

Electric dipole moments in $^{230,232}\text{U}$ and implications for tetrahedral shapes

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The nuclei ^{230}U and ^{232}U were populated in the compound nucleus reactions $^{232}\text{Th}(\alpha,6n)$ and $^{232}\text{Th}(\alpha,4n)$, respectively. Gamma rays from these nuclei were observed in coincidence with a recoil detector. A comprehensive set of in-band $E2$ transitions were observed in the lowest lying negative-parity band of ^{232}U while one $E2$ transition was also observed for ^{230}U . These allowed $B(E1; I^- \rightarrow I^+ - 1)/B(E2; I^- \rightarrow I^- - 2)$ ratios to be extracted and compared with systematics. The values are similar to those of their Th and Ra isotones. The possibility of a tetrahedral shape for the negative-parity U bands appears difficult to reconcile with the measured Q_2 values for the isotope ^{226}Ra .

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The actinide region has long attracted interest because of the possibility of encountering reflection-asymmetric shapes [1]. Below $N \sim 140$, the presence of alternating parity bands with sizable electric dipole moments D_0 , has been taken as evidence of rotation-induced octupole deformation; whereas above $N = 140$, the D_0 values are reduced and the positive- and negative-parity branches develop a sizable energy splitting. In this regime, the negative-parity bands have been interpreted in terms of octupole vibrations [1–3]. In recent years, a unified understanding of the above phenomena in terms of octupole phonons and their condensates has been suggested [4,5]. However, another prediction is the possible existence of tetrahedral shapes in the actinide region [6]. Such deformations are characterized by a triaxial octupole deformation Y_{32} , together with a near-zero quadrupole deformation Y_{20} . As a result, negative-parity bands with missing in-band $E2$ transitions are candidates for the rotation of a tetrahedral shape. Until recently, apart from the alpha cluster states in ^{16}O [7], the most favored region to observe tetrahedral states was in the mass-160 region [8]. However, recent measurements of quadrupole moments of low-lying negative-parity bands in this region have not supported the tetrahedral hypothesis [9,10]. This leaves the actinide region as the next frontier. Of particular interest are $^{230-234}\text{U}$, as it is in the lowest-lying, negative-parity bands of these nuclei in which, with the exception of some tentative lines in ^{230}U , no in-band $E2$ transitions have as yet been observed [11,12].

Ideally, a measurement of the quadrupole moments of these bands should be used as a simple test of the possibility of the existence of tetrahedral shapes. These nuclei can be accessed

using (α, xn) reactions on Th targets; however, such reactions suffer from severe competition from fission and low-recoil velocities, which complicate lifetime measurements based on Doppler-shift methods. We have, therefore, rather measured $B(E1)/B(E2)$ ratios in $^{230,232}\text{U}$, which can be compared with systematics in this region to obtain information on the likelihood of tetrahedral shapes in these nuclei. The reactions used were $^{232}\text{Th}(\alpha,6n)$ and $^{232}\text{Th}(\alpha,4n)$ at energies of 61 and 42 MeV, respectively, with the beams being supplied by the Separated Sector Cyclotron (SSC) of the iThemba LABS facility. A recoil detector, shown schematically in Fig. 1, was employed to select evaporation residues from the intense background of fission events. The detector, to be described fully elsewhere [13], is a development of earlier detectors [14,15] which operate chiefly on a time-of-flight principle. In our detector, recoiling residues, after multiple scattering inside the target, leave the target to strike an aluminized mylar foil, located 145 mm downstream. A hole in the foil allows the beam to pass through without producing an excessive background. Electrons liberated from the foil by the impact of the recoils are accelerated to a few hundred keV by a system of acceleration grids, and are then directed, by a perpendicular magnetic field, in semi-circular orbits to a $75 \times 93 \text{ mm}^2$ microchannel-plate detector placed directly above the foil. The time-of-flight was measured relative to the beam pulses from the SSC; these had a typical separation of 340 ns, which was long enough to detect recoils with energies as low as 270 keV. The initial energies of the recoils were 1100 and 700 keV at the two beam energies. To allow such low-energy recoils to escape the target, the thorium of the target was limited to a thickness of $120 \mu\text{g}/\text{cm}^2$, and

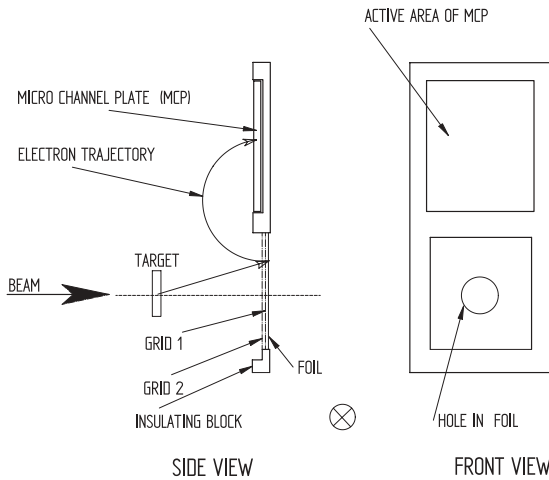


FIG. 1. Schematic showing recoil detector assembly. A magnetic field points into the page.

was supported by a $20\text{-}\mu\text{g}/\text{cm}^2$ foil of carbon on the upstream side. Gamma rays were detected using the AFRODITE array of up to 9 clover and 8 segmented LEPS HPGc detectors in coincidence with evaporation residues identified by the recoil detector. Different combinations of HPGc detectors were used over the course of the experiment, while beam intensities were typically between 15 and 40 pA.

The data were sorted into recoil-gated $\gamma\text{-}\gamma$ matrices of 7×10^6 and 1.8×10^6 events for ^{232}U and ^{230}U , respectively. Partial level schemes deduced [16] from these matrices are shown in Fig. 2. The ground-state bands of both nuclei have been extended in spin by two transitions compared to the earlier works [11,12]. In ^{232}U , the negative-parity sequence has also been extended by four units of spin and, more importantly, the in-band $E2$ transitions have been identified for the first

time. They can be seen in the spectra shown in Fig. 3. In addition, several new, but weak, transitions of the type $I^- \rightarrow I^+ + 1$ were identified. The data for ^{230}U were insufficient to add a similar comprehensive set of in-band $E2$ transitions for the negative-parity band. However, at least one such transition was observed, from the 17^- state (see Fig. 4) and another tentative transition from the 15^- level. The $B(E1; I^- \rightarrow I^+ - 1)/B(E2; I^- \rightarrow I^- - 2)$ ratios extracted from the data, together with magnitudes of the electric dipole moments $|D_0|$, are listed in Table I.

The latter were obtained from the $B(E1)/B(E2)$ ratios using the strong-coupling limit of the rotational model:

$$D_0 \approx \left[\frac{5(I-1) B(E1; I \rightarrow I-1)}{8(2I-1) B(E2; I \rightarrow I-2)} \right]^{1/2} Q_0.$$

This expression is justified if the Coriolis mixing of the band is small. In principle, the ratio of $B(E1)$ values, $B(E1; I^- \rightarrow I^+ + 1)/B(E1; I^- \rightarrow I^+ - 1)$, can be used to obtain a measure of the K -mixing present in the negative-parity band [17]. While our values of this ratio were not determined accurately, they are consistent with a pure $K = 0$ assignment. The quadrupole moment Q_0 of the negative-parity band, in contradiction with the tetrahedral assumption, was assumed to be the same as that measured [18] for the ground-state band.

To understand our results in the context of the systematics of the region, we have created two plots. In the first, shown in Fig. 5, the energies of the ground-state bands and negative-parity side bands for the $N = 138$ and $N = 140$ isotones of Ra, Th, and U nuclei are compared by aligning the energies of the 1^- states of the negative-parity bands. Remarkably, as first noted by Zeyen *et al.* [11], one observes that, for $N = 138$, the bands are practically identical—the corresponding transition energies are indeed the same to within a few keV. The same correspondence is also seen for $N = 140$ between the ^{232}U

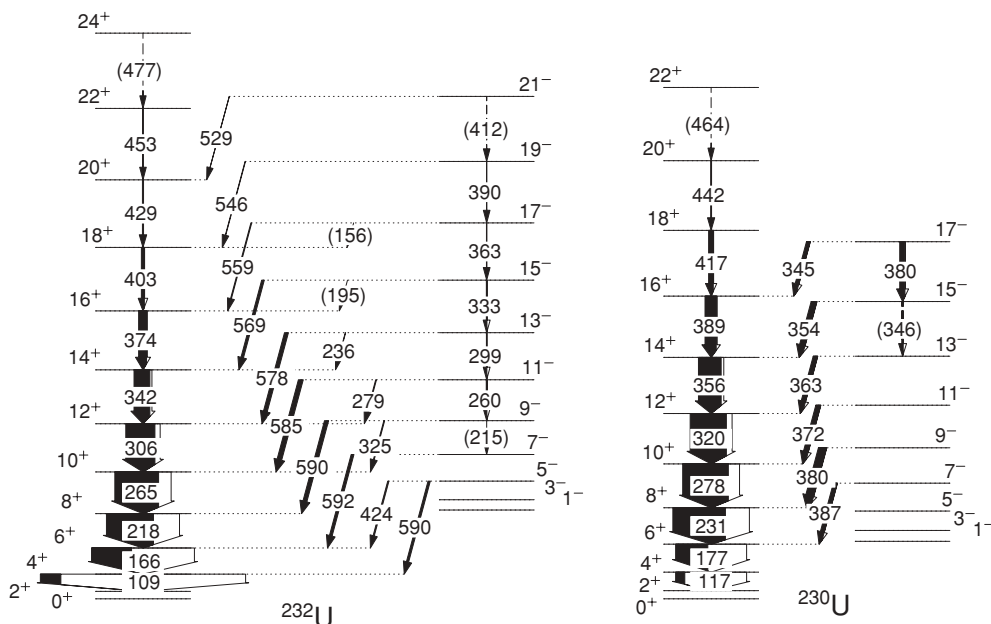


FIG. 2. Partial level schemes of $^{230,232}\text{U}$ showing γ rays observed in the present work. Arrow width indicates relative intensity.

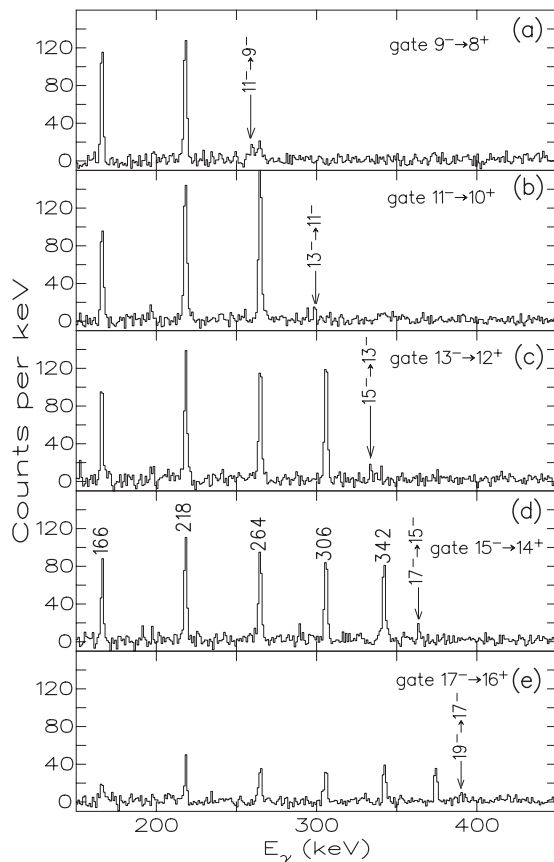


FIG. 3. Coincidence spectra gated by selected $E1$, $I \rightarrow I - 1$ transitions in ^{232}U showing in-band $E2$ transitions.

and ^{230}Th , although ^{228}Ra is only identical with these two at lower values of spin.

In the second plot, (Fig. 6), the magnitudes $|D_0|$ of the electric dipole moments of Ra, Th, and U nuclei, averaged over high-spin values (typically $I > 7$) are compared as a function of neutron number. Our value of $|D_0|$ for ^{230}U is in good agreement with the tentative value obtained by Ackermann *et al.* [12]. In the same figure, we have also plotted the value of the staggering function

$$\Delta E(I) = [E(I) - 0.5(E(I-1) + E(I+1))]/E_{2^+}$$

evaluated for $I = 11$. This function gives a measure of the splitting between the positive- and negative-parity bands and is normalized by the energy of the first 2^+ state [3]. An

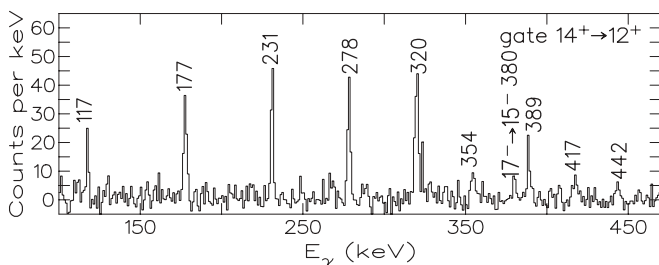


FIG. 4. Coincidence spectrum gated on the 356-keV transition in ^{230}U showing the in-band, 380-keV $E2$ transition.

TABLE I. $B(E1)/B(E2)$ ratios and dipole moments as a function of spin for the negative-parity bands of $^{230,232}\text{U}$.

Isotope	$I_i (\hbar)$	$\frac{B(E1; I_i \rightarrow I_i - 1)}{B(E2; I_i \rightarrow I_i - 2)}$ (fm^{-2})	$ D_0 $ ($e\text{-fm}$)
^{230}U	15	$> 9 \times 10^{-8}$	> 0.15
	17	$1.0(8) \times 10^{-7}$	0.16(6)
^{232}U	9	$> 55 \times 10^{-8}$	> 0.12
	11	$3.3(5) \times 10^{-8}$	0.095(9)
	13	$9.5(20) \times 10^{-8}$	0.161(19)
	15	$7.4(10) \times 10^{-8}$	0.144(12)
	17	$4.8(7) \times 10^{-8}$	0.116(11)
	19	$2.1(7) \times 10^{-8}$	0.077(13)
	21	$> 4 \times 10^{-8}$	> 0.12

anticorrelation is observed between the staggering function and the dipole moments. Thus, the region traditionally associated with octupole deformation, $N < 138$, is characterized by large dipole moments and negligible splitting between the bands, while the vibrational region, $N > 140$, is characterized by large splittings and small dipole moments.

It can be seen that the $N = 138$ and $N = 140$ isotones lie near a transitional region, where the large dipole moments of nuclei with $N < 138$ give way to the small values above $N = 140$ in the vibrational region. Just as the energies of the negative-parity bands of $^{230,232}\text{U}$ are similar to those of their isotones, so too are the values of $|D_0|$. An important point in connection with the possibility that the negative-parity bands of $^{230,232}\text{U}$ could be tetrahedral rotors is that the Coulomb excitation of the $N = 138$ isotone ^{226}Ra has been

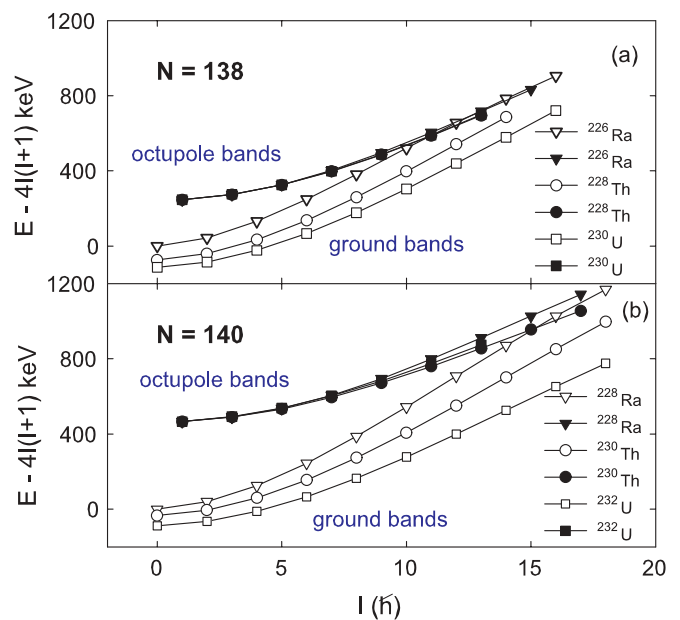


FIG. 5. (Color online) Energies of negative-parity octupole bands (solid symbols) compared with the energies of ground bands (open symbols) for (a) $N = 138$ and (b) $N = 140$. A rigid-rotor reference has been subtracted and the energies of the bands have been aligned with the 1^- states of the octupole bands. Data from Refs. [11,12,19].

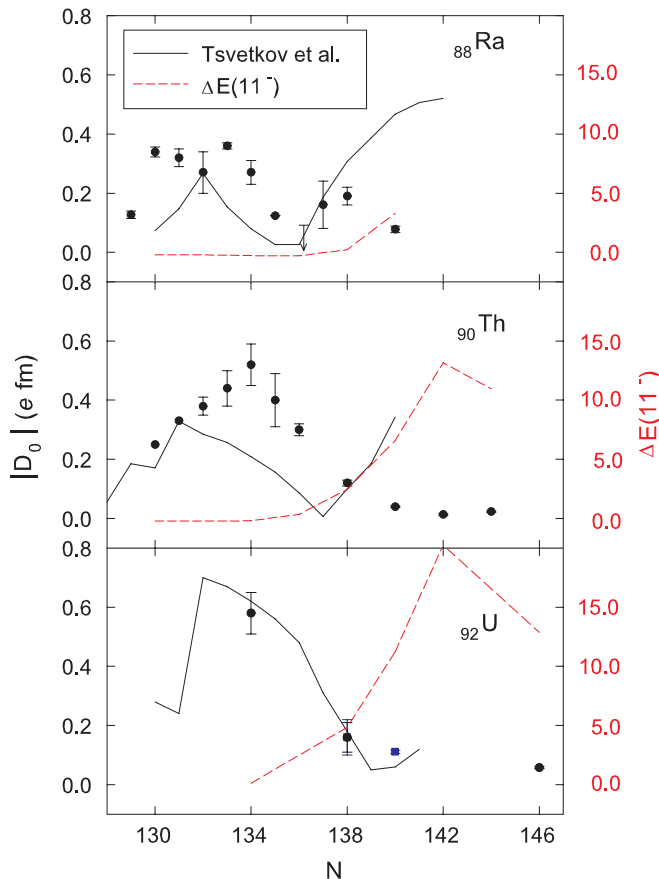


FIG. 6. (Color online) Systematics of experimental electric dipole moments (left hand scale) compared with calculations of Tsvetkov *et al.* [20] for Ra, Th, and U isotopes. Experimental $|D_0|$ (black circles) from the compilations of Refs. [1,21] and [22] ($^{222,226,228}\text{Ra}$, ^{234}Th), [12] (^{232}Th , ^{230}U), [23] (^{226}U), and [24] (^{238}U). Present $|D_0|$ (blue squares). Right hand scale; experimental staggering function $\Delta E(I)$ as described in the text.

comprehensively studied by Wollersheim *et al.* [19]. In this work, quadrupole and octupole moments of both the ground and negative-parity bands were measured and found to be equal, with values of $Q_2 = 750 \text{ fm}^2$ and $Q_3 = 3100 \text{ fm}^3$, respectively. Although the Q_3 value indicates an octupole deformation, the large value of Q_2 is inconsistent with that of a tetrahedral shape. Given the similarity in other properties of the $^{230,232}\text{U}$ bands with those of ^{226}Ra (see Figs. 5 and 6), it would be remarkable if the quadrupole moments of the negative-parity bands $^{230,232}\text{U}$ were zero and hence tetrahedral. We rather conclude that the present data for $^{230,232}\text{U}$ fall inside the

systematics of nuclei lying in the transitional region between those of stable octupole deformation and octupole vibrators.

The present data also allow a plausible explanation, without resorting to the tetrahedral assumption, of why the in-band $E2$ transitions were not observed in the past. In Fig. 5, we see that, with increasing proton number, the negative-parity bands move further away from the yrast line, thus making their population in fusion-evaporation reactions less favorable. Furthermore, with essentially constant $B(E1)/B(E2)$ ratios between the isotones, and fixed $E2$ transition energies, the $E2$ transition rates would remain constant along these isotones, (assuming approximately similar quadrupole moments, as is the case for the ground bands [18]). However, the $E1$ decay energy would increase, causing the $E1$ transition rates to increase as E_γ^3 . Thus, not only would moving the negative-parity band further from the yrast line make it more difficult to populate, but less of its population would be retained within the $E2$ transitions of the band due to the increasing $E1$ decay rate.

Finally, in Fig. 6, we compare the measured values of $|D_0|$ with the recent Skyrme Hartree-Fock calculations of Tsvetkov *et al.* [20]. These calculations are in substantial agreement with earlier theoretical works [21,25] for Ra and Th isotopes. Furthermore, Tsvetkov *et al.* also included triaxial quadrupole and octupole deformations, and extended the calculations of $|D_0|$ to U isotopes, for which they are in good agreement. However, the calculations do not extend heavier than ^{233}U and appear to overestimate the data for the heavier Ra and Th isotopes. It is interesting to note that the calculations of Tsvetkov *et al.* also predict increasing triaxial deformations with increasing spin—qualitatively in agreement with the heart-shaped deformations implied by the octupole condensate picture discussed by Frauendorf [4].

In summary, the close similarity in the energies and electric dipole moments of the negative-parity bands of the $N = 138$ and $N = 140$ isotones suggest a similar underlying structure, which by comparison with ^{226}Ra in particular, imply an octupole vibrational picture at odds with the tetrahedral interpretation.

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