

Hidden asymmetry and forward-backward correlations

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A model-independent method of studying the forward-backward correlations in symmetric high-energy processes is developed. The method allows a systematic study of the properties of various particle sources and allows one to uncover asymmetric structures hidden in symmetric hadron-hadron and nucleus-nucleus inelastic reactions.

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I. INTRODUCTION

New data from the CERN Large Hadron Collider (LHC) on soft particle production in pp collisions [1–3] open a new chapter in the long history of this problem, thus reviving some questions that were raised already many years ago. One of these questions concerns forward-backward correlations that were extensively studied in the framework of specific models [4] and shown to be useful in discriminating between various mechanisms of particle production.

In the present article we develop a systematic, model-independent method to study forward-backward correlations and show that it may be effective in addressing the issue (debated since the early seventies) of the number and nature of quasi-independent sources of particles contributing to particle production in various rapidity regions. Specifically, restricting ourselves to symmetric reactions (like pp or Au-Au collisions), we ask what are the contributions from symmetric and asymmetric sources and how do we measure them experimentally. The problem may be interesting since various models differ substantially in this respect.

The simplest hypothesis is to say that there is just one symmetric source. This is the case of the Landau hydrodynamic model [5] and its recent modifications [6] where particle production is governed by the evolution of a fluid. A similar conclusion follows from the simple multiperipheral model [7], which suggests just one symmetric source in the form of the multiperipheral chain. This idea was then reformulated in the parton model [8] and in the bremsstrahlung mechanism [9]. One may of course also consider many symmetric sources.

A more sophisticated possibility, taking into account the color structure of the colliding systems, was formulated in the dual-parton model [10]. Here the number of sources depends on the energy of the collision and on the type of projectiles. For p - p collisions, at relatively low energies there are two asymmetric sources (chains), spanning a valence diquark from one projectile and the valence quark from the other one. For nucleus-nucleus collisions the picture is similar, but the number of asymmetric sources fluctuates, depending on the number of participants in the two colliding nuclei. As energy increases, contributions from symmetric chains, spanning the sea quarks and the antiquarks, come into play.

Simpler ideas have also been put forward. In the wounded nucleon model [11] particles are emitted independently from the two colliding nucleons, thus creating two asymmetric sources [12]. A similar idea, applied to the constituent quarks and diquarks, is proposed in Ref. [13]. In the Fritjof model [14] there are also two sources, essentially two large diffractively produced clusters (each one related to one of the colliding hadrons).

In the present article we consider only symmetric collisions (e.g., p - p or Au-Au). We show that in this case a systematic study of forward-backward correlations allows one to distinguish between the various possibilities listed above and to obtain information about some characteristic properties of the sources.

Studies of forward-backward correlations in specific models have a long history (see, e.g., Refs. [4,10]). Our work was mostly triggered by a recent series of papers by Bzdak [15–18], suggesting that a strong asymmetric component is present not only in p - p [4] and d -Au collisions [12] but also in Au-Au collisions. This observation raises interesting questions about the hydrodynamic evolution of the quark-gluon plasma believed to be produced at the BNL Relativistic Heavy Ion Collider (RHIC) [19].

In the next section we formulate the problem in terms of generating functions. In Sec. III the relations that allow one to test various hypotheses in p - p collisions are given. Symmetric nucleus-nucleus collisions are discussed in Sec. IV. Some comments on recent STAR measurements [20] are given in Sec. V. Our conclusions are listed in the last section.

II. GENERAL FORMULATION

If particles emerge from M independent sources, the generating function for the particle distribution in a phase-space region G is a product of generating functions describing distributions of particles from individual sources.

Restricting the discussion to rapidity spectra,¹ we consider two sources that are asymmetric with respect to $y = y_{c.m.} = 0$ and a third symmetric source. Their generating functions are denoted by ϕ_L , ϕ_R , and ϕ_C .

¹Our discussion applies to any variable symmetric with respect to an axis or a plane.

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We discuss multiplicity distributions in two intervals of rapidity, denoted by Δ_L and Δ_R , situated symmetrically with respect to $y = 0$.

Consider first the sum $[\Delta_L + \Delta_R]$. The generating function of the multiplicity distribution in $[\Delta_L + \Delta_R]$ can be written as

$$\Phi(z; w_L, w_R, w_C) \equiv \sum_n P(n) z^n = [\phi_L(z)]^{w_L} [\phi_R(z)]^{w_R} [\phi_C(z)]^{w_C}, \quad (1)$$

where w_L , w_R , and w_C are numbers of the relevant sources.

Assuming now that the splitting between Δ_L and Δ_R of particles from each source is random (i.e., it follows the binomial distribution), we have for the joint distribution in Δ_L and Δ_R

$$\phi_m(z_L, z_R) \equiv \sum_{n_L, n_R} P_m(n_L, n_R) z_L^{n_L} z_R^{n_R} = \phi_m(p_{Lm} z_L + p_{Rm} z_R), \quad (2)$$

where p_{Lm} and p_{Rm} are probabilities that a particle emitted from the source labeled m ends up in Δ_L or in Δ_R . Thus $p_{Lm} + p_{Rm} = 1$. Symmetry implies

$$\begin{aligned} p_{LL} = p_{RR} &\equiv p_+; & p_{LR} = p_{RL} &\equiv p_- = 1 - p_+; \\ p_{LC} = p_{RC} &= \frac{1}{2}. \end{aligned} \quad (3)$$

Consequently,

$$\begin{aligned} \Phi(z_L, z_R; w_L, w_R, w_C) &= [p_+(z_L + z_R) + p_-(z_L - z_R)]^{w_L} [\phi(p_-(z_L + z_R))]^{w_R} \\ &\times [\phi_C(z_L/2 + z_R/2)]^{w_C}. \end{aligned} \quad (4)$$

For symmetric collisions we have $w_L = w_R \equiv w$ and $\phi_L(z) = \phi_R(z) \equiv \phi(z)$. We also note that the distribution of particles in one of the considered intervals, say Δ_L , is evaluated from $\Psi(z) = \Phi(z_L = z, z_R = 1)$.

From these formulas one can evaluate all moments of the joint distribution in Δ_L and Δ_R , as well as the moments of the distribution in one of the intervals, in terms of the moments of the distributions describing the sources.

When only symmetric or only asymmetric sources are present, one can derive relations between the joint moments (describing the forward-backward correlations) in terms of the moments characterizing the distribution in one of the intervals. These relations provide demanding tests for these hypotheses, allowing one to distinguish between various mechanisms of particle production. When both symmetric and asymmetric sources contribute, the relations allow one to obtain information on distributions characterizing the sources themselves.

In the next two sections we discuss some of these relations.

III. RELATIONS BETWEEN CUMULANTS

In this section we derive relations between the cumulants f_{ik} of the joint distributions in Δ_+ and Δ_- and the cumulants f_i of the distribution in one of the intervals.

In terms of the generating functions $\Phi(z_L, z_R)$ and $\Psi(z)$, the cumulants are defined as

$$\begin{aligned} f_{kl} &= \frac{\partial^{k+l} \{\log \Phi(z_L, z_R)\}}{\partial^k z_L \partial^l z_R} \Big|_{z_L=z_R=1}; \\ f_{i0} \equiv f_i &= \frac{d^i \{\log \Psi(z)\}}{dz^i} \Big|_{z=1}. \end{aligned} \quad (5)$$

Since the logarithm changes the products in $\Phi(z_L, z_R)$ and $\Psi(z)$ into sums, we immediately obtain

$$\begin{aligned} f_{k+l} - f_{kl} &= \frac{1}{2} [p_+^{k+l} + p_-^{k+l} - p_+^k p_-^l - p_+^l p_-^k] \bar{f}_{k+l} \\ &= g_k g_l \epsilon^2 \bar{f}_{k+l}, \end{aligned} \quad (6)$$

where

$$2 \bar{f}_i = \frac{d^i \{\log [\phi(z)]^{2w}\}}{dz^i} \Big|_{z=1} \quad (7)$$

are the cumulants of the particle distribution in $[\Delta_+ + \Delta_-]$ coming from the two asymmetric sources and

$$\epsilon = p_+ - p_-; \quad g_k = \frac{p_+^k - p_-^k}{p_+ - p_-} = \frac{1}{2^{k-1}} \sum_{j=0}^{k/2} \binom{k}{2j+1} \epsilon^{2j}. \quad (8)$$

Note that in Eq. (6) the dependence on the number of sources and the contributions from the symmetric sources drop out.

The cumulants f_i and f_{kl} can be determined from the standard factorial moments $F_{i0} = F_{0i} \equiv F_i$ and from the joint factorial moments in two intervals:

$$\begin{aligned} F_{kl} &\equiv \langle n_L \cdots (n_L - k + 1) n_R \cdots (n_R - l + 1) \rangle \\ &= \frac{\partial^{k+l} \Phi(z_L, z_R)}{\partial^k z_L \partial^l z_R} \Big|_{z_L=z_R=1}. \end{aligned} \quad (9)$$

The relevant relations become rather involved at high orders. The first few are listed in the Appendix.

One sees that, generally, the result depends on two functions (ϕ and ϕ_C) and one parameter (p_+ or p_-). The resulting relations involve not only the observed moments but also the moments of the distributions characterizing the asymmetric sources. Thus, if $p_+ \neq p_-$ (i.e., if the sources are indeed asymmetric), one can— from the measured F_{kl} and F_{k+l} — obtain information about \bar{f}_{k+l} characterizing the distributions of particles from asymmetric sources.

If there are only asymmetric sources we have obviously $\bar{f}_{k+l} = f_{k+l}$, so that all quantities entering the relations (6) can be measured. In other words, in this case, Eq. (6) represents identities between the measurable quantities that must be satisfied if the symmetric sources are not present.

The relation for $k = l = 1$ is of particular interest, because it allows one to determine the parameter $p_+ = 1 - p_-$ and thus to determine the distribution in rapidity of the particles from a single asymmetric source [15]. This determination is, of course, valid only if the two-sources idea is satisfied by data. The other relations can be used to verify this assumption.

When only the symmetric source is present, all moments ϕ_i vanish and we obtain $f_{kl} = f_{k+l}$, implying

$$F_{kl} = F_{k+l}, \quad (10)$$

a really strong constraint. Naturally, an identical result is obtained when $p_+ = p_-$, that is, when the asymmetric sources become symmetric.

IV. NUCLEUS-NUCLEUS COLLISIONS

Investigation of forward-backward correlations in nucleus-nucleus collisions may be of particular interest, because it allows one to verify to what extent the asymmetric components survive the period of thermalization and hydrodynamical expansion that are believed to determine the outcome of the collision.

To apply our analysis to this case, one has to take into account that the number of sources may fluctuate, depending on the number of collisions and/or wounded nucleons [17,18,21,22]. Denoting the number of left (right) movers by w_L (w_R) and the number of symmetric sources by w_C , we obtain for the generating function of the joint distribution in Δ_L and Δ_R

$$\begin{aligned} \Phi_w(z_L, z_R) \\ = \sum_{w_L, w_R, w_C} W(w_L, w_R, w_C) \Phi(z_L, z_R; w_L, w_R, w_C), \end{aligned} \quad (11)$$

where $W(w_L, w_R, w_C)$ is the probability distribution of the number of sources and $\Phi(z_L, z_R; w_L, w_R, w_C)$ is given by Eq. (4). The generating function of the distribution in one of the intervals, say Δ_L , is $\Psi_w(z) = \Phi_w(z_L = z, z_R = 1)$.

Fluctuations in the number of sources imply that the generating function $\Phi_w(z_L, z_R)$ is no longer a product of the functions describing the sources.² Therefore relations between cumulants become rather involved. It turns out that somewhat simpler relations are obtained for the factorial moments. For symmetric processes, where $W(w_L, w_R, w_C) = W(w_R, w_L, w_C)$, the simplest ones read

$$\begin{aligned} F_2 - F_{11} &= \epsilon^2 \langle L_2 - L_1 R_1 \rangle; \\ F_3 - F_{12} &= \epsilon^2 \{ \langle L_3 - L_2 R_1 \rangle + \langle C_1 [L_2 - L_1 R_1] \rangle \}; \\ F_4 - F_{22} &= \epsilon^2 \{ \langle L_4 - L_2 R_2 \rangle + 2 \langle C_1 [L_3 - L_2 R_1] \rangle \\ &\quad + \langle C_2 [R_2 - L_1 R_1] \rangle \}; \\ F_4 - F_{13} &= \epsilon^2 \{ (1 - p_+ p_-) \langle L_4 \rangle - \epsilon^2 \langle L_3 R_1 \rangle \\ &\quad - 3 p_+ p_- \langle L_2 R_2 \rangle \} + \frac{3}{2} \epsilon^2 \langle C_1 [L_3 - L_2 R_1] \rangle \\ &\quad + \frac{3}{4} \epsilon^2 \langle C_2 [L_2 - L_1 R_1] \rangle \end{aligned} \quad (12)$$

²Needless to say that, if there are no correlations between numbers of sources, that is, if the probability $W(w_L, w_R, w_C)$ is a product of three factors, the situation reduces to the one described in the previous section.

where $\langle \dots \rangle$ denotes the average over the number of sources and

$$\begin{aligned} L_i &= \frac{d^i \{ [\phi(z)]^{w_L} \}}{dz^i} \Big|_{z=1}, \quad R_i = \frac{d^i \{ [\phi(z)]^{w_R} \}}{dz^i} \Big|_{z=1}, \\ C_i &= \frac{d^i \{ [\phi_C(z)]^{w_C} \}}{dz^i} \Big|_{z=1} \end{aligned} \quad (13)$$

are moments of the distribution in $\Delta_L + \Delta_R$ produced by left, right, and symmetric sources, respectively. They cannot be directly measured, so Eq. (12) can be used to obtain information about them. Such analysis faces, however, a difficulty: One can see that the right-hand side of Eq. (12) depends not only on the structure of the sources but also on the correlation between the number of various sources. To disentangle these two effects it is necessary to study processes with various nuclei and at various centralities [17].

If the symmetric sources are absent, Eq. (12) simplifies to

$$\begin{aligned} F_2 - F_{11} &= \epsilon^2 [\langle L_2 \rangle - \langle L_1 R_1 \rangle]; \\ F_3 - F_{12} &= \epsilon^2 [\langle L_3 \rangle - \langle L_2 R_1 \rangle]; \\ F_4 - F_{22} &= \epsilon^2 [\langle L_4 \rangle - \langle L_2 R_2 \rangle]; \\ F_4 - F_{13} &= \epsilon^2 [(1 - p_+ p_-) \langle L_4 \rangle - \epsilon^2 \langle L_3 R_1 \rangle \\ &\quad - 3 p_+ p_- \langle L_2 R_2 \rangle]. \end{aligned} \quad (14)$$

Finally, if only symmetric sources are present we return to the simple relation, Eq. (10).

V. THREE INTERVALS

Fluctuations of the number of sources in nucleus-nucleus collisions are difficult to control because even precise determination of centrality of the collision is not sufficient to guarantee a fixed number of sources [17,18,21,22]. To improve this, the STAR Collaboration measured correlations in three intervals [20]. Apart from the intervals Δ_L and Δ_R , one adds the third interval Δ_C , located centrally around $y_{c.m.} = 0$ and not overlapping with Δ_L and Δ_R . The correlations between Δ_L and Δ_R are measured under the constraint that a fixed number of particles, n_C , was found in Δ_C . Particle multiplicity in Δ_C is obviously related to the number of sources and therefore one may expect it to be helpful in estimating this number on an event-by-event basis.

The extensive general discussion of this data is given in Ref. [22]. These data are also analyzed in Refs. [17] and [21] within specific models.

We would like to add three comments.

- (i) The relations derived in previous sections remain intact if one adds the condition that a certain number of particles is observed in the central interval Δ_C . This should be clear from the derivation: replacing the probabilities in Eqs. (1) and (2) by conditional probabilities (fixing the number of particles in Δ_C) does not change at all our argument.
- (ii) When symmetric and asymmetric sources are present, restricting n_C has only a limited effect on the reduction of fluctuations of the number of sources. This can be seen by considering the distribution of particles in Δ_C .

The relevant generating function is

$$\Phi_C(z_C) = [\phi(1 - p_C + p_C z_C)]^{w_L + w_R} \times [\phi_C(1 - q_C + q_C z_C)]^{w_C}, \quad (15)$$

where p_C is the probability that a particle from an asymmetric source lands in Δ_C and q_C is the probability that a particle from a symmetric source lands there.

One can see from Eq. (15) that the particle distribution in Δ_C does not depend on the difference $w_- = w_L - w_R$ and thus fluctuations of w_- are not restricted by Eq. (15). Moreover, Eq. (15) implies that the distribution of n_C depends on both $w_+ = w_L + w_R$ and w_C . Because the forward-backward correlations induced by fluctuations of asymmetric sources are, generally, weaker than those induced by the symmetric ones, it is important to separate the two contributions. We conclude that although measurements at a fixed n_C may be helpful, probably some additional model assumptions are necessary to disentangle this problem. Let us also note that in Refs. [21,22] only the symmetric sources are discussed and therefore this aspect of the problem does not appear.

- (iii) Using the methods of Secs. III and IV, relations can also be derived for the joint moments of the distribution in the three intervals. The only difference is that one has to consider the generating function of three variables.

VI. SUMMARY AND COMMENTS

A systematic, model-independent method of studying the forward-backward correlations in particle production is developed. It is shown that it provides a useful tool for determining the structure of the sources of particles created in high-energy collisions. In particular, it may be used to uncover left-right asymmetric components present in these processes and to study their properties. This point is of interest because, as explained in the Introduction, various mechanisms of particle production differ in their predictions for the presence and/or intensity of such asymmetric sources both in (p - p) and A - A collisions.

The existing data [23] show reasonable agreement with the idea that just two asymmetric sources dominate the observed correlations in p - p collisions [4,10,16]. A similar conclusion was obtained recently in the analysis of the Au-Au collisions [24], where data could also be explained without any symmetric contribution being present [18]. This seems not to be the case [22] for the more restrictive STAR data [20].³ These conclusions have, of course, important consequences for selecting the possible mechanisms of particle production.

As is indicated in Sec. V, the analysis of the heavy-ion data may require additional information about the correlation between the various particle sources. Even in this complicated situation the method proposed here can, however, clearly distinguish whether symmetric or asymmetric sources dominate the process in question.

New data from the LHC [1–3] show that the multiplicity in the central rapidity region increases with energy much faster than expected from simple extrapolations of the trends observed at lower energies. One possible explanation is that, as predicted in some models [10], a new symmetric source of particles may be excited at these high energies. Studying the forward-backward correlations using methods developed in the present article should be helpful in verification of this idea and, possibly, in identification of this new component, as well as in investigation of its properties.

A question that may be studied by the methods proposed in this article is the comparison of the forward-backward correlations in p - p and heavy-ion collisions. Such comparative studies at energies available at the LHC would allow one to obtain information about the *longitudinal* dynamics of quark-gluon plasma, a problem that is barely touched by the existing analyses. Forward-backward correlations are created at the very early stage of the collision (see, e.g., Refs. [22]) and, apparently, survive the evolution of the system. It remains, however, an interesting question to what extent they are distorted during this evolution.

Finally, let us emphasize that our approach is not restricted to rapidity distributions: it can be used to study correlations in other variables as well. One interesting possibility is to repeat the standard analysis in rapidity, restricting, however, the azimuthal angle (keeping of course forward-backward symmetry). This may provide useful information on correlations in the directed flow.

Another attractive possibility is to consider correlations in transverse momentum at a fixed and/or opposite rapidity. Because the p_t distribution in a minimum bias sample is now becoming accessible in a rather broad range [1–3], the full potential of the method can be explored. Such investigation may help in disentangling the jet structure in the low and medium p_t regions.

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APPENDIX

Some explicit relations between cumulants and factorial moments (for symmetric collisions) are listed below:

$$\begin{aligned} f_1 &= F_1; & f_{11} &= F_{11} - F_1^2; \\ f_{21} &= F_{21} - 2F_{11}F_1 - F_2F_1 + 2F_1^3; \\ f_{22} &= F_{22} - 4F_{21}F_1 - 2F_{11}^2 - F_2^2 + 4[F_2 + 2F_{11}]F_1^2 - 6F_1^4; \\ f_{31} &= F_{31} - 3F_{21}f_1 - F_3F_1 - 3F_{11}F_2 \\ &\quad + 6[F_{11} + F_2]F_1^2 - 6F_1^4; \\ f_2 &= F_2 - F_1^2; & f_3 &= F_3 - 3F_2F_1 + 2F_1^3; \\ f_4 &= F_4 - 4F_3F_1 - 3F_2^2 + 12F_2F_1^2 - 6F_1^4. \end{aligned} \quad (A1)$$

³See, however, Refs. [17,21].

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