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Microscopic examination of N_pN_n dominance in the evolution of nuclear structure

M. L. Liu, Y. H. Zhang, X. H. Zhou, J. J. He, F. Ma, Y. D. Fang, and W. T. Guo
Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, People's Republic of China
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The proton-neutron interaction in determining the evolution of nuclear structure has been studied by using the Brillouin-Wigner perturbation expansion. The particle-hole and particle-particle p-n interactions are unifiedly described in the theory. The obtained formulas of level energies and excitation energies scaled in the small- and large-N_pN_n limits can well explain the linearity of the extracted proton-neutron interaction energies and the attenuation of the 2_1^+ excitation energies against the valence nucleon product $N_pN_n$for five mass regions from $A = 100$–200.

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One of the most interesting discoveries in nuclei is the evolution of nuclear structure from the spherical multinucleon shell model or vibrational states to the deformed collective rotation as the nucleus moves away from the closed shells on the nuclear chart. It has been long recognized that the proton-neutron (p-n) interaction is primarily responsible for configuration mixing in nuclei and therefore plays a key role in the onset of nuclear deformation and collectivity [1,2]; Federman and Pittel showed by the shell model calculation the importance of the p-n interaction in developing nuclear deformation in heavy nuclei [3]. Since the product N_pN_n of valence nucleon numbers N_p and N_n roughly estimates the integrated p-n interaction strength, the observables, such as the excitation energies $E_x(2_1^+)$ of the first 2^+ states, the ratios $\frac{E_x(4_1^+)}{E_x(2_1^+)}$ and so on, are smooth functions of N_pN_n and thus the product N_pN_n should be dominated in the evolution of nuclear structure [4–6]. Here $N_p$and N_n are counted as the numbers of particles above the highest filled major shells or holes if the Fermi levels are beyond the midshell within the highest major shells [4–6]. The realistic p-n interaction has two principal components. The monopole term is responsible for the nuclear bulk properties and the residual part obtained after extracting the monopole term is dominated by the long-range quadrupole interaction [7]. The extracted monopole and evaluated quadrupole p-n interaction energies, which displayed approximate linearity and saturation phenomenon, gave a direct support to the N_pN_n scheme [6,8]. Although phenomenological studies of the N_pN_n scheme have been carried out over two decades, a microscopic examination of N_pN_n dominance in the evolution of nuclear structure has so far been missing. In this study, we present the first result of the Brillouin-Wigner perturbation expansion applied to the microscopic study of N_pN_n dominance in the evolution of nuclear level energies, where the particle-hole and particle-particle p-n interactions are unifiedly described in the theory [9].

We assume that the total Hamiltonian H of the even-even nucleus with N_p and N_n valence nucleons can be separated into two parts $H = H_0 + V$. One part H_0 is the sum of the valence-proton and valence-neutron Hamiltonians and the other part V represents the p-n interaction energy. Let eigenstates $|\Psi_i\rangle$ of H_0 with eigenvalues E_i form a complete set, in which $|\Psi_0\rangle$ with eigenvalue $E_J^{(0)}$ is the unperturbed yrast state with angular momentum J. If the particle-particle p-n force is attractive, the particle-hole interaction is repulsive. We therefore adopt the particle-hole (particle-particle) p-n interaction energy to be positive (negative). The Schrödinger's equation takes the forms $\pm(E - H_0)|\Psi\rangle = \pm V|\Psi\rangle$ for the respective particle-hole and particle-particle systems. By using the projection operator $|\Psi_0\rangle\langle\Psi_0|$ and its supplement $P = 1 - |\Psi_0\rangle\langle\Psi_0|$, the Schrödinger's equation can be transformed into an integral form [9]

$$\begin{aligned} |\Psi\rangle &= |\Psi_0\rangle - \frac{1}{H_0 - E} PV|\Psi\rangle, \\ \pm E &= \pm\langle\Psi_0|H_0|\Psi_0\rangle \pm \langle\Psi_0|V|\Psi\rangle. \end{aligned} \tag{1}$$

The perturbed energy of the yrast state can be written as the Brillouin-Wigner perturbation expansion [9]

$$\begin{aligned} \pm E_J &= \pm\langle\Psi_0|H_0|\Psi_0\rangle \pm \sum_{k=0}^{\infty}\langle\Psi_0|V \\ &\quad \times \left(\frac{-1}{H_0 - E_J} PV\right)^k |\Psi_0\rangle = \pm E_J^{(0)} \\ &\quad \pm \sum_{m=1,3,\ldots} E_J^{(m)} \mp \sum_{m=2,4,\ldots} E_J^{(m)}, \end{aligned} \tag{2}$$

where $E_J^{(0)} = \langle\Psi_0|H_0|\Psi_0\rangle$, $E_J^{(m)} = \langle\Psi_0|V(\sum_i \frac{|\Psi_i\rangle\langle\Psi_i|}{E_i - E_J} V)^{m-1}|\Psi_0\rangle$ $(E_i > E_J)$ and the sum is taken over the eigenstates except the yrast one.

When H_0 is dominated by the pairing forces, the eigenstates $|\Psi_i\rangle$ can be approximately described by the seniority $\upsilon = \upsilon_{\max} - i$, where the maximum seniority $\upsilon_{\max} = N_p + N_n$ state has energy zero. Thus, $E_i - E_J$ approximately depends on $N_p + N_n$ rather than N_pN_n. If the p-n interaction energies among the eigenstates $|\Psi_0\rangle$, $|\Psi_i\rangle$, and $|\Psi_l\rangle$ are roughly estimated as $\langle\Psi_0|V|\Psi_0\rangle = \overline{\chi}N_pN_n + \chi_J N_pN_n$, $\langle\Psi_0|V|\Psi_i\rangle = \sqrt{\chi_J\chi_i}N_pN_n$ and $\langle\Psi_l|V|\Psi_i\rangle = \sqrt{\chi_l\chi_i}N_pN_n$ ($\chi \geqslant 0$, $i, l \neq 0$), then the perturbed energy can be denoted by

$$\begin{aligned} \pm E_J &= \pm E_J^{(0)} \pm \overline{\chi}N_pN_n \pm \chi_J N_pN_n \\ &\quad \times \left[\sum_{m=0,2,\ldots}(\lambda' N_pN_n)^m - \sum_{m=1,3,\ldots}(\lambda' N_pN_n)^m\right], \end{aligned} \tag{3}$$

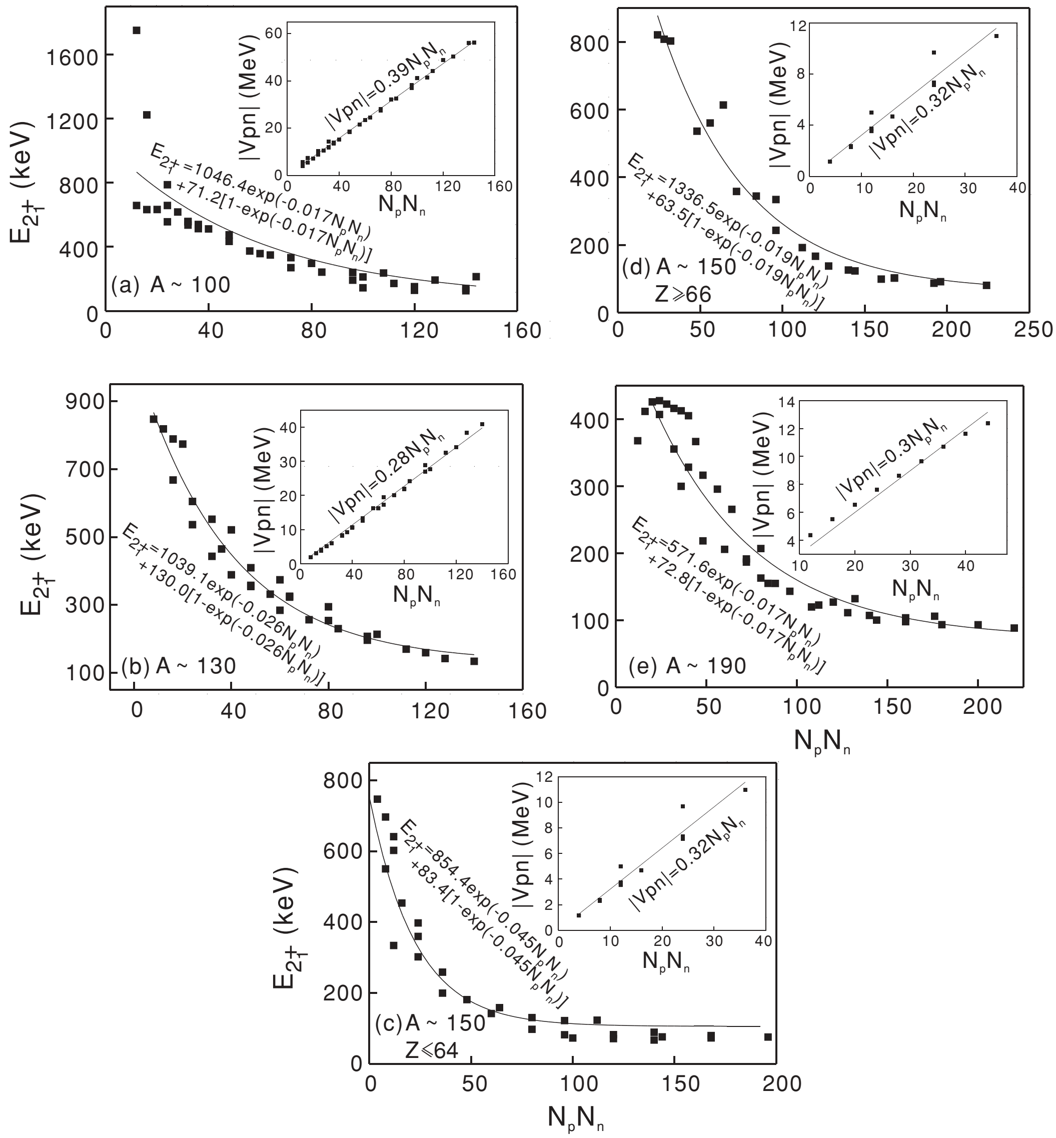

FIG. 1. Plots of the proton-neutron interaction energies and the first 2^+ excitation energies against the valence nucleon product N_pN_n for five mass regions from $A = 100$–200.

where $\lambda' = \sum_i \frac{\chi_i}{E_i - E_J}$. Here $\overline{\chi}N_pN_n$ and $\chi_J N_pN_n$ represent the monopole and quadrupole *p-n* interaction energies, respectively. As $\lambda' N_pN_n < 1$, the geometric series is convergent and the $\pm\overline{\chi}N_pN_n \pm \frac{\chi_J N_pN_n}{1+\lambda' N_pN_n}$ can therefore describe the linearity and saturation phenomenon of the *p-n* interaction energy.

However, the infinite power series is not convergent any more as $\lambda' N_pN_n \geqslant 1$. In fact, the linear scaling of the *p-n* interaction energy with particle number $\langle\Psi_i|V|\Psi_i\rangle = \chi_i N_pN_n$ is too simple to reflect the saturation phenomenon. The saturated phenomenon of the *p-n* interaction energy suggests that χ_i or $\sum_i \frac{\chi_i}{E_i-E_J}$ also depends on N_pN_n. We proceed by assuming that the expression $\sum_i \frac{\chi_i}{E_i-E_J} N_pN_n$ can also be expanded as an infinite power series in λN_pN_n, where λ ($\geqslant 0$) depends on J, E_J, and $N_p + N_n$. By substituting in series (3) for $\sum_i \frac{\chi_i}{E_i-E_J} N_pN_n$, we can obtain a new infinite power series for $\pm E_J$. In order to sufficiently assure the global convergence of the infinite power series for $\pm E_J$, we suggest the m-order term takes the form $\frac{\chi_J}{\lambda}\frac{(\lambda N_pN_n)^m}{m!}$. Thus, the perturbed energy of the yrast state can be written as

$$\begin{aligned}\pm E_J &= \pm E_J^{(0)} \pm \overline{\chi}N_pN_n \pm \frac{\chi_J}{\lambda} \sum_{m=1,3,\ldots} \frac{(\lambda N_pN_n)^m}{m!} \mp \frac{\chi_J}{\lambda} \\ &\quad \times \sum_{m=0,2,\ldots} \frac{(\lambda N_pN_n)^m}{m!} \pm \frac{\chi_J}{\lambda} = \pm\overline{\chi}N_pN_n \pm E_J^{(0)} \\ &\quad \times \exp(-\lambda N_pN_n) \pm \left(E_J^{(0)} + \frac{\chi_J}{\lambda}\right) \\ &\quad \times [1 - \exp(-\lambda N_pN_n)]. \end{aligned} \tag{4}$$

By comparing Eq. (4) and the geometric series, we can hence infer that the concrete form for $\sum_{i\neq 0} \frac{\chi_i}{E_i-E_J} N_pN_n$ is $\lambda N_pN_n + \frac{\lambda N_pN_n \exp(-\lambda N_pN_n)}{1-\exp(-\lambda N_pN_n)} - 1$, which includes a linear term λN_pN_n and a saturated term $\frac{\lambda N_pN_n \exp(-\lambda N_pN_n)}{1-\exp(-\lambda N_pN_n)} - 1$. The exact solution of Eq. (4) is almost impossible. An intermediate

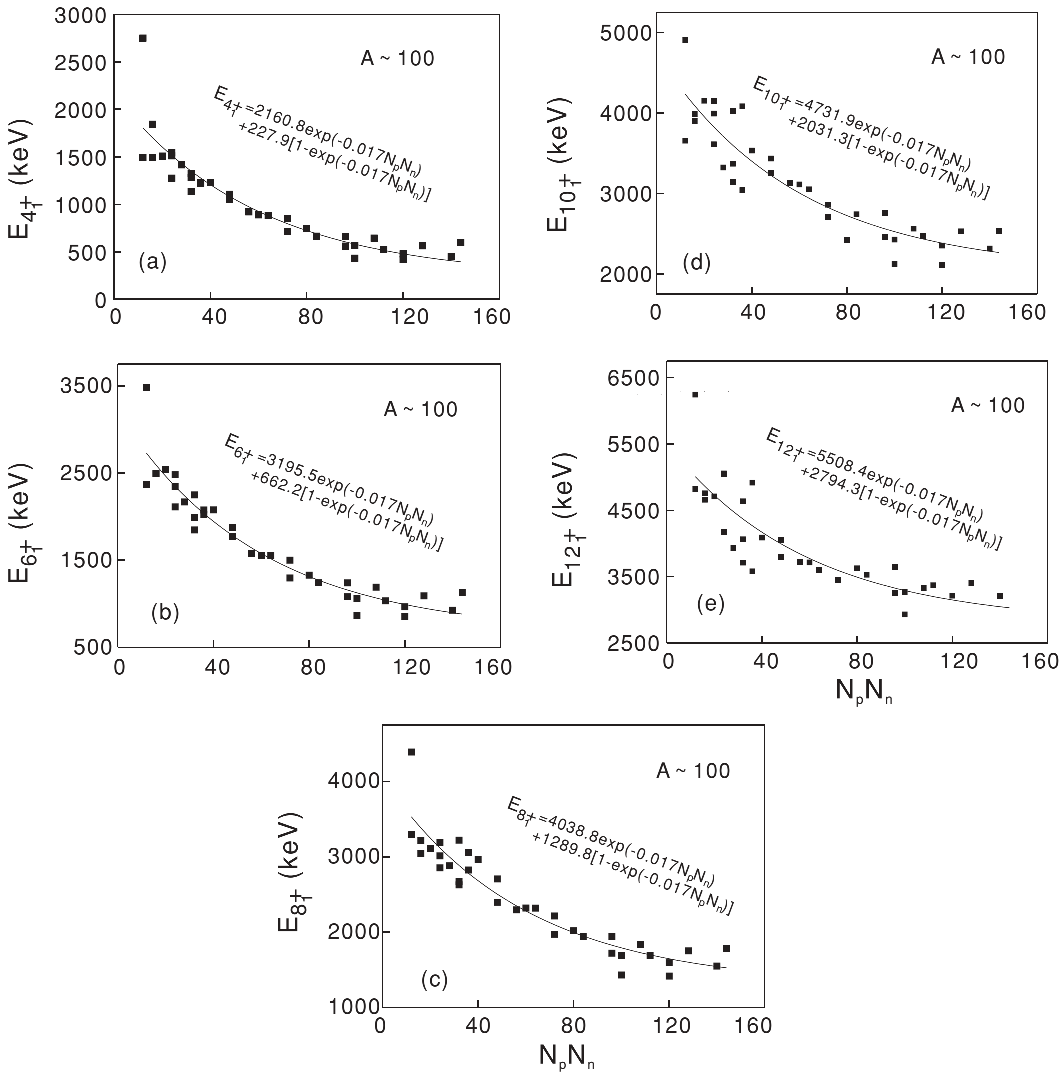

FIG. 2. Plots of the yrast state excitation energies against the valence nucleon product N_pN_n for $A \sim 100$ mass region.

approximation can be obtained by iteration for given initial value $\lambda = \lambda(J, E_J, N_p + N_n)$ until E_J approaches to the minimum. The unperturbed yrast state energy can be decomposed into the sum of the ground state energy and the excitation energy $\pm E_J^{(0)} = \pm E_0^{(0)} + \Delta E_J^{(0)}$. Substituting $\pm E_0^{(0)} + \Delta E_J^{(0)}$ and $\chi_0 \mp \Delta\chi_J$ for $\pm E_J^{(0)}$ and $\pm\chi_J$, we obtain the perturbed energy of the yrast state

$$\pm E_J = \pm\overline{\chi} N_p N_n \pm E_0^{(0)} \exp(-\lambda N_p N_n) \pm \left(E_0^{(0)} + \frac{\chi_0}{\lambda}\right) \times [1 - \exp(-\lambda N_p N_n)] + \Delta E_J^{(0)} \exp(-\lambda N_p N_n) + \left(\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda}\right)[1 - \exp(-\lambda N_p N_n)], \tag{5}$$

which is decomposed into the sum of the ground state energy and the excitation energy. In the small- and large-N_pN_n limits, $\Delta E_J^{(0)}$ and $\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda}$ represent the pairing-force-like and rotational spectra, respectively. If $\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda}$ is replaced by $\frac{\hbar^2}{2\Im} J(J+1)$, then the excitation energy becomes

$$E_x(J) = \Delta E_J^{(0)} \exp(-\lambda N_p N_n) + \frac{\hbar^2}{2\Im} J(J+1)[1 - \exp(-\lambda N_p N_n)], \tag{6}$$

where $\Im$ is the moment of inertia. It is clear from the replacement $\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda} = \frac{\hbar^2}{2\Im} J(J+1)$ that the decomposition of χ_J into $\chi_0 \mp \Delta\chi_J$ is to ensure the p-n interaction $\frac{\Delta\chi_J}{\lambda}$ effectively counteracting the like-nucleon correlations $\Delta E_J^{(0)}$, which is crucial in the development of nuclear rotation.

We review the N_pN_n schemes of five mass regions from $A = 100$–200 [5,8], shown in Fig. 1. The data are taken from Refs. [10,11]. The excitation energies of the first 2^+ states display exponential attenuation and can be well fitted by $\Delta E_2^{(0)} \exp(-\lambda N_p N_n) + \frac{\hbar^2}{2\Im} 2(2+1)[1 - \exp(-\lambda N_p N_n)]$. It is ~ 1.0 MeV for one broken-pair state of nucleus near closed shells and descends to $\approx$100 keV for a typical rotor by the p-n interaction counteracting the like-nucleon correlations. The p-n interaction energy for the ground

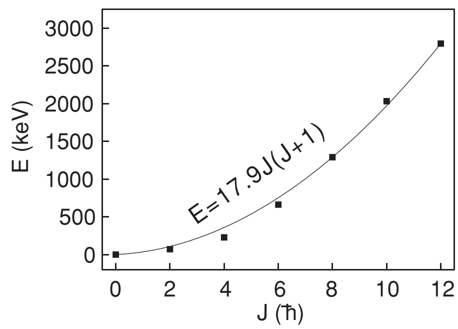

FIG. 3. The yrast state excitation energy in the large-N_pN_n limit as a function of the yrast state spin for $A \sim 100$ mass region.

state is extracted from the difference in binding energies $|V_{pn}(N_pN_n)| = |B(Z_0 + N_p, N_0 + N_n) + B(Z_0, N_0) - B(Z_0, N_0 + N_n) - B(Z_0 + N_p, N_0)|$, where (Z_0, N_0) specify the nearest magic numbers. In principle, the p-n interaction energy should be described by $\overline{\chi} N_pN_n + \frac{\chi_0}{\lambda}[1 - \exp(-\lambda N_pN_n)]$. However, the p-n interaction energy indicated in Fig. 1 approximately exhibits a linearly increasing trend against the valence nucleon product N_pN_n. This implies that the saturation term is smaller in magnitude than the linear one. The p-n interaction amounts to a maximum value of 390 keV for the spin-orbit-partner $g_{9/2}$ proton and $g_{7/2}$ neutron in the 100 mass region and goes to the minimum value of 280 keV for $A \sim 130$ nuclei, in agreement with the estimates in Ref. [5].

In order to test the arbitrarily introduced $J(J+1)$ law, we also plotted the data of the other yrast states up to 12^+ for the five mass regions. As a typical example, Fig. 2 exhibits the data of $A \sim 100$ nuclei [10]. The N_pN_n schemes of the other mass regions for higher-spin yrast states have similar behavior. The yrast state excitation energies are fitted by the formula $\Delta E_J^{(0)} \exp(-\lambda N_pN_n) + (\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda})[1 - \exp(-\lambda N_pN_n)]$, where we assume that λ is independent on the yrast state J. The fitted curve was obtained by iteration until most of the points fall around the curve. In the large-N_pN_n limit, the excitation energy $\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda}$ as a function of the yrast state spin J is indicated in Fig. 3. It is obvious that the excitation energy $\Delta E_J^{(0)} - \frac{\Delta\chi_J}{\lambda}$ basically obeys the $J(J+1)$ rule.

In summary, we present the first result of the Brillouin-Wigner perturbation expansion applied to the microscopic study of N_pN_n dominance in the evolution of nuclear level energies, where the particle-hole and particle-particle p-n interaction energies are described in a unified way. The linearity and saturation phenomenon of the p-n interaction energies can be seen from the obtained level-energy formula. The obtained formulas of level energies and excitation energies of yrast states can well explain the N_pN_n schemes of five mass regions from $A = 100$–200, i.e., the linearity of the extracted proton-neutron interaction energies and the exponential attenuation of the 2_1^+ excitation energies.

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