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Enhanced $E1$ transitions and $\alpha + {}^{208}\mathrm{Pb}(3^-)$ clustering in ^{212}Po

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We formulate a model for ^{212}Po, based on the coupled-channels of $\alpha + {}^{208}\mathrm{Pb}(0^+)$ and $\alpha + {}^{208}\mathrm{Pb}(3^-)$ in which the α-Pb interaction contains scalar, quadrupole, and octupole terms. The model reproduces the recently observed enhanced $E1$ transitions from the several new negative-parity levels to the yrast states. Because these data are hard to understand in the shell model, this success gives a strong support for a unique role of $\alpha + {}^{208}\mathrm{Pb}(3^-)$ clustering in ^{212}Po.

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Alpha clustering plays a vital role in the structure of light nuclei [1]. Its importance or persistence has continuously been studied in heavier nuclei around ^{44}Ti [2]. Whether or not the α clustering persists throughout the nuclear chart is one of the fundamental questions in nuclear many-body systems. The nucleus ^{212}Po serves as a testing ground to answer this question as it contains two neutrons and two protons outside the doubly magic nucleus ^{208}Pb. Though the yrast band structure in ^{212}Po is certainly reproduced well in a shell-model calculation [3], understanding the α-decay width of the ground state and the electric quadrupole ($E2$) transition rates among the band members definitely calls for a model that includes explicitly the degree of α clustering. The large α-decay width can successfully be accounted for in a microscopic model that combines the shell model and the $\alpha + {}^{208}\mathrm{Pb}(0^+)$ cluster model [4]. The enhancement of the α-decay width and the reduced transition rates $B(E2)$ is also explained by phenomenological models that assume the $\alpha + {}^{208}\mathrm{Pb}(0^+)$ cluster structure for the yrast band [5–7].

New information that appears closely relevant to the α clustering has very recently been obtained by the α-transfer reaction of $^{208}\mathrm{Pb}(^{18}\mathrm{O}, {}^{14}\mathrm{C})$ [8]. According to Ref. [8], several negative-parity levels with even angular momentum J have been observed in ^{212}Po at unexpectedly low excitation energies, and moreover they have been found to decay through enhanced electric dipole ($E1$) transitions to the yrast band states with the same J value. The enhancement is on the order of 10^{-3} Weisskopf units (W.u.). These data appear to be hard to understand in the shell model. The excitation energies of first 4^-, 6^-, and 8^- states predicted by the shell model are $E_x = 2.31$, 2.63, and 2.78 MeV, respectively [3], which are compared to the observed values of 1.74, 1.79, and 1.75 MeV [8]. In addition, the experiment reveals the second negative-parity band shifted from the first band by only about 0.2 MeV.

These findings pose an intriguing nuclear structure problem. The purpose of this rapid communication is to show the important role of $\alpha + {}^{208}\mathrm{Pb}(3^-)$ clustering by analyzing how such low-excited negative-parity states appear and what mechanism is responsible for the enhanced $E1$ transitions. The view of an α cluster moving around the ^{208}Pb core leads to only natural-parity states with parity $\pi = (-1)^L$ provided the core remains in its 0^+ ground state, where L is the α-^{208}Pb relative orbital angular momentum. The situation changes, however, if the excitation of ^{208}Pb is allowed. The first excited state of ^{208}Pb, a famous octupole vibrational state, has spin $I = 3$ and negative parity. An α-cluster motion around the 3^- core can produce both natural- and unnatural-parity states.

The state of ^{212}Po is described in an $\alpha + {}^{208}\mathrm{Pb}$ coupled-channels approximation:

$$\Psi_{JM} = \sum_{IL} C^J_{IL} \psi_{ILJM}, \tag{1}$$

$$\psi_{ILJM} = \phi(\alpha)[\phi_I(\mathrm{Pb}) Y_L(\hat{\boldsymbol{r}})]_{JM} \chi^J_{IL}(r), \tag{2}$$

where C^J_{IL} is an expansion coefficient and $\chi^J_{IL}(r)$ in the basis ψ_{ILJM} is a normalized radial function for the relative motion. The $I = 0$ component is considered small in the negative-parity states because its energy is considerably high [7], and it is neglected in what follows. This is reasonable in view of our purpose because that component gives no contribution to the unnatural negative-parity states. It should be noted that the $E1$ transition occurs only through the $I = 3$ components in the present model. The model predicts two 0^+, one 1^+, and four 2^+ states, in agreement with experiment.

The system is described with the Hamiltonian

$$H = T + V + H_C, \tag{3}$$

where T is the kinetic energy between α and ^{208}Pb, and H_C the intrinsic Hamiltonian for the Pb core and α. The interaction potential V between α and ^{208}Pb is assumed to contain scalar, quadrupole, and octupole terms:

$$V = V_0(r) + V_2(r)(Y_2(\mathrm{Pb}) \cdot Y_2(\hat{\boldsymbol{r}})) + V_3(r)(Y_3(\mathrm{Pb}) \cdot Y_3(\hat{\boldsymbol{r}})). \tag{4}$$

Here $(Y_l(\mathrm{Pb}) \cdot Y_l(\hat{\boldsymbol{r}}))$ stands for the scalar product. The scalar term V_0 acting on the diagonal channels represents the dominant part of the interaction including the Coulomb potential. In contrast to V_0, the octupole term couples the channels between $I = 0$ and $I = 3$, while the quadrupole term couples the channels among $I = 3$ only.

Assuming that the reduced matrix element involving the Pb wave functions is given as

$$\langle\phi_{I'}(\mathrm{Pb})||Y_\ell(\mathrm{Pb})||\phi_I(\mathrm{Pb})\rangle = \kappa_I\kappa_{I'}\sqrt{\frac{(2\ell+1)(2I+1)}{4\pi}}\langle\ell 0I0|I'0\rangle, \quad (5)$$

with an appropriate constant κ_I that characterizes the wave function $\phi_I(\mathrm{Pb})$, we obtain the matrix element of H in the basis (2):

$$\langle\psi_{I'L'JM}|H|\psi_{ILJM}\rangle = E^J_{IL}\delta_{I,I'}\delta_{L,L'} + \sum_{\ell=2,3}(-1)^{\ell+I+L}\sqrt{(2I+1)(2L'+1)} \times\langle I0I'0|\ell 0\rangle\langle L0L'0|\ell 0\rangle U(I\ell JL';I'L)\beta^J_{\ell II'}, \quad (6)$$

with

$$E^J_{IL} = \langle\chi^J_{IL}|T_L + V_0|\chi^J_{IL}\rangle + \epsilon_I, \quad (7)$$

$$\beta^J_{\ell II'} = (\kappa_{I'}/4\pi)\langle\chi^J_{I'L'}|V_\ell|\chi^J_{IL}\rangle, \quad (8)$$

where $T_L = \langle Y_{LM}(\hat{\boldsymbol{r}})|T|Y_{LM}(\hat{\boldsymbol{r}})\rangle$, ϵ_I is the excitation energy of ^{208}Pb(I), and U is a Racah coefficient in unitary form. As noted before, the diagonal matrix element of the quadrupole term is not included in the calculation because the term is assumed to be responsible only for the coupling among the $I = 3$ components. We suppress constant energy from H_C in Eq. (6) and measure the energy from the ground state of ^{212}Po.

Even though the potential form factor $V_\ell(r)$ is obtained in some way, solving the coupled-channels problem would be complicated involving the unknown functions χ^J_{IL}. It is possible to avoid this complication by assuming that those χ^J_{IL} which are expected to belong to the same number of oscillator quanta, $Q = 2N + L$, are virtually common for all L [9], where N is the node number of the oscillator function. This approximation is basically due to the Pauli principle. In the lowest shell-model configurations, the four nucleons altogether have $Q = 22$. Since they are assumed to form a $0\hbar\omega$ α cluster, all the oscillator quanta are distributed solely to the $\alpha + {}^{208}\mathrm{Pb}(0^+)$ relative motion. Hence $\chi^{J=L}_{0L}$ belonging to $Q = 22$ are assumed to be independent of L. Similarly, χ^J_{3L} with odd L are generated from $Q = 21$ (not from $Q = 23$) [10] because exciting the Pb(0^+) core to the 3^- state costs at least $1\hbar\omega$, and these χ^J_{3L} are taken to be the same for different values of J and L. Under this approximation the Hamiltonian matrix (6) for the positive parity is determined completely by six parameters, A^+_0, B^+_0, A^+_3, B^+_3, β^+_2, and β^+_3, where the first four parameters specify E^J_{IL} as

$$E^J_{0L} = A^+_0 + B^+_0 L, \quad E^J_{3L} = A^+_3 + B^+_3 L, \quad (9)$$

and β^+_2 and β^+_3 stand for β^J_{233} and $\beta^J_{303} = \beta^J_{330}$, respectively. Similarly, χ^J_{3L} for the negative parity all belong to $Q = 22$, and the Hamiltonian matrix is specified by three parameters, A^-_3, B^-_3, and β^-_2.

The six parameters for the positive parity are determined to fit the low-lying levels [11], 0^+(0, 1.801), 1^+(1.621), 2^+(0.727, 1.513, 1.679, 1.806), 4^+(1.133), 6^+(1.356), 8^+(1.476), and 10^+(1.834), where the numbers in the parentheses denote the excitation energies in MeV. The resulting values are, in units of MeV,

$$\beta^+_2 = -0.208, \quad \beta^+_3 = -0.1592, \quad A^+_0 = 0.1156, \quad B^+_0 = 0.412, \quad A^+_3 = 1.385, \quad B^+_3 = 0.078, \quad (10)$$

which give the χ^2 value per datum, $\chi^2/N = 0.0025$ MeV^2. The ground-state energy turns out to be only 6 keV above the experiment. In what follows, we ignore this small discrepancy. The β^+_2 value is about 60% of the one used in Ref. [9]. If a rotational energy, $L(L+1)$, is used in Eq. (9) instead of the linear term, the fit to the experiment becomes worse.

Determination of the parameters for the negative parity is hampered by the fact that there are not so many confirmed negative-parity states and that not all the low-lying negative-parity states may be observed yet. We find that the choice of $\beta^-_2 = \beta^+_2$ produces the lowest 4^-, 6^-, and 8^- states at the energies close to experiment. The optimal parameters are determined by fitting the states, 4^-(1.744, 1.946), 6^-(1.788, 2.017), and 8^-(1.753). The values obtained are

$$\beta^-_2 = -0.182, \quad A^-_3 = 1.92, \quad B^-_3 = 0.012, \quad (11)$$

which give $\chi^2/N = 0.000\,59$ MeV^2. The quadrupole strength for the negative parity is about 10% weaker than the one for the positive parity. This is reasonable because χ^J_{IL}'s are different for the positive- and negative-parity states.

In the present model, both $E1$ and $E2$ transition operators are reduced to the one contributed by the α-Pb relative motion, that is, the $E\lambda$ ($\lambda = 1, 2$) operator is proportional to $r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$. The transition rate is calculated using the formula

$$\langle\psi_{I'L'J'}||r^\lambda Y_\lambda(\hat{\boldsymbol{r}})||\psi_{ILJ}\rangle = \delta_{I,I'}(-1)^{L+L'}\sqrt{\frac{(2J'+1)(2L+1)}{4\pi}}\langle L0L'0|\lambda 0\rangle \times U(ILJ'\lambda;JL')\langle\chi^{J'}_{IL'}|r^\lambda|\chi^J_{IL}\rangle. \quad (12)$$

As mentioned already, the $E1$ matrix element has a nonvanishing contribution between the $I = 3$ components only, and it requires the radial matrix element $D = \langle\chi^{J'}_{3L'}|r|\chi^J_{3L}\rangle$. The $E2$ operator connects separately both $I = 0$ and $I = 3$ components, and its matrix element is specified by two factors, $Q_0 = \langle\chi^{L'}_{0L'}|r^2|\chi^L_{0L}\rangle$ and $Q_3 = \langle\chi^{J'}_{3L'}|r^2|\chi^J_{3L}\rangle$. We assume these radial factors to be independent of the angular momentum labels, following the argument made for the Hamiltonian matrix element. The $B(E2; J \to J-2)$ rate of the yrast band can then be expressed as

$$B(E2; J \to J-2) = e^2|a_0Q_0 + a_3Q_3|^2, \quad (13)$$

where the coefficients a_0 and a_3 indicate the contributions of the $I = 0$ and $I = 3$ components, respectively.

Figures 1 and 2 compare the calculated positive- and negative-parity levels with $J \leqslant 11$ to experiment. The levels obtained above the excitation energy of 2.5 MeV are not shown. All the known positive-parity states are reproduced very well. The theory predicts four states with 3^+, 5^+, 4^+, and 7^+ below the 1^+ state and a 3^+ state by 46 keV above it. Four α-decaying levels are observed around the 1^+(1.621) state

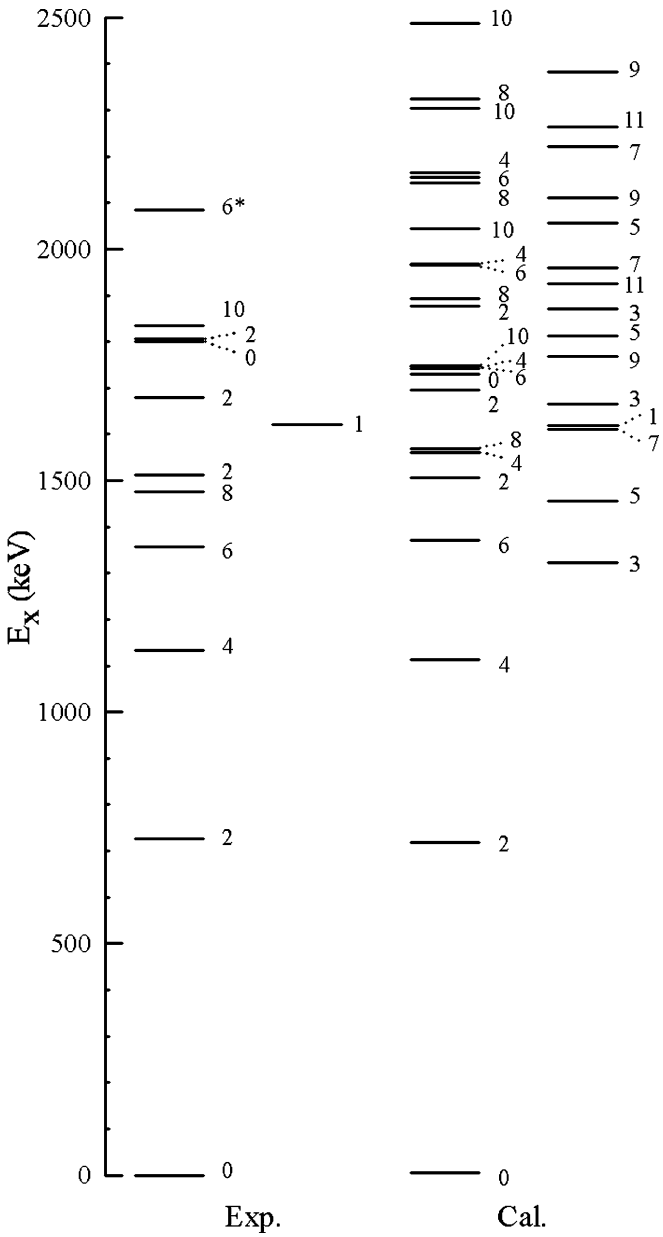

FIG. 1. Positive-parity levels of ^{212}Po. The natural- and unnatural-parity states are shown in the left and right columns, respectively. The asterisk indicates that no parity confirmation has been made yet. See Refs. [8,11] for experiment.

but their J^π values are unknown [11,12]. Because the lowest negative-parity level is at 1.68 MeV in our model, it is likely that the J^π values of these observed levels are 3^+, 5^+, 7^+, and 3^+. The possibility of 4^+ is discarded because then it could decay to the yrast band members by γ radiation, which is in contradiction to experiment. The remaining other states cannot decay to the yrast band members by $E2$ (though the $E2$ decay among them could be, in principle, possible) but dominantly α decay to the 3^- state of ^{208}Pb. As seen in Fig. 2, the calculation predicts many more negative-parity states at low excitation energies in addition to the double negative-parity bands. For example, three 10^- states are predicted at 1.80, 2.04, and 2.27 MeV but the observed 10^- state appears only at

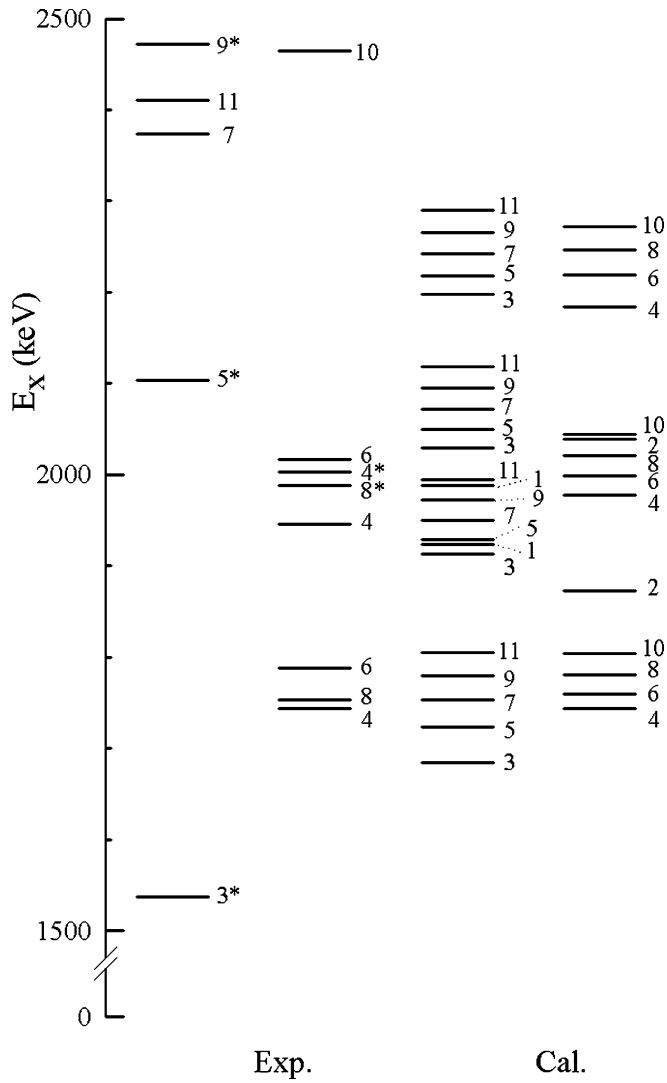

FIG. 2. The same as Fig. 1 but for the negative parity.

2.465 MeV. It is hard to identify which 10^- state corresponds to the experimental one. Thus a comparison between theory and experiment has ambiguity in the $10^- \to 10^+$ $E1$ transition. No such ambiguity occurs, however, for the $E1$ transitions from the 4^-, 6^-, and 8^- states.

Table I lists the probability of finding the (IL) component for the positive-parity states. It should be noted that the $I = 0$ channel does not occupy a main component in the high $J(\geqq 4)$ members of the yrast band, which is in a sharp contrast to the α-cluster description employed in Refs. [5–7]. As will be shown later, the structure change in the main component of the yrast band leads to improvement in the prediction of the $B(E2)$ values among the band members.

To evaluate the $B(E2)$ rate, we set $Q_I = \frac{3}{5}R_I^2$ with an appropriate radius R_I. For $I = 0$ we take $R_0 = 8.5$ fm whose choice is consistent with the one used to evaluate the α-decay width [4,5,7]. For $I = 3$ we take $R_3 = 7.16$ fm as estimated from the radius of ^{212}Po. Table II lists the resulting $B(E2)$ values together with a_0 and a_3 values. It should be noted that both components contribute destructively to the $B(E2)$ value.

TABLE I. Probability, given in %, of finding the (IL) component in the positive-parity state of ^{212}Po.

J^π	(IL)				
	$(0\,J)$	$(3\,J-3)$	$(3\,J-1)$	$(3\,J+1)$	$(3\,J+3)$
0_1^+	93.6				6.4
0_2^+	6.4				93.6
2_1^+	77.8		8.5	7.1	6.7
2_2^+	7.9		81.8	3.6	6.7
2_3^+	4.0		6.8	89.3	0.0
2_4^+	10.4		3.0	0.1	86.6
4_1^+	23.9	33.9	28.3	10.0	3.8
6_1^+	7.9	44.7	34.5	10.7	2.2
8_1^+	3.6	48.1	36.2	10.7	1.5
10_1^+	2.0	49.5	36.7	10.6	1.2

Despite this cancellation, the $B(E2)$ values are still enhanced compared to the W.u., except for the $4^+ \to 2^+$ transition. The calculated $B(E2)$ value from the 8^+ state is much closer to the recent, accurate experimental value [8] than the ones obtained in the pure $\alpha + {}^{208}\mathrm{Pb}(0^+)$ model [5,7]. Though the calculation of Ref. [6] is also based on a similar $\alpha + {}^{208}\mathrm{Pb}(0^+)$ model, the adopted radius parameter R (6.74 fm) for the α-Pb potential is considerably smaller than the others, thus leading to the smaller $B(E2)$ values. The present model reduces the $B(E2)$ rates overestimated in the $\alpha + {}^{208}\mathrm{Pb}(0^+)$ cluster model by introducing the excitation of the Pb core to the 3^- state. In the case of the $B(E2; 4^+ \to 2^+)$ rate, both contributions almost cancel each other, which is a consequence of the transient character of the 4^+ member as indicated in Table I.

The $0_2^+(1.801)$ state is known to decay to the ground state by the monopole $(E0)$ transition. The decay rate is calculated easily if the charge radius of the $^{208}\mathrm{Pb}(3^-)$ state is assumed to be the same as that of its ground state. The resulting $E0$ matrix element reads

$$|\langle 0_2^+|E0|0_1^+\rangle| = e\sqrt{P_0 P_3}|Q_0 - Q_3|, \tag{14}$$

where P_0 and P_3 are the probabilities of the $I = 0$ and $I = 3$ components in the ground state. The $E0$ matrix element turns out to be 5.13 e fm^2, which is compared to the experimental value of 1.9 e fm^2 [13]. It is noteworthy that this matrix element is a measure of the admixture of the $I = 3$ component as well as the difference between Q_0 and Q_3.

TABLE III. $B(E1; J^- \to J_1^+)$ values of ^{212}Po in units of 10^{-3} W.u. The experimental data are taken from Ref. [8].

J^-	J^+	$B(E1)$		J^-	J^+	$B(E1)$	
		Cal.	Exp.			Cal.	Exp.
2_1^-	2_1^+	0.64		6_3^-	6_1^+	0.58	
2_2^-	2_1^+	0.12		8_1^-	8_1^+	1.80	20 ± 10
4_1^-	4_1^+	4.81	2.5 ± 0.7	8_2^-	8_1^+	0.90	
4_2^-	4_1^+	1.09	1.1 ± 0.3	8_3^-	8_1^+	0.35	
4_3^-	4_1^+	0.70		10_1^-	10_1^+	1.22	1.8 ± 0.5
6_1^-	6_1^+	2.88	6.6 ± 1.2	10_2^-	10_1^+	0.66	
6_2^-	6_1^+	1.19	1.9 ± 0.6	10_3^-	10_1^+	0.22	

Table III lists the $B(E1)$ values calculated using $D = \frac{3}{4}R$ with $R = 7.6$ fm, in comparison with experiment. We choose R to be approximately an average of R_0 and R_3 that are used to estimate the $B(E2)$ rates because $\chi^J_{I=3\,L}$ for the negative parity extends further than the one for the positive parity. As seen from the table, the order of magnitude of the $B(E1)$ value is about 10^{-4} to 10^{-3} W.u., which is in fair agreement with experiment [8]. The $B(E1)$ value is proportional to R^2 but our result should not change at least qualitatively within a reasonable choice of R. It is thus possible to obtain the enhanced $E1$ strength of the order of 10^{-3} W.u. by including the 3^- excitation of the Pb core. The largest $B(E1)$ value observed for the $8^-(1.751) \to 8^+(1.475)$ transition, though the error bar is significantly large, is still 1 order of magnitude larger than the calculation.

To summarize, we have included the ground and 3^- states of ^{208}Pb in the $\alpha +$ Pb model to describe the low-lying levels of ^{212}Po. The α-Pb interaction contains the scalar, quadrupole, and octupole terms. Solving the coupled equations is greatly simplified by introducing the assumption that comes from the Pauli principle. The calculated spectra are in fair agreement with experiment. Especially, the double band structure of the negative-parity states is very well reproduced. The theory can account for the enhanced $E1$ transitions from these negative-parity states to the yrast states with the same spin. With the effect of channel coupling, the $E2$ transition rates, which are too large in the $\alpha + {}^{208}\mathrm{Pb}(0^+)$ model, are found to improve considerably. The $\alpha + {}^{208}\mathrm{Pb}(3^-)$ clustering is concluded to play a vital role in reproducing the enhanced $E1$ transitions of the newly observed negative-parity states.

TABLE II. $B(E2; J \to J-2)$ values, in units of W.u., between the yrast band members in ^{212}Po. See Eq. (13) for a_0 and a_3.

J	a_0	a_3	$B(E2)$					
			Cal.				Exp.	
			Present	[5]	[6]	[7]	[8]	[11]
2	−0.470	0.0645	4.51	7.41	4.58	7.71		
4	0.284	−0.253	0.27	10.3	6.32	10.8		
6	0.0951	−0.571	2.40	10.6	6.50	11.2	13.5 ± 3.6	3.9 ± 1.1
8	0.0375	−0.662	4.67	10.0	6.12	10.7	4.6 ± 0.09	2.30 ± 0.9
10	−0.0190	0.694	5.60	9.16	5.42	9.71		2.2 ± 0.6

It will be interesting to explore this type of interplay between the clustering and enhanced $E1$ transition in other nuclei such as Te isotopes ($Z = 52$) as well as ^{225}Ra and ^{225}Ac, as noted in Ref. [8].

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[10] This argument is exemplified in the case of the low-lying levels in ^{20}Ne, a system containing two neutrons and two protons outside the doubly magic nucleus ^{16}O, analogously to ^{212}Po. The yrast levels are described well in terms of the SU(3) shell model or $\alpha + {}^{16}$O cluster model. The $\alpha + {}^{16}$O(0^+) cluster model constrains its α-^{16}O relative motion to have $Q = 8$, leading to the SU(3) (80) state. If we use the $\alpha + {}^{16}$O(3^-) model and represent the ^{16}O(3^-) state with the SU(3) (21) one-particle, one-hole configuration, Q could be 7, and this $\alpha + {}^{16}$O(3^-) cluster model produces the SU(3) coupled state of (21) $\times$ (70), which contains not only (80) but also (42).
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