

Impact of hierarchy upon the values of neutrino mixing parameters

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A neutrino-oscillation analysis is performed of the more finely binned Super-Kamiokande (Super-K) atmospheric, MINOS, and CHOOZ data to examine the effect of neutrino hierarchy in this data set on the value of θ_{13} and the deviation of θ_{23} from maximal mixing. Exact oscillation probabilities are used, thus incorporating all powers of θ_{13} and $\varepsilon := \theta_{23} - \pi/4$. The extracted oscillation parameters are found to be dependent on the hierarchy, particularly for θ_{13} . We find, at a 90% confidence level, that these parameters are $\Delta_{32} = 2.44^{+0.26}_{-0.20}$ and $2.48^{+0.25}_{-0.22} \times 10^{-3} \text{ eV}^2$, $\varepsilon = \theta_{23} - \pi/4 = 0.06^{+0.06}_{-0.16}$ and $0.06^{+0.08}_{-0.17}$, and $\theta_{13} = -0.07^{+0.18}_{-0.11}$ and $-0.13^{+0.23}_{-0.16}$, for the normal and inverted hierarchies, respectively. The inverted hierarchy is preferred at a statistically insignificant level of 0.3σ .

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The field of neutrino oscillations has progressed rapidly over the past 15 years. The data can largely be understood by the oscillation of the three known neutrinos [1,2]. Oscillation phenomenology invokes a unitary matrix that relates the flavor basis, in which the neutrinos are created or destroyed, to the mass basis, in which the neutrinos propagate through vacuum. This matrix can be written in terms of three real mixing angles, θ_{12} , θ_{23} , and θ_{13} , and a phase δ that determines CP violation. Oscillations also require nonzero neutrino mass differences, being dependent on the difference of the square of the masses, $\Delta_{ij} := m_i^2 - m_j^2$, where m_i is the mass of neutrino i . A recent analysis [3] reported the present knowledge of the values of the oscillation parameters. In particular, we note that only an upper limit exists on the size of the “reactor” mixing angle $|\theta_{13}| = 0.19^{+0.12}_{-0.19} (1\sigma)$, a constraint that arises, in part, from the null oscillation result of the CHOOZ experiment [4]. A nonzero value of θ_{13} is requisite for the existence of CP violation in neutrino oscillations; hence, we presently have no knowledge of the value of δ . We also have no knowledge of the ordering of the neutrino mass eigenstate: namely, is m_3 greater or less than m_1 and m_2 ? The former (latter) situation is termed the normal (inverted) hierarchy.

Recently, hints of nonzero θ_{13} were reported as a means by which to ease the tension between the determination of the oscillation parameters θ_{12} and Δ_{21} by the solar and KamLAND experiments [3,5–7]. At a smaller significance, analyses of atmospheric data also hint at nonzero θ_{13} [8,9], although one study shows that the significance of the results are dependent on the precise nature of the statistical analysis [10]. Furthermore, an analysis of the updated atmospheric data finds no preference for nonzero θ_{13} [11]; however, this analysis employs approximate oscillation formulas and cannot be directly compared to these other works. Should these various hints be cleanly confirmed in current neutrino oscillation experiments [12–14], then one might be able to attack the issue of CP violation in the lepton sector. The situation, however, is confounded by the existence of various degeneracies among the parameters [15–17], which can render ambiguous their extraction from experiments. To break the degeneracies, one needs to combine the results of experiments (e.g., superbeams)

operating at different energies and/or baselines [18–30]. In the near future, the T2K [31] and NO ν A [32] experiments may be able to determine the hierarchy [33–36]. Atmospheric neutrino experiments can certainly be of some use in unraveling these unknowns as upgoing neutrinos travel through the Earth over a large range of baselines and energies [28,37–48]. In Ref. [49], with fixed solar parameters, we analyzed the determination of the remaining oscillation parameters from the atmospheric, MINOS [50], CHOOZ [4], and K2K [51] experiments assuming CP is conserved (i.e., taking $\delta = 0, \pi$); this work, unlike others, included the more finely binned data from Super-Kamiokande (Super-K) [52]. With that data, we now analyze the effect of the hierarchy on our parameter extraction and explore any preference for a particular hierarchy by the data.

From Ref. [53], we note that vacuum oscillations are invariant under the map

$$\begin{aligned} \theta_{12} &\mapsto \pi/2 - \theta_{12}, & \Delta_{21} &\mapsto \Delta_{21}, \\ \theta_{13} &\mapsto -\theta_{13}, & \Delta_{32} &\mapsto -\Delta_{31}, \\ \theta_{23} &\mapsto \theta_{23}, & \Delta_{31} &\mapsto -\Delta_{32}. \end{aligned} \quad (1)$$

In the limit of vanishing θ_{13} , we see that both hierarchies produce identical oscillation probabilities, provided the proper adjustment is made to θ_{12} . Of course, matter effects [54,55] in the solar sector break the symmetry between these hierarchies requiring $\theta_{12} \leq \pi/4$, provided one assumes $\Delta_{21} > 0$. To remain consistent with the solar data, a broken symmetry is introduced where one does not implement the transformation of θ_{12} or θ_{13} in Eq. (1) but does make the transformation of the mass-squared differences. Given that the octant of θ_{12} is known from matter effects, it is, in principle, possible to distinguish the hierarchy through the precision measurement of vacuum oscillation channels [56–59]; however, for $\theta_{13} \sim 0$, the hierarchy is difficult to discern in part because of the large separation between mass-squared differences, $\Delta_{21} \ll |\Delta_{32}|$. If θ_{13} is sufficiently nonzero, then matter effects provide the most promising avenue by which one might determine neutrino hierarchy.

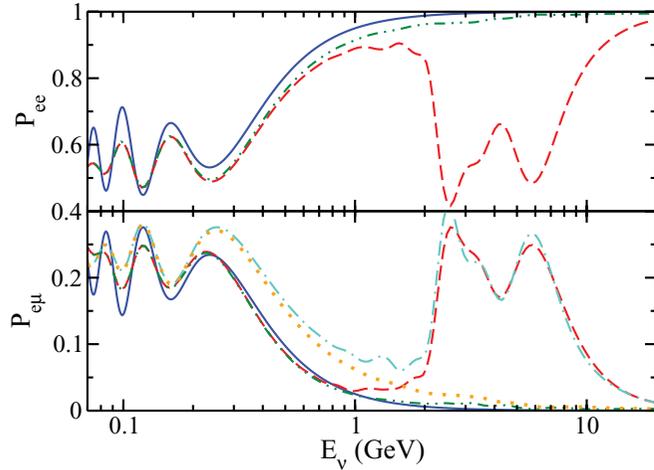


FIG. 1. (Color online) The oscillation probabilities \mathcal{P}_{ee} and $\mathcal{P}_{e\mu}$ vs neutrino energy for bin I ($-1.0 < \cos \varphi < -0.8$) of the Super-K experiment using a two-density model of the Earth. The (blue) solid curves are for both hierarchies and for $\theta_{13} = 0$; the (red) dashed curves are for the NH and $\theta_{13} = +0.15$; the (green) dot-dot-dashed curves are for the IH and $\theta_{13} = +0.15$. For $\mathcal{P}_{e\mu}$, the (turquoise) dot-dashed curve is the NH and $\theta_{13} = -0.15$; the (orange) dotted curve is the IH and $\theta_{13} = -0.15$.

For nonzero θ_{13} , resonant enhancement of the oscillation probability $\mathcal{P}_{e\mu}$ can occur over long baselines that traverse the Earth's mantle and/or core for energies around 3–7 GeV [60]. Using a two-density model of the Earth [61] given by a mantle density of 4.5 g/cm³ and a core density of 11.5 g/cm³ with radius 3486 km, this enhancement is apparent in Fig. 1 which shows the \mathcal{P}_{ee} and $\mathcal{P}_{e\mu}$ oscillation channels for neutrinos traveling roughly the entire diameter of the earth. The resonance at 6 GeV (2.5 GeV) arises from the mantle (core) density. Resonances only occur for neutrinos in the normal hierarchy (NH); however, a similar resonance occurs for antineutrinos in the inverted hierarchy (IH). If θ_{13} were significantly nonzero, then one could use the presence or absence of the resonance to discern which hierarchy is realized by nature. To do so, one would either need to have a relatively pure neutrino (or antineutrino) source or a detector, such as a magnetized iron calorimeter, which could distinguish neutrinos from antineutrinos [28,37,38,44,45]. Unfortunately, the atmospheric neutrino and antineutrino spectra are roughly equivalent, and water Cerenkov detectors, like Super-K, cannot resolve neutrinos from antineutrinos. However, there is some sensitivity to hierarchy as the neutrino cross section is around a factor of 2 greater than the antineutrino cross section over the relevant energy range.

In Refs. [9,49], we found that atmospheric neutrino data had an interesting effect on the extraction of θ_{13} from the global data set. Treating the allowed parameter range of θ_{13} as a continuous set (i.e., allowing negative values for the mixing angle, which is equivalent [62,63] to setting $\delta = \pi$), it was shown that the atmospheric data placed a stringent upper bound on θ_{13} . The atmospheric data only weakly bound θ_{13} from below with the CHOOZ data providing the dominant constraint. As such, the data show a statistically insignificant

preference for $\theta_{13} < 0$. These constraints were traced to an excess of e -like events in the sub-GeV data set, a region where the oscillations owing to the “solar” mass-squared difference are no longer trivial over very long baselines. Analytical expressions from previous studies of atmospheric neutrinos at these baselines and energies show that mass hierarchy has at most a 10% impact on the effective value of θ_{13} in matter [64,65]. This is apparent in Fig. 1. Below 1 GeV, for both \mathcal{P}_{ee} and $\mathcal{P}_{e\mu}$, the NH (red dashed curve) and IH (green dot-dot-dashed curve) for $\theta_{13} = +0.15$ nearly overlap. \mathcal{P}_{ee} is a function of θ_{13}^2 and thus the $\theta_{13} = -0.15$ curve is identical to the $+0.15$ curve. For $\mathcal{P}_{e\mu}$ and $\theta_{13} = -0.15$, the two hierarchies, the NH (turquoise dot-dashed curve) and the IH (orange dotted curve), also yield a nearly identical oscillation probability. Hierarchy should have little effect on the asymmetric nature of the bounds on θ_{13} , because this originates from the low-energy data.

From 1 GeV up to 20 GeV, we see the resonant enhancement of oscillations for the NH and nonzero θ_{13} , the (red) dashed curve, $\theta_{13} = +0.15$, and the (turquoise) dot-dashed curve, $\theta_{13} = -0.15$. The resonance is absent for the IH for $\theta_{13} = +0.15$, the (green) dot-dot-dashed curve, and $\theta_{13} = -0.15$, the (orange) dotted curve. Also, we see that the linear in θ_{13} terms are small in this region. Thus, the resonances provide information about the hierarchy and the magnitude of θ_{13} , but not its sign.

Turning to the data, in Fig. 2 we plot $\Delta\chi^2$ versus the three parameters we vary: (a) Δ_{32} , (b) $\varepsilon := \theta_{23} - \pi/4$, and (c) θ_{13} . Here, we set $\Delta\chi^2 := \chi^2 - \chi_{\min}^2$ with χ_{\min}^2 , the minimum value of χ^2 for the IH. The analysis tools used here are described in detail in Ref. [49]. This analysis incorporates the exact oscillation probabilities, and it makes use of the more finely binned atmospheric data [52]. We also include the MINOS [50] and CHOOZ [4] data. We fix the solar parameters at their best fit values [3]: $\Delta_{21} = 7.65 \times 10^{-5}$ eV² and $\theta_{12} = 0.584$. We are able to do this because, as can be inferred from Fig. 1, the solar data are at sufficiently low energies that it makes no distinction between the hierarchies. To obtain the χ^2 curves, we minimize the remaining varied parameters.

Because MINOS provides the strongest constraints on Δ_{32} and it does not have significant matter effects, we expect

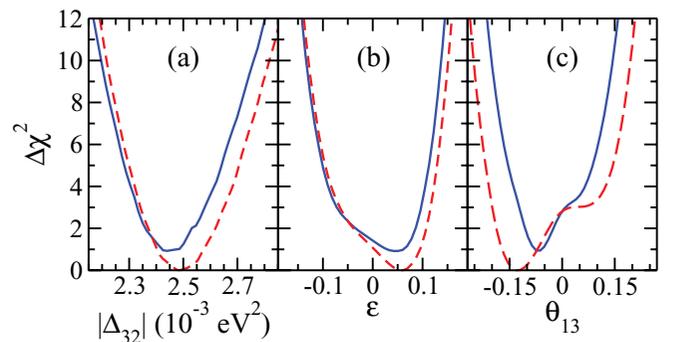


FIG. 2. (Color online) $\Delta\chi^2$ vs the three varied parameters for an analysis that utilizes the Super-K atmospheric, MINOS, and CHOOZ data. The solid (blue) curve is for the NH and the dashed (red) curve is for the IH.

small differences between the hierarchies, which we find. The minimum value of $|\Delta_{32}|$ for the IH occurs at a slightly higher value than for the NH. The resulting values at the 90% confidence level ($\Delta\chi^2 = 6.25$ for a three-parameter fit) for $|\Delta_{32}|$ are $|\Delta_{32}| = 2.44_{-0.20}^{+0.26}$ eV² and $2.48_{-0.22}^{+0.25} \times 10^{-3}$ eV² for the NH and IH, respectively. The fact that $|\Delta_{32}|$ in the IH is larger than that in the NH is not surprising, as suggested by the mappings between the mass-squared differences in Eq. (1). The difference in χ^2 between the minima for the two hierarchies is 0.9, which is a 0.3σ effect for a three-parameter fit.

Also in Fig. 2, we present $\Delta\chi^2$ versus ε . As we vary all three parameters, the difference in χ^2 between the two minima corresponding to the two hierarchies is the same in all three panels. The best-fit values are $\varepsilon = 0.06_{-0.16}^{+0.06}$ and $0.06_{-0.17}^{+0.08}$ for the NH and IH, respectively. This indicates a preference for θ_{23} to lie in the second octant (ε is positive) by $\Delta\chi^2$ of 0.6 and 1.2, or 0.2σ and 0.4σ , for the NH and IH, respectively.

In Fig. 2(c), we present $\Delta\chi^2$ versus θ_{13} . There are differences between the hierarchies in the location of the minima and in the errors for θ_{13} . Here we find the best-fit values for θ_{13} are $-0.07_{-0.11}^{+0.18}$ and $-0.13_{-0.16}^{+0.23}$ for the NH and IH, respectively. This implies that θ_{13} is negative and nonzero by $\Delta\chi^2$ of 1.8 and 2.8, or 0.5σ and 0.8σ , for the NH and IH, respectively. In Ref. [65], we showed that a negative value for θ_{13} allows θ_{23} to lie in the second octant while still maintaining an excess of sub-GeV e -like events.

These results follow what we expect from Fig. 1. We find a preference for a negative θ_{13} independent of hierarchy as this arises from the excess of sub-GeV e -like events seen in the data; in this region, the hierarchy question is not relevant. In extracting this mixing angle from the data, the main difference between the hierarchies is that the minimum value of χ^2 shifts further from zero and the error bars for the IH are larger. The larger error bars were previously noted [52,66]. The origin of this and the other differences between the hierarchies arises from the high-energy Mikheyev-Smirnov-Wolfenstein (MSW) resonances present for NH neutrinos and IH antineutrinos for nonzero θ_{13} . Owing to the difference in cross sections, neutrinos have a greater effect on the data than antineutrinos. Because the atmospheric data are less sensitive to antineutrinos, a larger value of $|\theta_{13}|$ is needed for the IH to account for the data in the resonance region. For the normal

hierarchy, which has resonances in the dominant neutrino channel, the atmospheric data are able to reasonably bound θ_{13} ; however, for the IH, bounds from the atmospheric data on θ_{13} are less stringent.

We found that currently available data have statistically insignificant implications for four important variables: the magnitude of θ_{13} , the sign of θ_{13} , the octant of θ_{23} , and neutrino hierarchy. What might be required for atmospheric data to provide some significant hint of hierarchy? The answer depends on the value of θ_{13} , because this controls the size of the matter resonances. We can provide a rough estimate by looking at the difference between the hierarchies as predicted by the theory for each data bin for the fully contained atmospheric data. The upgoing neutrinos provide the greatest impact; for these data bins, we find that the theoretical difference between the hierarchy values is about one half of the present statistical error of the experimental result for $\theta_{13} = \pm 0.15$. If θ_{13} is in fact this large, then a reduction of the statistical error bars by a factor of 2, or an increase in the total number of events by a factor of 4, could begin to produce statistically significant indications of hierarchy.

We investigated the question of how neutrino hierarchy affects the extraction of θ_{13} from atmospheric and long-baseline experiments. We included exact expressions for the oscillation probabilities, which necessarily contain all linear and higher-order terms in θ_{13} and ε . We also used the more finely binned Super-K atmospheric data. The IH is preferred at a statistically insignificant level. The extracted value of θ_{13} differs between the two hierarchies, with $\theta_{13} = -0.07_{-0.11}^{+0.18}$ and $-0.13_{-0.16}^{+0.23}$ for the NH and IH, respectively. The error on θ_{13} is smaller in the NH because the presence of the high-energy resonances in the dominant neutrino channel leads to the Super-K atmospheric data restricting the value of θ_{13} , an effect that is absent for neutrinos in the IH. The extracted values for θ_{23} and Δ_{32} are $\varepsilon = 0.06_{-0.16}^{+0.06}$ and $0.06_{-0.17}^{+0.08}$, $\Delta_{32} = 2.44_{-0.20}^{+0.26}$ and $2.48_{-0.22}^{+0.25} \times 10^{-3}$ eV², for the NH and IH, respectively.

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