Splitting of the one-body potential in spin-polarized isospin-symmetric nuclear matter

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(Received 27 May 2010; published 16 August 2010)

Spin-polarized symmetric nuclear matter is studied within the Dirac-Brueckner-Hartree-Fock approach. We pay particular attention to the difference between the one-body potentials of upward and downward polarized nucleons. This is formally analogous to the Lane potential for isospin-asymmetric nuclear matter. We point out the necessity for additional information on this fundamentally important quantity and suggest ways to constrain it.

DOI: 10.1103/PhysRevC.82.027307 PACS number(s): 21.65.−f, 21.30.Fe

Introduction. To describe the properties of spin-symmetric and isospin-symmetric nuclear matter still presents considerable intellectual challenges. For instance, the physical pictures of the underlying one-particle fields are very different in relativistic and nonrelativistic approaches. In relativistic models, saturation mechanisms are introduced through negative energy Dirac states, whereas nonrelativistic approaches must be implemented with three-body forces (TBF) to correctly describe saturation properties. Although the relation between the two philosophies seems to be understood in terms of TBF of the Z-diagram type, saturation details can be quite different in the two frameworks.

When other aspects are considered, such as spin-polarized and/or isospin-polarized states of nuclear matter, conclusions become even more model dependent, and available constraints are very limited. The magnetic properties of neutron/nuclear matter have been studied extensively with a variety of theoretical methods [1–28]. We also note the study reported in Ref. [29], where the possibility of phase transitions into spin-ordered states of symmetric nuclear matter was explored based on the Gogny interaction [30] and the Fermi liquid formalism. There, the appearance of an antiferromagnetic state (with opposite spins for neutrons and protons) was predicted, whereas the transition to a ferromagnetic state was not observed. This seems to be in contrast with predictions based on the Skyrme interaction [31], which favor the ferromagnetic spin ordering. On the other hand, it was later shown [32] that the state with oppositely directed spins for neutrons and protons can also be realized with the SLy4 Skyrme interaction. Furthermore, with the SkI3 Skyrme interaction, a phase transition from the state with antiparallel spins of neutrons and protons to the state with parallel spins is predicted in isospin-asymmetric matter at some critical density [32].

In a previous calculation [33], we have investigated spin-polarized pure neutron matter (NM). Lately, such a system has gathered much attention in relation to the issue of possible ferromagnetic instabilities. Also, the possibility of strong magnetic fields in the interior of neutron stars makes the study of polarized NM important and timely.

Although these are very exciting issues, there are other motivations for studies of polarized matter. Here, for instance, we will focus on the spin degrees of freedom of symmetric nuclear matter (SNM), by having a terrestrial scenario as a possible laboratory in mind. We will pay particular attention to the spin-dependent *symmetry potential*, namely, the gradient between the single-nucleon potentials for upward and downward polarized nucleons. The interest around this quantity arises because of its natural interpretation as a spin-dependent nuclear optical potential, defined in perfect formal analogy with the Lane potential [34] for the isospin degree of freedom in isospin-asymmetric nuclear matter (IANM).

With concern for optical potential analyses, to the best of our knowledge, spin degrees of freedom have not been given much attention, possibly because of the increased difficulties in obtaining empirical constraints as compared to the unpolarized system. Another way to access information related to the spin dependence of the nuclear interaction in nuclear matter is the study of collective modes such as spin giant resonances. However, those are not easily observed with sufficient strength [28,35].

What makes the issue of spin dependence particularly interesting is that spin degrees of freedom and relativity are inherently tied to each other. Thus, comparison between relativistic and nonrelativistic predictions should be insightful.

This paper is organized as follows: after reviewing the main aspects of the formalism, we demonstrate the splitting of the one-body potential in spin-asymmetric matter and discuss its significance. Lastly we summarize our conclusions.

Formalism. Our calculation is microscopic and treats the nucleons relativistically. Within the Dirac-Brueckner-Hartree-Fock (DBHF) method, the interactions of the nucleons with the nuclear medium are expressed as self-energy corrections to the nucleon propagator. That is, the nucleons are regarded as dressed quasiparticles. Relativistic effects lead to an intrinsically density-dependent interaction, which is consistent with the contribution from TBF typically employed in nonrelativistic approaches. The advantage of the DBHF approximation is the absence of phenomenological TBF to be extrapolated at higher densities from their values determined through observables at normal density.

The starting point of any microscopic calculation of nuclear structure or reactions is a realistic free-space nucleon-nucleon interaction. A realistic and quantitative model for the nuclear force with reasonable foundations in theory is the one-boson-exchange model [36]. Our standard framework consists of the Bonn B potential together with the DBHF approach to nuclear matter. A detailed description of our application of the DBHF

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method to nuclear matter, NM, and asymmetric matter can be found in a recent review of our work [37].

Similar to what we have performed to describe isospin asymmetries of nuclear matter, the single-particle potential is the solution of a set of coupled equations,

$$U_u = U_{ud} + U_{uu}, \tag{1}$$

$$U_d = U_{du} + U_{dd} \tag{2}$$

to be solved self-consistently along with the two-nucleon G matrix.

In the preceding equations, u and d refer to spin-up and spin-down polarizations, respectively, and each $U_{\sigma\sigma'}$ term contains the appropriate (spin-dependent) part of the interaction, $G_{\sigma\sigma'}$. More specifically,

$$U_\sigma(\vec{p}) = \sum_{\sigma'=u,d} \sum_{q \leqslant k_F^{\sigma'}} \langle \sigma, \sigma' | G(\vec{p}, \vec{q}) | \sigma, \sigma' \rangle, \tag{3}$$

where the second summation indicates integration over the Fermi sea of spin-up (or spin-down) nucleons, and

$$\begin{aligned}&\langle \sigma, \sigma' | G(\vec{p}, \vec{q}) | \sigma, \sigma' \rangle \\ &= \sum_{L,L',S,J,M,M_L} \left\langle \frac{1}{2}\sigma; \frac{1}{2}\sigma' | S(\sigma+\sigma') \right\rangle \left\langle \frac{1}{2}\sigma; \frac{1}{2}\sigma' | S(\sigma+\sigma') \right\rangle \\ &\quad \times \langle L M_L; S(\sigma+\sigma') | J M \rangle \langle L' M_L; S(\sigma+\sigma') | J M \rangle \\ &\quad \times i^{L'-L} Y^*_{L',M_L}(\hat{k}_{\rm rel}) Y_{L,M_L}(\hat{k}_{\rm rel}) \langle L S J | G(k_{\rm rel}, K_{\rm c.m.}) | L' S J \rangle.\end{aligned} \tag{4}$$

The notation $\langle j_1 m_1; j_2 m_2 | j_3 m_3 \rangle$ is used for the Clebsch-Gordan coefficients. Clearly, the need to separate the interaction by spin components brings along angular dependence, with the result that the single-particle potential also depends on the direction of the momentum. Notice that to solve the G-matrix equation requires knowledge of the single-particle potential, which, in turn, requires knowledge of the interaction. Hence, Eqs. (1) and (2) together with the G-matrix equation constitute a self-consistency problem, which is handled, technically, exactly the same way as previously performed for the case of isospin asymmetry [37,38]. The Pauli operator for scattering of two particles with unequal Fermi momenta, contained in the kernel of the G-matrix equation, is also defined in perfect analogy with the isospin-asymmetric one [38],

$$Q_{\sigma\sigma'}\left(p, q, k_F^\sigma, k_F^{\sigma'}\right) = \begin{cases} 1, & \text{if } p > k_F^\sigma \text{ and } q > k_F^{\sigma'}, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The Pauli operator is then expressed in terms of relative and center-of-mass coordinates $k_{\rm rel}$ and $K_{\rm c.m.}$ and angle averaged in the usual way.

The Splitting of the One-Body Potential in Spin-Polarized Nuclear Matter. Figure 1 displays the average potential energy of nucleons $\langle U_{u/d} \rangle$, in polarized SNM as a function of the degree of spin asymmetry, described by the spin-asymmetry parameter $\alpha = \frac{\rho_u - \rho_d}{\rho_u + \rho_d}$ ($\alpha > 0$, by allowing the spin-up species to increase in density). The *average* Fermi momentum is related to the *total* density in the usual way $\rho = \frac{2k_F^3}{3\pi^2}$. The splitting becomes more pronounced with increasing density; compare the left and right panels in the figure.

As mentioned earlier, these potentials become direction dependent in the presence of spin asymmetry, although we found such dependence (on the polar angle θ) to be very mild. In Fig. 1, the potentials are averaged with respect to both magnitude and direction of the momenta.

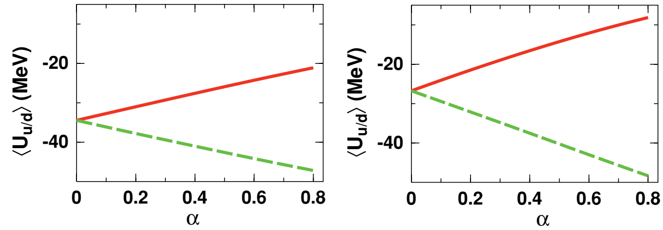

FIG. 1. (Color online) The spin splitting of the *average* potential energy in polarized nuclear matter as a function of the spin-asymmetry parameter. The average is taken over three-dimensional momenta. The (average) Fermi momentum is equal to 1.4 fm^{-1} (left frame) and 1.6 fm^{-1} (right frame).

From the approximate linear relation apparent in Fig. 1, one can write

$$\langle U_{u/d}(\rho, \alpha) \rangle \approx \langle U(\rho) \rangle_0 \pm \left\langle U^S_{\rm sym}(\rho) \right\rangle \alpha, \tag{6}$$

where $\langle U_{\rm sym} \rangle$ plays the role of an (average) spin-symmetry potential:

$$\langle U_{\rm sym} \rangle = (\langle U_u \rangle - \langle U_d \rangle)/2\alpha. \tag{7}$$

For a more direct connection with an actual physical experiment, one would write (by suppressing, for simplicity, ρ and α dependences),

$$U_{u/d} = U_0 + \mathcal{U}_\sigma \frac{\vec{s} \cdot \vec{\Sigma}}{A}, \tag{8}$$

where $\vec{s}$ and $\vec{\Sigma}$ are the projectile spin and the expectation value of the target spin operator, respectively, and A is the mass number of the target. (The momentum dependence may or may not be taken into account, depending on the particular analysis.) Because

$$\frac{\vec{s} \cdot \vec{\Sigma}}{A} = \frac{1}{A} \frac{1}{2} \sigma_z \left(\frac{1}{2} N_u - \frac{1}{2} N_d \right), \tag{9}$$

and $(N_u - N_d)/A$ is easily identified as the parameter α in the neutron-rich nucleus, one can establish an obvious relation between $\mathcal{U}_\sigma$ of Eq. (8) and $U^S_{\rm sym}$ defined as in Eq. (6) (without average if the momentum dependence is being analyzed). In practice, a spin-unsaturated nucleus will also have a net isospin, which means that $\mathcal{U}_\sigma$, $\mathcal{U}_\tau$, and $\mathcal{U}_{\sigma\tau}$ would all have to be considered. Comparison with some older analyses, (mostly based on proton scattering on ^{27}Al and ^{59}Co and neutron scattering on ^{59}Co), was performed in Refs. [39,40].

Next, we display the momentum dependence of $U_u(k)$ and $U_d(k)$ at some fixed values of α and for fixed density, see Fig. 2. The polar angle is also kept fixed (at the value of $\theta = 0$) in view of the mild angular dependence mentioned earlier. Again, we see how the spin-up and the spin-down potentials become more repulsive and more attractive, respectively. It is interesting to analyze the reasons for this behavior, as it sheds light on the similarity between spin and isospin asymmetries. First, let us assume, for the sake of simplification, that k_F^u is much larger than k_F^d so that U_u and U_d get the largest contributions from the U_{uu} and the U_{du} terms, respectively

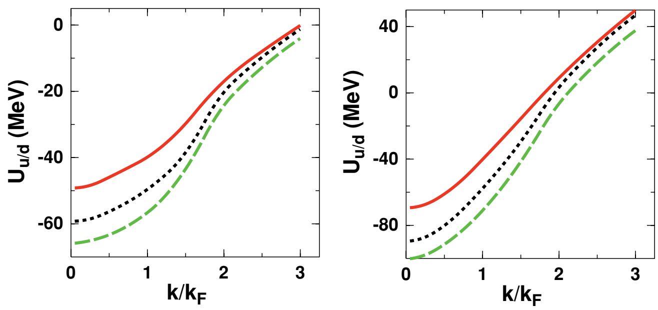

FIG. 2. (Color online) The momentum dependence of the single-nucleon potential with spin up (highest curve) and spin down (lowest curve) in spin-asymmetric matter with $\alpha = 0.6$. The middle curve displays the potential in spin-saturated matter. The Fermi momentum is equal to 1.1 fm^{-1} in the left frame and 1.4 fm^{-1} in the right frame.

(which have the same larger integration limit k_F^u). Thus, $U_u - U_d \approx U_{uu} - U_{du}$. The U_{uu} term receives contribution only from the $S = 1$, $M_S = \sigma + \sigma' = +1$ matrix elements. By moving on to the U_{du} term, it receives contributions, with equal weights, from $S = 0$, $M_S = 0$ and from $S = 1$, $M_S = 0$ matrix elements. When all of the appropriate weighting factors are taken into account, the interaction among nucleons with like-spin projections turns out to be *more repulsive* than the one among nucleons with opposite-spin components. Thus, the scenario becomes analogous to the case of isospin-asymmetric nuclear matter, where the interaction among like nucleons (with total isospin equal to 1), is more repulsive than the one among neutrons and protons. (It may be useful to mention that all arguments would remain invariant upon exchange of u and d labels. The physical source of the splitting we observe is in the different nature of the nuclear force between nucleons with parallel or antiparallel spins.)

The spin-symmetry potential $U^S_{\text{sym}} = (U_u - U_d)/(2\alpha)$ is displayed in Fig. 3 as a function of the momentum. It is remarkably similar, both qualitatively and quantitatively, to the symmetry potential for IANM [37] shown on the right-hand side of the figure $U^I_{\text{sym}} = (U_n - U_p)/(2\alpha)$. We notice the rise of the potential (around the Fermi momentum), a structure also found in the DBHF calculation of Ref. [41], which uses the Bonn A nucleon-nucleon potential [36]. It is interesting to observe, from Fig. 8 of Ref. [41], that none of the phenomenological models shows an enhancement. Instead, all predictions based on phenomenological potentials decrease

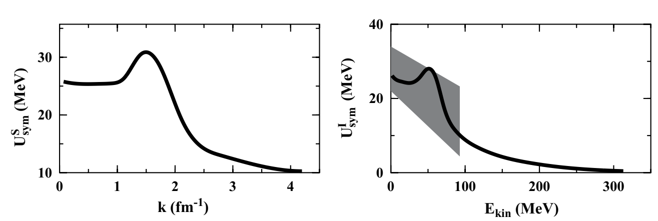

FIG. 3. Left frame: the spin symmetry potential as a function of the momentum. The Fermi momentum is equal to 1.4 fm^{-1}. The polar angle θ is taken to be equal to zero. The frame on the right shows the symmetry potential in isospin-asymmetric (unpolarized) matter at the same density [37]. The shaded area inside the right panel represents empirical constraints from Ref. [34].

(or increase) monotonically. The BHF calculation of Ref. [42], based on the Argonne potential and also shown in Fig. 8 of Ref. [41], is flat at first and then starts to drop with energy. Thus, the enhancement seems to be characteristic of the momentum structure of meson-theoretic potentials. We also point out that the *difference* between the spin-up and spin-down potentials is very sensitive to the details of the momentum dependence of the individual potentials. In fact, the seemingly large peak originates from rather small slope variations of the single-particle potentials *relative to each other*.

With concern for the symmetry potential for IANM, it has been shown that, by starting from a phenomenological formalism for the single-nucleon potential, it is possible to predict opposite tendencies for the energy dependence of the symmetry potential (while still maintaining nearly the same value of the symmetry energy [43–45]), which results in very different predictions of some heavy-ion (HI) observables. In the case of IANM, the difference in the potentials felt by neutrons and protons is effective in separating the neutron-proton dynamics. Observables such as isospin transport/diffusion have been identified as sensitive to the isospin asymmetry of the collision. A similar scenario can be visualized for polarized matter, and a similar model dependence can be expected with regard to U^S_{sym}. However, the bulk of nuclear matter, which results from the HI collision is, on the average, spin saturated, unless, of course, polarized HI beams are available. To the best of our knowledge, polarized secondary beams can be obtained from projectile fragmentation, as first proposed by Kubo *et al.* [46], and are part of RIKEN plans.

Finally, we notice that the approximately linear dependence (vs α) manifest from Fig. 1, along with a similar behavior of the kinetic energy, implies the well-known parabolic form for the EoS:

$$\langle e(\rho, \alpha) \rangle \approx \langle e_0(\rho) \rangle + \langle e^S_{\text{sym}}(\rho) \rangle \alpha^2 . \tag{10}$$

This is demonstrated in Fig. 4, where the energy-particle (averaged over spin-up and spin-down nucleons) is compared with the parabolic approximation.

Before closing, we again stress the importance of more and better empirical constraints to gain insight into the spin-dependent part of the nucleon-nucleus optical potential and, thus, the spin-dependent nuclear effective interaction. Valuable

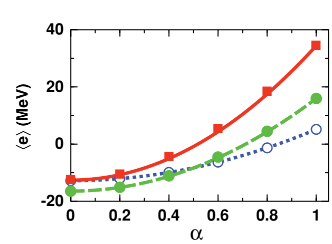

FIG. 4. (Color online) The energy/particle in polarized SNM at three fixed densities as a function of the spin-asymmetry parameter. The solid, dashed, and dotted lines are the parabolic approximation Eq. (10) to the calculated values shown by the squares, solid circles, and open circles, respectively. The various densities are in units of fm^{-3} and correspond to values of the average Fermi momentum equal to 1.1 (dotted line), 1.4 (dashed line), and 1.6 (solid line) fm^{-1}.

information can also come from HI collisions, provided polarized heavy targets are available.

Within the Landau theory of a Fermi liquid, the effective quasiparticle interaction is represented in terms of functions associated with the various spin and isospin operators. The Landau parameters are the lowest-order terms in the Legendre polynomial expansions of those functions. In particular, the strength of the interaction associated with the $\sigma_1 \cdot \sigma_2$ operator is represented, to lowest order, by the g_0 Migdal-Landau parameter, which drives nuclear matter instabilities against spin fluctuations. Thus, stringent constraints on the latter, which includes its density dependence, would provide much-needed insight into spin-spin correlations and the possibility for such instabilities. Because the expectation value of the $\sigma_1 \cdot \sigma_2$ operator is equal to -3 in the singlet states and $+1$ in the triplet states, values of g_0, which decrease with increasing density, would signify that the spin-spin force turns less attractive in the singlet configuration and less repulsive in the triplet one. Thus, there may come a point (in terms of density) when the state with aligned spins is energetically more favorable than the unpolarized one, which results in spin instability.

Conclusions. We continue our broad analysis of various phases of nuclear matter. Here, we specifically address the splitting of the single-nucleon potential in spin-polarized, but isospin-symmetric nuclear matter. The behavior of the predictions is perfectly parallel to the one encountered in IANM. We point out that additional constraints are crucial for a better understanding of the polarizability of nuclear matter. Spin-unsaturated and isospin-unsaturated phases of nuclear matter are also interesting systems, which we plan to study, although computationally more involved.

As usual, we adopt the microscopic approach for our nuclear matter calculations. With concern for our many-body method, we find DBHF to be a good starting point to look beyond the normal states of nuclear matter, which it successfully describes. The main strength of this method is its inherent ability to effectively incorporate crucial TBF contributions through relativistic effects (see Ref. [37] and references therein).

Support from the US Department of Energy under Grant No. DE-FG02-03ER41270 is acknowledged.

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