

Temperature dependence of the effective bag constant and the radius of a nucleon in the global color symmetry model of QCD

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We study the temperature dependence of the effective bag constant, the mass, and the radius of a nucleon in the formalism of the simple global color symmetry model in the Dyson-Schwinger equation approach of QCD with a Gaussian-type effective gluon propagator. We obtain that, as the temperature is lower than a critical value, the effective bag constant and the mass decrease and the radius increases with the temperature increasing. As the critical temperature is reached, the effective bag constant and the mass vanish and the radius tends to infinity. At the same time, the chiral quark condensate disappears. These phenomena indicate that the deconfinement and the chiral symmetry restoration phase transitions can take place at high temperature. The dependence of the critical temperature on the interaction strength parameter in the effective gluon propagator of the approach is given.

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I. INTRODUCTION

The phase transitions of quantum chromodynamics (QCD), for example the evolution between chiral symmetry breaking and its restoration, the color (or simply quark) confinement and deconfinement, have been the most active topics in nuclear and particle physics in recent years [1]. Even though recent investigation has provided hints that the phase transitions can be driven by the intrinsic characteristics, such as the running coupling strength and the current quark mass, of the system (see, for example, Refs. [2–6]), the more promising and much better believed is that the QCD may undergo phase transitions into a chirally symmetric and color deconfined phase at high temperature and/or density [7].

To demonstrate the phase transitions, one usually implements the variation behaviors of not only the features of QCD vacuum and the strong interaction matter but also the properties of hadrons at finite temperature and/or density. On theoretical side, one needs in principle QCD, which has been widely accepted as the fundamental theory of strong interaction, to carry out the investigation. However, as a basic theory, QCD still suffers from difficulties in the low energy region, which relates directly to strong interaction matter and hadrons. Then, besides the approaches of lattice simulations, QCD sum rules, instanton model(s), and Dyson-Schwinger equations and several models, such as the bag model [8], quark-meson coupling model (QMC) [9–11], Nambu-Jona-Lasinio (NJL) model [12], Polyakov-loop improved NJL model [13], global color symmetry model (GCM) [14], and so on, have been developed. The NJL model has been widely used since it preserves the feature of chiral symmetry and its dynamical breaking and is easy to carry out numerical calculation. Even though, with the Polyakov-loop improvement, the quark confinement effect is included at statistical level, the commonly accepted one still only takes into account the point (contact) interactions among quarks. The bag model is the one which

handles hadrons as bubbles of perturbative vacuum immersed in the physical vacuum. However, all nonperturbative physics is included in a quantity—bag constant, which is dealt with a phenomenological parameter in the model. And the QMC model involves the similar problem. For the GCM, since it can take the result of the Dyson-Schwinger equation (DSE) [15] approach as input, it manifests well the properties of chiral symmetry and its dynamical breaking. Because the bag constant in the model is taken as the difference between the energy densities of the perturbative and the physical vacuums, the color confinement effect is also handled well in some sense. The GCM is then believed to be a quite sophisticated model which involves as many characteristics of QCD as possible. And the NJL model, the QMC model, and the bag model can be treated as the special cases of the GCM.

Due to its solid QCD foundation, the GCM has been widely taken to study not only the properties of nucleon and some mesons [14,16–18] but also the QCD vacuum structure [19,20]. It has also been extended to investigate the properties of strong interaction matter at finite temperature and/or density and those of some hadrons in the matter [21–26]. For the property of nucleon in finite density matter, it has been discussed with various models for the effective gluon propagator in the DSE and the variation behaviors of the mass, the radius, and the bag constant of the nucleon have been given explicitly [23]. However, in the case of finite temperature, only the changing feature of the bag constant has been discussed with the Munczek-Nemirovsky model [27] for the effective gluon propagator of the DSE [22]. We will then, in this paper, discuss some of the properties of a nucleon at finite temperature with a sophisticated effective gluon propagator in the DSE.

The paper is organized as follows. In Sec. II, we describe briefly the formalism of the GCM soliton model. In Sec. III, we describe the algorithm to carry out the numerical calculation of the GCM soliton at finite temperature and the obtained results. Finally we give a brief summary and some remarks in Sec. IV.

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II. BRIEF DESCRIPTION OF GCM

The original action in the global color symmetry model (GCM) defined in Euclidean metric is expressed as [14]

$$S_{\text{GCM}}(\bar{q}, q) = \int d^4x \bar{q}(x)(i\gamma \cdot p + m_0)q(x) + \frac{g^2}{2} \iint d^4x d^4y j_\mu^a(x) D_{\mu\nu}^{ab}(x-y) j_\nu^b(y), \quad (1)$$

where $j_\mu^a(x) = \bar{q}(x) \frac{\lambda^a}{2} \gamma_\mu q(x)$ is the local quark current, $D_{\mu\nu}^{ab}(x-y)$ is the full gluon propagator, m_0 is the current quark mass, g is the quark-gluon coupling constant. The Euclidean metric is such that $a \cdot b = a_\mu b_\mu$, and $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. Taking the gluon propagator to be color diagonal in the Feynman-like gauge, i.e., $D^{ab}_{\mu\nu}(x-y) = \delta^{ab} \delta_{\mu\nu} D(x-y)$, and applying the Fierz transformation to reorder the quark fields, one can rewrite the action as

$$S_{\text{GCM}}[B^\theta(x, y)] = \iint d^4x d^4y \bar{q}(x)[(i\gamma \cdot p + m_0)\delta(x-y) + \Lambda^\theta B^\theta(x, y)]q(x) + \iint d^4x d^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g^2 D(x-y)}, \quad (2)$$

where $\{\Lambda^\theta\}$ are direct products of Lorentz, flavor, and color matrices of quarks which produce the scalar, vector, and pseudoscalar terms labeled by θ . $B^\theta(x, y)$ are bilocal Bose fields. Theoretically, it can be proved that the GCM is valid in any gauge even though one takes the Feynman-like gauge in deriving the above expression [28].

By integrating the quark fields, one gets the action

$$S_{\text{GCM}}[B^\theta(x, y)] = -\text{Tr} \ln G^{-1}[B^\theta(x, y)] + \iint d^4x d^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g^2 D(x-y)}, \quad (3)$$

where the inverse of the quark propagator can be written as

$$G^{-1}(x, y) = (i\gamma \cdot p + m_0)\delta(x-y) + \Lambda^\theta B^\theta(x, y). \quad (4)$$

Generally, the bilocal fields can be expanded as

$$B^\theta(x, y) = B_0^\theta(x, y) + \sum_i \Gamma_0^\theta(x, y) \phi_i^\theta \left(\frac{x+y}{2} \right), \quad (5)$$

where the first term is the translation invariant vacuum configuration. The second term stands for the fluctuations of the vacuum which can be identified as effective meson fields since the θ stands for the quantum number of Bose fields. In the lowest order, one takes the Goldstone mode, $\phi_0^\theta = \{\sigma, \vec{\pi}\}$, which is thought of as the most important low energy degree of freedom. The vacuum configuration can be determined by the saddle point condition $\delta S_{\text{GCM}}/\delta B_0^\theta = 0$. One has then

$$B_0^\theta(x, y) = g^2 D(x-y) \text{tr}[G(y, x)\Lambda^\theta], \quad (6)$$

and the quark self-energy $\Sigma(x-y) = \Lambda^\theta B_0^\theta(x, y)$. The equation of quark self-energy in momentum space coincides with

that of the truncated Dyson-Schwinger equation (DSE)

$$\begin{aligned} \Sigma(p) &= \int d^4x \Lambda^\theta B_0^\theta(x, y) e^{iq \cdot x} \\ &= g^2 \int \frac{d^4q}{(2\pi)^4} t_{\mu\nu} D(p-q) \frac{\lambda^a}{2} \gamma_\mu \\ &\quad \times \frac{1}{i\gamma \cdot (q+m) + \Sigma(q)} \gamma_\mu \frac{\lambda^a}{2}, \end{aligned} \quad (7)$$

where $t_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu / k^2$, with $k = p - q$, γ^μ is the color SU(3) matrix. Generally, the quark self-energy function can be decomposed as

$$\begin{aligned} \Sigma(p) &= S^{-1}(p) - S_0^{-1}(p), \\ S^{-1}(p) &= i\gamma \cdot p A(p^2) + B(p^2), \\ S_0^{-1}(p) &= i\gamma \cdot p + m_0, \end{aligned} \quad (8)$$

and $A(p^2)$, $B(p^2)$ are scalar functions of p^2 .

Recalling the configuration of the bilocal fields in Eq. (5) and considering the Bethe-Salpeter amplitude of the mesons and the partial conservation of axial-vector current, one can prove [29] that, when considering the most important low energy degree of freedom, i.e., the Goldstone mode $\phi_0^\theta = \{\sigma, \vec{\pi}\}$, Eq. (5) can be rewritten as

$$\begin{aligned} &\Lambda^\theta [B^\theta(x, y) - B_0^\theta(x, y)] \\ &= B(x-y) \left[\sigma \left(\frac{x+y}{2} \right) + i\gamma_5 \vec{\tau} \cdot \vec{\pi} \left(\frac{x+y}{2} \right) \right], \end{aligned} \quad (9)$$

where B is just the scalar part of the inverse of the quark propagator which can be determined by solving the quark's DSE.

With a nontopological-soliton ansatz [30], the action of the GCM soliton with quarks in chiral limit ($m_0 = 0$) can be given [14,17,18] as

$$S_{\text{GCM}} = \bar{q} \{ i\gamma \cdot p - \alpha [\sigma(x) + i\vec{\pi}(x) \cdot \vec{\tau} \gamma_5] \} q + \int \left[\frac{f_\sigma^2}{2} (\partial_\mu \sigma)^2 + \frac{f_\pi^2}{2} (\partial_\mu \vec{\pi})^2 - V(\sigma, \pi) \right] d^4z, \quad (10)$$

with

$$\begin{aligned} V(\sigma, \pi) &\approx -12 \int \frac{d^4p}{(2\pi)^4} \left\{ \ln \left[\frac{A^2(p)p^2 + (\sigma^2 + \vec{\pi}^2)B^2(p)}{A^2(p)p^2 + B^2(p)} \right] \right. \\ &\quad \left. - \frac{(\sigma^2 + \vec{\pi}^2 - 1)B^2(p)}{A^2(p)p^2 + B^2(p)} \right\}, \end{aligned} \quad (11)$$

and the quark meson coupling constant α is given as

$$\alpha(x) = \int \frac{d^4p}{(2\pi)^4} B(p) e^{-ip \cdot x}.$$

It is evident that such a quark meson coupling constant is just the vacuum configuration of the bilocal fields and is independent of the meson fields.

With the stationary condition of the soliton, one has the equations of motion for the quarks and mesons as

$$\{ i\gamma \cdot p - \alpha [\sigma(x) + i\vec{\pi}(x) \cdot \vec{\tau} \gamma_5] \} q = 0, \quad (12)$$

$$-\vec{\nabla}^2 \sigma(\vec{r}) + \frac{\delta V}{\delta \sigma(\vec{r})} + Q_\sigma(\vec{r}) = 0, \quad (13)$$

$$-\vec{\nabla}^2 \vec{\pi}(\vec{r}) + \frac{\delta V}{\delta \vec{\pi}(\vec{r})} + Q_{\vec{\pi}}(\vec{r}) = 0, \quad (14)$$

where Q_σ and $Q_{\vec{\pi}}$ are the source terms contributed from the valence quarks, and can be written as

$$Q_\sigma(\vec{R}) = \sum_{j=1}^3 \frac{1}{Z_j} \int d^3x d^3y \bar{u}_j(\vec{x}) B(\vec{x} - \vec{y}) \times \delta\left(\frac{\vec{x} + \vec{y}}{2} - \vec{R}\right) u_j(\vec{y}), \quad (15)$$

$$Q_{\vec{\pi}}(\vec{R}) = \sum_{j=1}^3 \frac{1}{Z_j} \int d^3x d^3y \bar{u}_j(\vec{x}) B(\vec{x} - \vec{y}) i \gamma_5 \vec{\tau} \times \delta\left(\frac{\vec{x} + \vec{y}}{2} - \vec{R}\right) u_j(\vec{y}), \quad (16)$$

with Z_j being the renormalization constant [17]

$$Z_j = - \int d^3p d^3q \bar{u}_j(\vec{p}) \frac{\partial G^{-1}(i\epsilon_j; \vec{p}, \vec{q})}{\partial \epsilon_j} u_j(\vec{q}). \quad (17)$$

The quark field and σ , π meson fields can be determined by solving Eqs. (12)–(14) self-consistently. As a consequence, the corresponding eigenenergies can be obtained. It is apparent that the meson fields corresponding to the vacuum configuration can be simply taken as $\sigma = 1$, $\pi = 0$ due to the (normalized with f_π^2) restriction $\pi^2 + \sigma^2 = 1$. The vacuum configuration is a minimum of $V(\sigma, \pi)$ and $V(1, 0) = 0$. In light of the nontopological-soliton ansatz [14,30], one can approximate the soliton as a chiral bag with bag constant

$$\mathcal{B} = V(\sigma, \vec{\pi}) - V(1, 0) = 12 \int \frac{d^4p}{(2\pi)^4} \times \left\{ \ln \left[\frac{A^2(p)p^2 + B^2(p)}{A^2(p)p^2 + (\sigma^2 + \vec{\pi}^2)B^2(p)} \right] + \frac{(\sigma^2 + \vec{\pi}^2 - 1)B^2(p)}{A^2(p)p^2 + B^2(p)} \right\}. \quad (18)$$

With the correction from the motion of center of mass, the zero-point effect, and the color-electronic and color-magnetic interactions being taken into account, the total energy of a bag (involving three valence quarks) is given as

$$E_B(R) = 3\varepsilon_j(R) + \frac{4}{3}\pi R^3 \mathcal{B} - \frac{Z_0}{R}, \quad (19)$$

where $\varepsilon_j(R)$ is the energy eigenvalue of the quark's equation of motion, R is the radius of the bag, Z_0/R denotes the contribution of the corrections of the motion of the center of mass, zero-point energy, and other effects with Z_0 being a parameter. Just the same as that in Ref. [14], the bag is identified as a nucleon in the present work. With the equilibrium condition

$$\frac{dE_B(R)}{dR} = 0,$$

we can obtain the radius of a nucleon.

III. ALGORITHM AND NUMERICAL RESULTS

A. Algorithm

From the description in last section, we know that the property of a nucleon (for instance, its mass, radius, and bag constant) is determined by the solutions of the equations of motion of the quarks and (chiral) mesons in the soliton. To solve the equations of motion, one needs the solutions $A(p^2)$ and $B(p^2)$ of the quark's Dyson-Schwinger equation. Then after solving the quark's DSE in Eq. (7), or more explicitly [with the help of the decomposition in Eq. (8)] the coupled equations

$$[A(p^2) - 1]p^2 = g^2 C_F \int \frac{d^4q}{(2\pi)^4} D(p - q) \text{tr} \times [i\gamma \cdot p t_{\mu\nu} \gamma_\mu S(q) \Gamma_\nu(p, q)], \quad (20)$$

$$B(p^2) - m_0 = g^2 C_F \int \frac{d^4q}{(2\pi)^4} D(p - q) \text{tr} \times [t_{\mu\nu} \gamma_\mu S(q) \Gamma_\nu(p, q)],$$

where C_F is the eigenvalue of the quadratic Casimir operator in the fundamental representation of the color symmetry group [for $SU(N_c)$, $C_F = (N_c^2 - 1)/2N_c$, it reads 4/3 at $N_c = 3$].

To investigate the temperature dependence of the property of a nucleon, one should at first discuss the form of the quark's DSE at finite temperature.

It has been well known that the appearance of (nonzero) temperature T in the QCD reduces the $O(4)$ symmetry to $O(3)$. Thus the quark's four-momentum p should be rewritten as $p = (\vec{p}, \omega_n)$, where $\omega_n = (2n + 1)\pi T$ ($n \in \mathbb{Z}$) are the discrete Matsubara frequencies of the quark, and the four-dimensional integral needs to be replaced by [31]

$$\int \frac{d^4p}{(2\pi)^4} \rightarrow T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3}. \quad (21)$$

The decomposition of the dressed quark propagator needs to be rewritten as

$$S^{-1}(\vec{p}, \omega_n) = i\vec{\gamma} \cdot \vec{p} A(|\vec{p}|, \omega_n) + i\gamma_4 \omega_n C(|\vec{p}|, \omega_n) + B(|\vec{p}|, \omega_n). \quad (22)$$

Furthermore, the gluon propagator at finite temperature can be generally expressed as [31]

$$D_{\mu\nu}(\vec{k}, \omega_n) = \frac{D_T(\vec{k}, \omega_n)}{k^2} P_{\mu\nu}^T(k) + \frac{D_L(\vec{k}, \omega_n)}{k^2} P_{\mu\nu}^L(k), \quad (23)$$

with the transverse and longitudinal projectors

$$P_{\mu\nu}^T(k) = \begin{cases} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \delta_{i\mu} \delta_{j\nu}, & \mu, \nu = 1, 2, 3, \\ 0, & \mu \text{ or/and } \nu = 4, \end{cases}$$

$$P_{\mu\nu}^L(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - P_{\mu\nu}^T(k). \quad (24)$$

Due to the lack of detailed information about the gluon propagator at finite temperature, one usually naively assumes that the transverse and longitudinal parts of the gluon propagator are equal and independent of temperature (see, for example, Ref. [32]), i.e., one has approximately $D_T(\vec{k}, \omega_n) = D_L(\vec{k}, \omega_n) = D(k)$. In this assumption and bare

vertex approximation, we get three coupled integral equations for the functions $A(\vec{p}, \omega_n)$, $C(\vec{p}, \omega_n)$, and $B(\vec{p}, \omega_n)$ as

$$\begin{aligned} A(|\vec{p}|, \omega_n) &= 1 + g^2 T \frac{C_F}{\vec{p}^2} \sum_m \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{D(k)}{k^2} \frac{1}{\Delta} \{ [(\vec{p} \cdot \vec{q})k^2 \\ &\quad + 2(\vec{p} \cdot \vec{k})(\vec{q} \cdot \vec{k})] A(|\vec{q}|, \omega_m) \\ &\quad + 2\omega_m \Omega_k (\vec{p} \cdot \vec{k}) C(|\vec{q}|, \omega_m) \}, \\ C(|\vec{p}|, \omega_n) &= 1 + g^2 T \frac{C_F}{\omega_n^2} \sum_m \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{D(k)}{k^2} \frac{1}{\Delta} \\ &\quad \times \{ [\omega_n \omega_m k^2 + 2\omega_n \omega_m \Omega_k^2] C(|\vec{q}|, \omega_m) \\ &\quad + 2\omega_n \Omega_k (\vec{p} \cdot \vec{k}) A(|\vec{q}|, \omega_m) \}, \\ B(|\vec{p}|, \omega_n) &= m_0 + g^2 T C_F \sum_m \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{D(k)}{\Delta} 3B(|\vec{q}|, \omega_m), \end{aligned} \quad (25)$$

where $k^2 \equiv \vec{k}^2 + \Omega_k^2$, $\Omega_k \equiv \omega_n - \omega_m$, and $\Delta = \vec{q}^2 A^2 + \omega_m^2 C^2 + B^2$.

As mentioned in last section, in view of the nontopological soliton ansatz, one can take a nucleon as a soliton bag. In the chiral limit ($m_0 = 0$), one has found that there exist two types of solutions for the quark's DSE. One is the Nambu-Goldstone solution which corresponds to the chiral symmetry spontaneously broken phase. The other is the Wigner solution which represents the state with the chiral symmetry. One gets then the effective bag constant as the pressure difference between the Nambu-Goldstone solution and the Wigner solution, which reads

$$\begin{aligned} \mathcal{B}(T) &\equiv P[\mathcal{G}^{NG}] - P[\mathcal{G}^W] \\ &= 4N_c \sum_m \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left[\frac{\Delta_{NG}}{\Delta_W} \right] \right. \\ &\quad \left. + \frac{\vec{p}^2 A_{NG} + \omega_m^2 C_{NG}}{\Delta_{NG}} - \frac{\vec{p}^2 A_W + \omega_m^2 C_W}{\Delta_W} \right\}, \end{aligned} \quad (26)$$

where $\Delta_{NG} \equiv \vec{p}^2 A_{NG}^2 + \omega_m^2 C_{NG}^2 + B_{NG}^2$, $\Delta_W \equiv \vec{p}^2 A_W^2 + \omega_m^2 C_W^2$, and A_{NG} , C_{NG} , B_{NG} , A_W , C_W denotes the Nambu-Goldstone solution, the Wigner solution for the DSE, respectively.

With the solutions of the DSE as input, we can have the explicit expression of the quark's equation of motion at finite temperature as

$$\begin{aligned} [i\vec{\gamma} \cdot \vec{p} A(\vec{p}, T) + i\gamma_4 \omega_n C(\vec{p}, T) + B(\vec{p}, T)] u_j(\vec{p}, T) \\ + \int \frac{d^3 k}{(2\pi)^3} B \left(\frac{p+k}{2}, T \right) [\hat{\sigma}(\vec{p} - \vec{k}) \\ + i\gamma_5 \vec{\tau} \cdot \vec{\pi}(\vec{p} - \vec{k})] u_j(\vec{k}, T) = 0, \end{aligned} \quad (27)$$

where $\hat{\sigma} \equiv \sigma - 1$. After solving the related eigenequation we can obtain the eigenenergy of the quark at finite temperature, $\varepsilon_j(T)$. One can in turn study the properties of a nucleon at finite temperature by extending the GCM soliton model described in last section.

In the case of zero temperature, one usually takes only the lowest energy for the $\varepsilon_j(R)$ in Eq. (18). At finite temperature, due to the influence of temperature, the contribution of the excited states of the quarks must be included. Then the energy

of the bag, i.e., the mass of a nucleon, at finite temperature T should be written as

$$M(T) = E_B(T) = 3\overline{\varepsilon_j(T)} - \frac{Z_0}{R} + \frac{4}{3}\pi R^3 \mathcal{B}(T), \quad (28)$$

where $\overline{\varepsilon_j(T)}$ is the average of a quark's energies at all possible states.

In the spirit of the most simple approximation of the GCM [14] (mentioned at the end of the last section), the quarks in the bag (GCM soliton) at finite temperature can also be regarded as the free one which satisfies the Dirac equation, and the energy eigenvalue can be expressed as

$$\varepsilon_j(T) = \frac{\kappa_j}{R(T)}, \quad (29)$$

where j denotes the quantum number labeling the energy level. Since quarks are fermions, we take the Fermi-Dirac statistics to evaluate the average energy of each quark in the bag at finite temperature, and have

$$\overline{\varepsilon_j(T)} = N \sum_{j=0}^{\infty} \frac{\varepsilon_j(T)}{1 + e^{\varepsilon_j(T)/T}}, \quad (30)$$

where N is the degeneracy of quarks.

Then, by solving the stability condition $\frac{dM(T)}{dR} = 0$, i.e.,

$$\frac{dE_B(T)}{dR} = 3 \frac{d\overline{\varepsilon_j(T)}}{dR} + \frac{Z_0}{R^2} + 4\pi R^2 \mathcal{B}(T) = 0, \quad (31)$$

we can obtain the (stable) nucleon radius $R(T)$ and the mass of a nucleon (the energy of the bag) $M(T)$.

It is apparent that after solving the quark's DSE and in turn the equations of motion of the quark and meson fields at zero and nonzero temperature, we can obtain the bag constant, the mass and the radius of a nucleon in the corresponding circumstance, and discuss the variation characteristic of the property with respect to temperature. To solve the quark's DSE, we take a simplified form of the effective gluon propagator in Ref. [33]:

$$g^2 D(k) = 4\pi^2 D \frac{k^2}{\omega^6} \exp\left(-\frac{k^2}{\omega^2}\right), \quad (32)$$

where D and ω are dimensional parameters that can be determined by fitting empirical data. Such an effective gluon propagator is naturally an extension of the Munczek-Nemirovsky model [27] and consistent with those given in lattice QCD calculations (see, for instance, Refs. [34,35]) and solving the coupled DSEs of quark, gluon, and ghost (see, for instance, Ref. [36]). It has also been shown to be successful in describing many hadron properties [15,33,37–39].

B. Calculation and numerical results

We first solve the quark's DSE at chiral limit ($m_0 = 0$) and zero temperature with parameters $D = 1.0 \text{ GeV}^2$ and $\omega = 0.5 \text{ GeV}$, which have been widely used (see, for example, Refs. [38,39]). The obtained result of the Nambu-Goldstone solution of the DSE at zero temperature is displayed in Fig. 1. It shows evidently that our result reproduces exactly that given in Ref. [37].

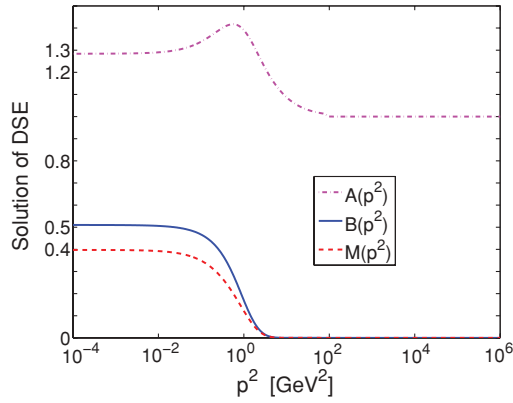


FIG. 1. (Color online) Calculated result of the Nambu-Goldstone solution of the quark's DSE at zero temperature, with parameters in the effective gluon propagator $\omega = 0.5 \text{ GeV}$, $D = 1 \text{ GeV}^2$.

We then solve the quark's DSE at nonzero temperature with the same effective gluon propagator. Figure 2 illustrates the obtained results of the functions $A(|\vec{p}|, T)$, $C(|\vec{p}|, T)$, $B(|\vec{p}|, T)$ in the Nambu-Goldstone solution and the mass function $M(|\vec{p}|, T)$ at a temperature $T = 30 \text{ MeV}$, as an example of those at nonzero temperature. We find from the figure that, just as expected, as the temperature is lower, the functions $A(|\vec{p}|, T)$, $C(|\vec{p}|, T)$, $B(|\vec{p}|, T)$ and $M(|\vec{p}|, T)$ have the correct zero temperature limit. Hence, even though we have not included explicitly the temperature effect in the effective gluon propagator, the calculated result can demonstrate the temperature dependence of quark propagator with quite high precision. In addition, it can be noticed from Fig. 2 that functions $A(|\vec{p}|, \omega_0)$ and $C(|\vec{p}|, \omega_0)$ have the same behavior when the temperature is low, just as that of the quark propagator at low chemical potential [40]. To study the Matsubara frequency dependence of the quark propagator, we illustrate the variation behaviors of the functions $A(|\vec{p}|, \omega_n)$ and $B(|\vec{p}|, \omega_n)$ at a temperature $T = 30 \text{ MeV}$ in Fig. 3 [since Fig. 2 has shown that the functions $A(|\vec{p}|, \omega_n)$ and $C(|\vec{p}|, \omega_n)$ at $T = 30 \text{ MeV}$ are almost exactly equal to each other, we do

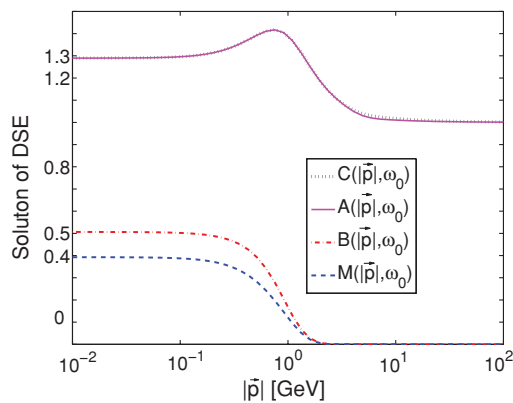


FIG. 2. (Color online) Calculated results of the functions $A(|\vec{p}|, T)$, $C(|\vec{p}|, T)$, $B(|\vec{p}|, T)$, and $M(|\vec{p}|, T)$ in the Nambu-Goldstone solution of the quark propagator at a temperature $T = 30 \text{ MeV}$, with parameters in the effective gluon propagator $\omega = 0.5 \text{ GeV}$, $D = 1 \text{ GeV}^2$.

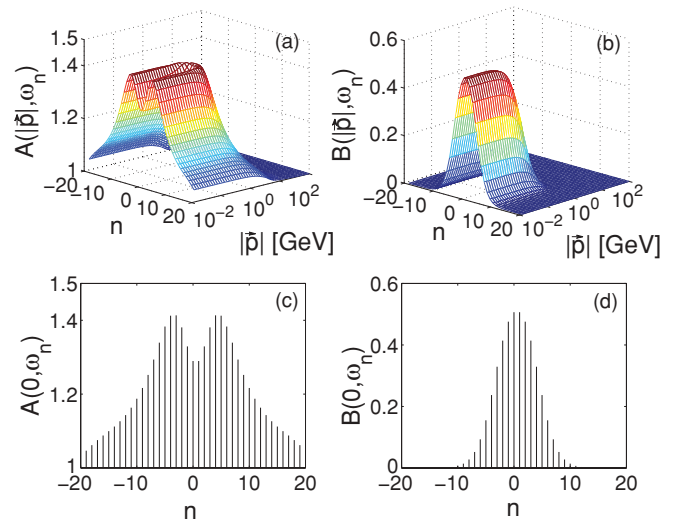


FIG. 3. (Color online) Calculated results of the functions $A(|\vec{p}|, \omega_n)$ and $B(|\vec{p}|, \omega_n)$, where $\omega_n = (2n + 1)\pi T$, in the Nambu-Goldstone solution of the quark propagator in chiral limit and at a temperature $T = 30 \text{ MeV}$ [(a), (b), respectively] and the special case at $|\vec{p}| = 0$ [(c), (d), respectively]. The calculations are also carried out with parameters $\omega = 0.5 \text{ GeV}$, $D = 1 \text{ GeV}^2$ in the effective gluon propagator.

not show the function $C(|\vec{p}|, \omega_n)$ as a representative. From Fig. 3, we can find that, when the temperature is low, both functions A and B involve an approximate bilateral symmetry about n [since for a fixed temperature, ω_n is proportional to n due to the definition $\omega_n = (2n + 1)\pi T$], and the function B decreases rapidly as the Matsubara frequency increases. It gives us a posterior knowledge that when we carry out the summation of the Matsubara frequencies ω_n , it is not necessary to do that up to a very large number of n .

To demonstrate the temperature dependence of the quark propagators explicitly, we illustrate the calculated variation behaviors of the Wigner solution at zero momentum and zero mode Matsubara frequency [i.e., the functions $A_W(|\vec{p}| = 0, \omega_0)$ and $C_W(|\vec{p}| = 0, \omega_0)$] and the Nambu-Goldstone solution under the same conditions [i.e., the functions $A_{NG}(|\vec{p}| = 0, \omega_0)$, $C_{NG}(|\vec{p}| = 0, \omega_0)$, and $B_{NG}(|\vec{p}| = 0, \omega_0)$] with respect to temperature in Fig. 4 as a representative. From Fig. 4, we can find that, when the temperature is low, the functions A_{NG} and C_{NG} coincide with each other very well. So do the A_W and C_W except that the temperature for the deviation between them to appear is lower. It indicates that only as the temperature is quite high, the breaking from $O(4)$ symmetry to $O(3)$ symmetry (especially, for the physical state, i.e., the Nambu-Goldstone state) becomes obvious. Moreover, the decreasing feature of the function B_{NG} is a manifestation of gradual restoration of chiral symmetry.

With the solutions of the quark's DSE as input, we solve the equations of motion of the quark and the chiral meson fields, and then obtain the property of a nucleon at zero and nonzero temperature. The obtained property of a nucleon at zero temperature is bag constant $\mathcal{B}(0) = (162 \text{ MeV})^4$, mass $M(0) = 939 \text{ MeV}$ (with the parameter Z_0 is fixed as 3.08), and radius $R(0) = 0.85 \text{ fm}$. It should be noted that the presently

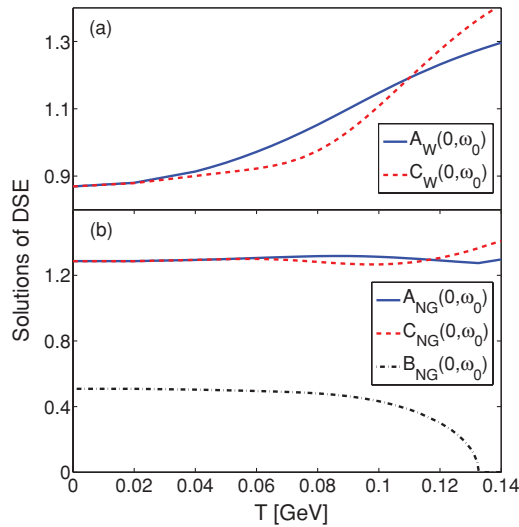


FIG. 4. (Color online) Calculated results of the temperature dependence of the Wigner solution of the quark's DSE at zero momentum and zero mode Matsubara frequency [i.e., the functions $A_W(|\vec{p}| = 0, \omega_0)$ and $C_W(|\vec{p}| = 0, \omega_0)$] (a) and the Nambu-Goldstone solution under the same conditions [i.e., the functions $A_{NG}(|\vec{p}| = 0, \omega_0)$, $C_{NG}(|\vec{p}| = 0, \omega_0)$ and $B_{NG}(|\vec{p}| = 0, \omega_0)$] (b). The calculations are also carried out with parameters $\omega = 0.5$ GeV, $D = 1$ GeV² in the effective gluon propagator.

fixed value of the parameter Z_0 , 3.08, is larger than the usually taken one, 1.84. Such a large value arises from the fact that the term $-Z_0/R$ is a combination of the contributions from not only the zero-point energy but also those of the color-electronic and color-magnetic interactions, the correction on the motion of center-of-mass and other effects. And it is consistent with the most recent result [41] and our previous results [23]. The gained variation behaviors of the nucleon's bag constant, mass, and radius with respect to temperature are illustrated in Figs. 5, 6, 7, respectively. From Figs. 5–7, one can notice that, with the increasing of temperature if it is below a critical one, the bag constant and the mass of a nucleon decrease, and the radius of the nucleon increases. At critical temperature $T = 133$ MeV, the bag constant and the mass of the nucleon vanish and the radius tends to be infinite. It manifests that

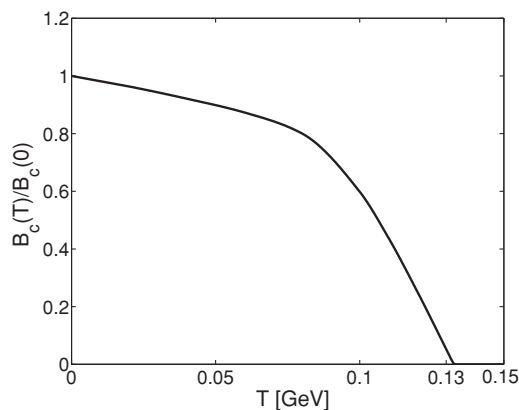


FIG. 5. Calculated result of the variation behavior of the bag constant of a nucleon with respect to temperature.

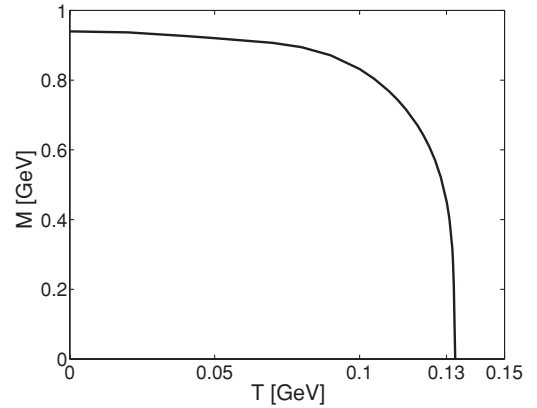


FIG. 6. Calculated result of the variation behavior of the mass of a nucleon with respect to temperature.

the nucleon can no longer exist as a bag soliton, so that the quark deconfinement happens. Admittedly, such a obtained critical temperature may be model-dependent, however, the gradual variation features of the nucleon's property indicate that the deconfinement phase transition process is that, with the increase of temperature, the nucleons in the matter touches with each other at first due to the increase of the radius, then the fields of the ingredients of the nucleons mixed with each other and the bound strength gets weaker simultaneously. As the bound (the bag constant) vanishes, the deconfinement phase transition occurs. Therefore, the temperature driven deconfinement process may be, in fact, a crossover but not a low order phase transition.

When discussing the QCD phase transition, one usually interests in the chiral symmetry and its dynamical breaking, too, and takes the chiral quark condensate as a order parameter in the case of chiral limit, which is defined as

$$-\langle \bar{q}q \rangle = N_c T \sum_{n=-\infty}^{n=+\infty} \int \frac{d^3 p}{(2\pi)^3} \text{tr}[S(\vec{p}, \omega_n)]. \quad (33)$$

We then calculate the temperature dependence of the chiral quark condensate. The obtained result is shown in Fig. 8. The figure displays evidently that the chiral quark condensate decreases gradually with the increase of temperature and

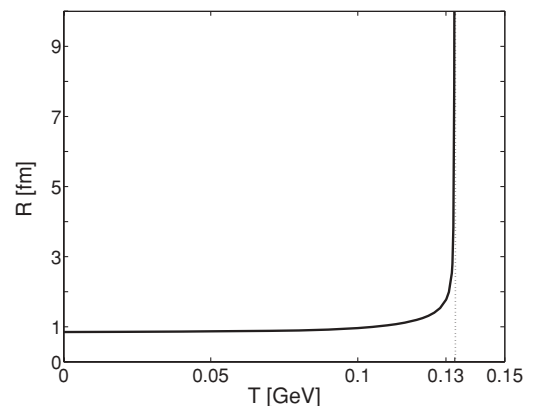


FIG. 7. Calculated result of the variation behavior of the radius of a nucleon with respect to temperature.

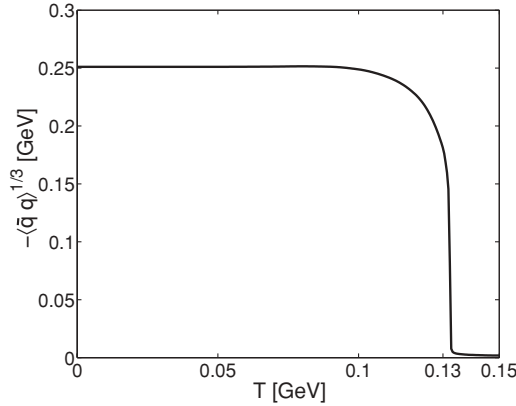


FIG. 8. Calculated result of the variation behavior of the chiral quark condensate with respect to temperature.

vanishes at a critical temperature. It indicates that dynamical chiral symmetry breaking effect gets weaker and weaker with the increase of temperature and the chiral symmetry can be restored as the temperature reaches the critical one. In the case with parameters $\omega = 0.5 \text{ GeV}$, $D = 1 \text{ GeV}^2$ in the effective gluon propagator, the critical temperature for the chiral symmetry to be restored is also approximately 133 MeV, the same as that for the deconfinement.

As mentioned above, the critical temperature for the QCD phase transitions to happen may be parameter dependent. To demonstrate the parameter dependence explicitly, we carry out a series calculations with various values of the coupling strength parameter D and the screening width parameter ω in the effective gluon propagator. Due to the good behavior of the Gaussian-type gluon propagator, we can scale the T_c and D by ω with definition $T'_c \equiv T_c/\omega$ and $D' \equiv D/\omega^2$. The obtained variation behavior of the T'_c with respect to the D' is displayed in Fig. 9. One can find easily from the figure that the critical temperature increases when the coupling strength gets larger. Furthermore, there exists a critical scaled coupling strength D' , below which the critical temperature for the deconfinement maintains zero. In fact, below the critical coupling strength,

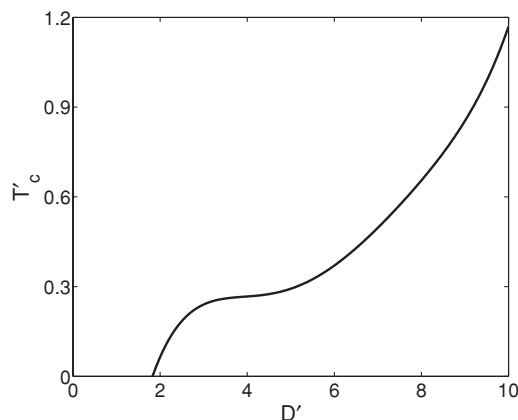


FIG. 9. Calculated result of the variation behavior of the scaled critical temperature with respect to the scaled strength parameter D' in the effective gluon propagator, where $T'_c = T_c/\omega$ and $D' = D/\omega^2$ are the scaled quantities.

the quark's DS equation does not have a Nambu-Goldstone solution [4,5]. In other words, there exists only quarks in chiral symmetry. In turn, we have only deconfined quarks, but no nucleons (more general, hadrons) even if the temperature is zero.

IV. SUMMARY AND REMARKS

In this paper we have calculated the temperature dependence of the quark propagator by solving the quark DSE with a Gaussian-type effective gluon propagator. Based on the calculations, we investigated the temperature dependence of the bag constant, the mass and the radius of a nucleon in the framework of the GCM soliton model. It shows that, as the temperature is lower than a critical value, the bag constant and the mass decrease and the radius increases with the increasing of the temperature. In the case with parameters $\omega = 0.5 \text{ GeV}$, $D = 1 \text{ GeV}^2$ in the effective gluon propagator, the critical temperature is found to be about 133 MeV. At the critical temperature, the bag constant and the mass decrease to zero and the radius increases to infinity. It means that the nucleon can no longer exist as a bag soliton, so that the deconfinement phase happens. This indicates evidently that the quark deconfinement phase transition can take place at high temperature. Moreover, we give the dependence of the critical temperature on the interaction strength parameter D in the effective gluon propagator. It shows that, as the interaction strength parameter is larger than a critical value, the critical temperature increases with the increasing of the strength parameter.

Even though the temperature dependence of some of the properties of nucleon is given with some approximations and model parameters in the present work, the qualitative behavior would be universal and it is the first one given with quite a sophisticated approximation of QCD. Of course, there are various aspects to be improved. For example, we take the commonly used effective gluon propagator, which is independent of temperature, to solve the quark DSE and make use of the preliminary GCM soliton model [14,16,23]. It is necessary to implement the real GCM soliton model [17,18] with solving at first the coupled DSEs of the quark, gluon, and ghost, and then the coupled equations of the quark and the chiral fields, with the inclusion of the temperature effect. In more detail, for the quark gluon interaction vertex, we take simply the bare vertex γ_μ in our present work, in fact more realistic vertex functions, such as the BC vertex [42], which has been shown to be able to improve the calculation of meson properties greatly [39], even the BC vertex together with the transverse part being included simultaneously [43–45], should be implemented to make the calculation with much more solid QCD foundation. It is also necessary to notice the dimensionless quantity Z_0 which takes account of the contributions of the zero-point effect, the color-electronic and color-magnetic interactions, the motion of center of mass, and all the others. In our present work, we handle it, in the commonly taken way, as a free parameter to be fixed by the property of a nucleon in free space. In fact, all the aspects of the Z_0 are quite complicated and have recently been paid great attentions (see for example, Refs. [46,47])

tried to evaluate the zero-point effect part from the gluon field fluctuations directly). Furthermore, extending the result of the thermal Casimir effect in ideal metal rectangular boxes [48], we infer that the Z_0 (at least, the zero-point effect part) may depend on temperature. It would then be interesting to study the temperature dependence of the parameter Z_0 . The related investigations are in progress.

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