Analytical description of odd-*A* nuclei near the critical point of the spherical to axially deformed shape transition

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A coupling scheme for even-even nuclei with the X(5) critical point symmetry coupled to a single valence nucleon in a j orbit is proposed to approximately describe the critical point phenomena of spherical to axially deformed shape (phase) transition in odd-A nuclear systems. The corresponding scheme, which can be solved analytically, is called the X$(5/(2j+1))$ model. A special case with $j = 1/2$ is analyzed in detail to show its level structure and transition patterns. It is further shown that ^{189}Au and ^{155}Tb may be possible X$(5/(2j+1))$ symmetry candidates with $j = 1/2$ and $j = 3/2$, respectively.

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I. INTRODUCTION

Recently, quantum phase transitions in nuclei [1–17] have attracted a lot of interest. A new class of symmetries, called critical point symmetries [18,19], were introduced by Iachello nearly a decade ago to describe shape (phase) transitions properties of even-even nuclei. Most importantly, the critical point of spherical to γ-unstable shape transition [18], called E(5), and the critical point from spherical to axially deformed shape [19], called X(5), that were proposed have been confirmed by experiment. Very recently, the concept of critical point symmetry has also been applied to odd-A systems. The first case of a critical point Bose-Fermi symmetry [20], called E(5/4), was developed by Iachello to analytically describe a γ-soft critical point E(5) even-even system coupled with a particle in a $j = 3/2$ orbit. ^{135}Ba was suggested as an empirical example of the E(5/4) symmetry [21]. Another critical point Bose-Fermi symmetry, called E(5/12), was developed by Alonso, Arias, and Vitturi [22], which provided a systematic way to describe criticality in odd-A nuclei since it extended the case of a single-j orbit into a multi-j case with $j = 1/2, 3/2, 5/2$. Both the E(5/4) and E(5/12) models were developed to describe odd-A nuclei near the critical point of the spherical to gamma unstable shape (phase) transition. Most notably, systems with critical point symmetries, such as E(5/4) or E(5/12), can be studied analytically in these two models.

To analyze odd-A nuclei adjacent to even-even nuclei with the X(5) critical point symmetry, such as even-even nuclei with $N = 90$ that have already been confirmed by experiment [23,24], in this article, in a manner similar to what is used in the E$(5/(2j+1))$ scheme, we propose the X$(5/(2j+1))$ critical point symmetry model to describe odd-A nuclei near the critical point of spherical to axially deformed shape (phase) transitions. In Sec. II, a coupling scheme of an even-even system with the X(5) symmetry coupled to a single valence particle in a j orbit is proposed, which leads to the X$(5/(2j+1))$ model. In Sec. III, as a typical example, a $j = 1/2$ case is analyzed in detail. Two possible X$(5/(2j+1))$ symmetry candidates are also suggested. A short summary is given in Sec. IV.

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II. THE X$(5/(2j+1))$ MODEL

As shown in Refs. [20,25], the E(5/4) model is constructed by considering the case of a collective core with the E(5) symmetry coupled with a single particle in a $j = 3/2$ orbit via five-dimensional "spin-orbit" interaction $\hat{\Sigma} \cdot \hat{L}$, which is a scalar of the Spin(5) or SO(5) group [20,25]. The corresponding Hamiltonian can be written as

$$H_{\mathrm{E}(5/4)} = H_{\mathrm{E}(5)} + g(\beta)\,[2\hat{\Sigma} \cdot \hat{L} + 5/2], \tag{1}$$

where $g(\beta)$ is the interaction strength and the constant 5/2 is added to simplify the results. As is well known, the E(5) model describes a system within a five-dimension infinite well and the eigenfunctions of the Hamiltonian $H_{\mathrm{E}(5)}$ of the E(5) model correspond to a set of half-integer order Bessel functions. It follows that the spectrum of the E(5) model is determined by zeros of those Bessel functions. As for the E(5/4) model, the interaction $g(\beta)[2\hat{\Sigma} \cdot \hat{L} + 5/2]$ included in $H_{\mathrm{E}(5/4)}$ only changes the orders of the Bessel function associated with eigenfunctions of the Hamiltonian (1) as compared to those of the E(5) model if one takes $g(\beta) \sim 1/\beta^2$ as adopted in Ref. [20].

To establish an X$(5/(2j+1))$ model in a way that is similar to the coupling scheme leading to the E(5/4) model, we introduce a physically relevant spin-orbit interaction $f(\beta)\,(\hat{L} \cdot \hat{j})$ into the model Hamiltonian when the even-even core is near the X(5) critical point, where $\hat{L}$ is the angular momentum operator of the X(5) model, $\hat{j}$ is the total angular momentum operator of the single particle, and $f(\beta)$ is the coupling strength. Thus, the Hamiltonian of the X$(5/(2j+1))$ model is

$$H = H_{\mathrm{X}(5)} + f(\beta)\hat{L} \cdot \hat{j}, \tag{2}$$

where $H_{X(5)}$ is the Hamiltonian of the X(5) model. Hence, the explicit form of the Hamiltonian (2) is given by

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{\hat{L}_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)}\right] + V(\beta,\gamma) + f(\beta)\hat{L}\cdot\hat{j}, \tag{3}$$

with $V(\beta,\gamma) = V(\beta) + V(\gamma)$, in which the potential $V(\beta)$ is taken to be an infinite square well with $V(\beta) = 0$ for $\beta \leqslant \beta_W$ and $V(\beta) = \infty$ for $\beta \geqslant \beta_W$, whereas the potential $V(\gamma)$ is assumed to be harmonic around $\gamma = 0$. As was shown in Ref. [19], since the potential $V(\gamma)$ has a minimum around $\gamma = 0$, the rotational energy term in Eq. (3) can be approximated with the following form:

$$\sum_k \frac{\hat{L}_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)} \approx \frac{4}{3}\left(\hat{L}_1^2 + \hat{L}_2^2 + \hat{L}_3^2\right) + \hat{L}_3^2\left(\frac{1}{\sin^2\gamma} - \frac{4}{3}\right). \tag{4}$$

The spin-orbit coupling strength in Eq. (2) is also assumed to be $f(\beta) = \frac{\hbar^2 f}{B\beta^2}$, where f is a parameter. In a similar manner to what was done in Ref. [19], the eigenfunctions of Eq. (2) can be obtained by coupling the even-even core part to the single-particle wave function $\chi_{jm_j}(\eta)$ with

$$\Psi_{JLj}^K(\beta,\gamma,\vartheta,\eta) = F(\beta,\gamma)\sum_{M_L m_j}\langle LM_L jm_j|JM_J\rangle D_{M_L K}^L(\vartheta)\chi_{jm_j}(\eta), \tag{5}$$

where $D_{M_L K}^L(\vartheta)$ is the Wigner function of the Euler angles $\vartheta_i (i = 1,2,3)$, and η represents coordinates of the spin-j particle. Further, the spin-orbit interaction term can be expressed as

$$\frac{\hbar^2 f}{B\beta^2}\hat{L}\cdot\hat{j} = \frac{\hbar^2 f}{2B\beta^2}[\hat{J}\cdot\hat{J} - \hat{L}\cdot\hat{L} - \hat{j}\cdot\hat{j}], \tag{6}$$

with $\hat{J} = \hat{L} + \hat{j}$. One can solve the eigenvalue equation $H\Psi = E\Psi$ to get the the energy spectrum of the X(5/(2j + 1)) model. By introducing the reduced energy $\epsilon = 2BE/\hbar^2$ and reduced potential $u(\beta,\gamma) = 2BV(\beta,\gamma)/\hbar^2$, the corresponding Schrödinger equation can be simplified as

$$\left\{-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} + \frac{1}{\beta^2}\left[\left(\frac{1}{3} - f\right)L(L+1) + fJ(J+1) - fj(j+1) + \frac{1}{4}K^2\left(\frac{1}{\sin^2\gamma} - \frac{4}{3}\right)\right] + u(\beta,\gamma)\right\}F(\beta,\gamma) = \epsilon F(\beta,\gamma). \tag{7}$$

Since $u(\beta,\gamma) = u(\beta) + \upsilon(\gamma)$ is assumed, the above equation can be approximately separated as

$$\left\{-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2}\left[\left(\frac{1}{3} - f\right)L(L+1) + fJ(J+1) - fj(j+1)\right] + u(\beta)\right\}\xi_{JLj}(\beta) = \epsilon_\beta\xi_{JLj}(\beta), \tag{8}$$

$$\left[-\frac{1}{\langle\beta^2\rangle\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} + \frac{1}{4 < \beta^2 >}K^2 \times\left(\frac{1}{\sin^2\gamma} - \frac{4}{3}\right) + \upsilon(\gamma)\right]\phi_K(\gamma) = \epsilon_\gamma\phi_K(\gamma), \tag{9}$$

with $F(\beta,\gamma) = \xi(\beta)\phi(\gamma)$, where $\epsilon = \epsilon_\beta + \epsilon_\gamma$, and $\langle\beta^2\rangle$ is the average of β^2 over $\xi(\beta)$.

It can be seen that Eq. (9) is the same as the γ part of the differential equation in the X(5) model. We also take $\upsilon(\gamma) = \frac{2B}{\hbar^2}V(\gamma)$ to be the harmonic potential adopted in Ref. [19], in which the corresponding analytical solution is given with eigenvalues

$$\epsilon_\gamma = E_0 + An_\gamma + CK^2, \tag{10}$$

where E_0, A, and C are constants depending on the concrete form of $\upsilon(\gamma)$, the related quantum numbers n_γ, K, and L may take the following values:

$$\begin{aligned} &n_\gamma = 0, \quad K = 0; \quad n_\gamma = 1, \quad K = \pm 2,\\ &n_\gamma = 2, \quad K = 0, \quad \pm 4, \ldots, \end{aligned} \tag{11}$$

and $L = 0, 2, 4, \ldots$ when $K = 0$, while $L = K, K+1, K+2, \ldots$ when $K \neq 0$. More details about solutions of Eq. (9) are shown in Ref. [19]. Furthermore, since $u(\beta)$ is an infinite well potential, Eq. (8) is also analytically solvable. Inside the well, one can transform Eq. (8) into the Bessel equation

$$\varphi'' + \frac{\varphi\prime}{z} + \left[1 - \frac{v^2}{z^2}\right]\varphi = 0, \tag{12}$$

with

$$v = \left[\left(\frac{1}{3} - f\right)L(L+1) + fJ(J+1) - fj(j+1) + \frac{9}{4}\right]^{1/2}, \tag{13}$$

where we have taken $\varphi = \beta^{3/2}\xi(\beta)$ and $z = \beta\sqrt{\epsilon_\beta}$. Considering the boundary condition $\varphi(\beta_W) = 0$, one can determine the relevant eigenvalues

$$\epsilon_\beta \equiv \epsilon_{s,JLj} = (k_{s,JLj})^2, \quad k_{s,JLj} = \frac{x_{s,JLj}}{\beta_W}, \tag{14}$$

for given j, L, and J, where $x_{s,JLj}$ is the sth zero of the Bessel function $J_v(k_{s,JLj}\beta)$. While the eigenfunctions of Eq. (8) are

$$\xi_{s,JLj}(\beta) = c_{s,JLj}\beta^{-3/2}J_v(k_{s,JLj}\beta), \tag{15}$$

where $c_{s,JLj}$ is the normalization constant.

E2 transition rates can be calculated by taking the transition operator with $T^{E(2)} = T_B(\alpha_u) + T_F(\eta)$, where $T_B(\alpha_u)$ only operates on the collective part, and $T_F(\eta)$ operates on the single-particle part [20,25]. In the following, we set the

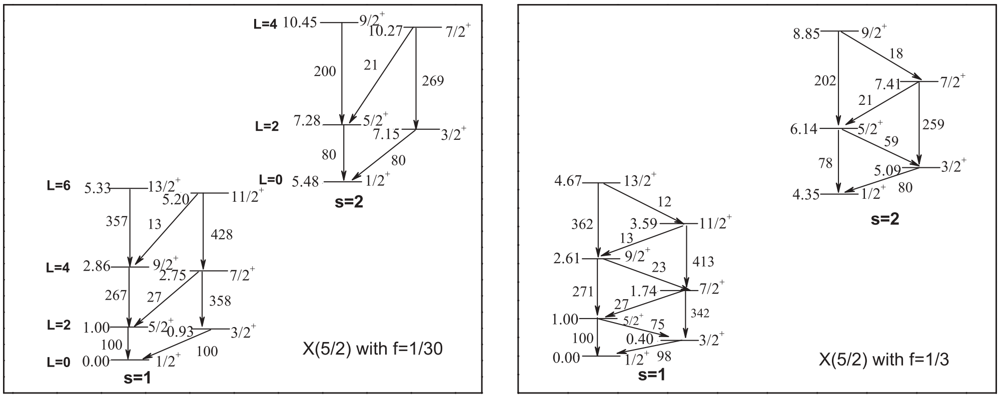

FIG. 1. Some low-lying energy levels and interband E2 transition rates of the X(5/2) model for $f = 1/30$ (left panel) and $f = 1/3$ (right panel), respectively. Only states with $n_\gamma = 0$ are shown.

single-particle part $T_F(\eta) = 0$ for simplicity. The explicit form of $T_B(\alpha_u)$ is

$$T_B = t\beta \left[D^{(2)}_{u,0} \cos\gamma + \frac{1}{\sqrt{2}} \left(D^{(2)}_{u,2} + D^{(2)}_{u,-2} \right) \sin\gamma \right], \qquad (16)$$

where t is the effective charge. As a result, E2 transition rates within the same K band are mainly determined by $\xi_{s,JLj}(\beta)$ under the condition that $\gamma \approx 0$ as shown in Ref. [19] because the eigenfunctions in γ only contribute a $\delta_{n_\gamma n'_\gamma}$ to matrix elements of the E2 operator for $\Delta K = 0$ transitions due to the orthonormality condition [26].

III. LEVEL AND E2 TRANSITION PATTERNS AND COMPARISONS WITH EXPERIMENTAL RESULTS

In a manner similar to the E(5/4) model, and as shown in the previous section, the X(5/(2j + 1)) model—where the X(5/(2j + 1)) notation does not imply any special group theoretical significance, is built from an even-even core with the X(5) symmetry coupled to a particle in a j orbit with $m_j = -j, -j+1, \ldots, j-1, j$.

To show the level and E2 transition patterns in the X(5/(2j + 1)) model, in the following, we consider the case with $j = 1/2$ as a typical example. Energy levels and E2 transition rates in a weak ($f = 1/30$) as well as a strong ($f = 1/3$) spin-orbit coupling situation are calculated, which are shown in Fig. 1. It should be noted that both the energy levels and E2 transition rates can be calculated analytically up to an overall scaling factor. In Fig. 1, the energy levels are normalized to the energy of the first $5/2^+_1$ excited state, while B(E2) values are normalized to $B(\mathrm{E2}; 5/2^+_1 \to 1/2^+_1)$.

As is shown in the left panel of Fig. 1, there are two obvious features in the weak spin-orbit coupling case, namely, energy levels with the same L are nearly degenerated, and the B(E2) values of the transitions with $\Delta J = 2$ are much larger than those of the transitions with $\Delta J = 1$ except for $B(\mathrm{E2}; 3/2^+_1 \to 1/2^+_1)$, which takes the same value as that of the nearest $5/2^+_1 \to 1/2^+_1$ transition. In the strong coupling case shown in the right panel of Fig. 1, the degeneracy in energy levels with the same L is broken since the Bessel function orders in this case are mainly determined by the total angular momentum quantum number J, especially for the case with $f = 1/3$. The contribution of the quantum L to the Bessel function order in this case is completely canceled as clearly shown in Eq. (8). This is similar to the corresponding situation in the E(5/4) model [20], in which the contribution of the SO(5) quantum number τ to the order of Bessel function is also canceled in the case with $k = 1$ given in Ref. [20]. B(E2) transition rates seem to be almost spin-orbit coupling strength independent. As a result, whether E2 transitions between states with $\Delta J = 2$ are much stronger than those with $\Delta J = 1$, and whether strengths of the first two transitions are comparable can be regarded as important signals of the X(5/2) symmetry in odd-A nuclei near the X(5) critical point.

Examples of odd-A nuclei with the X(5/(2j + 1)) symmetry may be found near the X(5) critical point, where the single valence nucleon outside an even-even core is mainly confined to a single j orbit. Specifically, since the ^{186}Pt nucleus lies near the X(3) symmetry point [27], a γ-rigid version of the X(5) symmetry, it seems reasonable to assume that ^{187}Au and ^{189}Au may be good candidates for this critical symmetry in odd-A nuclei. However, since parities of low-lying states in the ^{187}Au nucleus have not all been assigned in the experiment [28], we can only check for consistency to see whether the known results are consistent with the X(5/(2j + 1)) scheme. And indeed, we found that ^{189}Au seems to be a possible X(5/2) symmetry candidate with $f = 1/7$ since the valence proton outside the even-even core in this case occupies the $3s_{1/2}$ orbit, and the ground state of ^{189}Au is just $1/2^+_1$. As is well known, spectra of odd-A nuclei are much more complex and difficult to describe than those of even-even nuclei. In the following, we only choose the lowest states with $J = |L-j|, |L-j+1|, \ldots, |L+j-1|, |L+j|$ and $L = 0, 2, 4, 6$ to be compared with the theoretical calculation since the X(5/(2j + 1)) model can only be used to describe the collective states satisfying the angular momentum coupling rule according to the interaction $f(\beta)\hat{L} \cdot \hat{j}$ in Eq. (2). For example, the ground-state band of ^{189}Au just satisfies the previous condition, which is compared with the X(5/2) model results with $f = 1/7$ and shown in Fig. 2. As can be seen from Fig. 2, the theory agrees quite well with the experimental

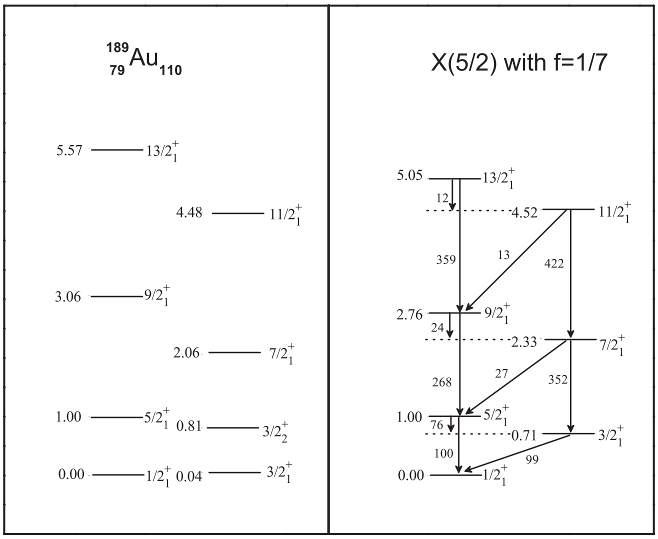

FIG. 2. The ground-state bands of ^{189}Au [29] and spectrum of the X(5/2) model with $f = 1/7$.

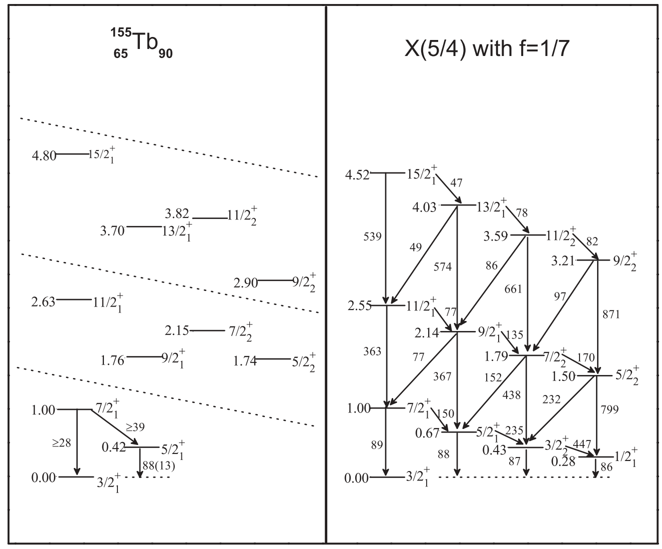

FIG. 3. The low-lying structure of ^{155}Tb [34] and spectrum of the X(5/4) model with $f = 1/7$, where the lowest positive parity states with $J = |L - 3/2|, |L - 1/2|, |L + 1/2|, |L + 3/2|$ and $L = 0, 2, 4, 6$ are shown.

results [29], except for the $3/2_1^+$ state. The experimental $3/2_1^+$ state is nearly degenerate with the $1/2_1^+$ ground state. A possible reason is that the $3s_{1/2}$ orbit is near the adjacent $2d_{3/2}$ orbit in this nucleus since the two orbits are generally adjacent in both spherical and deformed cases [22,30]. As a consequence, the $3/2_1^+$ state may be a single-particle excitation of the $2d_{3/2}$ orbit, while the $3/2_2^+$ state may come from a collective excitation based upon the $1/2_1^+$ level. Therefore, the $3/2_2^+$ state confirmed in the experiment is recognized to be the $3/2_1^+$ state in the X(5/2) model, while the $3/2_1^+$ state from the experiment is regarded as an intruder state due to a single-particle excitation from the $2d_{3/2}$ orbit. It should be noted that there are some other low-lying levels of ^{189}Au below the $13/2_1^+$ level not shown in Fig. 2, such as $5/2_2^+, 7/2_2^+, 9/2_2^+$, and $11/2_2^+$. These levels lie higher but close to the $5/2_1^+, 7/2_1^+$, $9/2_1^+$, and $11/2_1^+$ levels, respectively, as shown in Ref. [29]. They may belong to an excited band built on the $3/2_1^+$ state similar to the ground-state band built on the $1/2_1^+$ state, which seems another experimental signal showing the $3/2_1^+$ state possibly being a single-particle excitation. Further verification of our interpretation needs more experimental results, such as E2 transition rates, and so on. Actually, a similar situation also occurs in ^{135}Ba according to the analysis shown in Ref. [21], where the $1/2_1^+$ state assigned in the experiment is considered to come from a single-particle excitation based on the $2d_{3/2}$ orbit while the $1/2_2^+$ state is taken as a collective excitation to be compared with the lowest $1/2^+$ state in the E(5/4) model [20]. Moreover, a complete description of the low-lying spectrum including negative parity states, which are also not shown in Fig. 2, and some other positive parity states, which cannot be arranged in the $X(5/(2j + 1))$ model confined to a single j orbit, requires us to enlarge the model space. Further extension to include either spherical multi-j or deformed single-particle orbits may be necessary to fully describe this odd-A nucleus, but such schemes will become more complicated. In addition, ^{185}Ir was described by coupling an odd proton hole to the even-even ^{186}Pt [31], which is a typical nucleus with the X(5) symmetry [27] as mentioned previously. However, it is more difficult to realize the highly irregular low-lying levels of ^{185}Ir [32] in the $X(5/2j + 1)$ model with such a simple coupling scheme as used in this article. Furthermore, the quadrupole-type interaction may also need to be considered to describe the low-lying spectrum of ^{185}Ir as analyzed in Ref. [31]. These possible issues will be studied further in our future work.

Since ^{154}Gd is another nucleus with the X(5) symmetry [33], the nearest odd-A nucleus ^{155}Tb is then considered as a candidate for the X(5/4) model because the last valence proton in ^{155}Tb occupies the $2d_{3/2}$ orbit. As shown in Fig. 3, the X(5/4) model agrees generally well with the experimental results [34], but there are two obvious exceptions. First, the collective $3/2_2^+$ and $1/2_1^+$ levels are not observed in the experiments. Second, the $9/2_1^+$ and $13/2_1^+$ levels from the X(5/4) model are noticeably high relative to those observed in the experiment. An improvement may be possible by including additional single-particle orbits resulting in a multi-j model, but that also goes beyond the scope of the present analysis. The first three E2 transition rates of ^{155}Tb were also calculated, with the effective charge fit to the experimental value of the $B(E2; 5/2_1^+ \to 3/2_1^+)$. As is shown in Fig. 3, the values of the first three B(E2) values, which are the only experimentally available results [34], are also fitted well by the X(5/4) model. It should be noted that we have fixed the coupling strength $f = 1/7$ in the $X(5/(2j + 1))$ model for both $j = 1/2$ and $j = 3/2$ cases when comparing with the experimental data. Similar to ^{189}Au, the explanation of some other low-lying positive parity levels lying between the $9/2_2^+$ and $15/2_1^+$ levels in ^{155}Tb not shown in Fig. 3 also needs to extend the model to include multi-j orbits. Anyway, most low-lying levels of ^{155}Tb can be regarded as those built from angular momentum coupling of the single-particle in the $2d_{3/2}$ orbit with the X(5) core in the X(5/4) model.

IV. SUMMARY AND CONCLUSION

In summary, the proposed simple $X(5/(2j+1))$ model seems suitable for approximately describing most low-lying levels in odd-A nuclei near the X(5) critical point symmetry associated with spherical to axially deformed phase (shape) transitions in neighboring even-even systems. Typical level structures and E2 transition patterns in the model is provided with $j = 1/2$. It was also shown that ^{189}Au and ^{155}Tb may be approximate $X(5/(2j+1))$ symmetry candidates with $j = 1/2$ and $j = 3/2$, respectively. However, additional experimental B(E2) data and more accurate adopted level schemes for these nuclei are needed to confirm the predictions. In addition, our analysis suggests that a multi-orbit scheme, similar to the E(5/12) model, may be necessary to improve and extend the theory. The latter, which can be constructed similarly, will be the topic of our future work. It seems a comparison of the $X(5/(2j+1))$ model with a transitional description based on the interacting boson-fermion model [30] may also be interesting, with the same angular momentum coupling scheme adopted for the case with an X(3) even-even core [27]. Related work on this is also in progress.

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