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Binding energy of a holographic deuteron and tritium in anti-de-Sitter space/conformal field theory (AdS/CFT)

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In the large 't Hooft coupling limit, the hadronic size of baryon is small and the nucleon-nucleon potential is obtained from massless pseudoscalar exchanges and an infinite tower of spin-one mesons exchanges. In this article we use the holographic nucleon-nucleon interaction and obtain the corresponding potential and binding energy for deuteron and tritium nuclei. The obtained potentials are repulsive at short distances and clearly become zero by increasing the distance as we expected.

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I. INTRODUCTION

One application of anti-de-Sitter space/conformal field theory (AdS/CFT) duality is in low-energy hadron dynamics [1] that is referred to as holographic QCD or AdS/QCD. This method expresses two related issues from opposite directions, one from string theory [2,3] and the other from the low-energy chiral effective-field theory of mesons and baryons [4,5]. From the view point of string theory the duality of strongly coupled dynamics of QCD and the bulk sector in a controlled weak coupling limit of the gravity theory is an interested subject. However, from the low-energy chiral effective-field theory outlook the purpose is whether holographic QCD can make clear predictions on processes that are difficult to study by using QCD proper.

There are many remarkable examples [6–11] such as chiral dynamics of hadrons, in particular, baryons at low energy that are typically of strong-coupling QCD and very difficult to explain by using QCD techniques.

Between the holographic models of QCD suggested recently, the Sakai and Sugimoto (SS) model [2] is one of the most interesting and realistic models because of the accurate results of this model. For example, the predicted results from the SS model on the glueball spectrum of pure QCD are in good agreement with lattice simulation [12]. Also, this model successfully described baryons and their interactions with mesons [2,13,14]. This is a $D_4 - D_8$ model that involves a large number of colors, N_c large 't Hooft coupling λ, and quenching of fermions.

The holographic baryon in the $D_4 - D_8$ model is equivalent with the skyrmion of chiral perturbation theory. Actually, Holographic baryon is a direct uplift of skyrmion in the holographic picture, but considering baryon as a solution is not a proper way to obtain the nucleon-nucleon interaction. It arises because finding a suitable configuration is impossible for such complicated solitons.

To study the interaction of baryons in large distances, where the interbaryon distance is large compared with the size of the considering baryons as point-like particles is a good approximation. In this case, the interactions can be all ascribed by exchange of light particles such as mesons and one can find the baryon-baryon interaction with the Feynman diagrams using cubic interaction vertices including baryon currents and light mesons [15].

Fortunately from the $D_4 - D_8$ holographic QCD model, all nucleon-meson coupling constants, at least in large λN_c, are obtained [13]. Also some of these coupling constants such as the axial coupling to pions g_A and vector meson couplings $g_{\rho\mathcal{N}\mathcal{N}}$ and $g_{\omega\mathcal{N}\mathcal{N}}$ are in good agreement with the experimental data. Recently, the nucleon-nucleon potential in the holographic picture was studied using the meson exchange method [16].

In this article we are going to calculate the binding energy of light nuclei such as deuteron and tritium using the AdS/QCD. In order to do this, we apply the $D_4 - D_8$ model, nucleon-meson interaction, and nucleon-nucleon potential. Then we consider the exchange of pions, isospin singlet vector mesons, isospin triplet vector mesons, and triplet axial-vector mesons in this potential. The minimum of the resultant potential is considered as nuclear binding energy. Finally we obtain the radius of these nuclei in the holographic picture.

II. $D_4 - D_8$ HOLOGRAPHIC QCD

In this model, N_c stack of D_4 and $N_f D_8$ branes are considered in the background of Type II A superstring [2] and the flavor symmetries of the quark sector are embedded into a $U(N_f)$ gauge symmetry in $R^{1+3} \times I$. By restricting to the modes that are localized near the origin of the fifth direction, which is topologically an interval, we can arrive at the four-dimensional low-energy physics. Also the massless part of this model is pure $U(N_c)$ Yang-Mills theory because fermions permit an anti-periodic boundary condition.

In large N_c limit, the dynamics of D_4 brane is dual to a closed string theory in the curved background with flux in accordance with the general AdS/CFT idea. In the large 't Hooft coupling limit ($\lambda = g_{\rm YM}^2 N_c \gg 1$) and neglecting the

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gravitational backreaction from the D_8 branes the metric is [17]

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} [\eta_{\mu\nu} dx^\mu dx^\nu + f(U)\, d\tau^2] + \left(\frac{R}{U}\right)^{3/2} \left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right], \quad (1)$$

where $R^3 = \pi g_s N_c l_s^3$ and $f(U) = 1 - U_{KK}^3/U^3$. The coordinate τ is compactified as $\tau = \tau + \delta\tau$ with $\delta\tau = 4\pi R^{3/2}/(3U_{KK}^{1/2})$.

The effective action on a D_8 brane embedding in a D_4 background has the following form:

$$S_{D8} = -\mu_8 \int \mathrm{d}^9 x\, e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} + \mu_8 \int C_3 \wedge \mathrm{Tr}\, e^{2\pi\alpha' F}, \quad (2)$$

with $\mu_8 = \frac{2\pi}{(2\pi l_s)^9}$. By introducing the conformal coordinate w instead of the holographic coordinate U as

$$w = \int_{U_{KK}}^{U} \frac{R^{3/2} dU'}{\sqrt{U'^3 - U_{KK}^3}}, \quad (3)$$

the noncompact five-dimensional (5D) part of the metric is conformally flat, then the induced metric on the D_8 brane has the following form:

$$g_{8+1} = \frac{U^{3/2}(w)}{R^{3/2}}(dw^2 + \eta_{\mu\nu} dx^\mu dx^\nu) + \frac{R^{3/2}}{U^{1/2}(w)} d\Omega_4^2, \quad (4)$$

where $U_{KK} = 2/9 g_{\rm YM}^2 N_c M_{KK} l_s^2$. M_{KK}, λ, and N_c determine all the physical scales such as the QCD scale and the pion decay constant.

In the low-energy limit, the world-volume dynamics of the multi-D_8 brane system give the following Yang-Mills action with a Chern-Simons term as

$$\frac{1}{4} \int_{4+1} \sqrt{-g_{4+1}}\, \frac{e^{-\Phi} V_{S^4}}{2\pi(2\pi l_s)^5}\, \mathrm{tr} F_{\hat{m}\hat{n}} F^{\hat{m}\hat{n}} + \frac{N_c}{24\pi^2} \int_{4+1} \omega_5(\mathcal{A}), \quad (5)$$

where V_{S^4} is the S^4 volume while the dilaton is

$$e^{-\Phi} = \frac{1}{g_s}\left(\frac{R}{U}\right)^{3/4}, \quad (6)$$

and $d\omega_5(A) = \mathrm{tr}F^3$.

III. BARYON HOLOGRAPHY AND NUCLEON-NUCLEON POTENTIAL

Witten introduced a D_4 brane wrapping the compact S^4 as a baryon vertex on the 5D space-time [18]. It is shown that a D_4 brane wrapping S^4 looks like an object carrying electric charge with respect to the gauge field on D_8 and it is possible to say that D_4 brane spread inside D_8 brane as an instanton. The size of this instanton is determined by minimizing its total energy [13,14], which is combined mass and Coulomb energy,

$$\rho_{\rm baryon} \sim \frac{9.6}{M_{KK}\sqrt{g_{\rm YM}^2 N}}. \quad (7)$$

Thus, in the large 't Hooft coupling limit, instantonic baryon is a small object in five dimensions and baryon can be considered as a point-like quantum field in 5D. In consequence, there should be couplings between this quantum field and the 5D gauge fields moreover the standard Dirac kinetic and a position-dependent mass term [19].

The action involving the baryon field and the gauge field in the conformal coordinate (x^μ, w) is written as

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w) \bar{\mathcal{B}}\mathcal{B} + g_5(w) \frac{\rho_{\rm baryon}^2}{e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B} \right] - \int d^4x dw \frac{1}{4e^2(w)} \mathrm{tr} F_{mn} F^{mn}, \quad (8)$$

$g_5(w)$ is an unknown function of w that is evaluated only at $w = 0$, where $g_5(0) = 2\pi^2/3$.

Since the four-dimensional (4D) low-energy physics is found by restricting to the modes that are localized near the origin of the fifth direction w, the physical 4D nucleons will arise as the lowest eigenmodes of the 5D baryon along the w coordinate. Thus the 5D action, Eq. (8), must be reduced to four dimensions. It can be done by applying the mode expansion for the baryon field and the gauge field and plugging these into the baryon action.

On one hand, the gauge field A_μ, in the $A_5 = 0$ gauge, has the following mode expansion:

$$A_\mu(x, w) = i\alpha_\mu(x)\psi_0(w) + i\beta_\mu(x) + \sum_n a_\mu^{(n)}(x)\psi_{(n)}(w). \quad (9)$$

The eigenmode analysis was done by Sakai and Sugimoto in Ref. [2] previously. We only note that $\psi_{(2k+1)}(w)$ is even, while $\psi_{(2k)}(w)$ is odd under $w \to -w$, corresponding to the vector and axial-vector mesons, respectively. Also the eigenfunctions $\psi_{(n)}$ obey the following equation according to [2]

$$-K^{-1/3}\partial_\omega[K^{1/3}\partial_\omega \psi_{(n)}] = \left(U_{KK}^2 M_{KK}^2\right)\lambda_n \psi_{(n)}, \quad (10)$$

where $K = (\frac{U}{U_{KK}})^3$. Also they satisfy the orthonormality condition

$$\int dw \frac{e^{-\Phi} V_{S^4}}{4\pi(2\pi l_s)^5} \psi_{(n)}(w)^* \psi_{(m)}(w) = \delta_{nm}. \quad (11)$$

We have to solve Eq. (10) with the normalization condition given by Eq. (11) to find the eigenfunction $\psi_{(n)}$. These equations were solved numerically by means of a shooting method. The corresponding computations are given in Ref. [2] in detail.

On the other hand, the nucleon field can be expanded as $\mathcal{B}_{L,R}(x^\mu, w) = B_{L,R}(x^\mu) f_{L,R}(w)$, where $\gamma^5 B_{L,R} = \pm B_{L,R}$ are 4D chiral components. $f_{L,R}(w)$ are profile functions that satisfy the following conditions in the interval $w \in [-w_{\max}, w_{\max}]$

$$\begin{aligned} \partial_w f_L(w) + m_b(w) f_L(w) &= m_B f_R(w), \\ -\partial_w f_R(w) + m_b(w) f_R(w) &= m_B f_L(w). \end{aligned} \tag{12}$$

The eigenvalue m_B is the mass of the nucleon mode $B(x)$, where the 4D Dirac field for the nucleon is $B = \binom{B_L}{B_R}$ and the eigenfunctions $f_{L,R}(w)$ are normalized as

$$\int_{-w_{\max}}^{w_{\max}} dw\ |f_L(w)|^2 = \int_{-w_{\max}}^{w_{\max}} dw\ |f_R(w)|^2 = 1. \tag{13}$$

Using the properties $f_L(w) = \pm f_R(-w)$ as well as $\psi_0(w)$ and $\psi_n(w)$ under $w \to -w$ and by plugging into the mode expansion of the gauge field, the 4D effective action is achieved for the nucleon

$$\int dx^4 (-i\bar{B}\gamma^\mu \partial_\mu B - i m_B \bar{B} B + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}}), \tag{14}$$

where the nucleon coupling to vector mesons $\mathcal{L}_{\text{vector}}$ and axial mesons $\mathcal{L}_{\text{axial}}$ are

$$\begin{aligned} \mathcal{L}_{\text{vector}} &= -i\bar{B}\gamma^\mu \beta_\mu B - \sum_{k\geqslant 0} g_V^{(k)} \bar{B}\gamma^\mu a_\mu^{(2k+1)} B, \\ \mathcal{L}_{\text{axial}} &= -\frac{i g_A}{2} \bar{B}\gamma^\mu \gamma^5 \alpha_\mu B - \sum_{k\geqslant 1} g_A^{(k)} \bar{B}\gamma^\mu\gamma^5 a_\mu^{(2k)} B, \end{aligned} \tag{15}$$

where various coupling constants $g_{A,V}^{(k)}$ as well as the pion-nucleon axial coupling g_A are calculated by suitable wave-function overlap integrals. These coupling constants were studied in Ref. [13] in detail.

Finally, in general, the one-boson exchange nucleon-nucleon potential is written as [16]

$$V_\pi + V_{\eta'} + \sum_{k=1}^{\infty} V_{\rho^{(k)}} + \sum_{k=1}^{\infty} V_{\omega^{(k)}} + \sum_{k=1}^{\infty} V_{a^{(k)}} + \sum_{k=1}^{\infty} V_{f^{(k)}}, \tag{16}$$

that is a sum of the pseudoscalar, vector, and axial-vector mesons exchange terms, respectively.

But only following four classes of these couplings have a leading contribution in nucleon-nucleon potential

$$\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \sim g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \sim \frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \sim g_{a^{(k)}\mathcal{N}\mathcal{N}}. \tag{17}$$

In the $D_4 - D_8$ holography model, the pion mass is zero then one pion exchange potential (OPEP) in this sense has the following form

$$V_\pi = \frac{1}{4\pi}\left(\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}}\right)^2 \frac{1}{M_{KK}^2 r^3} S_{12} \vec{\tau}_1 \cdot \vec{\tau}_2. \tag{18}$$

Also, the holographic potentials for the isospin singlet vector mesons $\omega^{(k)}$, isospin triplet vecto mesons $\rho^{(k)}$, and the triplet axial-vector mesons $a^{(k)}$ are

$$V_{\omega^{(k)}} = \frac{1}{4\pi}\ [g_{\omega^{(k)}\mathcal{N}\mathcal{N}}]^2\ m_{\omega^{(k)}}\ y_0[m_{\omega^{(k)}} r], \tag{19}$$

and

$$\begin{aligned} V_{\rho^{(k)}} \simeq{}& \frac{1}{4\pi}\left[\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}}\right]^2 \frac{m_{\rho^{(k)}}^3}{3M_{KK}^2}\{2y_0[m_{\rho^{(k)}} r]\vec{\sigma}_1\cdot\vec{\sigma}_2 \\ &- y_2(m_{\rho^{(k)}} r) S_{12}(\hat{r})\}\vec{\tau}_1\cdot\vec{\tau}_2, \end{aligned} \tag{20}$$

$$\begin{aligned} V_{a^{(k)}} \simeq{}& \frac{1}{4\pi}\ (g_{a^{(k)}\mathcal{N}\mathcal{N}})^2\ \frac{m_{a^{(k)}}}{3}\{-2y_0[m_{a^{(k)}} r]\vec{\sigma}_1\cdot\vec{\sigma}_2 \\ &+ y_2[m_{a^{(k)}} r) S_{12}(\hat{r})\}\vec{\tau}_1\cdot\vec{\tau}_2, \end{aligned} \tag{21}$$

respectively.

In the above equations, level p is determined by distance scale and

$$S_{12} = 3(\vec{\sigma}\cdot\hat{r})(\sigma_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2, \tag{22}$$

$$y_0(x) = \frac{e^{-x}}{x}, \quad y_2(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\frac{e^{-x}}{x}. \tag{23}$$

The masses of all mesons are of order M_{KK} and $m_{\rho^{(k)}} = m_{\omega^{(k)}} < m_{a^{(k)}}$.

In general, for large limited λ, in the smallest distance $1/\sqrt{\lambda} M_{KK}$, the one-meson exchange potential is satisfied. Also, $p \simeq \sqrt{\lambda/10}$ is an acceptable value for this potential.

In the large λN_c limit, the coupling constants are given by [13]

$$\begin{aligned} \frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &\simeq 8.43\sqrt{\frac{N_c}{\lambda}}, \\ g_{\omega^{(k)}\mathcal{N}\mathcal{N}} &\simeq \sqrt{2.3^3.\pi^3}\hat{\psi}_{(2k-1)}(0)\sqrt{\frac{N_c}{\lambda}} = \xi_k\sqrt{\frac{N_c}{\lambda}}, \\ \frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &\simeq \sqrt{\frac{2^2.3^2.\pi^3}{5}}\hat{\psi}_{(2k-1)}(0)\sqrt{\frac{N_c}{\lambda}} = \zeta_k\sqrt{\frac{N_c}{\lambda}}, \\ g_{a^{(k)}\mathcal{N}\mathcal{N}} &\simeq \sqrt{\frac{2^2.3^2.\pi^3}{5}}\hat{\psi}'_{(2k)}(0)\sqrt{\frac{N_c}{\lambda}} = \chi_k\sqrt{\frac{N_c}{\lambda}}, \end{aligned} \tag{24}$$

where the coefficients, ξ_k, ζ_k, and χ_k are calculated using the ψ values by numerical methods. The value of these coefficients are given in Ref. [16], and listed in Table I.

TABLE I. Numerical results for $\hat{\psi}_{(2k-1)}(0)$, $\hat{\psi}'_{(2k)}(0)$, ξ_k, ζ_k, and χ_k for spin-one mesons interacting with nucleons [16].

k	$\hat{\psi}_{(2k-1)}(0)$	ξ_k	ζ_k	$\hat{\psi}'_{(2k)}(0)$	χ_k
1	0.5973	24.44	8.925	0.629	9.40
2	0.5450	22.30	8.143	1.10	16.4
3	0.5328	21.81	7.961	1.56	23.3
4	0.5288	21.64	7.901	2.02	30.1
5	0.5270	21.57	7.874	2.47	36.9
6	0.5261	21.52	7.860	2.93	43.8
7	0.5255	21.50	7.852	3.38	50.5
8	0.5251	21.48	7.846	3.83	57.3
9	0.5249	21.48	7.843	4.29	64.1
10	0.5247	21.47	7.840	4.74	70.9

IV. BINDING ENERGIES OF DEUTERON AND TRITIUM NUCLEI

Here we aim to calculate the binding energy of deuteron and tritium nuclei using the holographic nucleon-nucleon potential represented in Sec. III.

To calculate the binding energy of deuteron, the following potential is considered

$$V_{\text{deuteron}}^{\text{holography}} = V_C + \left(V_T^{\sigma} \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^S S_{12}\right) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (25)$$

where

$$V_C = \sum_{k=1}^{10} \frac{1}{4\pi} \; [g_{\omega^{(k)}\mathcal{N}N}]^2 \; m_{\omega^{(k)}} \; y_0[m_{\omega^{(k)}} r] m, \quad (26)$$

$$V_T^{\sigma} = \sum_{k=1}^{10} \frac{1}{4\pi} \left[\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}N} M_{KK}}{2m_{\mathcal{N}}}\right]^2 \frac{m^3_{\rho^{(k)}}}{3M^2_{KK}} \{2y_0[m_{\rho^{(k)}} r]\} + \sum_{k=1}^{10} \frac{1}{4\pi} [g_{a^{(k)}\mathcal{N}N}]^2 \; \frac{m_{a^{(k)}}}{3} \; \{-2y_0[m_{a^{(k)}} r]\}, \quad (27)$$

and

$$V_T^S = \frac{1}{4\pi} \left(\frac{g_{\pi\mathcal{N}N} M_{KK}}{2m_{\mathcal{N}}}\right)^2 \frac{1}{M^2_{KK} r^3} + \sum_{k=1}^{10} \frac{1}{4\pi} \left(\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}N} M_{KK}}{2m_{\mathcal{N}}}\right)^2 \frac{m^3_{\rho^{(k)}}}{3M^2_{KK}} \{-y_2[m_{\rho^{(k)}} r]\} + \sum_{k=1}^{10} \frac{1}{4\pi} [g_{a^{(k)}\mathcal{N}N}]^2 \; \frac{m_{a^{(k)}}}{3} \; \{y_2[m_{a^{(k)}} r]\}. \quad (28)$$

The values of coupling constants for different amounts of k, along with the mass of the vector and axial-vector mesons (in units of M_{KK} and for large $\lambda \, N_c$) are presented in Table II. In these computations, we choose $\lambda = 400$, $m_{\mathcal{N}} = 0.55$ GeV and $N_c = 3$ for realistic QCD.

The deuteron nucleus consists of one proton and one neutron, thus by superselection rules we have

$$S_{12} = 2, \quad \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, \quad \vec{\tau}_1 \cdot \vec{\tau}_2 = -3. \quad (29)$$

The deuteron potential for the large N_c with $p = 10$ is calculated and is shown in Fig. 1. As it is clear from this figure, the deuteron potential has a minimum point at 4.41 M_{KK}. For distance, r less than the $r_{\min}$ potential increases rapidly and becomes infinity at $r = 0$ as expected. The minimum value of the potential is $-1.9645 \, M_{KK} \, N_c/4\pi\lambda$. So the binding energy of deuteron is obtained roughly -2.204 MeV that is consistent with the experimental nuclear data.

Also, tritium consist of three nucleons, two neutrons and one proton, so we suppose the following form for its potential

$$\begin{aligned} V_{\text{Tritium}}^{\text{holography}} &= V_{12} + V_{13} + V_{23} \\ &= V_C + \left(V_T^{\sigma} \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^S S_{12}\right) \vec{\tau}_1 \cdot \vec{\tau}_2 \\ &\quad + V_C + \left(V_T^{\sigma} \vec{\sigma}_1 \cdot \vec{\sigma}_3 + V_T^S S_{13}\right) \vec{\tau}_1 \cdot \vec{\tau}_3 \\ &\quad + V_C + \left(V_T^{\sigma} \vec{\sigma}_2 \cdot \vec{\sigma}_3 + V_T^S S_{23}\right) \vec{\tau}_2 \cdot \vec{\tau}_3. \end{aligned} \quad (30)$$

TABLE II. Numerical results for masses and coupling constants for spin-one mesons interacting with nucleons in the large λN_c limit. We choose $\lambda = 400$, $m_{\mathcal{N}} = 0.55$ GeV, and $N_c = 3$ for realistic QCD.

k	$m_{\omega^{(k)}}$	$m_{a^{(k)}}$	$g_{\omega^{(k)}\mathcal{N}\mathcal{N}}$	$\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}$	$g_{a^{(k)}\mathcal{N}\mathcal{N}}$
1	0.818	1.25	2.1165	0.7055	0.8140
2	1.69	2.13	1.9312	0.6437	1.4202
3	2.57	3.00	1.8888	0.6296	2.0178
4	3.44	3.87	1.8740	0.6246	2.6067
5	4.30	4.73	1.8680	0.6226	3.1956
6	5.17	5.59	1.8636	0.6212	3.7931
7	6.03	6.46	1.8619	0.6206	4.3734
8	6.89	7.32	1.8602	0.6200	4.9623
9	7.75	8.19	1.8602	0.6200	5.5512
10	8.62	9.05	1.8593	0.6197	6.1401

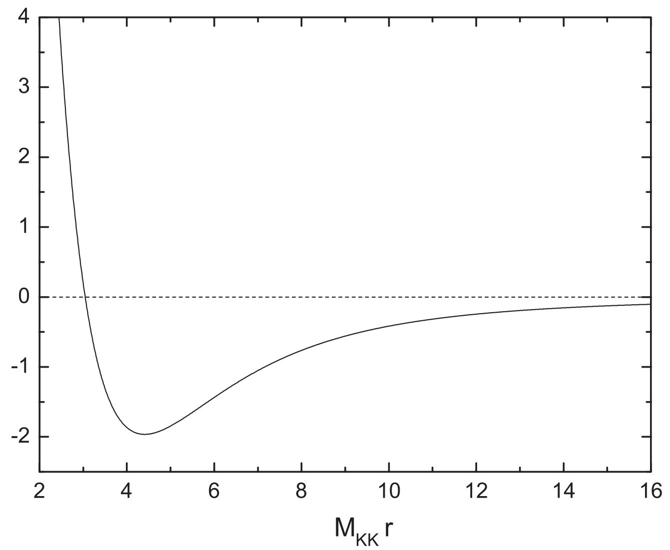

FIG. 1. The deuteron potential in large $\lambda \, N_c$ limit and $p = 10$. The horizontal axis is $r \, M_{KK}$, while the vertical axis is deuteron potential in unit of $M_{KK} \, N_c/4\pi\lambda$.

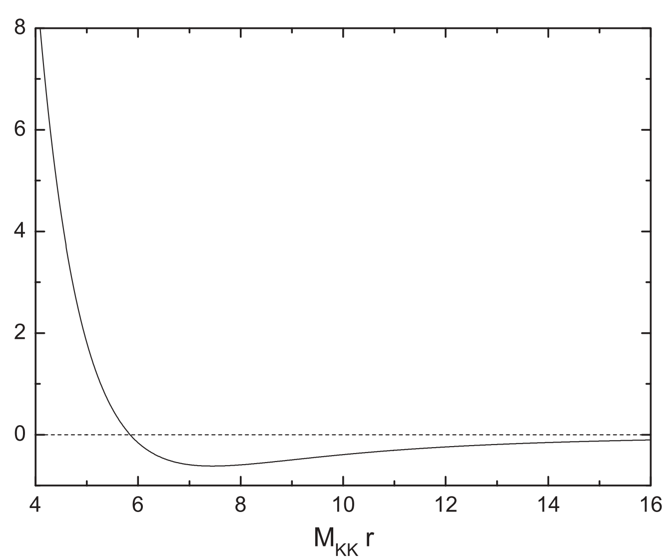

FIG. 2. The tritium potential in large $\lambda \, N_c$ limit and $p = 10$. The horizontal axis is $r \, M_{KK}$, while the vertical axis is deuteron potential in unit of $M_{KK} \, N_c/4\pi\lambda$.

The superselection rules for this three-nucleon system implies that

$$\begin{aligned} S_{12} &= 2, \quad \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, \quad \vec{\tau}_1 \cdot \vec{\tau}_2 = -3, \\ S_{13} &= 0, \quad \vec{\sigma}_1 \cdot \vec{\sigma}_3 = -3, \quad \vec{\tau}_1 \cdot \vec{\tau}_3 = -3, \\ S_{23} &= 0, \quad \vec{\sigma}_2 \cdot \vec{\sigma}_3 = -3, \quad \vec{\tau}_2 \cdot \vec{\tau}_3 = 1. \end{aligned} \tag{31}$$

The holographic potential of tritium in terms of $M_{KK}\, r$ is shown in Fig. 2. This potential also has a minimum that occurs in 7.46 M_{KK}. The value of potential in its minimum is $-0.617\, M_{KK}\, N_c/4\pi\lambda$, so the binding energy of tritium is equal to -1.034 MeV. This figure also shows the repulsive behavior of potential at short distances.

V. CONCLUSION

In this investigation we calculate the deuteron and tritium binding energy using the QCD holography model. Here we use the nucleon-nucleon interaction in the $D_4 - D_8$ model in the base of one-boson exchange picture. This potential involves only the exchanges of pions, isospin singlet mesons, isospin triplet mesons, and triplet axial-vector mesons. We selected the $\lambda = 400$ and at least 10 terms of infinite tower of spin one mesons are considered.

We depicted the deuteron and tritium potentials in terms of $M_{KK} r$ and in unit of $M_{KK} N_c/4\pi\lambda$. As it is indicated in Figs. 1 and 2, these potentials have repulsive behavior at short distances and became roughly zero at large $M_{KK} r$. The deuteron potential contains a shallow minimum in depth $\sim -13.84\, M_{KK}\, N_c/\lambda$ around the $r\, M_{KK} = 4.41$. Also the potential of tritium nuclei reach a more shallow minimum around $r\, M_{KK} = 7.46$ with depth $\sim -4.35\, M_{KK}\, N_c/\lambda$. Thus by using these results binding energies of deuteron and tritium nuclei approximated by -2.204 and -1.039 MeV, respectively.

This method can be improved to calculate binding energies of heavier nuclei by considering exchange of heavier mesons.

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