Medium effects of magnetic moments of baryons on neutron stars under strong magnetic fields

C. Y. Ryu,[1,*] K. S. Kim,[2,†] and Myung-Ki Cheoun[1,‡]
[1]*Department of Physics, Soongsil University, Seoul 156-743, Korea*
[2]*School of Liberal Arts and Science, Korea Aerospace University, Koyang 412-791, Korea*
(Received 23 May 2010; published 16 August 2010)

We investigate medium effects caused by density-dependent magnetic moments of baryons on neutron stars under strong magnetic fields. If we allow the variation of anomalous magnetic moments (AMMs) of baryons in dense matter under strong magnetic fields, AMMs of nucleons are enhanced to be larger than those of hyperons. The enhancement naturally causes the chemical potentials of the baryons to be large and leads to the increase of the proton fraction. Consequently, it causes the suppression of hyperons, resulting in stiffness of the equation of state. Under the presumed strong magnetic fields, we evaluate the relevant particle populations, the equation of state, and the maximum masses of neutron stars by including density-dependent AMMs and compare them with those obtained from AMMs in free space.

DOI: 10.1103/PhysRevC.82.025804 PACS number(s): 25.30.Pt, 21.65.Cd, 24.10.Jv

I. INTRODUCTION

Recently, strong magnetic fields were observed at the surface of soft γ ray repeaters, called magnetars. The magnitude of the fields was estimated as on the order of 10^{14}–10^{15} G [1]. In the interior of neutron stars, according to the scalar virial theorem, the magnetic field strength could be about 10^{18} G. Such strong magnetic fields may affect the structure of a neutron star, changing properties such as the populations of particles, the equation of state (EOS), and mass-radius relations. Many studies of neutron stars with strong magnetic fields have been reported in several papers, including the electromagnetic interaction, the Landau quantization of charged particles, and anomalous magnetic moments (AMMs) of baryons [2–8]. But the roles of the relevant particles' AMMs in a strong magnetic field are still uncertain because the properties of AMMs in nuclear matter are not fully scrutinized yet.

In contrast, medium effects of the electromagnetic (EM) form factors for nucleons have been mainly investigated in electron scattering both experimentally [9–11] and theoretically [12,13]. From these results, one may expect the effect of the EM form factors to increase by about 20%–40%. In particular, various possible variations of the AMMs of baryons in nuclear matter have been studied extensively by many different theoretical models [14–21]. However, there are still remained some ambiguities about the density dependence of the AMMs stemming from the model dependence of baryons in nuclear matter. Furthermore, experimental data also show large error bars. For example, the AMM of the Λ hyperon in the ${}^{7}_{\Lambda}$Li nucleus recently measured at BNL [22] still showed large error bars. Further experiments are expected to deduce more clearly the AMM properties in nuclear matter.

The authors of Ref. [21] studied the medium dependence of the AMMs of baryons in symmetric nuclear matter by using two different models, the quark-meson coupling (QMC) [23] and the modified quark-meson coupling (MQMC) models [24]. In the QMC model, the density dependence of the AMMs of baryons is very small, while the AMM values of a proton and a Λ hyperon in the MQMC model are enhanced by about 25% and about 10%, respectively, at saturation density. Such large enhancements in the MQMC model are quite feasible because the AMM of a baryon generally depends strongly on the bag radius.

In the sense, the MQMC model could effectively take the increased effect of nucleons into account, by increasing the bag radius by about 20% at saturation density. But in the QMC model the bag radius is rarely changed to make the change of AMMs very small. Therefore, the MQMC model can provide us with a theoretical framework to discuss medium effects of AMMs.

In this work, under the assumption that the AMM values of baryons may considerably depend on the medium, we apply the effects to a neutron star. The calculation of the AMMs of baryons in a medium is done by considering only SU(6) quark wave functions obtained by using the MQMC model. Further possible consequences of the effects under strong magnetic fields are also discussed using observational quantities of neutron stars.

Since quantum hadrodynamics (QHD), which is a systematically developed model for finite nuclei and nuclear matter, provides us with results very similar to those obtained by the MQMC model for the structure of a neutron star, we employ the QHD model for a neutron star under strong magnetic fields by including the electromagnetic potential, the Landau quantization of charged particles, and the AMM values of baryons [3,5,7,8]. But to extract the density dependence of the AMMs, we adopt the MQMC model because the model can be more easily applied than QHD to describe the AMMs in nuclear matter and successfully generate the AMM values of baryon octets in nuclear matter.

This paper is organized as follows. In Sec. II, the QHD model for dense matter under a strong magnetic field is briefly introduced by focusing on the role of the AMM in the magnetic field. Results and discussion are presented in Sec. III. The summary and conclusions are given in Sec. IV.

*cyryu@skku.edu
†kyungsik@kau.ac.kr
‡Corresponding author: cheoun@ssu.ac.kr

II. THEORY

The Lagrangian density of the QHD model for dense matter in the presence of strong magnetic fields, which is introduced by the vector potential A^μ due to magnetic fields, can be represented in terms of octet baryons, leptons, and five meson fields as follows:

$$\begin{aligned}\mathcal{L} = &\sum_b \bar{\psi}_b \bigg(i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - M_b^*(\sigma, \sigma^*) - g_{\omega b}\gamma_\mu \omega^\mu \\ &- g_{\phi b}\gamma_\mu \phi^\mu - g_{\rho b}\gamma_\mu \vec{\tau}\cdot\rho^\mu - \frac{1}{2}\kappa_b \sigma_{\mu\nu}F^{\mu\nu}\bigg)\psi_b \\ &+ \sum_l \bar{\psi}_l (i\gamma_\mu\partial^\mu - q_l\gamma_\mu A^\mu - m_l)\psi_l + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma \\ &- \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) + \frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} \\ &- \frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2 w_\mu w^\mu - \frac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu \\ &- \frac{1}{4}R_{i\mu\nu}R_i^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \end{aligned} \tag{1}$$

where b and l denote the octet baryons and the leptons (e^- and μ^-), respectively. The effective mass of a baryon, M_b^*, is simply given by $M_b^* = M_b - g_{\sigma b}\sigma - g_{\sigma^* b}\sigma^*$, where M_b is the free mass of a baryon in vacuum. The σ, ω, and ρ meson fields describe nucleon-nucleon (N-N) and nucleon-hyperon (N-Y) interactions. The Y-Y interaction is mediated by the σ^* and ϕ meson fields. $U(\sigma)$ is the self-interaction of the σ field given by $U(\sigma) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4$. $W_{\mu\nu}$, $R_{i\mu\nu}$, $\Phi_{\mu\nu}$, and $F_{\mu\nu}$ represent the field tensors of the ω, ρ, ϕ, and photon fields, respectively.

The AMMs of baryons interact with an external magnetic field in the form $\kappa_b\sigma_{\mu\nu}F^{\mu\nu}$, where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and κ_b is the AMM of a baryon. For a proton, $\kappa_p = 1.7928\mu_N$ with the nucleon magneton defined as $\mu_N = e/2M_p$ in natural units. Therefore, one may expect two possibilities for the medium dependence of the AMM. The first one comes from the strength of the AMM and the second may stem from the possible variation of the baryon mass at the nucleon magneton in the nuclear medium.

But the nucleon magneton is given by the mass of a proton in free space because it is usually treated as a unit for the magnetic moment of the Dirac particle. It means that the density dependence of the AMM is taken into account by the AMM strength. That is, in this work, the κ_b strength depends on the density but μ_N does not. The medium dependence of the AMM on the density is evaluated from the MQMC model in our previous paper [21], where baryons are treated as MIT bags, and all κ_b are calculated from SU(6) quark wave functions and the bag radius depending on the medium.

The Dirac equations of octet baryons and leptons in the mean field approximation are given by

$$\begin{aligned}\big(i\gamma_\mu\partial^\mu - q_b\gamma_\mu A^\mu - M_b^*(\sigma,\sigma^*) - g_{\omega b}\gamma^0\omega_0 - g_{\phi b}\gamma^0\phi_0 \\ - g_{\rho b}\gamma^0\tau_3\rho_{30} - \tfrac{1}{2}\kappa_b\sigma_{\mu\nu}F^{\mu\nu}\big)\psi_b = 0,\end{aligned} \tag{2}$$

$$(i\gamma_\mu\partial^\mu - q_l\gamma_\mu A^\mu - m_l)\psi_l = 0, \tag{3}$$

where $A_\mu = (0, 0, Bx, 0)$ refers to the constant magnetic field B, which is assumed to be along the z axis. The energy spectra of baryons and leptons are given by

$$\begin{aligned} E_b^C &= \sqrt{k_z^2 + \left(\sqrt{M_b^{*2} + 2\nu|q_b|B} - s\kappa_b B\right)^2} + g_{\omega b}\omega_0 \\ &\quad + g_{\phi b}\phi_0 + g_{\rho b}I_3^b\rho_{30}, \\ E_b^N &= \sqrt{k_z^2 + \left(\sqrt{M_b^{*2} + k_x^2 + k_y^2} - s\kappa_b B\right)^2} + g_{\omega b}\omega_0 \\ &\quad + g_{\phi b}\phi_0 + g_{\rho b}I_3^b\rho_{30}, \\ E_l &= \sqrt{k_z^2 + m_l^2 + 2\nu|q_l|B}, \end{aligned} \tag{4}$$

where E_b^C and E_b^N represent the energies of a charged baryon and a neutral baryon, respectively. The Landau quantization of a charged particle caused by magnetic fields is denoted as $\nu = n + 1/2 - \mathrm{sgn}(q)s/2 = 0, 1, 2, \ldots,$ with electric charge q and spin up (down) $s = 1(-1)$. The equations of the meson fields are given by

$$\begin{aligned} m_\sigma^2\sigma + \frac{\partial U(\sigma)}{\partial\sigma} &= g_{\sigma b}\sum_b \rho_s^b, \quad m_{\sigma^*}^2\sigma^* = g_{\sigma^* b}\sum_b\rho_s^b, \\ m_\omega^2\omega_0 &= g_{\omega b}\sum_b\rho_v^b, \quad m_\phi^2\phi_0 = g_{\phi b}\sum_b\rho_v^b, \\ m_\rho^2\rho_{30} &= g_{\rho b}\sum_b I_3^b\rho_v^b, \end{aligned} \tag{5}$$

where ρ_s and ρ_v are the scalar and the vector densities under magnetic fields, respectively. Detailed expressions for these quantities are given in Refs. [3,7]. The chemical potentials of baryons and leptons are, respectively, given by

$$\mu_b = E_f^b + g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\rho b}I_z^b\rho_{30}, \tag{6}$$

$$\mu_l = \sqrt{k_f^2 + m_l^2 + 2\nu|q_l|B}, \tag{7}$$

where E_f^b is the Fermi energy of a baryon and k_f is the Fermi momentum of a lepton. For charged particles, the E_f^b is written as

$$E_f^{b\,2} = k_f^{b\,2} + \left(\sqrt{m_b^{*2} + 2\nu|q_b|B} - s\kappa_b B\right)^2, \tag{8}$$

where k_f^b is the Fermi momentum of a baryon. Since the Landau quantization does not appear for neutral baryons, the Fermi energy is simply given by

$$E_f^{b\,2} = k_f^{b\,2} + (m_b^* - s\kappa_b B)^2. \tag{9}$$

We exploit three constraints for calculating the properties of a neutron star: baryon number conservation, charge neutrality, and chemical equilibrium. The meson field equations in Eq. (5) are solved with the chemical potentials of baryons and leptons under the above three constraints. The total energy density is given by $\varepsilon_{\rm tot} = \varepsilon_m + \varepsilon_f$, where the energy density for matter fields is given by

$$\begin{aligned}\varepsilon_m = &\sum_b\varepsilon_b + \sum_l\varepsilon_l + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega^2 \\ &+ \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\rho^2\rho^2 + U(\sigma),\end{aligned} \tag{10}$$

and the energy density owing to the magnetic field is given by $\varepsilon_f = B^2/2$. The total pressure can also be written as

$$P_{\rm tot} = P_m + \frac{1}{2}B^2, \qquad (11)$$

where the pressure due to matter fields is obtained from $P_m = \sum_i \mu_i \rho_v^i - \varepsilon_m$. The relation between mass and radius for a static and spherical symmetric neutron star is generated by calculating the Tolman-Oppenheimer-Volkoff (TOV) equations with the equation of state above.

III. RESULTS AND DISCUSSION

We use the parameter set in Ref. [25] for the coupling constants $g_{\sigma N}$, $g_{\omega N}$, and $g_{\rho N}$, where N denotes the nucleon. For the coupling constants of hyperons in the nuclear medium, $g_{\omega Y}$ is determined by the quark counting rule, and $g_{\sigma Y}$ is fitted to reproduce the potential of each hyperon at saturation density, whose strengths are given by $U_\Lambda = -30$ MeV, $U_\Sigma = 30$ MeV, and $U_\Xi = -15$ MeV. For density-dependent AMM values of baryons, we use the values obtained from our previous calculation done using the MQMC model [21]. Since the magnetic fields may also depend on density, we take the density-dependent magnetic fields used in Refs. [2,8]:

$$B(\rho/\rho_0) = B^{\rm surf} + B_0\{1 - \exp[-\beta(\rho/\rho_0)^\gamma]\}, \qquad (12)$$

where $B^{\rm surf}$ is the magnetic field at the surface of a neutron star, which is taken as 10^{15} G from observations, and B_0 represents the magnetic field saturated at high densities.

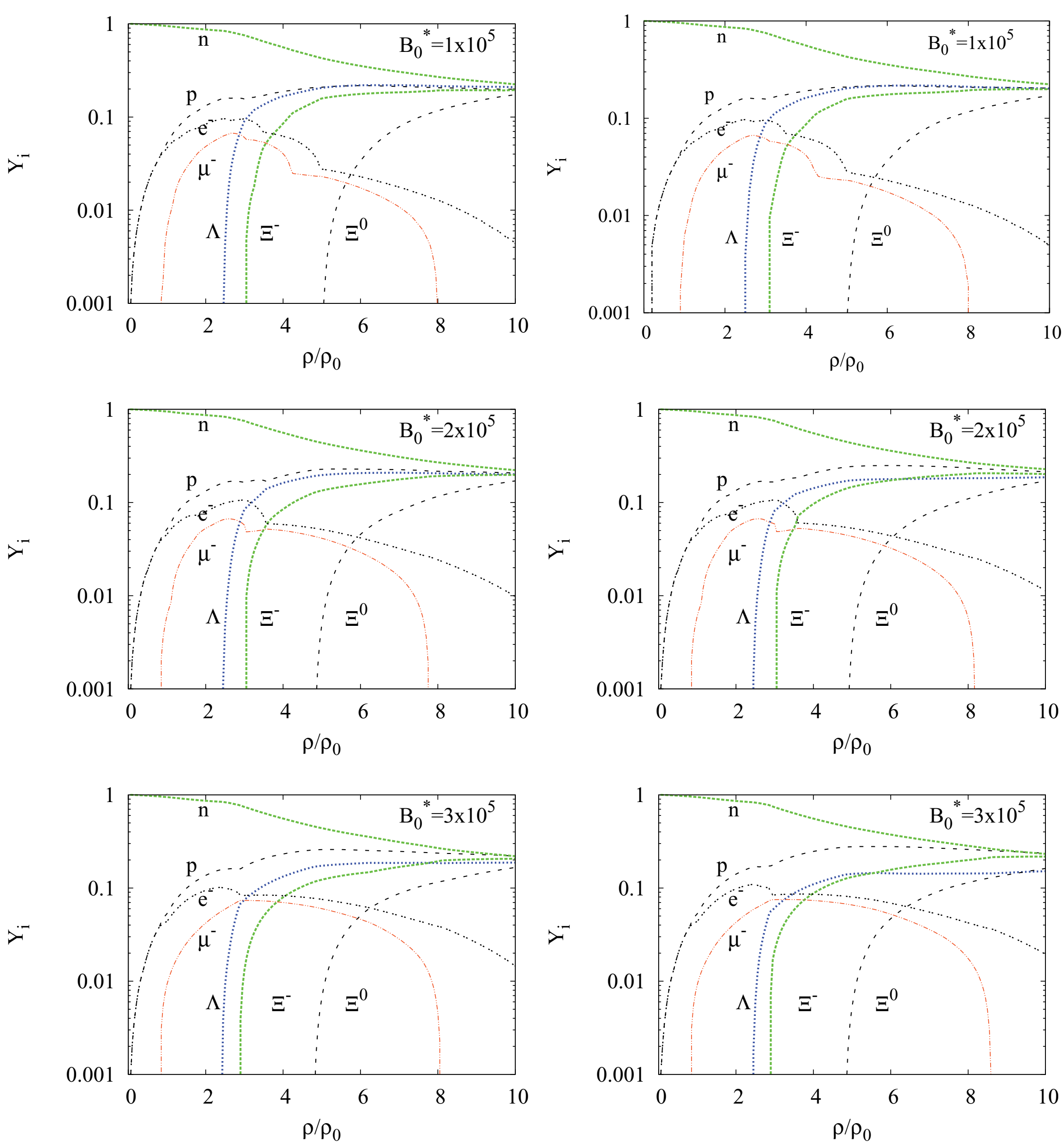

FIG. 1. (Color online) Populations of particles in a neutron star for the slowly varying magnetic field ($\beta = 0.05$ and $\gamma = 2$). Left panels denote results for constant AMMs in free space and right panels are for density-dependent AMMs obtained from the MQMC model. For more direct comparison, all results for p and Ξ^- are summarized in the left-hand side (LHS) of Fig. 3.

In the present work, we use two different sets, slowly ($\beta = 0.05$ and $\gamma = 2$) and quickly ($\beta = 0.02$ and $\gamma = 3$) varying magnetic fields. Since the magnetic field is usually written in units of the critical field for the electron, $B_e^c = 4.414 \times 10^{13}$ G, the B and the B_0 in Eq. (12) can be written as $B^* = B/B_e^c$ and $B_0^* = B_0/B_e^c$. Here, we regard the B_0^* as a free parameter and investigate the medium effects of AMMs in a neutron star for three different magnetic fields given by $B_0^* = 1 \times 10^5$, 2×10^5, and 3×10^5.

A. Medium effects on the populations of particles

Before presenting medium effects of density-dependent AMMs, we briefly discuss the effects of a magnetic field on a neutron star. A strong magnetic field affects charged particles through the EM interaction term (eB), which leads to the Landau quantization, and all baryons by the AMM term ($\kappa_b B$).

The quantum numbers for the Landau levels have positive values $\nu = 0, 1, 2, \ldots$, so that the magnetic field increases the energies of charged particles. Consequently, the chemical potentials of charged particles are increased by the magnetic field.

In contrast, the AMM term gives rise to spin splitting, so that the energy level is divided into two levels: one higher and the other lower. Since the chemical potential is the Fermi surface energy of a particle, the AMM term enlarges the chemical potential of baryons with increasing magnetic fields.

If we allow variations of AMMs in a nuclear medium, the AMM values of relevant baryons are usually increased. According to our previous results from the MQMC model [21], for example, the AMM enhancements of a proton, Λ, and Ξ are about 25%, 10%, and 5%, respectively, at saturation density.

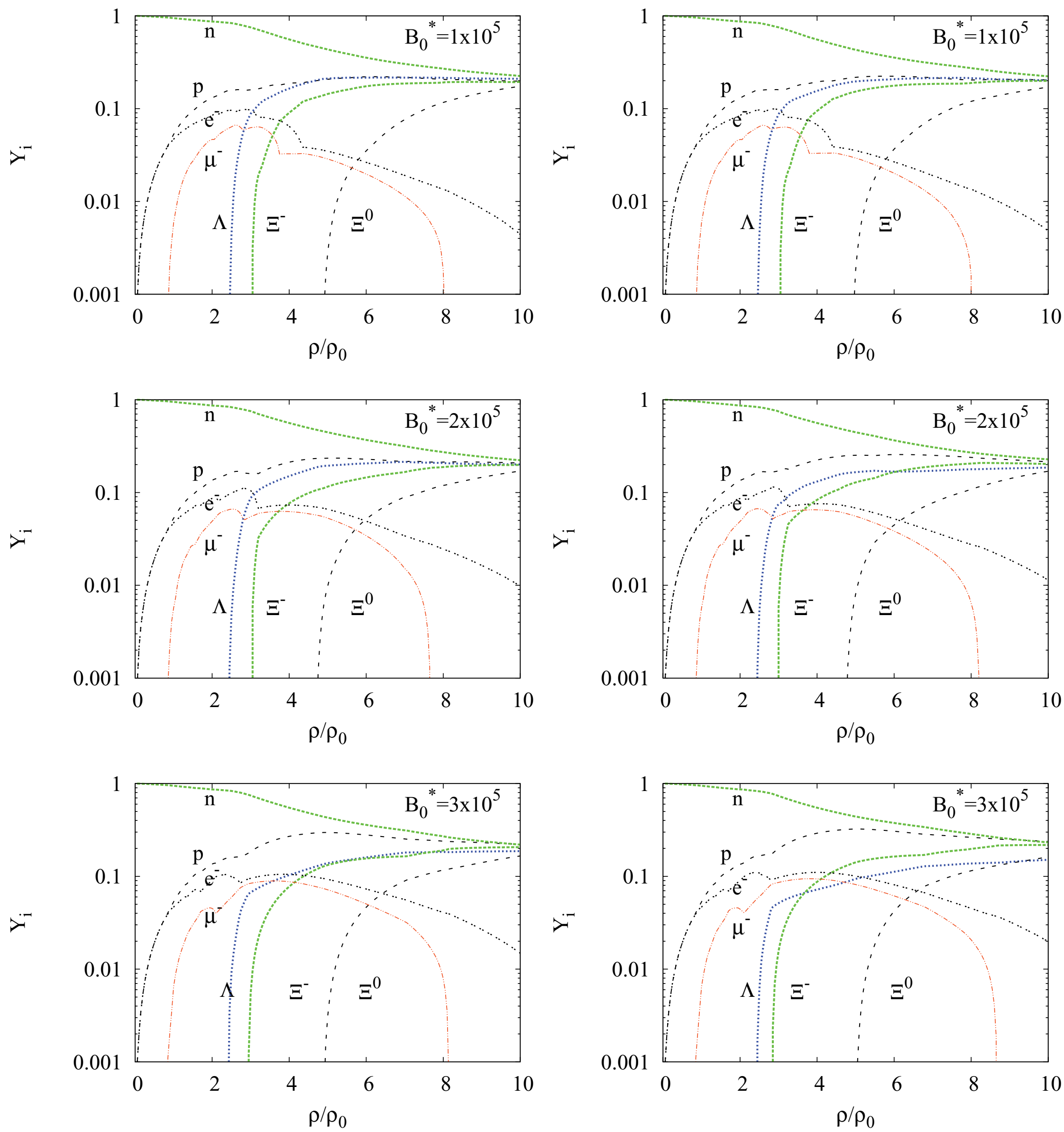

FIG. 2. (Color online) Same as Fig. 1 but for the quickly varying magnetic field ($\beta = 0.02$ and $\gamma = 3$). For more direct comparison, all results for p and Ξ^- are summarized in the right-hand side (RHS) of Fig. 3.

Therefore, medium effects because of density-dependent AMMs cause the chemical potentials of relevant baryons to become larger in addition to the enlargement by the effect of the magnetic fields.

In Figs. 1 and 2, the populations of baryons and leptons for the slowly (Fig. 1) and quickly (Fig. 2) varying magnetic fields are presented for various B_0^* values. The left panels are results for constant AMMs and the right panels those for density-dependent AMMs. The populations of protons and electrons are enhanced with higher B^* from the upper to the lower figures. If we look at the electron population at $\rho/\rho_0 = 10$, the enhancement is easily discerned. In particular, the population of electrons is larger than that of protons because the Bohr magneton μ_e is about 2000 times larger than the nucleon magneton μ_N. This effect is fully ascribed to the increased magnetic fields.

The difference between left and right panels shows medium effects due to density-dependent AMMs. One can see the increase of electron population from the left to the right panels. The higher the magnetic field, the larger medium effect appears.

In order to clearly demonstrate both effects, i.e., the magnetic field effects and medium effects due to density-dependent AMMs, the effects together in Fig. 3. We show populations of protons and a Ξ^- for two different B_0^* fields, and for the constant and the density-dependent AMM values. Since both effects increase the chemical potentials of charged particles, populations of both particles are clearly increased.

The magnetic field effect seems to play a major role in increasing populations compared to the medium effect. But in the $\rho/\rho_0 = 6$–8 region the medium effect due to density-dependent AMMs can be competitive with the magnetic field effect. The medium effect is almost the same as that of the magnetic field increased by one unit.

The enhancement of the proton fraction gives rise to the suppression of other baryons because of baryon number conservation. This means that there suppression of neutrons and Λ hyperons appears, as shown in Figs. 1 and 2.

However, the threshold density for Ξ^- is pushed to a higher density with stronger magnetic field as shown in Fig. 3. However, the abundance of Ξ^- is not changed as much in comparison with Λ as shown in Figs. 1 and 2. Since the Ξ^- hyperon is a charged particle, the population is increased by the magnetic field, while baryon number conservation and charge neutrality lead to suppression of the population. Therefore, the behavior of the Ξ^- population is balanced by the effects of the magnetic field and the conditions of a neutron star.

The difference between the slowly (Fig. 1) and quickly (Fig. 2) varying magnetic fields is the slope of the magnetic field in the region of middle densities. Therefore, this difference just corresponds to the increase of the magnetic field strength B_0^* at the same density. However, the effect of the difference is not remarkable because the exponential term is small compared with the effect of the B_0^* term, irrespective of the γ and β values used here.

B. Medium effects on the EOS, mass, and radius

Magnetic fields and density-dependent AMMs also affect the EOS and the maximum mass of a neutron star. As shown in Fig. 4, the EOS in dense matter becomes stiffer with the increase of the chemical potentials and the suppression of hyperons by the magnetic field. As a result, the maximum masses of neutron stars are increased. The pressure caused by matter fields also strongly depends on the strength of the magnetic fields, but weakly depends on the density-dependent AMMs as shown in Fig. 4. For the quickly varying magnetic field, the slopes of the EOSs between $\rho/\rho_0 = 3$ and 5 are rapidly changed because magnetic fields cause the EOS to become stiffer more quickly.

But the effects of density-dependent AMMs are smaller than those of the magnetic field strength, just as for the populations. In a relatively small magnetic field ($B_0^* = 1 \times 10^5$ G), the density dependence of AMMs rarely affects the EOS. However, in strong magnetic fields ($B_0^* \geqslant 2 \times 10^5$), the contribution of the density-dependent AMMs in the nuclear medium appears explicitly. For example, the increase of the pressure P_m is about 37 MeV fm^{-3} at $\rho = 6\rho_0$ for the fast case in $B_0^* = 3 \times 10^5$.

The mass-radius relations of neutron stars obtained from the TOV equations are shown in Fig. 5. The masses of neutron stars, $M_{\rm star}$, which are obtained from total energy density and total pressure ($\varepsilon_{\rm tot}$,$P_{\rm tot}$), depend very strongly

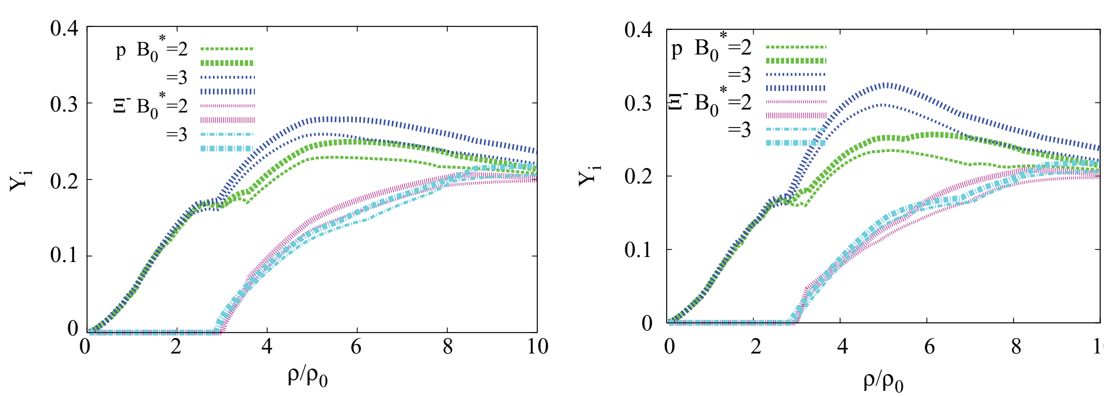

FIG. 3. (Color online) Populations of p and Ξ^- in a neutron star for both slowly (LHS) and quickly (RHS) varying magnetic fields. Thick lines represent results for density-dependent AMM and thin lines are for constant AMM. B_0^* values are given in units of 10^5.

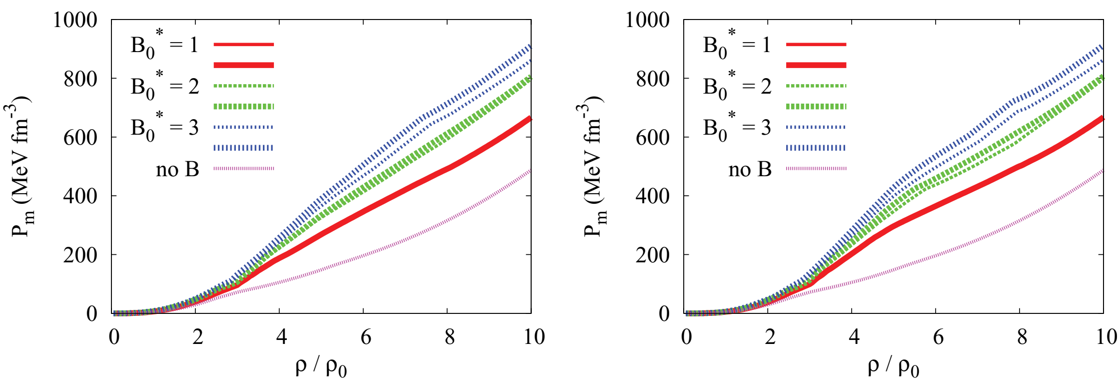

FIG. 4. (Color online) Equation of state (slowly varying magnetic field in the LHS and quickly in the RHS). Thick lines represent results for density-dependent AMM and thin lines are for constant AMM. B_0^* values are given in units of 10^5.

on the strength of magnetic fields. But the contribution of density-dependent AMMs is indiscernible; it is about $0.1M_\odot$, maximally, even in the largest magnetic fields. Since there are no direct data for mass-radius relations of magnetars, we compare our results with the observed neutron stars in the next section.

C. Comparison with observations

Neutron stars and heavy ion collisions may provide valuable constraints for the nuclear EOS [26]. Recent data reported higher masses and larger radii for neutron stars. For instance, $M = (2.0 \pm 0.1)M_\odot$ for 4U 1636-536 was reported in Ref. [27], and the authors in Ref. [28] recently investigated seven neutron stars, six binaries and an isolated neutron star (RX J1865-3754), showing $M = (1.9\text{–}2.3)M_\odot$ and $R =$ 11–13 km. Pulsar I of the globular cluster Terzan 5 (Ter 5 I) shows a lower mass limit $M \geqslant 1.68M_\odot$ at 95% confidence level [29]. Another constraint deduced independently of the given models is obtained from XTE J1739-285 [30], which presents a constrained curve for the ratio between mass and radius.

Thus we compare our results with Ter 5 I and XTE J1739-285 in Fig. 5. In the hyperonic star without magnetic fields ("no B" in Fig. 5), the maximum mass is about $1.59M_\odot$, which does not satisfy the mass limit ($1.68M_\odot$) of Ter 5 I. In addition, the constraint from XTE J1739-285 runs though an unstable region. When magnetic fields are introduced, the LHS in Fig. 5 for the slowly varying field shows that the line from XTE J1739-285 also goes through the unstable region. However, results for the quickly varying magnetic field can satisfy the constraint of XTE J1739-285 and explain masses of neutron stars as 2–3)$M_\odot$ with magnetic fields for hyperonic stars.

In order to detail the effects of density-dependent AMMs, in Table I, the central density (ρ_c), maximum masses, and magnetic fields at central density (B_c^*) for the quickly varying magnetic field are tabulated for both constant and density-dependent AMMs. The effects of density-dependent AMMs are negligible in small magnetic fields. But as the magnetic fields increase, the effect also increases, and the maximum mass is increased by about $0.07M_\odot$ for $B_0^* = 3 \times 10^5$ in the quickly varying magnetic fields.

Finally, one can derive the limit of magnetic fields in the interior of a neutron star and the limit of density-dependent AMMs in the medium. The allowed strength of the magnetic fields is usually constrained by the scalar virial theorem [4,31]. It is given by the approximate relation $B \sim 2 \times 10^8 (M/M_\odot)(R_\odot/R)^2$ G for a nonrotating star. For the star

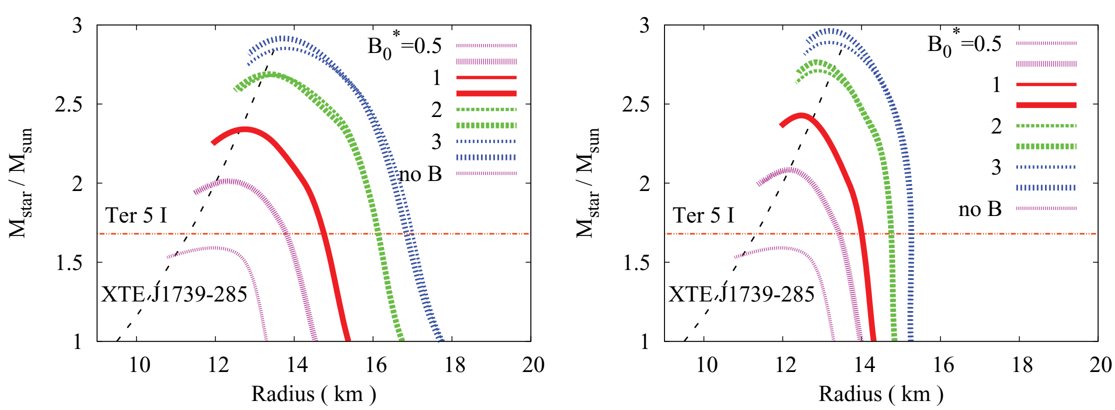

FIG. 5. (Color online) Mass-radius relations. Thick lines represent results for density-dependent AMM and thin lines are for constant AMM. LHS is for the slowly varying magnetic field and RHS is for the quickly varying one. B_0^* values are given in units of 10^5.

TABLE I. The central density (ρ_c), maximum masses ($M/M_\odot$), and central magnetic field (B_c^*) for various B_0^* in both constant and changing AMMs. The results are obtained in quickly varying magnetic fields.

	B_0^*	ρ_c	$M_{\rm star}/M_\odot$	B_c^*
Constant AMM	5×10^4	6.05	2.08	4.94×10^4
	1×10^5	6.05	2.43	9.88×10^4
	2×10^5	5.55	2.71	1.93×10^5
	3×10^5	4.90	2.88	2.71×10^5
Density-dependent	5×10^4	6.05	2.08	4.94×10^4
AMM	1×10^5	6.05	2.42	9.88×10^4
	2×10^5	5.75	2.76	1.95×10^5
	3×10^5	5.00	2.96	2.75×10^5

with $R \approx 10$ km and $M \sim M_\odot$, we obtain $B \sim 10^{18}$ G from the above relation.

In a model-independent calculation for the maximum mass of neutron star, the limit of maximum mass is about $M = (3\text{–}5)M_\odot$ [31]. Furthermore, the observations show that there is no neutron star in the large-mass region which exceeds $3M_\odot$. In these results, for the fast case at $B_0^* = 3 \times 10^5$ G, the maximum mass of the star is $2.96M_\odot$ ($2.89M_\odot$) for density-dependent (constant) AMM and the central magnetic fields is about $B = 2.75\ (2.71) \times 10^5 B_e^c$ G. We can thus conclude that the upper limit of magnetic fields might be $B \approx 3 \times 10^5 B_e^c$ G in a neutron star with hyperons in this work, although detailed numbers depend on the model and parameters.

According to the model dependence of the AMM in other calculations [14–20], the largest enhancement is by about 40% for nucleons at saturation density [15], but the other models show enhancement of about 10%–25%. Thus the enhancement of 25% in this work corresponds to the maximum enhancement except in Ref. [15]. If we employ much larger enhancement for the AMM, like the value in Ref. [15], the contribution of the density-dependent AMM in the medium may become larger. However, all populations, the EOS, and the maximum mass should depend on the strength of magnetic field very strongly, so that the contribution from varying the AMM still remains in a subsidiary role. Thus the effect of a density-dependent AMM might be maximal around $0.1M_\odot$.

IV. SUMMARY

We investigate the effect of a density-dependent AMM of baryons in a neutron star under strong magnetic fields by using the QHD model, which includes a baryon octet and leptons. By exploiting the density-dependent AMM values of baryons obtained from the MQMC model, we calculate the populations of particles, the EOS, and the mass-radius relations for slowly and quickly varying magnetic fields. The strength of the magnetic field is expressed as the EM interaction of all charged particles and the AMM of the baryon octet.

In the populations of particles, all charged particles experience Landau quantization and its effect depends strongly on the strength of magnetic fields. Increase of the magnetic fields enhances the chemical potentials of all charged particles. In particular, since the proton is the lightest particle among baryons, the fraction of protons is enlarged by the magnetic field. As a result, hyperons are suppressed to satisfy the conservation of baryon number. The EOS becomes stiffer and thus the maximum mass of the neutron star also becomes larger.

The mass-radius relations of neutron stars obtained from magnetic fields are compared with observational data. The mass-radius relation with the quickly varying magnetic fields satisfy the constraint given by XTE J1739-285. The effect of density-dependent AMMs appears in very high magnetic fields, causing an increase of the maximum mass of the star by about $0.1M_\odot$ in $B_0^* = 3 \times 10^5$.

We assume a constant magnetic field along the z axis for a nonrotating star. However, a real neutron star under a strong magnetic field rotates very rapidly, and the magnetic fields may be generated by the rotation of matter fields [32]. Thus the calculations should be consistent with each other, that is, the matter fields in a rotating star create a magnetic field and the magnetic field produced affects the matter fields. We shall study this self-consistent approach for the magnetic field next.

ACKNOWLEDGMENTS

This work was supported by the Korea Science and Engineering Foundation (KOSEF) funded by the Korean Government (Grant No. R01-2007-000-20569) and the Soongsil University Research Fund. The work of C.Y.R. was supported by the Korea Research Foundation funded by the Korean Government (MOEHRD) (Grant No. KRF-2008-214-C00015).

[1] C. Y. Cardall, M. Prakash, and J. M. Lattimer, Astrophys. J. **554**, 322 (2001).
[2] D. Bandyopadhyay, S. Chakrabarty, and S. Pal, Phys. Rev. Lett. **79**, 2176 (1997).
[3] A. Broderick, M. Prakash, and J. M. Lattimer, Astrophys. J. **537**, 351 (2000).
[4] I.-S. Suh and G. J. Mathews, Astrophys. J. **546**, 1126 (2001).
[5] P. Dey, A. Bhattacharyya, and D. Bandyopadhyay, J. Phys. G **28**, 2179 (2002).
[6] P. Yue and H. Shen, Phys. Rev. C **74**, 045807 (2006).
[7] P. Yue, F. Yang, and H. Shen, Phys. Rev. C **79**, 025803 (2009).
[8] A. Rabhi, H. Pais, P. K. Panda, and C. Providencia, J. Phys. G **36**, 115204 (2009).
[9] J. J. Aubert *et al.*, Phys. Lett. B **123**, 275 (1983).
[10] P. J. Mulders, Phys. Rep. **185**, 83 (1990).
[11] S. Strauch *et al.*, AIP Conf. Proc. **967**, 135 (2007).
[12] K. S. Kim and M. K. Cheoun, Phys. Rev. C **67**, 034603 (2003).
[13] K. S. Kim and L. E. Wright, Phys. Rev. C **68**, 027601 (2003).
[14] Ulf-G. Meißner, Phys. Lett. B **220**, 1 (1989).
[15] Il-T. Cheon and M. T. Jeong, J. Phys. Soc. Jpn. **61**, 2726 (1992).
[16] M. R. Frank, B. K. Jennings, and G. A. Miller, Phys. Rev. C **54**, 920 (1996).

[17] D. H. Lu, K. Tsushima, A. W. Thomas, A. G. Williams, and K. Saito, Phys. Rev. C **60**, 068201 (1999).
[18] U. T. Yakhshiev, U.-G. Meißner, and A. Wirzba, Eur. Phys. J. A **16**, 569 (2003).
[19] J. R. Smith and G. A. Miller, Phys. Rev. C **70**, 065205 (2004).
[20] T. Horikawa and W. Bentz, Nucl. Phys. A **762**, 102 (2005).
[21] C. Y. Ryu, C. H. Hyun, T.-S. Park, and S. W. Hong, Phys. Lett. B **674**, 122 (2009).
[22] H. Tamura, AIP Conf. Proc. **915**, 70 (2007).
[23] P. A. M. Guichon, Phys. Lett. B **200**, 235 (1988).
[24] X. Jin and B. K. Jennings, Phys. Rev. C **54**, 1427 (1996).
[25] C. Y. Ryu, C. H. Hyun, S. W. Hong, and B. T. Kim, Phys. Rev. C **75**, 055804 (2007).
[26] T. Klähn *et al.*, Phys. Rev. C **74**, 035802 (2006).
[27] G. Lavagetto, I. Bombaci, A. D'Ai, I. Vidana, and N. R. Robba, arXiv:astro-ph/0612061 [astro-ph.HE].
[28] A. W. Steiner, J. M. Lattimer, and E. F. Brown, arXiv:1005.0811 [astro-ph.HE].
[29] S. Ransom *et al.*, Science **307**, 892 (2005).
[30] P. Kaaret *et al.*, arXiv:astro-ph/0611716 [astro-ph.HE].
[31] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (John Wiley & Sons, New York, 1983).
[32] N. Yasutake, K. Kiuchi, and K. Kotake, arXiv:0910.0327 [astro-ph.HE].