states with $J = 0$ and 1 in the outgoing p-^{3}H channel. The T-matrix elements involving PV transitions are obtained in first-order perturbation theory in the hadronic weak-interaction potential, while those connecting states of the same parity are derived from solutions of the strong-interaction Hamiltonian with the hyperspherical-harmonics method. The coupled-channel nature of the scattering problem is fully accounted for. Results are obtained corresponding to realistic or chiral two- and three-nucleon strong-interaction potentials in combination with either the DDH or pionless EFT model for the weak-interaction potential. The asymmetries, predicted with PV pion and vector-meson coupling constants corresponding (essentially) to the DDH "best values" set, range from –9.44 to –2.48 in units of 10^{-8}, depending on the input strong-interaction Hamiltonian. This large model dependence is a consequence of cancellations between long-range (pion) and short-range (vector-meson) contributions and is of course sensitive to the assumed values for the PV coupling constants.

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I. INTRODUCTION AND OVERVIEW

A number of experiments aimed at studying parity violation in low-energy processes involving few nucleon systems are being completed or are in an advanced stage of planning at cold neutron facilities, such as the Los Alamos Neutron Science Center, the NIST Center for Neutron Research, and the Spallation Neutron Source at Oak Ridge. The primary objective of this program is to determine the fundamental parameters of hadronic weak interactions, in particular, the strength of the long-range part of the parity-violating (PV) two-nucleon (NN) potential, mediated by one-pion exchange (OPE). While such a component is theoretically expected on the basis of the weak interactions between quarks and the spontaneously broken chiral symmetry of QCD, experimental evidence for its presence has proven to be elusive, and indeed current constraints are inconclusive (for a review see Ref. [1]).

In contrast, in the strong-interaction sector OPE dominates the NN potential at internucleon separations larger than 1.5 fm, and the spatial-spin-isospin correlations it induces leave their imprint on many nuclear properties. These include, for example, (i) the observed ordering of levels in light nuclei and, in particular, the observed absence of stable systems with mass number $A = 8$ [2], (ii) the single-particle energy spacings and shell structure of medium- and heavy-weight nuclei [3] and, in particular, the observed change in the energy gap between the $h_{11/2}$ and $g_{7/2}$ orbits in tin isotopes [4], and (iii) the relative magnitude of the momentum distributions of pp versus np pairs in nuclei [5], which leads to the strong suppression of $(e, e'pp)$ relative to $(e, e'np)$ knockout cross sections from ^{12}C, recently measured at Jefferson Lab [6].

The determination of the parameters that characterize parity violation in nuclei requires evaluating matrix elements of hadronic weak-interaction operators between eigenstates of the strong-interaction Hamiltonian. Thus, experiments in this field are especially reliant on theory for their analysis and interpretation. For this reason, over the last several years, we have embarked on a program aimed at developing a systematic framework for studying PV observables in few-nucleon systems, for which accurate—essentially exact—calculations are possible. Two earlier articles [7,8] have dealt with the two-nucleon system and provided a rather complete analysis of the longitudinal asymmetry in $\vec{p}$-p scattering [7] up to 300 MeV laboratory energies and of a variety of PV observables in the np system [8], including, among others, the neutron spin rotation in $\vec{n}$-p scattering and the photon angular asymmetry in the $\vec{n}$-p radiative capture at thermal neutron energies. In the next phase, we studied the spin rotation in $\vec{n}$-d [9] and $\vec{n}$-α [10] scattering at cold neutron energies.

Measurements are available for the following PV observables: the longitudinal analyzing power in $\vec{p}$-p [11–14] and $\vec{p}$-α [15] scattering; the photon asymmetry and photon circular polarization in, respectively, the ^{1}H$(\vec{n}, \gamma)^2$H [16,17] and ^{1}H$(n, \vec{\gamma})^2$H [18] radiative captures; and the neutron spin rotation in $\vec{n}$-α scattering [19,20]. There is also a set of experiments that is currently being planned, including measurements of the neutron spin rotation in $\vec{n}$-p [19] and $\vec{n}$-d [21] scattering and of the longitudinal asymmetry in the charge-exchange reaction ^{3}He$(\vec{n}, p)^3$H at cold neutron energies [22], the subject of the present article.

At vanishing neutron energies, the only channels entering the incoming n-^{3}He scattering state have quantum numbers $^{2S+1}L_J = {}^1S_0$ and $^3S_1$. In the outgoing p-^{3}H scattering state, the relevant channels are $^{2S+1}L_J = {}^1S_0$, $^3S_1$, and $^3D_1$ with positive parity and $^3P_0$, $^1P_1$, and $^3P_1$ with negative parity. We show (in Sec. II) that the PV observable in this process, that

is, the longitudinal analyzing power A_z, reads

$$A_z = a_z \cos\theta, \tag{1.1}$$

where θ is the angle between the proton momentum and the neutron beam direction, and the coefficient a_z can be expressed in terms of products of T-matrix elements involving (three) parity-conserving (PC) and (three) PV transitions as

$$\begin{aligned} a_z = -\frac{4}{\Sigma}\mathrm{Re}\big(&\sqrt{3}\,\overline{T}^{21,1}_{01,10}\,\overline{T}^{21,0\,*}_{00,00} - \overline{T}^{21,0}_{00,11}\,\overline{T}^{21,1\,*}_{01,01} \\ &+\sqrt{2}\,\overline{T}^{21,0}_{00,11}\,\overline{T}^{21,1\,*}_{01,21} + \sqrt{6}\,\overline{T}^{21,1}_{01,11}\,\overline{T}^{21,1\,*}_{01,01} \\ &+\sqrt{3}\,\overline{T}^{21,1}_{01,11}\,\overline{T}^{21,1\,*}_{01,21}\big) \end{aligned} \tag{1.2}$$

and

$$\Sigma = \left|\overline{T}^{21,0}_{00,00}\right|^2 + 3\left|\overline{T}^{21,1}_{01,01}\right|^2 + 3\left|\overline{T}^{21,1}_{01,21}\right|^2. \tag{1.3}$$

In $\overline{T}^{21,J}_{LS,L'S'}$ the label J specifies the total angular momentum, the superscript 21 denotes the charge-exchange transition n-^{3}He to p-^{3}H (as opposed, for example, to the elastic transition, which would be denoted by the superscript 22), the subscripts $LS\,(L'S')$ are the relative orbital angular momentum and channel spin of the n-^{3}He (p-^{3}H) clusters, and lastly the overline notes the inclusion of a convenient phase factor—see Eq. (2.12) below. The PC (PV) T-matrix elements have $L + L'$ even (odd), and the sum Σ in Eq. (1.3) is proportional to the ^{3}He$(n,p)^3$H cross section. We observe that a_z vanishes if only the channels $^1S_0$ and $^3P_0$ (with $J = 0$) are retained.

The T-matrix elements are related to the (real) R-matrix elements (Sec. III and Appendix A), and the latter for PC transitions are calculated via the Kohn variational principle with the hyperspherical-harmonics (HH) method [23,24] (Sec. V). We use strong-interaction Hamiltonian models, consisting of the Argonne v_{18} (AV18) [25] or chiral (N3LO) [26] two-nucleon potential in combination with the Urbana IX (UIX) [27] or chiral (N2LO) [28] three-nucleon potential. The HH calculation is a challenging one, for two reasons. The first is the coupled-channel nature of the scattering problem: even at vanishing energies for the incident neutron, the elastic n-^{3}He and charge-exchange p-^{3}H channels are both open. The second is the presence of a $J^\pi = 0^+$ resonant state (of zero total isospin) between the p-^{3}H and n-^{3}He thresholds, which slows down the convergence of the expansion and requires a large number of HH basis functions in order to achieve numerically stable results. Further discussion of this aspect of the calculations is in Sec. V, where we also present current predictions for the n-^{3}He scattering lengths corresponding to the Hamiltonian models mentioned earlier. They are in good agreement with the measured values.

The R-matrix elements involving PV transitions are computed in first-order perturbation theory with quantum Monte Carlo techniques (Sec. VI). We adopt as the PV potential the meson-exchange (DDH) model of Desplanques *et al.* [29] as well as the pionless effective-field-theory (EFT) model recently derived in Refs. [30,31] (Sec. IV), and we present results for the various components of the DDH and EFT potentials in combination with the AV18, AV18/UIX, N3LO, and N3LO/N2LO Hamiltonians in Sec. VII. Additional results for the R- and T-matrix elements, and combinations thereof entering the PV observable, are listed (for the AV18/UIX) in Appendix B for completeness. For the DDH model only, we also present predictions for a_z corresponding essentially—but see Sec. IV for further details—to the "best values" of the π-, ρ-, and ω-meson weak-interaction coupling constants [29]. These predictions range from –9.44 to –2.48 in units of 10^{-8} depending on whether the N3LO/N2LO or AV18/UIX Hamiltonian is considered and thus exhibit a significant model dependence due to cancellations (or lack thereof) between the pion and vector-meson contributions.

We conclude by observing that the EFT analysis presented in this work could be improved by employing chiral potentials in both the strong- and weak-interaction sectors. At order Q/Λ_χ, where Q is the low energy/momentum scale that characterizes the particular process of interest, and $\Lambda_\chi \simeq 1$ GeV is the chiral-symmetry-breaking scale, the PV potential contains seven low-energy constants (LECs), five of which are associated with four-nucleon contact terms and the remaining two with long-range OPE components [30]. When electromagnetic interactions are also introduced, another (unknown) LEC must be included—it is needed to fix the strength of a PV two-body current operator of pion range [30]. One can envisage, at least in principle, a suite of experiments involving $A = 2$–5 systems, which would constrain, in fact overconstrain, these eight LECs. Some of these have been mentioned earlier, additional ones include, for example, measurements of the photon asymmetries in the radiative captures ^{2}H$(\vec{n},\gamma)^3$H and ^{3}He$(\vec{n},\gamma)^4$He. These processes are strongly suppressed: the experimental values for the corresponding (PC) cross sections [32,33] are, respectively, almost 3 and 4 orders of magnitude smaller than those measured in ^{1}H$(n,\gamma)^2$H. One would naively expect relatively large PV asymmetries in these cases, possibly orders of magnitude larger than in the $A = 2$ system. Clearly, accurate theoretical estimates for them could be useful in motivating our experimental colleagues to carry out these extremely challenging measurements.

From a theoretical perspective, most of the methodological and technical developments needed to carry out the calculations are already in place. We have recently reported results [34] for the $A = 3$ and 4 (PC) captures, using wave functions obtained from the N3LO/N2LO Hamiltonian and electromagnetic currents derived in chiral EFT up to one loop [35], which are in excellent agreement with data. However, there is one aspect in the computation of the proposed PV threshold captures that still needs to be addressed: the determination of the small admixtures induced by the PV potential into the bound and continuum wave functions. Even a first-order perturbative treatment of those admixtures requires construction of the full Green's function for the strong (PC) Hamiltonian, an impractical task. However, it may be possible to generate them using correlated basis methods similar to those employed in Ref. [36].

II. THE PARITY-VIOLATING OBSERVABLE

The neutron energies in the reaction ^{3}He$(\vec{n},p)^3$H of interest here are in the meV range, and at these energies only two channels are open: the n-^{3}He elastic channel and the p-^{3}H charge-exchange channel. In the following, the index $\gamma = 1\,(2)$

is used to identify the p-^{3}H (n-^{3}He) clusters in the final (initial) state. In the absence of strong and weak interactions between the two clusters, the wave function in channel γ is written as

$$\Phi_\gamma^{m_3 m_1} = \frac{1}{\sqrt{4}} \sum_{p=1}^{4} \Psi_\gamma^{m_3}(ijk) \chi_\gamma^{m_1}(l)\, \phi_{\mathbf{q}_\gamma}(\mathbf{y}_p) \equiv \frac{1}{\sqrt{4}} \sum_{p=1}^{4} \Phi_{\gamma,\,p}^{m_3 m_1}, \tag{2.1}$$

where $\Psi_\gamma^{m_3}$ is the (antisymmetrized) trinucleon bound-state wave function in spin projection m_3, $\chi_\gamma^{m_1}$ is the nucleon spin-isospin state with spin and isospin projections m_1 and p for $\gamma = 1$ or n for $\gamma = 2$, respectively, and ϕ is the intercluster wave function, that is, a Coulomb wave function for $\gamma = 1$ or simply a plane wave $e^{i\,\mathbf{q}_2\cdot\mathbf{y}_p}$ for $\gamma = 2$. The separation between the center-of-mass positions of the two clusters is denoted by $\mathbf{y}_p$ with $\mathbf{y}_p = \mathbf{r}_l - \mathbf{R}_{ijk}$, and their relative momentum is specified by $\mathbf{q}_\gamma$, so that the energy E is given by

$$E = -B_\gamma + \frac{q_\gamma^2}{2\,\mu_\gamma}, \quad \frac{1}{\mu_\gamma} = \frac{1}{m_\gamma} + \frac{1}{M_\gamma}. \tag{2.2}$$

Here B_γ and M_γ are the binding energy and mass of ^{3}H (^{3}He) for $\gamma = 1(2)$, and m_γ is the proton (neutron) mass for $\gamma = 1(2)$. Lastly, the wave functions in Eq. (2.1) are antisymmetrized by summing over the four permutations p with $(ijk, l) \equiv (123, 4)$, $(124, 3)$, $(134, 2)$, and $(234, 1)$.

It is useful to expand the wave functions in Eq. (2.1) in partial waves as

$$\Phi_\gamma^{m_3 m_1} = \frac{1}{\sqrt{4}} \sum_{p=1}^{4} \sum_{LSJ} i^L\, Z_{m_3 m_1}^{L0SJJ_z}\, \Omega_{\gamma LS,\,p}^{JJ_z}\, \frac{\mathcal{F}_L^F(q_\gamma; y_p)}{q_\gamma\, y_p}, \tag{2.3}$$

where $\mathcal{F}_L^F(q_\gamma; y_p)$ reduces to a regular Coulomb function $F_L(q_\gamma; y_p)$ (multiplied by a phase factor we need not specify here) for $\gamma = 1$ or a spherical Bessel function $x\, j_L(x)$ for $\gamma = 2$, with $x = q_\gamma\, y_p$. The channel functions $\Omega_{\gamma LS,\,p}^{JJ_z}$ are defined as

$$\Omega_{\gamma LS,\,p}^{JJ_z} = \{Y_L(\hat{\mathbf{y}}_p) \otimes [\Psi_\gamma(ijk) \otimes \chi_\gamma(l)]_S\}_{JJ_z}, \tag{2.4}$$

while the Clebsch-Gordan coefficients associated with the recoupling of the angular momenta (and other factors) are lumped into

$$Z_{m_3 m_1}^{LMSJJ_z} = \sqrt{4\pi}\,\sqrt{2L+1}\,\langle 1/2, m_3\,; 1/2, m_1 | S, S_z\rangle \times \langle L, M; S, S_z | J, J_z\rangle. \tag{2.5}$$

The momentum $\mathbf{q}_\gamma$ has been taken to define the spin-quantization axis, that is, the z axis.

In the presence of intercluster interactions, the n-^{3}He wave function in the asymptotic region reads

$$\Psi_{\gamma=2}^{m_3 m_1} \simeq \frac{1}{\sqrt{4}} \sum_{p=1}^{4} \sum_{LSJ} i^L\, Z_{m_3 m_1}^{L0SJJ_z} \left[\Omega_{2LS,\,p}^{JJ_z}\, j_L(q_2 y_p) + \sum_{L'S'} T_{LS,\,L'S'}^{22,J}\, \Omega_{2L'S',\,p}^{JJ_z}\, \frac{e^{i(q_2 y_p - L'\pi/2)}}{y_p} + \sum_{L'S'} T_{LS,\,L'S'}^{21,J}\, \Omega_{1L'S',\,p}^{JJ_z}\, \frac{e^{i[q_1 y_p - L'\pi/2 - \eta_1 \ln(2\,q_1 y_p) + \sigma_{L'}]}}{y_p} \right] \tag{2.6}$$

and contains outgoing spherical waves in the n-^{3}He elastic channel ($\gamma = 2$) as well as in the p-^{3}H charge-exchange channel ($\gamma = 1$) multiplied by corresponding T-matrix elements $T_{LS,L'S'}^{\gamma\gamma',J}$. Here $\eta_1 = \alpha\mu_1/q_1$, where α is the fine structure constant and μ_1 is the p-^{3}H reduced mass defined previously, and σ_L is the Coulomb phase-shift. Thus Coulomb distortion in the p-^{3}H outgoing state is fully accounted for.

The probability amplitude $M_{m_3' m_1', m_3 m_1}$ to observe a p-^{3}H final state with spin projections m_1' and m_3', respectively, is obtained from

$$\left\langle \Phi_{\gamma=1,\,p=1}^{m_3' m_1'} \middle| \Psi_{\gamma=2}^{m_3 m_1} \right\rangle = \frac{1}{\sqrt{4}} M_{m_3' m_1', m_3 m_1} \frac{e^{i[q_1 y - \eta_1 \ln(2\,q_1 y)]}}{y}, \tag{2.7}$$

where we have assumed that the p-^{3}H state is in partition (123,4) corresponding to permutation $p = 1$, namely, the bound cluster consists of particles 123 and the proton is particle 4. For brevity, we have also set $\mathbf{y} \equiv \mathbf{y}_{p=1}$. Using the orthonormality of the channel functions $\Omega_{\gamma LS,\,p}^{JJ_z}$, we find

$$M_{m_3' m_1', m_3 m_1} = \frac{1}{\sqrt{4\pi}} \sum_{JLSL'S'} i^L (-i)^{L'} \frac{e^{i\,\sigma_{L'}}}{\sqrt{2L'+1}}\, Z_{m_3 m_1}^{L0SJJ_z} \times T_{LS,\,L'S'}^{21,J}\, Z_{m_3' m_1'}^{L'M'S'JJ_z}\, Y_{L'M'}(\hat{\mathbf{y}}), \tag{2.8}$$

where the Clebsch-Gordan coefficients require $J_z = S_z = m_3 + m_1$, $S_z' = m_3' + m_1'$, and $M' = J_z - S_z' = m_3 + m_1 - (m_3' + m_1')$.

The spin-averaged cross section follows from

$$\sigma_0 \equiv \frac{d\sigma}{d\Omega} = \frac{1}{4} \frac{\mu_2}{\mu_1} \frac{q_1}{q_2} \sum_{m_3, m_1} \sum_{m_3', m_1'} \left| M_{m_3' m_1', m_3 m_1} \right|^2, \tag{2.9}$$

since $(1/4)(q_1/\mu_1)\,|M_{m_3' m_1', m_3, m_1}|^2\, d\Omega$ is the flux of outgoing particles in the solid angle $d\Omega \equiv d\hat{\mathbf{y}}$ and $(1/4)(q_2/\mu_2)$ is the incident flux, where the factors $1/4$ originate from the normalization factors $1/\sqrt{4}$ in Eqs. (2.1) and (2.7). These cancel out in Eq. (2.9), leaving an extra $1/4$ coming from the average over the initial polarizations. The longitudinal asymmetry A_z is defined as

$$\sigma_0 A_z = \frac{1}{2} \frac{\mu_2}{\mu_1} \frac{q_1}{q_2} \sum_{m_3} \sum_{m_3', m_1'} \left[\left| M_{m_3' m_1', m_3 m_1 = +\frac{1}{2}} \right|^2 - \left| M_{m_3' m_1', m_3 m_1 = -\frac{1}{2}} \right|^2 \right]. \tag{2.10}$$

At meV energies it suffices to keep only $L = 0$ in the entrance channel, so that

$$M_{m_3' m_1', m_3 m_1} = \sum_{J=0,1} \sum_{L'S'} \langle 1/2, m_3; 1/2, m_1 | J, J_z\rangle \times \frac{\overline{T}_{0J,L'S'}^{21,J}}{\sqrt{2L'+1}} Z_{m_3' m_1'}^{L'M'S'JJ_z} Y_{L'M'}(\hat{\mathbf{y}}), \tag{2.11}$$

where we have defined

$$\overline{T}_{0J,L'S'}^{21,J} = (-i)^{L'}\, e^{i\,\sigma_{L'}}\, T_{0J,L'S'}^{21,J}. \tag{2.12}$$

After inserting the expression for $Z_{m_3' m_1'}^{L'M'S'JJ_z}$ and carrying out the sums over m_1, m_3 and m_1', m_3', we find the unpolarized

cross section to be given by

$$\sigma_0 = \frac{1}{4}\frac{\mu_2}{\mu_1}\frac{q_1}{q_2}\sum_{J=0,1}\sum_{L'S'}(2J+1)\left|\overline{T}^{21,J}_{0J,L'S'}\right|^2$$
$$= \frac{1}{4}\frac{\mu_2}{\mu_1}\frac{q_1}{q_2}\left[\left|T^{21,0}_{00,00}\right|^2 + 3\left|T^{21,1}_{01,01}\right|^2 + 3\left|T^{21,1}_{01,21}\right|^2\right], \quad (2.13)$$

where in the second line we have ignored T-matrix elements involving transitions to odd parity final states (and hence parity violating), since these are induced by hadronic weak interactions and consequently are much smaller than the PC T-matrices associated with strong interactions. We observe that the matrix elements $T^{21,J}$ (and $\overline{T}^{21,J}$) are finite in the limit $q_2 = 0$, and therefore σ_0 is divergent as q_2 goes to zero, as expected for a neutron-capture reaction.

The asymmetry A_z can be written as

$$\sigma_0 A_z = \frac{1}{2}\frac{\mu_2}{\mu_1}\frac{q_1}{q_2}\sum_{J_1,J_2=0,1}\sum_{L_1L_2S}\epsilon_{L_1L_2}\overline{T}^{21,J_1}_{0J_1,L_1S}\left[\overline{T}^{21,J_2}_{0J_2,L_2S}\right]^*$$
$$\times\sum_{|M|}C^{J_1J_2}_{L_1L_2S}(|M|)P^{|M|}_{L_1}(\theta)P^{|M|}_{L_2}(\theta), \quad (2.14)$$

where the $P^{|M|}_L(\theta)$'s are associated Legendre functions, θ is the angle of the outgoing proton momentum relative to the direction of the incident beam, the $C^{J_1J_2}_{L_1L_2S}(|M|)$'s denote combinations of Clebsch-Gordan coefficients, defined as

$$C^{J_1J_2}_{L_1L_2S}(|M|) = \frac{1}{2\pi}\sum_{J_z}\sum_{\mu=\pm|M|}\sqrt{\frac{(L_1-|M|)!(L_2-|M|)!}{(L_1+|M|)!(L_2+|M|)!}}$$
$$\times Z^{L_1\mu SJ_1J_z}_{m_3m_1=+1/2}Z^{L_2\mu SJ_2J_z}_{m_3m_1=+1/2}, \quad (2.15)$$

and lastly the phase factor $\epsilon_{L_1L_2}$,

$$\epsilon_{L_1L_2} \equiv \frac{1-(-)^{L_1+L_2}}{2}, \quad (2.16)$$

ensures that either L_1 or L_2 must be odd, which in turn implies that either $T^{21,J_1}_{0J_1,L_1S}$ or $T^{21,J_2}_{0J_2,L_2S}$ involves a PV transition, that is, a transition from an incoming positive-parity n-^{3}He state to an outgoing negative-parity p-^{3}H state. The nonvanishing C's for the relevant channels are listed in Table I, and evaluation of the sums in Eq. (2.14) allows one to express the PV asymmetry as in Eqs. (1.1)–(1.3).

TABLE I. The coefficients $C^{J_1J_2}_{L_1L_2S}(|M|)$ for the relevant channels.

J_1, J_2	L_1	L_2	S	$\lvert M\rvert$	$C^{J_1J_2}_{L_1L_2S}(\lvert M\rvert)$
0, 1	0	1	0	0	$-\sqrt{3}$
	1	0	1	0	$+1$
	1	2	1	0	$-\sqrt{2}$
	1	2	1	1	$-\sqrt{1/2}$
1, 0	1	0	0	0	$-\sqrt{3}$
	0	1	1	0	$+1$
	2	1	1	0	$-\sqrt{2}$
	2	1	1	1	$-\sqrt{1/2}$
1, 1	0	1	1	0	$-\sqrt{6}$
	2	1	1	0	$-\sqrt{3}$
	2	1	1	1	$-\sqrt{3/4}$
	1	0	1	0	$-\sqrt{6}$
	1	2	1	0	$-\sqrt{3}$
	1	2	1	1	$-\sqrt{3/4}$

III. T-MATRIX ELEMENTS

The calculation proceeds in two steps: we first determine, via the Kohn variational principle, the R-matrix elements and then relate these to the T-matrix elements. The wave function describing a scattering state with total angular momentum JJ_z in channel γLS is written as

$$\Psi^{JJ_z}_{\gamma,LS} = \Psi^{C,JJ_z}_{\gamma,LS} + \Psi^{F,JJ_z}_{\gamma,LS} + \sum_{\gamma'L'S'}R^{\gamma\gamma',J}_{LS,L'S'}\Psi^{G,JJ_z}_{\gamma',L'S'}, \quad (3.1)$$

where the asymptotic wave functions $\Psi^{\lambda,JJ_z}_{\gamma,LS}$ with $\lambda = F$ and G are defined as

$$\Psi^{\lambda,JJ_z}_{\gamma,LS} = \frac{D_\gamma}{\sqrt{4}}\sum_{p=1}^{4}\Omega^{JJ_z}_{\gamma LS,p}\frac{\mathcal{F}^\lambda_L(q_\gamma;y_p)}{q_\gamma y_p}, \quad (3.2)$$

and the superscript $\lambda = F$ is to denote the regular radial functions introduced earlier in Eq. (2.3), and $\lambda = G$ is to denote the irregular Coulomb or spherical Bessel functions, namely,

$$\gamma = 1\colon\ \mathcal{F}^G_L(x) = \widetilde{G}_L(\eta_1,x);\ \gamma = 2\colon\ \mathcal{F}^G_L(x) = -x\widetilde{y}_L(x). \quad (3.3)$$

The tilde over G_L and y_L indicates that they have been multiplied by short-range cutoffs to remove the singularity at the origin. Thus $\mathcal{F}^G_L$ is well behaved in all space. The normalization factor D_γ,

$$D_\gamma = \sqrt{\frac{2\mu_\gamma q_\gamma}{\kappa^3}}, \quad (3.4)$$

where $\kappa = \sqrt{3/2}$, is introduced for convenience—κ is a numerical factor relating the intercluster separation $\mathbf{y}_p$ to the Jacobi variable $\mathbf{x}_{1p}$, that is, $\mathbf{x}_{1p} = \kappa\,\mathbf{y}_p$ [see Eq. (5.1) below].

The wave functions $\Psi^{C,JJ_z}_{\gamma,LS}$ vanish in the asymptotic region and describe the dynamics of the interacting nucleons when they are close to each other, while the $R^{\gamma\gamma',J}_{LS,L'S'}$'s are the R-matrix elements. The latter, as well as the coefficients entering the expansion of $\Psi^{C,JJ_z}_{\gamma,LS}$ in terms of HH functions, are determined via the Kohn variational principle

$$\left[R^{\gamma\gamma',J}_{LS,L'S'}\right] = R^{\gamma'\gamma,J}_{L'S',LS} - \left\langle\Psi^{JJ_z}_{\gamma,LS}\middle|H-E\middle|\Psi^{JJ_z}_{\gamma',L'S'}\right\rangle, \quad (3.5)$$

as discussed in Sec. V.

The next step consists in relating the R-matrix elements to the T-matrix elements. To this end, it is convenient to simplify the notation by dropping the superscripts JJ_z and by introducing a single label α to denote the channel quantum numbers LS, so that the wave functions in Eq. (3.1)

corresponding to $\gamma = 1$ and 2 are written as

$$\Psi_{1,\alpha} = \Psi^C_{1,\alpha} + \Psi^F_{1,\alpha} + \sum_{\alpha'} R^{11}_{\alpha,\alpha'} \Psi^G_{1,\alpha'} + \sum_{\alpha'} R^{12}_{\alpha,\alpha'} \Psi^G_{2,\alpha'}, \quad (3.6)$$

$$\Psi_{2,\alpha} = \Psi^C_{2,\alpha} + \Psi^F_{2,\alpha} + \sum_{\alpha'} R^{21}_{\alpha,\alpha'} \Psi^G_{1,\alpha'} + \sum_{\alpha'} R^{22}_{\alpha,\alpha'} \Psi^G_{2,\alpha'}. \quad (3.7)$$

From these we form the linear combination

$$\Psi = \sum_{\alpha'} (U_{\alpha,\alpha'} \Psi_{1,\alpha'} + V_{\alpha,\alpha'} \Psi_{2,\alpha'}), \quad (3.8)$$

where the matrices U and V are determined below. Inserting the expressions above for $\Psi_{\gamma,\alpha}$ and rearranging terms lead to

$$\begin{aligned}\Psi = \Psi^C &+ \sum_{\alpha'} [U - i\,(U R^{11} + V R^{21})]_{\alpha,\alpha'} \Psi^F_{1,\alpha'} \\ &+ \sum_{\alpha'} (U R^{11} + V R^{21})_{\alpha,\alpha'} \left(\Psi^G_{1,\alpha'} + i\,\Psi^F_{1,\alpha'}\right) \\ &+ \sum_{\alpha'} [V - i\,(U R^{12} + V R^{22})]_{\alpha,\alpha'} \Psi^F_{2,\alpha'} \\ &+ \sum_{\alpha'} (U R^{12} + V R^{22})_{\alpha,\alpha'} \left(\Psi^G_{2,\alpha'} + i \Psi^F_{2,\alpha'}\right), \end{aligned} \quad (3.9)$$

where Ψ^C is a combination of internal parts of no interest here. We now require Ψ to consist, in the asymptotic region, of a plane wave in channel $\gamma = 2$ (or n-^{3}He) and of a purely outgoing wave in channel $\gamma = 1$ (or p-^{3}H). These requirements are satisfied by demanding that

$$U - i\,(U R^{11} + V R^{21}) = 0, \quad (3.10)$$

$$V - i\,(U R^{12} + V R^{22}) = I, \quad (3.11)$$

where I is the identity matrix. Comparing the resulting Ψ with the wave function given in Eq. (2.6), specifically its component in channel LSJ, allows one to express the T matrix as

$$\begin{aligned} T^{21,J}_{LS,L'S'} &= \frac{D_1}{D_2\, q_1} (U^J R^{11,J} + V^J R^{21,J})_{LS,L'S'} \\ &= -i\,\frac{D_1}{D_2\, q_1}\, U^J_{LS,L'S'}, \end{aligned} \quad (3.12)$$

where we have reinstated the LSJ labels. Finally the matrix U is obtained by solving the system in Eq. (3.11):

$$\begin{aligned} T^{21,J}_{LS,L'S'} = \frac{D_1}{D_2\, q_1} &\{[I - i R^{22,J} + R^{21,J}(I - i R^{11,J})^{-1} R^{12,J}]^{-1} \\ &\times R^{21,J}(I - i R^{11,J})^{-1}\}_{LS,L'S'}. \end{aligned} \quad (3.13)$$

In fact, we compute the R-matrix elements at zero energy, that is, in the limit $q_2 \to 0$, and define

$$\begin{aligned} \overline{R}^{12,J}_{LS,L'S'} &= \frac{R^{12,J}_{LS,L'S'}}{q_2^{L'+1/2}}, \quad \overline{R}^{21,J}_{LS,L'S'} = \frac{R^{21,J}_{LS,L'S'}}{q_2^{L+1/2}}, \\ \overline{R}^{22,J}_{LS,L'S'} &= \frac{R^{22,J}_{LS,L'S'}}{q_2^{L+L'+1}}, \end{aligned} \quad (3.14)$$

and it can be shown that the $\overline{R}$-matrix elements are finite in this limit. In particular, we note that the factor q_2^L follows from the small argument expansion of the spherical Bessel function j_L in $\Psi^{F,JJ_z}_{\gamma=2,LS}$, while the extra $q_2^{1/2}$ is due to the normalization D_2. At zero energy, we have

$$[I - i\,R^{22,J} + R^{21,J}(I - i\,R^{11,J})^{-1} R^{12,J}] \to I, \quad (3.15)$$

since $R^{22,J}$ and the product $R^{21,J} R^{12,J}$ are proportional to q_2 or higher powers of q_2. Furthermore, the relevant T-matrix elements entering the expression for the asymmetry A_z are those with quantum number $L = 0$ in channel $\gamma = 2$, and hence

$$T^{21,J}_{0J,L'S'} = \frac{1}{\sqrt{q_1}} \sum_{L''S''} \overline{R}^{21,J}_{0J,L''S''} (I - i\,R^{11,J})^{-1}_{L''S'',L'S'}, \quad (3.16)$$

with $J = 0$ and 1. Note that we have neglected the difference in the n-^{3}He and p-^{3}H reduced masses.

IV. THE PARITY-VIOLATING POTENTIAL

Two different models of the PV weak-interaction potentials are adopted in the calculations reported in the following. One is the model developed 30 years ago by Desplanques *et al.* [29] (and known as DDH): it is parametrized in terms of π-, ρ-, and ω-meson exchanges and involves in practice six weak pion and vector-meson coupling constants to the nucleon [37]. These were estimated within a quark model approach incorporating symmetry arguments and current algebra requirements [29,38]. Due to the inherent limitations of such an analysis, however, the coupling constants determined in this way have rather wide ranges of allowed values.

The other model for the PV potential considered in the present work is that formulated by Zhu *et al.* [30] in 2005 and reduced to its minimal form by Girlanda [31] in 2008, within an EFT approach in which only nucleon degrees of freedom are retained explicitly. At lowest-order Q/Λ_χ, where Q is the small momentum scale characterizing the low-energy PV process and $\Lambda_\chi \simeq 1$ GeV is the scale of chiral symmetry breaking, it is parametrized by a set of five contact four-nucleon terms.

The DDH and EFT PV two-nucleon potentials are conveniently written as

$$v^\alpha_{ij} = \sum_{n=1}^{12} c^\alpha_n\, O^{(n)}_{ij}, \quad \alpha = \text{DDH or EFT}, \quad (4.1)$$

where the parameters c^α_n and operators $O^{(n)}_{ij}$, $n = 1, \ldots, 12$, are listed in Table II. In this table the vector operators $\mathbf{X}^{(n)}_{ij,\pm}$ are defined as

$$\mathbf{X}^{(n)}_{ij,+} \equiv [\mathbf{p}_{ij}\,,\ f_n(r_{ij})]_+, \quad (4.2)$$

$$\mathbf{X}^{(n)}_{ij,-} \equiv \mathrm{i}\,[\mathbf{p}_{ij}\,,\ f_n(r_{ij})]_-, \quad (4.3)$$

where $[\ldots,\ \ldots]_\mp$ denotes the commutator $(-)$ or anticommutator $(+)$, and $\mathbf{p}_{ij}$ is the relative momentum operator, $\mathbf{p}_{ij} \equiv (\mathbf{p}_i - \mathbf{p}_j)/2$. In the DDH model, the functions $f_x(r)$, $x = \pi$, ρ, and ω, are Yukawa functions, suitably modified by the inclusion of monopole form factors,

$$f_x(r) = \frac{1}{4\pi\, r} \left\{ \mathrm{e}^{-m_x r} - \mathrm{e}^{-\Lambda_x r} \left[1 + \frac{\Lambda_x r}{2} \left(1 - \frac{m_x^2}{\Lambda_x^2} \right) \right] \right\}. \quad (4.4)$$

TABLE II. Components of the DDH and EFT models for the PV potential. The vector operators $\mathbf{X}^{(n)}_{ij,\mp}$ and functions $f_x(r)$, $x = \pi$, ρ, ω, and μ, are defined in Eqs. (4.2) and (4.3) and Eqs. (4.4) and (4.5), respectively. Only five operators and low-energy constants enter the pionless EFT interaction at the leading order, and in this article they have been chosen to correspond to rows 1, 4, 6, 8, and 9.

n	$c_n^{\rm DDH}$	$f_n^{\rm DDH}(r)$	$c_n^{\rm EFT}$	$f_n^{\rm EFT}(r)$	$O_{ij}^{(n)}$
1	$+\frac{g_\pi\, h^1_\pi}{2\sqrt{2}\,m}$	$f_\pi(r)$	$\frac{2\,\mu^2}{\Lambda_\chi^3}\,C_6$	$f_\mu(r)$	$(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z\,(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(1)}_{ij,-}$
2	$-\frac{g_\rho\, h^0_\rho}{m}$	$f_\rho(r)$	0	0	$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\,(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(2)}_{ij,+}$
3	$-\frac{g_\rho\, h^0_\rho(1+\kappa_\rho)}{m}$	$f_\rho(r)$	0	0	$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\,(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(3)}_{ij,-}$
4	$-\frac{g_\rho\, h^1_\rho}{2m}$	$f_\rho(r)$	$\frac{\mu^2}{\Lambda_\chi^3}\,(C_2 + C_4)$	$f_\mu(r)$	$(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)_z\,(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(4)}_{ij,+}$
5	$-\frac{g_\rho\, h^1_\rho(1+\kappa_\rho)}{2m}$	$f_\rho(r)$	0	0	$(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)_z\,(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(5)}_{ij,-}$
6	$-\frac{g_\rho\, h^2_\rho}{2\sqrt{6}\,m}$	$f_\rho(r)$	$-\frac{2\,\mu^2}{\Lambda_\chi^3}\,C_5$	$f_\mu(r)$	$(3\,\tau_{i,z}\tau_{j,z} - \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\,(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(6)}_{ij,+}$
7	$-\frac{g_\rho\, h^2_\rho(1+\kappa_\rho)}{2\sqrt{6}\,m}$	$f_\rho(r)$	0	0	$(3\,\tau_{i,z}\tau_{j,z} - \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\,(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(7)}_{ij,-}$
8	$-\frac{g_\omega\, h^0_\omega}{m}$	$f_\omega(r)$	$\frac{2\,\mu^2}{\Lambda_\chi^3}\,C_1$	$f_\mu(r)$	$(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(8)}_{ij,+}$
9	$-\frac{g_\omega\, h^0_\omega(1+\kappa_\omega)}{m}$	$f_\omega(r)$	$\frac{2\,\mu^2}{\Lambda_\chi^3}\,\tilde{C}_1$	$f_\mu(r)$	$(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(9)}_{ij,-}$
10	$-\frac{g_\omega\, h^1_\omega}{2m}$	$f_\omega(r)$	0	0	$(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)_z\,(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(10)}_{ij,+}$
11	$-\frac{g_\omega\, h^1_\omega(1+\kappa_\omega)}{2m}$	$f_\omega(r)$	0	0	$(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)_z\,(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(11)}_{ij,-}$
12	$-\frac{g_\omega h^1_\omega - g_\rho h^1_\rho}{2m}$	$f_\rho(r)$	0	0	$(\boldsymbol{\tau}_i - \boldsymbol{\tau}_j)_z\,(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot \mathbf{X}^{(12)}_{ij,+}$

In the EFT model, however, the short-distance behavior is described by a single function $f_\mu(r)$, which is itself taken as a Yukawa function with mass parameter μ,

$$f_\mu(r) = \frac{1}{4\pi\, r}\mathrm{e}^{-\mu r}, \tag{4.5}$$

with $\mu \simeq m_\pi$ as appropriate in the present formulation, in which pion degrees of freedom are integrated out.

In the potential $v_{ij}^{\rm DDH}$, the strong-interaction coupling constants of the π, ρ, and ω mesons to the nucleon are denoted as g_π, g_ρ, κ_ρ, g_ω, and κ_ω, while the weak-interaction ones are denoted as h^1_π, h^0_ρ, h^1_ρ, h^2_ρ, h^0_ω, and h^1_ω, where the superscripts 0, 1, and 2 specify the isoscalar, isovector, and isotensor content of the corresponding interaction components. In the EFT model, the five low-energy constants $C_1, \tilde{C}_1, C_2 + C_4, C_5$, and C_6 completely characterize $v_{ij}^{\rm EFT}$, to lowest order Q/Λ_χ.

The values for the coupling constants and short-range cutoffs in the DDH model are listed in Table III, while the mass μ in the EFT model is taken to be m_π. These values for coupling constants and cutoffs were also used in the DDH-based calculations of PV two-nucleon observables in Refs. [7,8] and neutron spin rotation in $\vec{n}\,d$ scattering [9]. In particular, we note that the linear combination of ρ- and ω-meson weak coupling constants corresponding to pp states has been taken from an earlier analysis of $\vec{p}\,p$ elastic scattering experiments [7]. The remaining couplings are the "best value" estimates suggested in Ref. [29].

TABLE III. Values used for the strong- and weak-interaction coupling constants and short-range cutoff parameters of the π, ρ, and ω mesons in the DDH potential.

	$g_\alpha^2/4\pi$	κ_α	$10^7 \times h^0_\alpha$	$10^7 \times h^1_\alpha$	$10^7 \times h^2_\alpha$	Λ_α (GeV/c)
π	13.2			4.56		1.72
ρ	0.840	6.1	−16.4	−2.77	−13.7	1.31
ω	20.0	0.0	3.23	1.94		1.50

In the analysis of the a_z observable to follow, we report results for the coefficients $I_n^{\rm DDH}$ and $I_n^{\rm EFT}$ in the expansion

$$a_z = \sum_{n=1}^{12} c_n^\alpha\, I_n^\alpha. \tag{4.6}$$

Thus we will not need to consider specific values (or range of values) for the strength parameters c_n^α. However, the I_n^α values depend on the masses (and short-range cutoffs Λ_x for the DDH model) occurring in the Yukawa functions.

V. THE HH WAVE FUNCTIONS

The "internal" wave function $\Psi^{C,JJ_z}_{\gamma,LS}$, see Eq. (3.1), is expanded in the HH basis. For four equal mass particles, a suitable choice for the Jacobi vectors is

$$\begin{aligned}
\mathbf{x}_{1p} &= \sqrt{\frac{3}{2}}\left(\mathbf{r}_l - \frac{\mathbf{r}_i + \mathbf{r}_j + \mathbf{r}_k}{3}\right),\\
\mathbf{x}_{2p} &= \sqrt{\frac{4}{3}}\left(\mathbf{r}_k - \frac{\mathbf{r}_i + \mathbf{r}_j}{2}\right),\\
\mathbf{x}_{3p} &= \mathbf{r}_j - \mathbf{r}_i,
\end{aligned} \tag{5.1}$$

where p specifies a given permutation corresponding to the ordering $(ijkl)$. By definition, the permutation $p = 1$ is chosen to correspond to (1234).

For the given Jacobi vectors, the hyperspherical coordinates include the so-called hyperradius ρ, defined by

$$\rho = \sqrt{x_{1p}^2 + x_{2p}^2 + x_{3p}^2} \quad \text{(independent of } p\text{)}, \tag{5.2}$$

and a set of angular variables which in the Zernike and Brinkman [39,40] representation are (i) the polar angles $\hat{\mathbf{x}}_{ip} \equiv (\theta_{ip}, \phi_{ip})$ of each Jacobi vector and (ii) the two additional "hyperspherical" angles ϕ_{2p} and ϕ_{3p}, defined as

$$\cos\phi_{2p} = \frac{x_{2p}}{\sqrt{x_{1p}^2 + x_{2p}^2}}, \quad \cos\phi_{3p} = \frac{x_{3p}}{\sqrt{x_{1p}^2 + x_{2p}^2 + x_{3p}^2}}, \tag{5.3}$$

where x_{jp} is the magnitude of the Jacobi vector $\mathbf{x}_{jp}$. The set of angular variables $\hat{\mathbf{x}}_{1p}$, $\hat{\mathbf{x}}_{2p}$, $\hat{\mathbf{x}}_{3p}$, ϕ_{2p}, and ϕ_{3p} is denoted hereafter as Ω_p. A generic HH function reads

$$\begin{aligned}
&\mathcal{H}^{K\Lambda M}_{\ell_1\ell_2\ell_3 L_2 n_2 n_3}(\Omega_p) \\
&\quad = \mathcal{N}^{\ell_1\ell_2\ell_3}_{n_2 n_3} \left\{ \left[Y_{\ell_1}(\hat{\mathbf{x}}_{1p}) \otimes Y_{\ell_2}(\hat{\mathbf{x}}_{2p}) \right]_{L_2} \otimes Y_{\ell_3}(\hat{\mathbf{x}}_{3p}) \right\}_{\Lambda M} \\
&\qquad \times \left(\sin\phi_{2p}\right)^{\ell_1} \left(\cos\phi_{2p}\right)^{\ell_2} \left(\sin\phi_{3p}\right)^{\ell_1+\ell_2+2n_2} \\
&\qquad \times \left(\cos\phi_{3p}\right)^{\ell_3} P_{n_2}^{\ell_1+1/2,\,\ell_2+1/2}\left(\cos 2\phi_{2p}\right) \\
&\qquad \times P_{n_3}^{\ell_1+\ell_2+2n_2+2,\,\ell_3+1/2}\left(\cos 2\phi_{3p}\right), \qquad (5.4)
\end{aligned}$$

where $P_n^{a,b}$ are Jacobi polynomials and the coefficients $\mathcal{N}^{\ell_1\ell_2\ell_3}_{n_2 n_3}$ are normalization factors. The quantity $K = \ell_1 + \ell_2 + \ell_3 + 2(n_2 + n_3)$ is the so-called grand angular quantum number. The HH functions are the eigenfunctions of the hyperangular part of the kinetic energy operator. Another important property is that $\rho^K \mathcal{H}^{K\Lambda M}_{\ell_1\ell_2\ell_3 L_2 n_2 n_3}(\Omega_p)$ are homogeneous polynomials of the particle coordinates of degree K.

A set of antisymmetrized hyperangular-spin-isospin states of grand angular quantum number K, total orbital angular momentum Λ, total spin Σ, and total isospin T (for the given values of total angular momentum J and parity π) can be constructed as follows:

$$\Psi^{K\Lambda\Sigma T}_{\mu} = \sum_{p=1}^{12} \Phi^{K\Lambda\Sigma T}_{\mu}(ijkl), \tag{5.5}$$

where the sum is over the 12 even permutations $p \equiv ijkl$, and

$$\begin{aligned}
&\Phi^{K\Lambda\Sigma T}_{\mu}(ijkl) \\
&\quad = \left(\mathcal{H}^{K\Lambda M}_{\ell_1\ell_2\ell_3 L_2 n_2 n_3}(\Omega_p) \otimes \{[(\chi_i \otimes \chi_j)_{S_a} \otimes \chi_k]_{S_b} \otimes \chi_l\}_{\Sigma} \right)_{JJ_z} \\
&\qquad \times \{[(\xi_i \otimes \xi_j)_{T_a} \otimes \xi_k]_{T_b} \otimes \xi_l\}_{TT_z}. \qquad (5.6)
\end{aligned}$$

Here, χ_i (ξ_i) denotes the spin (isospin) state of particle i. The total orbital angular momentum Λ of the HH function is coupled to the total spin Σ to give the total angular momentum JJ_z, whereas the parity π is $(-1)^{\ell_1+\ell_2+\ell_3}$. The quantum number T specifies the total isospin of the state, and μ labels the possible choices of hyperangular, spin, and isospin quantum numbers, namely,

$$\mu \equiv \{\ell_1, \ell_2, \ell_3, L_2, n_2, n_3, S_a, S_b, T_a, T_b\}, \tag{5.7}$$

compatible with the given values of K, Λ, Σ, T, J, and π. Another important classification scheme for the states is to group them in "channels": states belonging to the same channel have the same values of angular ($\ell_1, \ell_2, \ell_3, L_2, \Lambda$), spin ($S_a, S_b, \Sigma$), and isospin ($T_a, T_b, T$) quantum numbers, but different values of n_2 and n_3.

Each state $\Psi^{K\Lambda\Sigma T}_{\mu}$ entering the expansion of the four-nucleon wave function must be antisymmetric under the exchange of any pair of particles. Consequently, it is necessary to consider states such that

$$\Phi^{K\Lambda\Sigma T}_{\mu}(ijkl) = -\Phi^{K\Lambda\Sigma T}_{\mu}(jikl), \tag{5.8}$$

which is fulfilled when the condition

$$\ell_3 + S_a + T_a = \text{odd} \tag{5.9}$$

is satisfied.

The number $M_{K\Lambda\Sigma T}$ of antisymmetrized functions $\Psi^{K\Lambda\Sigma T}_{\mu}$ having given values of K, Λ, Σ, and T, but different combinations of quantum numbers μ—see Eq. (5.7)—is in general very large. In addition to the degeneracy of the HH basis, the four spins (isospins) can be coupled in different ways to total Σ (T). However, many of the states $\Psi^{K\Lambda\Sigma T}_{\mu}$, with μ ranging from 1 to $M_{K\Lambda\Sigma T}$, are linearly dependent. In the expansion of $\Psi^{C,JJ_z}_{\gamma,LS}$, it is necessary to include only the subset of linearly independent states, whose number is fortunately significantly smaller than $M_{K\Lambda\Sigma T}$.

The internal part of the wave function can be finally written as

$$\Psi^{C,JJ_z}_{\gamma,LS} = \sum_{K\Lambda\Sigma T} \sum_{\mu} u^{\gamma,LS}_{K\Lambda\Sigma T\mu}(\rho) \Psi^{K\Lambda\Sigma T}_{\mu}, \tag{5.10}$$

where the sum is restricted only to the linearly independent states. We have found it convenient to expand the "hyperradial" functions $u^{\gamma,LS}_{K\Lambda\Sigma T\mu}(\rho)$ in a complete set of functions, namely,

$$u^{\gamma,LS}_{K\Lambda\Sigma T\mu}(\rho) = \sum_{m=0}^{M-1} c^{\gamma,LS}_{K\Lambda\Sigma T\mu,m}\, g_m(\rho), \tag{5.11}$$

and have chosen

$$g_m(\rho) = \sqrt{\frac{m!}{(m+8)!}} \beta^{9/2} L_m^{(8)}(\beta\rho) \mathrm{e}^{-\beta\rho/2}, \tag{5.12}$$

where $L_l^{(8)}(\beta\rho)$ are Laguerre polynomials [41].

The c coefficients of the expansion (5.11) and the R-matrix elements of Eq. (3.1) are determined variationally via the Kohn variational principle. This principle states that the functional $[R^{\gamma\gamma',J}_{LS,L'S'}]$ defined in Eq. (3.5) is stationary with respect to variations in the $R^{\gamma\gamma',J}_{LS,L'S'}$ and $c^{\gamma,LS}_{K\Lambda\Sigma T\mu,m}$. By applying this principle, a linear set of equations for $R^{\gamma\gamma',J}_{LS,L'S'}$ and $c^{\gamma,LS}_{K\Lambda\Sigma T\mu,m}$ is obtained [23] and then solved using the Lanczos algorithm. The other parameter entering the expansion is the (nonlinear) parameter β [see Eq. (5.12)], used to describe the hyperradial functions $u^{\gamma,LS}_{K\Lambda\Sigma T\mu}(\rho)$. We have checked that, once a sufficient number M of functions $g_m(\rho)$ are employed ($M \approx 20$), the results are practically independent on β. In the present work we have used $\beta = 4$ fm^{-1}.

The application of the method has two main difficulties. The first is the accurate computation of the matrix elements of the Hamiltonian. By exploiting the properties of the HH functions, however, this task can be noticeably simplified, as discussed in Refs. [23,42]. The second difficulty is the slow convergence of the HH expansion. This problem has been overcome by dividing the set of states $\Psi^{K\Lambda\Sigma T}_{\mu}$ defined in Eq. (5.5) (in the following referred to simply as "HH states") in *classes*, depending on the value of $\mathcal{L} = \ell_1 + \ell_2 + \ell_3$, total isospin T, and n_2 and n_3. In the present article, we have

considered four different classes. Because for n-^{3}He scattering the asymptotic states do not have a definite total isospin (they are a superposition of $T = 0$ and $T = 1$ components), it is mandatory to include HH states with both $T = 0$ and 1. The contribution of $T = 2$ states is expected to be tiny and consequently they are ignored in the present article.

Following Refs. [42,43], in the first class we have included the $n_2 = 0$ HH states belonging to some special channels, for which the convergence has been found to be critical. The radial part of these HH states depends only on $\cos\phi_{3p} = r_{ij}/\rho$, and thus they take into account two-body correlations. The $n_2 > 0$ HH states belonging to the same channels are included in the second class, together with those having $\mathcal{L} \leqslant 2$. The other classes are then defined simply by grouping HH states belonging to channels with an increasing value of $\mathcal{L}$. In particular, for the construction of the positive (negative) parity "internal"wave function $\Psi^{C,JJ_z}_{\gamma,LS}$, classes 3 and 4 include all HH states with $\mathcal{L} = 4$ and 6 ($\mathcal{L} = 3$ and 5), respectively. The convergence of these last two classes is less critical, and consequently, only HH states with lower values of grand angular quantum number K need be considered. Moreover, the convergence with $\mathcal{L}$ is quite fast. In particular, we have found that, at the energy considered, the contribution of HH states with $\mathcal{L} > 6$ can be neglected.

The calculation is performed including in the expansion all HH states belonging to classes $i = 1, \ldots, 4$ with grand angular quantum number $K \leqslant K_i$, where $K_1, \ldots, K_4$ are a set of nonnegative integers. The convergence of a quantity of interest (for example, the phase shifts or the coefficient a_z defining the PV asymmetry) is then studied by increasing the values of K_i. A more complete study of the convergence will be presented elsewhere [24].

To exhibit the convergence pattern, we report in Table IV the calculated n-^{3}He scattering lengths. As is evident from Eq. (2.6), they are defined as

$$a_J = -\lim_{q_2 \to 0} T^{22,J}_{0J,0J}, \tag{5.13}$$

with both incoming and outgoing n-^{3}He clusters in relative S wave. Note that in general this scattering length is complex, since the channel p-^{3}H is always open, and therefore the unitarity condition imposes that $\mathrm{Im}\, a_J < 0$, because the total cross section is proportional to $\sum_{J=0,1}(2J+1)\,\mathrm{Im}\, T^{22,J}_{0J,0J}$. The results obtained for the singlet ($J = 0$) and triplet ($J = 1$) scattering lengths are reported in Table IV for all four potential models used in this work. The calculated n-^{3}He scattering lengths are compared with experimental values and the results of other calculations available in the literature.

Inspection of the table shows that the convergence for the triplet scattering length is very good and that there is reasonable agreement with available experimental values, and the results of other calculations, in particular those of the AGS method.

TABLE IV. Convergence of the n-^{3}He singlet and triplet scattering lengths corresponding to the inclusion, in the internal part of the wave function, of four different classes in which the HH basis has been subdivided. For the singlet scattering length, the line labeled "EXT" reports the extrapolated values obtained by examining the convergence pattern with increasing number of HH functions in the expansion. The calculated scattering lengths are compared with results obtained using the resonating group method (RGM), Faddeev-Yakubovsky (FY) equations, and Alt-Grassberger-Sandhas (AGS) equations, as well as with results of R-matrix analyses. The experimental values are reported in the rows labeled "EXP" (the imaginary parts are taken from Ref. [44]).

Triplet scattering length a_1 (fm)

Method	K_1	K_2	K_3	K_4	AV18	N3LO	AV18/UIX	N3LO/N2LO
HH	28	28	20	20	$3.56 - i\,0.0078$	$3.47 - i\,0.0047$	$3.39 - i\,0.0059$	$3.37 - i\,0.0042$
HH	30	30	22	22	$3.56 - i0.0077$	$3.46 - i0.0048$	$3.39 - i0.0059$	$3.37 - i0.0042$
RGM [44]					$3.45 - i0.0066$		$3.31 - i0.0051$	
FY [45]					$3.43 - i0.0082$	$3.56 - i0.0070$	$3.23 - i0.0054$	
AGS [46]					$3.51 - i0.0074$	$3.47 - i0.0068$		
R-matrix [44]						$3.29 \quad - i\,0.0012$		
EXP [47]						$3.28(5) - i\,0.001(2)$		
EXP [48]						$3.36(1)$		
EXP [49]						$3.48(2)$		

Singlet scattering length a_0 (fm)

Method	K_1	K_2	K_3	K_4	AV18	N3LO	AV18/UIX	N3LO/N2LO
HH	48	44	30	22	$7.34 - i\,6.27$	$7.38 - i\,5.23$	$7.90 - i\,3.65$	$4.45 - i\,9.02$
HH	50	46	32	24	$7.41 - i\,6.16$	$7.40 - i\,5.20$	$7.90 - i\,3.59$	$5.25 - i\,9.25$
HH			EXT		$7.69 - i\,5.70$	$7.57 - i\,4.97$	$7.89 - i\,3.44$	$6.02 - i\,9.48$
RGM [44]					$7.78 - i\,5.02$		$7.62 - i\,4.09$	
AGS [46]					$7.80 - i\,4.97$	$7.82 - i\,4.51$		
R-matrix [44]						$7.40 \quad - i\,4.449$		
EXP [47]						$7.37(6) - i\,4.448(5)$		
EXP [48]						$7.46(2)$		
EXP [49]						$7.57(3)$		

In the case of the singlet scattering length, the situation is more delicate, since in the channel $J^\pi = 0^+$ the n-^{3}He interaction is attractive and the wave function must be orthogonal to the ^{4}He bound state. Consequently, the convergence is more problematic, in particular for the N3LO/N2LO interaction model. In the row labeled "EXT,' we have reported the extrapolated values for this quantity obtained by analyzing the convergence pattern. For the AV18, N3LO, and AV18/UIX interaction models we observe reasonable agreement with the results of other calculations and the experimental data. The N3LO/N2LO values are significantly different from those obtained with the other interaction models, which is presumably related to the slow convergence observed in this case. A complete study of the n-^{3}He scattering lengths is in progress [24].

Recently, there has been a new measurement [50] for the quantity $\Delta a' = \mathrm{Re}(a_1 - a_0) = -4.20(3)$ fm. The calculated values of $\Delta a'$ with the AV18, N3LO, AV18/UIX, and N3LO/N2LO models are -4.13, -4.11, -4.50, and -2.65 fm, respectively. Again the N3LO/N2LO value stands out: it is off that obtained with the other interaction models and the measured value.

The convergence for the negative-parity states is similar to that discussed above. For the 0^- state, there is a close resonant state and the convergence is slow like in the 0^+ case. For the 1^- state, the resonance is far and we observe good convergence, like for the 1^+ state. Note, however, that in these cases the N3LO/N2LO convergence pattern is not different from that observed with the other models.

VI. CALCULATION

There is a total of two (four) states with $J = 0$ ($J = 1$): one (two) with positive parity having $LS = 00$ ($LS = 01, 21$) and one (two) with negative parity having $LS = 11$ ($LS = 10, 11$). The R-matrix elements $R^{\gamma\gamma',0}_{LS,LS}$ with $LS = 00$ or $LS = 11$ for $J = 0$ and $R^{\gamma\gamma',1}_{LS,L'S'}$ with $LS, L'S' = 01, 21$ or $LS, L'S' = 10, 11$ for $J = 1$, involving PC transitions induced by the strong interactions are calculated with the HH method, as described in the previous section. However, the R-matrix elements involving PV transitions are obtained in first-order perturbation theory as

$$R^{\gamma\gamma',J}_{LS,L'S'} = -\left\langle \Psi^{JJ_z}_{\gamma',L'S'} \middle| v^{PV} \middle| \Psi^{JJ_z}_{\gamma,LS} \right\rangle , \tag{6.1}$$

where $L + L'$ must be odd. Specifically, the R-matrix elements relevant for the calculation of the asymmetry are $R^{11,0}_{00,11}$ and $\overline{R}^{21,0}_{00,11}$ for $J = 0$ and $R^{11,1}_{01,10}$, $R^{11,1}_{01,11}$, $R^{11,1}_{21,10}$, $R^{11,1}_{21,11}$, $\overline{R}^{21,1}_{01,10}$, and $\overline{R}^{21,1}_{01,11}$ for $J = 1$. Quantum Monte Carlo (QMC) techniques are employed to evaluate these matrix elements (see below).

The asymmetry in Eq. (1.2) is expressed in terms of T-matrix elements, which are in turn derived from R-matrix elements via Eq. (3.16). This latter equation can be further simplified by retaining only linear terms in the PV R-matrix elements, and the resulting expressions for the PC $T^{21,0}_{00,00}$, $T^{21,1}_{01,01}$, and $T^{21,1}_{01,21}$ and PV $T^{21,0}_{00,11}$, $T^{21,1}_{01,10}$, and $T^{21,1}_{01,11}$ matrix elements are listed in Appendix A.

The QMC techniques used to evaluate the matrix element in Eq. (6.1) are similar to those discussed in Ref. [9] for the neutron spin rotation in $\vec{n}d$ scattering. The wave functions for an assigned spatial configuration specified by the set of Jacobi variables $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ are expanded on a basis of 16×6 spin-isospin states for the four nucleons as

$$\psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \sum_{a=1}^{96} \psi_a(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)|a\rangle, \tag{6.2}$$

where the components $\psi_a(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ are generally complex functions, and the basis states $|a\rangle = |(n\downarrow)_1(p\downarrow)_2(n\downarrow)_3(p\downarrow)_4\rangle$, $|(n\downarrow)_1(n\downarrow)_2(p\downarrow)_3(p\downarrow)_4\rangle$, and so on. Matrix elements of the PV potential components are written schematically as

$$\langle f|O|i\rangle = \sum_{a,b=1}^{96} \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 \psi^*_{f,a}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \times [O(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)]_{ab} \psi_{i,b}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) , \tag{6.3}$$

where $[O(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)]_{ab}$ denotes the matrix representing in configuration space any of the components in Table II. Note that the operators $\mathbf{X}^{(n)}_{ij,\mp}$ occurring in v^{PV}_{ij} are conveniently expressed as

$$\mathbf{X}^{(n)}_{ij,+} = -\mathrm{i}[2 f_n(r_{ij})\nabla_{ij} + \hat{\mathbf{r}}_{ij} f'_n(r_{ij})], \tag{6.4}$$

$$\mathbf{X}^{(n)}_{ij,-} = \hat{\mathbf{r}}_{ij} f'_n(r_{ij}), \tag{6.5}$$

where the gradient operator $\nabla_{ij} = (\nabla_i - \nabla_j)/2$ acts on the right (initial) wave function, and $f'(x) = df(x)/dx$. Gradients are discretized as

$$\nabla_{i,\alpha}\psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \simeq [\psi(\cdots \mathbf{r}_i + \delta\hat{\mathbf{e}}_\alpha \cdots) - \psi(\cdots \mathbf{r}_i - \delta\hat{\mathbf{e}}_\alpha \cdots)]/(2\delta), \tag{6.6}$$

where δ is a small increment and $\hat{\mathbf{e}}_\alpha$ is a unit vector in the α direction. Matrix multiplications in the spin-isospin space are performed exactly with the techniques developed in Ref. [51]. The problem is then reduced to the evaluation of the spatial integrals, which is efficiently carried out by a combination of Monte Carlo (MC) and standard quadrature techniques. We write

$$\langle f|O|i\rangle = \int d\hat{\mathbf{x}}_1 d\mathbf{x}_2 d\mathbf{x}_3 F(\hat{\mathbf{x}}_1, \mathbf{x}_2, \mathbf{x}_3) \simeq \frac{1}{N_c} \sum_{c=1}^{N_c} \frac{F(c)}{W(c)}, \tag{6.7}$$

where the c's denote collectively (uniformly sampled) directions $\hat{\mathbf{x}}_1$ and Jacobi coordinates $(\mathbf{x}_2, \mathbf{x}_3)$, and the probability density $W(c) = |\Psi(\mathbf{x}_2, \mathbf{x}_3)|^2/(4\pi)$—$\Psi(\mathbf{x}_2, \mathbf{x}_3)$ is the triton bound-state wave function normalized to one—is sampled via the Metropolis algorithm. For each such configuration c (total number N_c), the function F is obtained by Gaussian integrations over the x_1 variable, that is,

$$F(c) = \sum_{a,b=1}^{96} \int_0^\infty dx_1 x_1^2 \psi^*_{f,a}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \times [O(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)]_{ab} \psi_{i,b}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3). \tag{6.8}$$

Convergence in the x integrations requires of the order of 50 Gaussian points, distributed over a nonuniform grid extending beyond 20 fm, while N_c of the order of a 100 000 is sufficient to reduce the statistical errors in the MC integration on the PV T-matrix elements at the few percent level. In this respect, we note that these errors are computed directly, by accumulating, in the course of the random walk, values—and their squares—for the appropriate linear combinations of R-matrix elements, as given in Eqs. (A5), (A14), and (A15) of Appendix A. Because of correlations, the errors on the T-matrix elements obtained in this way are much smaller than those that would be inferred from the R-matrix elements by naive error propagation.

The present method turns out to be computationally intensive, particularly because of the large number of wave functions (and their derivatives) that have to be generated at each configuration $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$. The computer codes have been successfully tested by carrying out a calculation based on Gaussian wave functions for the initial and final states, as described in the following subsection.

A. Test calculation

To test the computer programs based on QMC techniques, we carried out a preliminary calculation using wave functions for which it is possible to evaluate the matrix elements of the PV potential also analytically. These (antisymmetric) wave functions are written as

$$\Psi^{JJ_z}_{\gamma,LS} = \frac{1}{4\pi}\sum_{p=1}^{4} e^{-\beta\rho^2} y_p^{L+2n_\beta} \times \{Y_L(\hat{\mathbf{y}}_p)\otimes[\phi_\gamma(ijk)\otimes\chi_\gamma(l)]_S\}_{JJ_z}, \tag{6.9}$$

where ϕ_γ (χ) represents a three-nucleon (single-nucleon) spin-isospin one-half state with isospin projection $-1/2$ ($+1/2$) for $\gamma = 1$ (p-^{3}H channel) and $+1/2$ ($-1/2$) for $\gamma = 2$ (n-^{3}He channel). Thus, as in the realistic case, the aforementioned wave functions do not have a definite total isospin T but, rather, are combinations of $T = 0$ and $T = 1$ states (having, of course, $T_z = 0$). The whole radial dependence is given by the factor $y_p^{L+2n_\beta} e^{-\beta\rho^2}$, where ρ is the hyperradius. The non-negative integer n_β and the real parameter β can be varied so as to obtain a family of wave functions. For the purpose of computing matrix elements of two-body operators, it is convenient to express the pieces in Eq. (6.9) corresponding to permutations $p \neq 1$ in terms of quantities relative to the permutation $p = 1$ or (123, 4). This can be accomplished by making use of the properties of Wigner coefficients:

$$\Psi^{JJ_z}_{\gamma,LS} = e^{-\beta\rho^2}\sum_{\mu} C^{LSJ}_{\gamma n_\beta;\mu} x_1^{n_1} x_2^{n_2} x_3^{n_3} \Big(\Big\{\Big[Y_{\ell_3}(\hat{\mathbf{x}}_3)\otimes(\chi_1\otimes\chi_2)_{S_2}\Big]_{j_3} \otimes\Big[Y_{\ell_2}(\hat{\mathbf{x}}_2)\otimes\chi_3\Big]_{j_2}\Big\}_{J_2}\otimes\Big[Y_{\ell_1}(\hat{\mathbf{x}}_1)\otimes\chi_4\Big]_{j_1}\Big)_{JJ_z} \times\{[(\xi_1\otimes\xi_2)_{T_2}\otimes\xi_3]_{T_3}\otimes\xi_4\}_T, \tag{6.10}$$

$$\mu \equiv \{\ell_1\ell_2\ell_3 n_1 n_2 n_3 j_1 j_2 j_3 J_2 S_2 T_2 T_3 T\},$$

where χ_i and ξ_i are the spin and isospin states of nucleon i, $\mathbf{x}_j$ are the Jacobi vectors corresponding to the permutation $p = 1$ and $n_1 + n_2 + n_3 = L + 2n_\beta$, and the C's denote combinations of Wigner coefficients. It is now relatively simple to evaluate the matrix of the PV potential $\sum_{i<j} v_{ij}^{\rm PV} = 6v_{12}^{\rm PV}$ [9] by expressing the wave functions as in Eq. (6.10).

TABLE V. Values of the quantum numbers and parameters for some of the test wave functions used in this work. See text for explanation.

State	J^π	LS	n_β	β
$\|1\rangle$	0^+	00	0	0.25
$\|2\rangle$	0^-	11	0	0.25

As an example, we report here the results obtained for two $J = 0$ wave functions. In Table V, we list the values of the quantum numbers LS and J^π, and parameters n_β and β, used in the actual calculation. The ket $|\gamma\rangle$ with $\gamma = 1$ (2) describes a "p-^{3}H" ("n-^{3}He") state. We compute the matrix elements in two ways: (i) by performing the analytical calculation via the transformation of Eq. (6.10) and (ii) by using the QMC techniques discussed earlier.

The values for the matrix elements $-\langle 1|O_{12}^{(n)}|2\rangle$ corresponding to the 12 components of the DDH potential (see Table II) are reported in Table VI. There is good agreement between the results of the two calculations. Note that the $n = 2$ contribution associated with an isoscalar operator as well as the $n = 6$ and $n = 7$ contributions corresponding to isotensor operators vanishes. The test wave functions consist of a superposition of $T = 0$ and $T = 1$ components, and therefore it is not immediately apparent why this should be so. The reason for this result becomes clear only after carrying out the decomposition of the wave functions as in Eq. (6.10). It comes about because of delicate cancellations among various terms. We find it reassuring that these same matrix elements are seen to vanish (within machine precision) with the QMC code. We have verified explicitly that the close agreement between the two calculations persists for the matrix elements involving other pairs of states, including those having $J = 1$.

TABLE VI. Results for the real part of the (adimensional) matrix element $-\langle 1|O_{12}^{(n)}|2\rangle$ calculated analytically and by using the QMC code. For the latter calculation, the statistical uncertainties are reported in parentheses and correspond to a (rather modest) set of 5k samples. The operators $O_{12}^{(n)}$ are those of the DDH potential, listed in Table II.

n	Analytical	QMC
1	−2.987	−3.020(15)
2	0.000	0.000
3	−0.333	−0.349(4)
4	−0.264	−0.281(4)
5	−0.222	−0.233(2)
6	0.000	0.000
7	0.000	0.000y
8	−0.335	−0.349(6)
9	−0.143	−0.147(2)
10	−0.335	−0.349(6)
11	−0.286	−0.294(3)
12	−0.264	−0.281(4)

TABLE VII. The coefficient $I_n^{\rm DDH}$ corresponding to the DDH potential components $O^{(n)}$ in combination with the AV18, AV18/UIX, N3LO, and N3LO/N2LO strong-interaction Hamiltonians. The statistical Monte Carlo errors are not shown, but are at the most 10% for the smallest contributions, and less than 2% for the largest. The $I_n^{\rm DDH}$ are in units of fm^{-1}.

n	$I_n^{\rm DDH}$			
	AV18	AV18/UIX	N3LO	N3LO/N2LO
1	-0.186	-0.189	-0.203	-0.113
2	-0.826×10^{-2}	-0.577×10^{-2}	-0.608×10^{-2}	-0.622×10^{-2}
3	$+0.811 \times 10^{-2}$	$+0.864 \times 10^{-2}$	$+0.333 \times 10^{-2}$	-0.693×10^{-2}
4	-0.620×10^{-2}	-0.794×10^{-2}	-0.970×10^{-2}	-0.753×10^{-2}
5	-0.800×10^{-2}	-0.976×10^{-2}	-0.102×10^{-1}	-0.781×10^{-2}
6	-0.359×10^{-3}	-0.170×10^{-3}	-0.942×10^{-3}	$+0.322 \times 10^{-3}$
7	$+0.631 \times 10^{-3}$	$+0.115 \times 10^{-2}$	-0.641×10^{-4}	$+0.703 \times 10^{-3}$
8	$+0.605 \times 10^{-2}$	$+0.404 \times 10^{-2}$	-0.699×10^{-3}	-0.794×10^{-2}
9	$+0.314 \times 10^{-2}$	$+0.289 \times 10^{-2}$	-0.171×10^{-2}	-0.577×10^{-2}
10	-0.689×10^{-2}	-0.887×10^{-2}	-0.115×10^{-1}	-0.902×10^{-2}
11	-0.930×10^{-2}	-0.113×10^{-1}	-0.123×10^{-1}	-0.940×10^{-2}
12	-0.801×10^{-2}	-0.979×10^{-2}	-0.115×10^{-1}	-0.606×10^{-2}

VII. RESULTS

The results for the coefficients I_n^α in Eq. (4.6), obtained with the (zero energy) n-^{3}He continuum wave functions corresponding to the AV18, AV18/UIX, N3LO, and N3LO/N2LO strong-interaction Hamiltonians, are reported for the DDH and pionless EFT PV potentials in Tables VII and VIII, respectively. The subscript n in I_n^α specifies the operators as listed in Table II, and the set of cutoff parameters entering the modified Yukawa functions are given in Table III.

A quick glance at Table VII makes it clear that (i) the contribution of the long-range component of the DDH potential due to pion exchange is at least a factor 15 larger than that of any of the short-range components induced by vector-meson exchanges and (ii) among the vector-meson exchange contributions the isoscalar ($n = 2, 3$ and $n = 8, 9$) and isovector ($n = 4, 5$ and $n = 8$–12) ones are comparable in magnitude and much larger than those due to isotensor ρ-meson exchanges ($n = 6, 7$). It is also clear that the pion-exchange contribution is fairly insensitive to the choice of input strong-interaction Hamiltonian (with or without the inclusion of a three-nucleon potential) used to generate the n-^{3}He even and odd parity states with $J = 0$ and 1. However, the N3LO/N2LO model stands out: the pion-range contribution is (in magnitude) substantially smaller than that calculated for the other models. Moreover, the isoscalar ρ-meson (ω-meson) contribution corresponding to $n = 3$ ($n = 8$) has the sign opposite that obtained for the other (AV18 and AV18/UIX) models.

TABLE VIII. The coefficient $I_n^{\rm EFT}$ corresponding to the pionless EFT potential components $O^{(n)}$ in combination with the AV18/UIX and N3LO/N2LO strong interaction Hamiltonians. Note that there are no potential components with $n = 2, 3, 5, 7, 10, 11$, and 12. The statistical Monte Carlo errors are not shown, but are typically less than 5%. The $I_n^{\rm EFT}$ are in units of fm^{-1}.

n	$I_n^{\rm EFT}$	
	AV18/UIX	N3LO/N2LO
1	-0.195	-0.119
4	-0.606	-0.391
6	-0.639×10^{-2}	$+0.179 \times 10^{-1}$
8	$+0.608$	-0.515×10^{-1}
9	$+0.301$	$+0.426 \times 10^{-1}$

It is useful to express the asymmetry as

$$a_z = h_\pi^1 C_\pi^1 + h_\rho^0 C_\rho^0 + h_\rho^1 C_\rho^1 + h_\rho^2 C_\rho^2 + h_\omega^0 C_\omega^0 + h_\omega^1 C_\omega^1, \tag{7.1}$$

where the h_α^i's, $\alpha = \pi$, ρ, and ω and $i = 0$, 1, and 2, denote the PV coupling constants in the DDH model along with the isospin content of the corresponding interaction. Moreover, the coefficients C_α^i are obtained from the $I_n^{\rm DDH}$'s and $c_n^{\rm DDH}$'s listed in Table II via

$$\begin{aligned}
C_\pi^1 &= +\frac{g_\pi}{2\sqrt{2}m} I_1^{\rm DDH}, \\
C_\rho^0 &= -\frac{g_\rho}{m} I_2^{\rm DDH} - \frac{g_\rho(1+\kappa_\rho)}{m} I_3^{\rm DDH}, \\
C_\rho^1 &= -\frac{g_\rho}{2m} I_4^{\rm DDH} - \frac{g_\rho(1+\kappa_\rho)}{2m} I_5^{\rm DDH} + \frac{g_\rho}{2m} I_{12}^{\rm DDH}, \\
C_\rho^2 &= -\frac{g_\rho}{2\sqrt{6}m} I_6^{\rm DDH} - \frac{g_\rho(1+\kappa_\rho)}{2\sqrt{6}m} I_7^{\rm DDH}, \\
C_\omega^0 &= -\frac{g_\omega}{m} I_8^{\rm DDH} - \frac{g_\omega(1+\kappa_\omega)}{m} I_9^{\rm DDH}, \\
C_\omega^1 &= -\frac{g_\omega}{2m} I_{10}^{\rm DDH} - \frac{g_\omega(1+\kappa_\omega)}{2m} I_{11}^{\rm DDH} - \frac{g_\omega}{2m} I_{12}^{\rm DDH}.
\end{aligned} \tag{7.2}$$

The calculated coefficients C_α^i are listed in Table IX and depend on the input Hamiltonian used to generate the continuum wave functions, as well as on the assumed values for the PC pion- and vector-meson coupling constants and associated cutoffs (see Table III).

The coefficients C_α^i follow from the linear combination given in Eq. (1.2). Isotensor ρ-exchange (C_ρ^2) is negligible. The isoscalar and isovector vector-meson exchanges give

TABLE IX. The coefficients C_α^i entering the PV observable a_z, corresponding to the AV18, AV18/UIX, N3LO, and N3LO/N2LO strong-interaction potentials and the DDH weak-interaction potential. The statistical errors due to the Monte Carlo integrations are indicated in parentheses and correspond to a sample consisting of ∼130k configurations.

	C_π^1	C_ρ^0	C_ρ^1	C_ρ^2	C_ω^0	C_ω^1
AV18	−0.1892(86)	−0.0364(40)	+0.0193(9)	−0.0006(1)	−0.0334(29)	+0.0413(10)
AV18/UIX	−0.1853(150)	−0.0380(70)	+0.0230(18)	−0.0011(1)	−0.0231(56)	+0.0500(20)
N3LO	−0.1989(87)	−0.0120(49)	+0.0242(9)	+0.0002(1)	+0.0080(30)	+0.0587(11)
N3LO/N2LO	−0.1110(75)	+0.0379(56)	+0.0194(10)	−0.0007(1)	+0.0457(36)	+0.0408(14)

contributions of the same magnitude, both of which are smaller than OPE. However, the OPE contribution seems to be significantly suppressed. For example, in the case of the neutron spin rotation in $\vec{n}$-d scattering this contribution is calculated to be at least a factor of ∼30 larger than that of any of the ρ and ω exchanges, which is not the case for the process under consideration. This may be due to the predominant isoscalar character of the $^1S_0$ and $^3P_0$ channels—see discussion in Appendix B. The N3LO/N2LO results should be considered as preliminary, since the HH solution for the 0^+ wave function has not yet fully converged (at least as far as the singlet scattering length is concerned, see Sec. V). This fact may explain why the inclusion of a three-nucleon potential like N2LO [28] should reduce C_π^1 by almost a factor of two relative to the other models. Finally we note that the "best values" for the PV couplings constants of the pion and ρ-meson are (in units of 10^{-7}), respectively, +4.56 and –16.4, and this leads to the large cancellation (and consequent model dependence) in the values predicted for a_z and referred to earlier in Sec. I.

To investigate the stability of the AV18/UIX and N3LO/N2LO results with respect to convergence in the internal part of the wave function, we present in Table X the coefficients C_α^i entering the PV observable a_z in Eq. (7.1) for two different choices of wave functions. The results labeled "wf2" were listed earlier in Table IX, except that those relative to the N3LO/N2LO model are based here on a smaller number of configurations. These results are obtained by including in the expansion of the internal parts of the $0^\pm$ and $1^\pm$ wave functions the maximum number of HH functions we have considered in the present work. The results corresponding to the row "wf1" are obtained by reducing this number: in practice, for each of the classes $K_1, \ldots, K_4$ we set $K_i(\text{wf1}) = K_i(\text{wf2}) - 2$ (see discussion in Sec. V). Note also that the Monte Carlo calculation of the "wf1" coefficients for the AV18/UIX model uses a factor 3 smaller number of configurations, and therefore the associated statistical errors are substantially larger. On the other hand, the "wf1" and "wf2" N3LO/N2LO results correspond to the same number of configurations and indeed the same random walk. Taking into account errors, we conclude that both AV18/UIX and N3LO/N2LO calculations have converged. This is not the case as far as the N3LO/N2LO singlet scattering length is concerned.

Therefore, the differences found between the N3LO/N2LO and the other models are presumably due to the fact that the HH expansion for the N3LO/N2LO wave functions (specifically the 0^+ wave function) has not fully converged. Consequently, in the following we restrict our discussion to the results obtained with the AV18, N3LO, and AV18/UIX models. In reference to the pion contribution, the calculated C_π^1 is rather insensitive to the choice of strong Hamiltonian. However, there is still a considerable model dependence in the results obtained for the individual contributions due to vector-meson exchanges. This model dependence, in turn, impacts very significantly predictions for the PV asymmetry a_z, as it can be surmised from Table XI. Of course, this is so under the assumption that the values for the strong- and weak-interaction coupling constants characterizing the DDH potential are those listed in Table III. For example, the combination of coupling constants corresponding to pion-exchange ($n = 1$) and isoscalar ρ-meson exchange ($n = 2$ and 3) are, respectively, $c_1^{\text{DDH}} = (4.48 \times 10^{-7})$ fm, $c_2^{\text{DDH}} = (11.2 \times 10^{-7})$ fm, and $c_3^{\text{DDH}} = (79.5 \times 10^{-7})$ fm—note that $c_3^{\text{DDH}} = (1 + \kappa_\rho) c_2^{\text{DDH}}$ and $\kappa_\rho = 6.1$ is the value adopted here for the tensor coupling of the ρ-meson to the nucleon [52]. Consequently, the contribution $c_3^{\text{DDH}} \times I_3^{\text{DDH}}$ is comparable in magnitude and opposite in sign to the pion-exchange contribution $c_1^{\text{DDH}} \times I_1^{\text{DDH}}$. In this respect, we note that the asymmetry a_z changes roughly from -27×10^{-8} to $+13 \times 10^{-8}$ as the six PV weak coupling constants entering the

TABLE X. The coefficients C_α^i entering the PV observable a_z, corresponding to the AV18/UIX and N3LO/N2LO strong-interaction potentials and the DDH weak-interaction potential for two sets of wave functions (see text for details). The statistical errors due to the Monte Carlo integrations are indicated in parentheses.

	C_π^1	C_ρ^0	C_ρ^1	C_ρ^2	C_ω^0	C_ω^1
AV18/UIX-wf1	−0.2077(281)	−0.0433(116)	+0.0242(29)	−0.0011(2)	−0.0232(77)	+0.0490(30)
AV18/UIX-wf2	−0.1853(150)	−0.0380(70)	+0.0230(18)	−0.0011(1)	−0.0231(56)	+0.0500(20)
N3LO/N2LO-wf1	−0.1118(29)	+0.0369(25)	+0.0200(8)	−0.0009(1)	+0.0390(23)	+0.0402(12)
N3LO/N2LO-wf2	−0.1050(35)	+0.0445(33)	+0.0189(9)	−0.0008(1)	+0.0454(31)	+0.0417(12)

TABLE XI. Cumulative contributions to a_z and associated errors (rows 1–12), obtained for the DDH PV potential with values for the coupling constants as listed in Table III. The four columns correspond to the different combinations of strong-interaction Hamiltonians adopted in the calculations. The last row shows the minimum and maximum (central) values that a_z can attain, because the PV couplings are varied over the allowed ranges in the original DDH formulation [29].

n	$10^8 \times a_z^{\rm DDH}$			
	AV18	AV18/UIX	N3LO	N3LO/N2LO
1	-8.33 ± 0.35	-8.45 ± 0.69	-9.07 ± 0.40	-5.06 ± 0.34
2	-9.26 ± 0.35	-9.09 ± 0.70	-9.75 ± 0.40	-5.76 ± 0.38
3	-2.80 ± 0.68	-2.22 ± 1.34	-7.10 ± 0.89	-11.3 ± 0.98
4	-2.86 ± 0.68	-2.30 ± 1.34	-7.20 ± 0.89	-11.3 ± 0.98
5	-3.40 ± 0.68	-2.95 ± 1.34	-7.88 ± 0.89	-11.9 ± 0.98
6	-3.41 ± 0.68	-2.95 ± 1.34	-7.90 ± 0.89	-11.9 ± 0.98
7	-3.32 ± 0.68	-2.80 ± 1.34	-7.91 ± 0.89	-11.8 ± 0.98
8	-3.97 ± 0.69	-3.23 ± 1.35	-7.83 ± 0.90	-10.9 ± 0.99
9	-4.31 ± 0.69	-3.55 ± 1.35	-7.65 ± 0.90	-10.3 ± 0.99
10	-4.09 ± 0.69	-3.26 ± 1.35	-7.28 ± 0.90	-10.0 ± 0.99
11	-3.79 ± 0.69	-2.89 ± 1.35	-6.88 ± 0.90	-9.70 ± 0.99
12	-3.45 ± 0.69	-2.48 ± 1.35	-6.40 ± 0.90	-9.44 ± 0.99
Variation	From -27.1 to $+13.3$	From -27.6 to $+13.8$	From -26.0 to $+3.68$	From -29.7 to $+6.66$

DDH model are varied over their respective allowed ranges determined in Ref. [29]. Thus, a_z could potentially be large enough to make its measurement (relatively) easy.

The coefficients $I_n^{\rm EFT}$ for the operators entering the pionless EFT PV potential, that is, $n = 1$, 4, 6, 8, and 9, are reported in Table VIII. The coefficients $I_n^{\rm EFT}$ for $n = 1$, 4, 8, and 9, corresponding to isoscalar and isovector structures, are all of the same order of magnitude, while that for $n = 6$ with isotensor character is much smaller. Note that the radial functions are taken to be the same for all n, $f_n^{\rm EFT}(r) = f_\mu(r)$. Of course, the $I_n^{\rm EFT}$'s will depend significantly on the value of the mass μ—either $\mu = m_\pi$, as appropriate in the present pionless EFT formulation, or $\mu = 1$ GeV, the scale of chiral symmetry breaking, as appropriate in the formulation in which pion degrees of freedom are explicitly retained. Indeed, in this latter formulation the leading-order component of $v^{\rm PV}$ has the same form as the pion-exchange term in DDH.

Finally, rough estimates have been made for the range of values allowed for the low-energy constants C_1, $C_2 + C_4$, C_5, $\tilde{C}_1$, and C_6 in Ref. [30]. However, at the present time a systematic program for their determination is yet to be carried out. In view of this, we refrain here from making EFT-based predictions for the longitudinal asymmetry.

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APPENDIX A: FROM R TO T MATRICES

Consider the case with $J = 0$ first. For the PC T matrix we have

$$T^{21,0}_{00,00} = \frac{1}{\sqrt{q_1}}\big[\,\overline{R}^{21,0}_{00,00}(I - iR^{11,0})^{-1}_{00,00} + \overline{R}^{21,0}_{00,11}(I - iR^{11,0})^{-1}_{11,00}\big], \tag{A1}$$

where $I - iR^{11}$ is a 2×2 matrix with very small off-diagonal elements, that is,

$$I - iR^{11,0} = \begin{pmatrix} a & \epsilon \\ \epsilon & b \end{pmatrix}, \quad a = 1 - i\,R^{11,0}_{00,00}, \quad \epsilon = -i\,R^{11,0}_{00,11}, \quad b = 1 - i\,R^{11,0}_{11,11}, \tag{A2}$$

with $|a|, |b| \gg |\epsilon|$. To first order in ϵ, we approximate

$$(I - i\,R^{11,0})^{-1} = \begin{pmatrix} 1/a & -\epsilon/ab \\ -\epsilon/ab & 1/b \end{pmatrix}, \tag{A3}$$

and hence

$$T^{21,0}_{00,00} = \frac{1}{\sqrt{q_1}}\frac{\overline{R}^{21,0}_{00,00}}{a}. \tag{A4}$$

Similarly, for the PV T-matrix element we find

$$T^{21,0}_{00,11} = \frac{1}{\sqrt{q_1}}\big[\,\overline{R}^{21,0}_{00,00}(I - i\,R^{11,0})^{-1}_{00,11} + \overline{R}^{21,0}_{00,11}(I - i\,R^{11,0})^{-1}_{11,11}\big] = \frac{1}{\sqrt{q_1}}\left(\frac{i\,\overline{R}^{21,0}_{00,00}\,R^{11,0}_{00,11}}{a\,b} + \frac{\overline{R}^{21,0}_{00,11}}{b}\right). \tag{A5}$$

The case $J = 1$ is somewhat more involved because the matrices are now 4×4. The matrix $(I - i\,R^{11,1})^{-1}$ is

written as

$$I - i\,R^{11,1} = \begin{pmatrix} A & \epsilon \\ \epsilon^T & B \end{pmatrix}, \tag{A6}$$

where A, ϵ, and B are 2×2 matrices,

$$A = \begin{pmatrix} 1 - i\,R^{11,1}_{01,01} & -i\,R^{11,1}_{01,21} \\ -i\,R^{11,1}_{21,01} & 1 - i\,R^{11,1}_{21,21} \end{pmatrix}, \tag{A7}$$

$$B = \begin{pmatrix} 1 - i\,R^{11,1}_{10,10} & -i\,R^{11,1}_{10,11} \\ -i\,R^{11,1}_{11,10} & 1 - i\,R^{11,1}_{11,11} \end{pmatrix}, \tag{A8}$$

$$\epsilon = \begin{pmatrix} -i\,R^{11,1}_{01,10} & -i\,R^{11,1}_{01,11} \\ -i\,R^{11,1}_{21,10} & -i\,R^{11,1}_{21,11} \end{pmatrix}. \tag{A9}$$

Note that A and B, as well as their inverse A^{-1} and B^{-1}, are symmetric. To first order in ϵ, it follows that

$$(I - i\,R^{11,1})^{-1} = \begin{pmatrix} A^{-1} & C \\ C^T & B^{-1} \end{pmatrix}, \tag{A10}$$

where the 2×2 matrix C and its transpose are defined as

$$C = -A^{-1}\epsilon B^{-1}, \quad C^T = -B^{-1}\epsilon^T A^{-1}. \tag{A11}$$

This shows that $(I - i\,R^{11,1})^{-1}$ is also symmetric in this approximation. The PC $T^{21,1}_{01,01}$ and $T^{21,1}_{01,21}$ and PV $T^{21,1}_{01,10}$ and $T^{21,1}_{01,11}$ matrix elements entering Eq. (3.16) are then given by

$$T^{21,1}_{01,01} = \frac{1}{\sqrt{q_1}}\Big[\overline{R}^{21,1}_{01,01}(A^{-1})_{01,01} + \overline{R}^{21,1}_{01,21}(A^{-1})_{21,01}\Big], \tag{A12}$$

$$T^{21,1}_{01,21} = \frac{1}{\sqrt{q_1}}\Big[\overline{R}^{21,1}_{01,01}(A^{-1})_{01,21} + \overline{R}^{21,1}_{01,21}(A^{-1})_{21,21}\Big], \tag{A13}$$

$$T^{21,1}_{01,10} = \frac{1}{\sqrt{q_1}}\Big[\overline{R}^{21,1}_{01,01}C_{01,10} + \overline{R}^{21,1}_{01,21}C_{21,10} + \overline{R}^{21,1}_{01,10} \times (B^{-1})_{10,10} + \overline{R}^{21,1}_{01,11}(B^{-1})_{11,10}\Big], \tag{A14}$$

$$T^{21,1}_{01,11} = \frac{1}{\sqrt{q_1}}\Big[\overline{R}^{21,1}_{01,01}C_{01,11} + \overline{R}^{21,1}_{01,21}C_{21,11} + \overline{R}^{21,1}_{01,10} \times (B^{-1})_{10,11} + \overline{R}^{21,1}_{01,11}(B^{-1})_{11,11}\Big]. \tag{A15}$$

APPENDIX B: NUMERICAL VALUES FOR R AND T MATRIX ELEMENTS

The set of Tables XII–XV are all relative to the AV18/UIX + DDH model and present results for the R-matrix elements involving PV transitions between states with $J = 0$ and $J = 1$, the corresponding T-matrix elements which follow from them and the parity-conserving (PC) R-matrix elements via Eqs. (A5) and (A14)–(A15), and lastly the coefficients $d_i^{(n)}$,

$$\begin{aligned} d_1^{(n)} &= \overline{T}^{21,1}_{01,10}(n)\,\overline{T}^{21,0\,*}_{00,00}, & d_2^{(n)} &= \overline{T}^{21,0}_{00,11}(n)\,\overline{T}^{21,1\,*}_{01,01}, \\ d_3^{(n)} &= \overline{T}^{21,0}_{00,11}(n)\,\overline{T}^{21,1\,*}_{01,21}, & d_4^{(n)} &= \overline{T}^{21,1}_{01,11}(n)\,\overline{T}^{21,1\,*}_{01,01}, \\ d_5^{(n)} &= \overline{T}^{21,1}_{01,11}(n)\,\overline{T}^{21,1\,*}_{01,21}, \end{aligned} \tag{B1}$$

TABLE XII. The PV R-matrix elements for $J = 0$ corresponding to the DDH potential components $O^{(n)}$ in combination with the AV18/UIX strong-interaction potentials at vanishing n-^{3}He energy. The statistical Monte Carlo errors are not shown, but are typically ∼1%–2% for the largest values and less than 10% for the smallest. The R-matrix element without (with) an overline is in units of fm^{-1} (fm$^{-1/2}$), see text for explanation.

n	$R^{11,0}_{00,11}$	$\overline{R}^{21,0}_{00,11}$
1	$+0.198 \times 10^{1}$	$+0.278 \times 10^{1}$
2	$+0.126$	$+0.305$
3	-0.149	-0.373
4	$+0.533 \times 10^{-1}$	$+0.530 \times 10^{-1}$
5	$+0.632 \times 10^{-1}$	$+0.691 \times 10^{-1}$
6	$+0.156 \times 10^{-2}$	$+0.154 \times 10^{-2}$
7	$+0.129 \times 10^{-2}$	$+0.163 \times 10^{-2}$
8	-0.211	-0.523
9	-0.797×10^{-1}	-0.203
10	$+0.589 \times 10^{-1}$	$+0.588 \times 10^{-1}$
11	$+0.720 \times 10^{-1}$	$+0.784 \times 10^{-1}$
12	$+0.154 \times 10^{-1}$	$+0.134 \times 10^{-1}$

where the $\overline{T}$-matrix elements are defined as in Eq. (2.12) and the label (n) on those involving PV transitions refers to the operator component $O^{(n)}$ in Table II. The I_n's discussed earlier follow from

$$I_n = -\frac{4}{\Sigma}\,\mathrm{Re}\big[\sqrt{3}\,d_1^{(n)} - d_2^{(n)} + \sqrt{2}\,d_3^{(n)} + \sqrt{6}\,d_4^{(n)} + \sqrt{3}\,d_5^{(n)}\big], \tag{B2}$$

where Σ has been defined in Eq. (1.3). A few words on units: Beause the operators $O^{(n)}$ do not include the c_n's, that is, the combinations of nucleon mass and strong- and weak-interaction coupling constants, the resulting R-matrix (T-matrix) elements involving PV transitions are in units of fm^{-1} (adimensional)—they would otherwise be adimensional (in units of fm). Further, because of the definition in Eq. (3.14), the $\overline{R}$-matrix elements have dimensions of fm$^{-1/2}$. Note, however, that the $\overline{T}$- and T-matrix elements only differ by a phase factor, and hence the former are also adimensional.

The R-matrix elements in the $J = 1$ states (Table XIII) are typically 2 orders of magnitude smaller than those in the $J = 0$ states (Table XII). Among the former, those with orbital angular momentum $L = 2$ in channel p-^{3}H ($\gamma = 1$) are much suppressed at the low energies of interest in the present work. Inspection of Table XII also shows that the (isovector) pion-exchange interaction ($n = 1$) is dominant, which suggests that the $J^\pi = 0^+$ and 0^- states in both n-^{3}He and p-^{3}H are not purely isoscalar, but rather have significant admixtures of isospin components $T > 0$.

To compute the d_i's in Table XV, one needs, in addition to the T-matrix elements listed in Table XIV, also the T-matrix elements associated with PC transitions. These have been calculated to be (at zero n-^{3}He energy) $T^{21,0}_{00,00} = (-1.356 + i4.482)$ fm, $T^{21,1}_{01,01} = (0.1679 - i0.6937)$ fm, and $T^{21,1}_{01,21} = (0.003\,497 - i0.000\,353\,5)$ fm. We conclude by noting that the $d_1^{(n)}$ and $d_2^{(n)}$ combinations give the leading contributions to I_n

TABLE XIII. Same as in Table XII, but for $J = 1$.

n	$R^{11,1}_{01,10}$	$R^{11,1}_{01,11}$	$R^{11,1}_{21,10}$	$R^{11,1}_{21,11}$	$\overline{R}^{21,1}_{01,10}$	$\overline{R}^{21,1}_{01,11}$
1	-0.160×10^{-1}	-0.930×10^{-1}	$+0.199 \times 10^{-2}$	$+0.365 \times 10^{-2}$	-0.535×10^{-1}	-0.106×10^{-1}
2	$+0.614 \times 10^{-3}$	$+0.131 \times 10^{-2}$	-0.216×10^{-4}	-0.627×10^{-4}	-0.339×10^{-2}	$+0.854 \times 10^{-2}$
3	-0.837×10^{-3}	-0.198×10^{-2}	-0.204×10^{-4}	$+0.941 \times 10^{-4}$	$+0.528 \times 10^{-2}$	$+0.604 \times 10^{-3}$
4	-0.188×10^{-3}	$+0.782 \times 10^{-3}$	-0.643×10^{-4}	-0.958×10^{-5}	-0.179×10^{-2}	$+0.161 \times 10^{-2}$
5	-0.317×10^{-3}	$+0.918 \times 10^{-3}$	-0.853×10^{-4}	-0.133×10^{-4}	-0.232×10^{-2}	$+0.186 \times 10^{-2}$
6	$+0.116 \times 10^{-3}$	$+0.159 \times 10^{-2}$	-0.404×10^{-4}	-0.777×10^{-4}	-0.257×10^{-3}	-0.427×10^{-2}
7	-0.186×10^{-4}	$+0.191 \times 10^{-2}$	-0.713×10^{-4}	-0.870×10^{-4}	$+0.617 \times 10^{-4}$	-0.518×10^{-2}
8	-0.769×10^{-3}	-0.181×10^{-2}	-0.217×10^{-4}	$+0.860 \times 10^{-4}$	$+0.506 \times 10^{-2}$	$+0.812 \times 10^{-3}$
9	-0.364×10^{-3}	-0.852×10^{-3}	-0.274×10^{-4}	$+0.382 \times 10^{-4}$	$+0.260 \times 10^{-2}$	$+0.448 \times 10^{-2}$
10	-0.211×10^{-3}	$+0.867 \times 10^{-3}$	-0.716×10^{-4}	-0.107×10^{-4}	-0.201×10^{-2}	$+0.179 \times 10^{-2}$
11	-0.367×10^{-3}	$+0.105 \times 10^{-2}$	-0.985×10^{-4}	-0.151×10^{-4}	-0.270×10^{-2}	$+0.214 \times 10^{-2}$
12	-0.543×10^{-3}	-0.144×10^{-2}	-0.699×10^{-5}	$+0.636 \times 10^{-4}$	-0.258×10^{-2}	-0.102×10^{-3}

TABLE XIV. The PV T-matrix elements corresponding to the DDH potential components $O^{(n)}$ in combination with the AV18/UIX strong-interaction potentials at vanishing n-^{3}He energy. The statistical Monte Carlo errors are not shown, but are typically less than 10%. The T-matrix elements are adimensional; see text for explanation.

n	$T^{21,0}_{00,11}$		$T^{21,1}_{01,10}$		$T^{21,1}_{01,11}$	
	Re	Im	Re	Im	Re	Im
1	-0.104×10^{1}	-0.302×10^{1}	-0.133	-0.134×10^{-2}	-0.316×10^{-1}	-0.168×10^{-1}
2	$+0.219$	-0.996×10^{-1}	-0.830×10^{-2}	$+0.143 \times 10^{-3}$	$+0.210 \times 10^{-1}$	$+0.123 \times 10^{-2}$
3	-0.289	$+0.108$	$+0.129 \times 10^{-1}$	-0.269×10^{-3}	$+0.136 \times 10^{-2}$	-0.293×10^{-3}
4	-0.767×10^{-1}	-0.971×10^{-1}	-0.442×10^{-2}	$+0.335 \times 10^{-5}$	$+0.401 \times 10^{-2}$	$+0.330 \times 10^{-3}$
5	-0.771×10^{-1}	-0.111	-0.573×10^{-2}	-0.682×10^{-5}	$+0.463 \times 10^{-2}$	$+0.385 \times 10^{-3}$
6	-0.226×10^{-2}	-0.285×10^{-2}	-0.625×10^{-3}	$+0.453 \times 10^{-4}$	-0.104×10^{-1}	-0.224×10^{-3}
7	-0.110×10^{-2}	-0.210×10^{-2}	$+0.150 \times 10^{-3}$	$+0.194 \times 10^{-4}$	-0.126×10^{-1}	-0.277×10^{-3}
8	-0.393	$+0.159$	$+0.124 \times 10^{-1}$	-0.253×10^{-3}	$+0.189 \times 10^{-2}$	-0.239×10^{-3}
9	-0.161	$+0.559 \times 10^{-1}$	$+0.637 \times 10^{-2}$	-0.144×10^{-3}	$+0.110 \times 10^{-1}$	$+0.364 \times 10^{-3}$
10	-0.843×10^{-1}	-0.107	-0.495×10^{-2}	$+0.375 \times 10^{-5}$	$+0.445 \times 10^{-2}$	$+0.367 \times 10^{-3}$
11	-0.887×10^{-1}	-0.126	-0.667×10^{-2}	-0.757×10^{-5}	$+0.533 \times 10^{-2}$	$+0.443 \times 10^{-3}$
12	-0.265×10^{-1}	-0.295×10^{-1}	-0.640×10^{-2}	-0.285×10^{-4}	-0.340×10^{-3}	-0.245×10^{-3}

TABLE XV. The real parts of the coefficients d_i ($i = 1, \ldots, 5$), and the coefficients I_n^{DDH}, corresponding to the DDH potential components $O^{(n)}$ in combination with the AV18/UIX strong-interaction potentials. The statistical Monte Carlo errors are not shown, but are typically less than 10%. The d_i are adimensional, while I_n^{DDH} are in units of fm^{-1}.

n	$\mathrm{Re}\, d_1^{(n)}$	$\mathrm{Re}\, d_2^{(n)}$	$\mathrm{Re}\, d_3^{(n)}$	$\mathrm{Re}\, d_4^{(n)}$	$\mathrm{Re}\, d_5^{(n)}$	I_n^{DDH}
1	$+0.617$	$+0.349 \times 10^{-1}$	$+0.107 \times 10^{-1}$	-0.414×10^{-2}	$+0.616 \times 10^{-4}$	-0.189
2	$+0.384 \times 10^{-1}$	$+0.436 \times 10^{-1}$	$+0.333 \times 10^{-3}$	$+0.345 \times 10^{-2}$	-0.597×10^{-5}	-0.577×10^{-2}
3	-0.598×10^{-1}	-0.559×10^{-1}	-0.356×10^{-3}	$+0.249 \times 10^{-3}$	$+0.923 \times 10^{-6}$	$+0.864 \times 10^{-2}$
4	$+0.205 \times 10^{-1}$	-0.614×10^{-2}	$+0.347 \times 10^{-3}$	$+0.651 \times 10^{-3}$	-0.147×10^{-5}	-0.794×10^{-2}
5	$+0.266 \times 10^{-1}$	-0.527×10^{-2}	$+0.395 \times 10^{-3}$	$+0.751 \times 10^{-3}$	-0.171×10^{-5}	-0.976×10^{-2}
6	$+0.287 \times 10^{-2}$	-0.183×10^{-3}	$+0.102 \times 10^{-4}$	-0.173×10^{-2}	$+0.159 \times 10^{-5}$	-0.170×10^{-3}
7	-0.709×10^{-3}	-0.378×10^{-4}	$+0.748 \times 10^{-5}$	-0.210×10^{-2}	$+0.195 \times 10^{-5}$	$+0.115 \times 10^{-2}$
8	-0.573×10^{-1}	-0.769×10^{-1}	-0.528×10^{-3}	$+0.333 \times 10^{-3}$	$+0.694 \times 10^{-6}$	$+0.404 \times 10^{-2}$
9	-0.294×10^{-1}	-0.309×10^{-1}	-0.184×10^{-3}	$+0.181 \times 10^{-2}$	-0.213×10^{-5}	$+0.289 \times 10^{-2}$
10	$+0.230 \times 10^{-1}$	-0.672×10^{-2}	$+0.383 \times 10^{-3}$	$+0.722 \times 10^{-3}$	-0.163×10^{-5}	-0.887×10^{-2}
11	$+0.310 \times 10^{-1}$	-0.612×10^{-2}	$+0.451 \times 10^{-3}$	$+0.864 \times 10^{-3}$	-0.197×10^{-5}	-0.113×10^{-1}
12	$+0.297 \times 10^{-1}$	-0.241×10^{-2}	$+0.106 \times 10^{-3}$	-0.401×10^{-4}	$+0.888 \times 10^{-6}$	-0.979×10^{-2}

and that, in the case of pion exchange, $d_1^{(1)}$ is in fact dominant. This fits in well with the expectation that the ${}^1S_0 \to {}^3P_0$ transition entering $\overline{T}^{21,0}_{00,11}$ in $d_2^{(1)}$ is predominantly isoscalar, while $d_1^{(1)}$ involves the transition ${}^3S_1 \to {}^1P_1$ in $\overline{T}^{21,1}_{01,10}$, which presumably has both isoscalar and isovector character. Indeed, the contributions of isoscalar ρ- and ω-exchange interactions are comparable in $d_1^{(2)}, d_2^{(3)}$ and $d_1^{(8)}, d_2^{(9)}$, respectively.

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