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Spin-$\frac{5}{2}$ fields in hadron physics

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We show that the Lagrangian of a free spin-$\frac{5}{2}$ field in the spinor-tensor representation with an auxiliary spinor field depends on three arbitrary parameters. The first two parameters are associated with the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sector of the theory while the last is related to the auxiliary degrees of freedom. We derive a corresponding propagator of the system which represents a (2×2) matrix in the $(\psi_{\mu\nu}, \xi)$ space. The diagonal terms stand for the propagation of the spin-$\frac{5}{2}$ and auxiliary fields whereas the nondiagonal ones correspond to the $\psi_{\mu\nu}$-ξ mixing. The resulting spin-$\frac{5}{2}$ propagator contains nonpole contributions coming from the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sector of the spinor-tensor representation. A general form of the interaction vertex involving the spin-$\frac{5}{2}$ field is discussed by the example of the $\pi N N^*_{\frac{5}{2}}$ coupling. It is demonstrated that lower spin degrees of freedom can be removed from the theory by use of higher-order derivative coupling.

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I. INTRODUCTION

The description of pion- and photon-induced reactions in the resonance energy region is faced with the problem of the proper treatment of higher-spin states. In 1941 Rarita and Schwinger (RS) suggested a set of equations that a field function of a higher spin should obey [1]. Another formulation has been developed by Fierz and Pauli [2] where an auxiliary field concept is used to derive subsidiary constraints on the field function.

Regardless of the procedure used, the Lagrangians obtained for free higher-spin fields turn out to be always dependent on arbitrary free parameters. For the spin-$\frac{3}{2}$ fields, this issue is widely discussed in the literature (see, e.g., [3–5] for the modern status of the problem). The case of the spin-$\frac{5}{2}$ fields is less studied. The first attempts were made in [6,7], where a theory of free fields was suggested. The authors of [7] deduced an equation of motion as a decomposition in terms of corresponding projection operators with additional algebraic constraints on parameters of the decomposition. Schwinger [6] derived a particular form of the spin-$\frac{5}{2}$ equation which coincides with the equation suggested in [7] for a specific choice of the parameters.

The free-particle propagator is a central quantity in most calculations in quantum field theory. In [7] the authors deduced a spin-$\frac{5}{2}$ propagator written in operator form. In practical calculations, however, one needs an explicit expression of the propagator. An attempt to construct a propagator only from the spin-$\frac{5}{2}$ projection operator has been made in [8]. We demonstrate that such a quantity is not consistent with the equation of motion for the spin-$\frac{5}{2}$ field. Another pathology is experienced with the propagator [9] and projector [10] used in calculations of the resonance production amplitudes: they do not satisfy the condition $[\gamma_0 G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}]^\dagger = \gamma_0 G^{\frac{5}{2}}_{\rho\sigma;\mu\nu}$, where $G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}$ is a spin-$\frac{5}{2}$ propagator, and consequently are not Hermitian. Therefore, it is important to derive the propagator and investigate its properties in detail. To our knowledge no such study has been done so far.

The aim of this paper is to deduce an explicit expression for the spin-$\frac{5}{2}$ propagator and study its properties. Guided by the properties of the free spin-$\frac{3}{2}$ RS theory, one would expect the equation of motion for the spin-$\frac{5}{2}$ field to have two arbitrary free parameters which define the nonpole spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ contributions to the full propagator. The coupling of the spin-$\frac{5}{2}$ field to the pion-nucleon final state, for example, is therefore defined up to two "off-shell" parameters [11] which scale the nonpole contributions to the physical observables. Hence, one can ask whether this arbitrariness can be removed from the theory.

The possibility of constructing consistent higher-spin massless theories has already been pointed out by Weinberg and Witten some time ago [12]. Pascalutsa and Timmermans showed that, by use of a gauge-invariant coupling for higher-spin fields, it is possible to remove the extra degrees of freedom [13] in a particular case of the RS theory that maintains gauge invariance in the massless limit.

As we demonstrated in [4], the demand for gauge invariance may not be enough to eliminate the extra degrees of freedom at the interaction vertex. The problem appears when the theory does not have a massless limit. However, a coupling that removes nonpole terms from the spin-$\frac{5}{2}$ propagator can be easily constructed by using higher-order derivatives. A corresponding interaction Lagragian was deduced in [4] for the case of spin-$\frac{3}{2}$ fields and can be easily extended to higher spins too.

The paper is organized as follows. In Sec. II we suggest an alternative form of the free spin-$\frac{5}{2}$ Lagrangian as compared to [7] and discuss its properties in detail. The presence of the auxiliary field complicates the derivation of the propagator. Therefore, in Sec. III we first demonstrate how the free propagator can be obtained for a vector field in the presence of an auxiliary one. The method is then applied to the

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spin-$\frac{5}{2}$ field in Sec. IV. The resulting spin-$\frac{5}{2}$ propagator contains contributions corresponding to the lower spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sector of the spin-tensor representation. In Sec. V we discuss how these degrees of freedom can be removed from the physical observables using the example of the pion-nucleon scattering amplitude. The results are summarized in Sec. VI.

II. FREE SPIN-$\frac{5}{2}$ FIELD

The field function of higher spins in the spinor-tensor representation is a solution of the set of equations suggested by Rarita and Schwinger in [1]. In a consistent theory the description of the free field is specified by setting up an appropriate Lagrange function $\mathcal{L}(\psi_{\mu\nu}, \partial_\rho\psi_{\mu\nu})$. The spin-$\frac{5}{2}$ Lagrangian in the presence of the auxiliary spinor field $\xi(x)$ can be written in the form

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(\mathrm{aux})}, \tag{1}$$

where the explicit expressions for $\mathcal{L}^{(1)}$, $\mathcal{L}^{(2)}$, and $\mathcal{L}^{(\mathrm{aux})}$ read

$$\begin{aligned}
\mathcal{L}^{(1)} &= ia\bar{\psi}_{\mu\nu}(x)[(\gamma^\mu g^{\nu\sigma} + \gamma^\nu g^{\mu\sigma})\overrightarrow{\partial^\rho} + (\gamma^\mu g^{\nu\rho} + \gamma^\nu g^{\mu\rho})\\
&\quad \times \overrightarrow{\partial^\sigma} - (\gamma^\rho g^{\nu\sigma} + \gamma^\sigma g^{\nu\rho})\overleftarrow{\partial^\mu} - (\gamma^\rho g^{\mu\sigma} + \gamma^\sigma g^{\mu\rho})\overleftarrow{\partial^\nu}]\\
&\quad \times \psi_{\rho\sigma}(x) + i\frac{F_1(a)}{2}\bar{\psi}_{\mu\nu}(x)\gamma^\lambda(\overrightarrow{\not\partial} - \overleftarrow{\not\partial})\gamma^\delta\psi_{\rho\sigma}(x)\\
&\quad \times (g^{\lambda\mu}g^{\delta\rho}g^{\nu\sigma} + g^{\lambda\nu}g^{\delta\rho}g^{\mu\sigma} + g^{\lambda\mu}g^{\delta\sigma}g^{\nu\rho} + g^{\lambda\nu}g^{\delta\sigma}g^{\mu\rho})\\
&\quad + mF_2(a)\,\bar{\psi}_{\mu\nu}(x)(\gamma^\mu\gamma^\rho g^{\nu\sigma} + \gamma^\nu\gamma^\rho g^{\mu\sigma} + \gamma^\mu\gamma^\sigma g^{\nu\rho}\\
&\quad + \gamma^\nu\gamma^\sigma g^{\mu\rho})\psi_{\rho\sigma}(x) + \bar{\psi}_{\mu\nu}(x)\left(\frac{i}{2}(\overrightarrow{\not\partial} - \overleftarrow{\not\partial}) - m\right)\\
&\quad \times \psi_{\rho\sigma}(x)\,(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}),\\
\mathcal{L}^{(2)} &= ib\,\bar{\psi}_{\mu\nu}(x)[g^{\mu\nu}(\gamma^\rho\overrightarrow{\partial^\sigma} + \gamma^\sigma\overrightarrow{\partial^\rho}) - (\gamma^\nu\overleftarrow{\partial^\mu} + \gamma^\mu\overleftarrow{\partial^\nu})g^{\rho\sigma}]\\
&\quad \times \psi_{\rho\sigma}(x) + i\frac{G_1(a,b)}{2}\,\bar{\psi}_{\mu\nu}(x)g^{\mu\nu}(\overrightarrow{\not\partial} - \overleftarrow{\not\partial})\\
&\quad \times g^{\rho\sigma}\psi_{\rho\sigma}(x) + m\,G_2(a,b)\,\bar{\psi}_{\mu\nu}(x)g^{\mu\nu}g^{\rho\sigma}\psi_{\rho\sigma}(x),\\
\mathcal{L}^{(\mathrm{aux})} &= mc[\bar{\psi}_{\mu\nu}(x)g^{\mu\nu}\xi(x) + \bar{\xi}(x)g^{\rho\sigma}\psi_{\rho\sigma}(x)]\\
&\quad + B(a,b,c)\,\bar{\xi}(x)\left(\frac{i}{2}(\overrightarrow{\not\partial} - \overleftarrow{\not\partial}) + 3m\right)\xi(x),
\end{aligned} \tag{2}$$

and $F_1(a)$, $F_2(a)$, $G_1(a,b)$, $G_2(a,b)$, and $B(a,b,c)$ are functions of the free real parameters a, b, and c (see Appendix B).

The Lagrangian equation (1) in general depends on only three independent real parameters a, b, and c. This formulation of the spin-$\frac{5}{2}$ theory is simpler than that of suggested in [7]. In fact, the Lagrangian in [7] is written as a decomposition in terms of projection operators with a number of free parameters. These parameters are subjected to additional subsidiary constraints that need to be resolved.

Independent variations of the $\psi_{\mu\nu}$ and ξ fields give two equations of motion, which in momentum space can be written in the following form:

$$\left[\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p) + \Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)\right]\psi^{\rho\sigma}(p) + cmg^{\mu\nu}\xi(p) = 0, \tag{3}$$

$$mcg^{\rho\sigma}\psi_{\rho\sigma}(p) + B(a,b,c)(\not p + 3m)\xi(p) = 0, \tag{4}$$

where the operators $\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)$ and $\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ are

$$\begin{aligned}
\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p) &= (\not p - m)(g_{\mu\sigma}g_{\nu\rho} + g_{\mu\rho}g_{\nu\sigma})\\
&\quad + a(\gamma_\mu p_\rho g_{\nu\sigma} + \gamma_\nu p_\rho g_{\mu\sigma} + \gamma_\mu p_\sigma g_{\nu\rho}\\
&\quad + \gamma_\nu p_\sigma g_{\mu\rho} + \gamma_\rho p_\mu g_{\nu\sigma} + \gamma_\sigma p_\mu g_{\nu\rho} + \gamma_\rho p_\nu g_{\mu\sigma}\\
&\quad + \gamma_\sigma p_\nu g_{\mu\rho}) + F_1(a)\,(\gamma_\mu\not p\gamma_\rho g_{\nu\sigma} + \gamma_\nu\not p\gamma_\rho g_{\mu\sigma}\\
&\quad + \gamma_\mu\not p\gamma_\sigma g_{\nu\rho} + \gamma_\nu\not p\gamma_\sigma g_{\mu\rho}) + mF_2(a)\,(\gamma_\mu\gamma_\rho g_{\nu\sigma}\\
&\quad + \gamma_\nu\gamma_\rho g_{\mu\sigma} + \gamma_\mu\gamma_\sigma g_{\nu\rho} + \gamma_\nu\gamma_\sigma g_{\mu\rho}),
\end{aligned} \tag{5}$$

$$\begin{aligned}
\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p) &= b\,(\gamma_\mu p_\nu g_{\rho\sigma} + \gamma_\nu p_\mu g_{\rho\sigma} + \gamma_\rho p_\sigma g_{\mu\nu} + \gamma_\sigma p_\rho g_{\mu\nu})\\
&\quad + [\not p G_1(a,b) + mG_2(a,b)]\,g_{\mu\nu}g_{\sigma\rho}.
\end{aligned} \tag{6}$$

The equations of motion (3) and (4) are written in the most general form and are consistent with those defined in [6,7]. For example, the equation suggested by Schwinger corresponds to the choice of parameters $a = -1$, $b = 1$, and $c = -2$. Note that the functions $F_1(a)$, $F_2(a)$, $G_1(a,b)$, and $G_2(a,b)$ do not contain the parameter c which reflects the independence of the spin-$\frac{5}{2}$ field on the auxiliary degrees of freedom. The RS constraints [1] follow from Eqs. (3) and (4) with the additional condition $\xi(p) = 0$ (see Appendix B).

It is interesting to note that the operator $\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)$ would give an equation of motion $\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)\psi^{\rho\sigma} = 0$ for the spin-$\frac{5}{2}$ fields, provided $g^{\mu\nu}\psi_{\mu\nu} = 0$, where the latter property is assumed *a priori*. However, the corresponding inverse operator $[\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)]^{-1}$ has additional nonphysical poles in the spin-$\frac{1}{2}$ sector. This indicates that the constraint $g^{\mu\nu}\psi_{\mu\nu} = 0$ should also follow from the equation of motion and cannot be assumed *a priori*. The second operator $\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ acts only in the spin-$\frac{1}{2}$ sector of the spin-tensor representation. This can be checked by a direct decomposition of the operator Eq. (6) in terms of projection operators given in Appendix A. The same conclusion can be drawn from the observation that $\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ is orthogonal to all $\mathcal{P}^{\frac{5}{2}}_{\rho\sigma;\tau\delta}(p)$ and $\mathcal{P}^{\frac{3}{2}}_{ij;\rho\sigma;\tau\delta}(p)$ projection operators, where $i, j = 1, 2$. Hence the parameter b is related only to the spin-$\frac{1}{2}$ degrees of freedom, whereas a scales both the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ ones.

In practical calculations, one needs to know a free propagator corresponding to the spin-$\frac{5}{2}$ field. The derivation of the propagator becomes complicated in the presence of the auxiliary degrees of freedom. To demonstrate the procedure, it is useful to consider first an example of the free vector field φ_μ in the presence of an auxiliary one. In the next section we outline a general procedure that can be applied to the spin-$\frac{5}{2}$ case.

III. FREE VECTOR FIELD

The idea of using auxiliary degrees of freedom to describe systems with higher spins was first utilized in the original work of Fierz and Pauli [2]. As is well known, however, there is no need for such complications in the case of spins $J \leqslant 2$ and $J \leqslant \frac{3}{2}$. For higher spins the use of auxiliary degrees of becomes inevitable [7]. Here we consider the case of the vector field φ^μ in the presence of an additional scalar field λ and derive the free propagator of the system. The Lagrangian of the (φ_μ, λ) system can be written as

$$\mathcal{L}^{\mathbb{v}} = -\tfrac{1}{2}(\partial^\mu \varphi_\nu)(\partial_\mu \varphi^\nu) + \tfrac{1}{2}m^2\varphi_\nu\varphi^\nu + am(\partial_\nu \varphi^\nu)\,\lambda - \tfrac{1}{2}a^2m^2\lambda^2, \tag{7}$$

where φ_μ is a vector, λ is an auxiliary scalar field, and a is an arbitrary free parameter. Independent variations of the vector and auxiliary fields produce two equations of motion

$$\begin{aligned}(\Box + m^2)\varphi_\mu - am\partial_\mu\,\lambda = 0,\\ am\partial_\mu\varphi^\mu - a^2m^2\lambda = 0.\end{aligned} \tag{8}$$

Diagonalization of the system (8) leads to the Proca equation for the vector field φ_μ, whereas the auxiliary field λ vanishes. Although $\lambda = 0$, the propagator of the system always contains a component associated with the auxiliary field and φ_μ-λ mixing terms.

To obtain a propagator for the system of fields (φ_μ, λ), it is convenient to rewrite Eq. (8) in the matrix form

$$\Lambda^{\mathbb{v}}_{\{\mu\nu\}}\Phi^{\mathbb{v}\{\nu\}} = 0, \tag{9}$$

where

$$\Lambda^{\mathbb{v}}_{\{\mu\nu\}} = \begin{pmatrix}(\Box + m^2)g_{\mu\nu} & -am\partial_\mu\\ am\partial_\nu & -a^2m^2\end{pmatrix} \quad \text{and} \quad \Phi^{\mathbb{v}\{\nu\}} = \begin{pmatrix}\varphi^\nu\\ \lambda\end{pmatrix}. \tag{10}$$

Since the system contains vector and scalar degrees of freedom, the Lorentz indices in curly brackets of Eqs. (9) and (10) are associated with corresponding tensor and vector elements of $\Lambda^{\mathbb{v}}_{\{\mu\nu\}}$ and $\Phi^{\mathbb{v}}_{\{\nu\}}$.

The inverse operator (propagator) can be obtained as a solution of the following equation:

$$\Lambda^{\mathbb{v}}{}_{\{\mu}{}^{\rho\}}\; G^{\mathbb{v}}_{\{\rho\nu\}} = I^{\mathbb{v}}_{\{\mu\nu\}}\,\delta^4(x - x'), \tag{11}$$

where the propagator $G^{\mathbb{v}}_{\{\mu\nu\}}$ and the unit matrix $I^{\mathbb{v}}_{\{\mu\nu\}}$ are defined as

$$G^{\mathbb{v}}_{\{\mu\nu\}} = \begin{pmatrix}G^{(\varphi\varphi)}_{\mu\nu} & G^{(\varphi\lambda)}_{\mu}\\ G^{(\lambda\varphi)}_{\nu} & G^{(\lambda\lambda)}\end{pmatrix} \quad \text{and} \quad I^{\mathbb{v}}_{\{\mu\nu\}} = \begin{pmatrix}g_{\mu\nu} & 0\\ 0 & 1\end{pmatrix}.$$

The four components of the matrix $G^{\mathbb{v}}_{\{\mu\nu\}}$ have simple physical meanings: $G^{(\varphi\varphi)}_{\mu\nu}$ and $G^{(\lambda\lambda)}$ stand for the propagator of the purely vector and auxiliary scalar fields, respectively, whereas the nondiagonal $G^{(\varphi\lambda)}_{\mu}$ and $G^{(\lambda\varphi)}_{\nu}$ terms are associated with the φ-λ mixing. The solution of Eq. (11) in the momentum space is

$$G^{\mathbb{v}}_{\{\mu\nu\}}(p) = \begin{pmatrix}\dfrac{\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)}{p^2 - m^2} & i\,\dfrac{p_\mu}{a\,m^3}\\[2ex] -i\,\dfrac{p_\nu}{a\,m^3} & \dfrac{(p^2 - m^2)}{a^2\,m^4}\end{pmatrix}. \tag{12}$$

The pole $(p^2 - m^2)^{-1}$ appears only in the vector component $G^{(\varphi\varphi)}_{\mu\nu}(p)$; this term completely coincides with the corresponding expression well known from quantum field theory. The remaining terms in the propagator depend on the free parameter a associated with the auxiliary field λ. From Eq. (12) one can conclude that $G^{(\lambda\lambda)}$ gives contributions only off shell, $p^2 \neq m^2$. Note that the scalar component of the propagating vector field φ_μ mixes with the λ field, which leads to the appearance of finite nondiagonal components in the propagator Eq. (12).

Despite the complications related to the introduction of the auxiliary field, the description in terms of the (φ_μ, λ) system is completely equivalent to the conventional description in terms of the pure vector field. It implies that physical observables do not depend on the free parameter a appearing in the the full propagator (12). For the free fields this conclusion immediately follows from the fact that the auxiliary field can be excluded from the upper equation (8). It also holds true in the case of interacting fields provided there is no coupling to auxiliary degrees of freedom.

IV. PROPAGATOR FOR THE FREE SPIN-$\frac{5}{2}$ FIELD

Similarly to the procedure described in Sec. III it is convenient to rewrite the set of Eqs. (3) and (4) in matrix form,

$$\begin{pmatrix}\Lambda^{(\psi\psi)}_{\mu\nu;\rho\sigma}(p) & mcg_{\mu\nu}\\ cmg_{\rho\sigma} & \Lambda^{(\xi\xi)}(p)\end{pmatrix}\begin{pmatrix}\psi^{\rho\sigma}(p)\\ \xi(p)\end{pmatrix} = 0, \tag{13}$$

where $\Lambda^{(\psi\psi)}_{\mu\nu;\rho\sigma}(p) = \Lambda^{(1)}_{\mu\nu;\rho\sigma}(p) + \Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ and $\Lambda^{(\xi\xi)}(p) = B(a,b,c)(\not{p} + 3m)$. While the auxiliary field vanishes on shell, the full propagator should also contain an off-shell part related to the auxiliary field $\xi(x)$. Hence, the full propagator of the system is

$$\mathcal{G}^{\{\tau\lambda;\rho\sigma\}}(p) = \begin{pmatrix}G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) & G^{\tau\lambda;}_{(\psi\xi)}(p)\\ G^{;\rho\sigma}_{(\xi\psi)}(p) & G_{(\xi\xi)}(p)\end{pmatrix} \tag{14}$$

and satisfies the equation

$$\begin{pmatrix}\Lambda^{(\psi\psi)}_{\mu\nu;\tau\lambda}(p) & mcg_{\mu\nu}\\ cmg_{\tau\lambda} & \Lambda^{(\xi\xi)}(p)\end{pmatrix}\begin{pmatrix}G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) & G^{\tau\lambda;}_{(\psi\xi)}(p)\\ G^{;\rho\sigma}_{(\xi\psi)}(p) & G_{(\xi\xi)}(p)\end{pmatrix} = \begin{pmatrix}I^{\rho\sigma}_{\mu\nu} & 0\\ 0 & 1\end{pmatrix}, \tag{15}$$

where $I^{\rho\sigma}_{\mu\nu} = g^\rho_\mu g^\sigma_\nu + g^\sigma_\mu g^\rho_\nu$. The diagonal terms $G^{\mu\nu;\rho\sigma}_{(\psi\psi)}(p)$ and $G_{(\xi\xi)}(p)$ are related to the fields ψ and ξ, respectively, whereas the nondiagonal ones stand for mixing between the auxiliary spinor field and the "off-shell" spin-$\frac{1}{2}$ component of the spin-$\frac{5}{2}$ field.

The propagator of the spin-$\frac{5}{2}$ field $G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) = G^{(\psi\psi)}_{\mu\nu;\rho\sigma}(p)$ is obtained as a solution of the set of equations

$$\begin{aligned}\Lambda^{(\psi\psi)}_{\mu\nu;\tau\lambda}(p)\,G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) + mcg_{\mu\nu}\,G^{;\rho\sigma}_{(\xi\psi)}(p) = I^{\rho\sigma}_{\mu\nu},\\ cmg_{\tau\lambda}G^{\tau\lambda;}_{(\psi\xi)}(p) + \Lambda^{(\xi\xi)}(p)\,G_{(\xi\xi)}(p) = 1.\end{aligned} \tag{16}$$

In the literature one sometimes encounters a propagator defined as

$$G'_{\rho\sigma;\tau\delta}(p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon}\mathcal{P}^{\frac{5}{2}}_{\rho\sigma;\tau\delta}(p), \tag{17}$$

where $\mathcal{P}^{\frac{5}{2}}_{\rho\sigma;\tau\delta}(p)$ is a spin-$\frac{5}{2}$ projection operator in the spinor-tensor representation (A1). However, the quantity defined above does not have an inverse and therefore cannot obey

Eqs. (16) for any choice of the free parameters. This can be shown by replacing the $G^{\tau\lambda\rho\sigma}_{(\psi\psi)}(p)$ in the upper equation (16) by the expression from Eq. (17) and multiplying both sides of the resulting equation from the right by a projection operator $\mathcal{P}^{\frac{3}{2}}_{22;\rho\sigma,\tau\delta}(p)$. Using the general properties of projection operators Eq. (A2), the expression obtained reduces to

$$mc\,g_{\mu\nu}\,G^{;\rho\sigma}_{(\xi\psi)}(p)\mathcal{P}^{\frac{3}{2}}_{22;\rho\sigma,\tau\delta}(p) = 2\mathcal{P}^{\frac{3}{2}}_{22;\mu\nu,\tau\delta}(p). \qquad (18)$$

This leads to a contradiction: From Eq. (18) it follows that $G^{\rho\sigma}_{(\xi\psi)}(p)\mathcal{P}^{\frac{3}{2}}_{22;\rho\sigma,\tau\delta}(p)$ cannot be zero but by multiplying both sides of the same equation by $g^{\mu\nu}$ and using the property $g^{\mu\nu}\mathcal{P}^{\frac{3}{2}}_{22;\mu\nu,\tau\delta}(p) = 0$, one can draw the opposite conclusion. Hence, the quantity defined in Eq. (17) does not obey Eqs. (16) and cannot be a spin-$\frac{5}{2}$ propagator.

In solving Eq. (16), we restrict ourselves to a solution with a specific choice of the parameters $a = -1$, $b = -1$, whereas c is kept arbitrary. This choice of parameters is discussed in Sec. V. The independence of $G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p)$ of the parameter c signifies that the auxiliary field does not contribute to the physical observables. With this specific choice of the free parameters, the resulting equations are

$$\begin{aligned}
&[(\not p - m)(g_{\mu\tau}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\tau}) - (\gamma_\mu p_\nu g_{\lambda\tau} + \gamma_\nu p_\mu g_{\lambda\tau}\\
&\quad + \gamma_\lambda p_\tau g_{\mu\nu} + \gamma_\tau p_\lambda g_{\mu\nu}) + (\not p + m)g_{\mu\nu}g_{\tau\lambda} - (\gamma_\mu p_\lambda g_{\nu\tau}\\
&\quad + \gamma_\nu p_\lambda g_{\mu\tau} + \gamma_\mu p_\tau g_{\nu\lambda} + \gamma_\nu p_\tau g_{\mu\lambda} + \gamma_\lambda p_\mu g_{\nu\tau} + \gamma_\tau p_\mu g_{\nu\lambda}\\
&\quad + \gamma_\lambda p_\nu g_{\mu\tau} + \gamma_\tau p_\nu g_{\mu\lambda}) + (\gamma_\mu \not p \gamma_\lambda g_{\nu\tau} + \gamma_\nu \not p \gamma_\lambda g_{\mu\tau}\\
&\quad + \gamma_\mu \not p \gamma_\tau g_{\nu\lambda} + \gamma_\nu \not p \gamma_\tau g_{\mu\lambda}) + m\,(\gamma_\mu\gamma_\lambda g_{\nu\tau} + \gamma_\nu\gamma_\lambda g_{\mu\tau}\\
&\quad + \gamma_\mu\gamma_\tau g_{\nu\lambda} + \gamma_\nu\gamma_\tau g_{\mu\lambda})]G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) + mc\,g_{\mu\nu}\,G^{;\rho\sigma}_{(\xi\psi)}(p)\\
&\quad = g^\rho_\mu\,g^\sigma_\nu + g^\sigma_\mu\,g^\rho_\nu,\\
&m g_{\tau\lambda}G^{\tau\lambda;}_{(\psi\xi)}(p) - \frac{6\,c^2}{5}(\not p + 3m)(p)\,G_{(\xi\xi)}(p) = 1. \qquad (19)
\end{aligned}$$

The obtained spin-$\frac{5}{2}$ propagator $G^{\mu\nu;\rho\sigma}_{\frac{5}{2}}(p) = G^{\mu\nu;\rho\sigma}_{(\psi\psi)}(p)$ can be written as a decomposition in terms of projection operators as follows [7]:

$$\begin{aligned}
G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) = \frac{1}{p^2 - m^2}\Bigg((\not p + m)\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p)\\
- \frac{p^2 - m^2}{m^2}\big[\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p) + \mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)\big]\Bigg), \qquad (20)
\end{aligned}$$

where $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$ stand for the contributions from the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sector of the spinor-vector representation (see Appendix C). As expected, the spin-$\frac{5}{2}$ propagator itself does not depend on the parameter c related to the spinor field ξ. This observation also holds for arbitrary values of a and b in Eq. (16), as we have checked by explicit calculations. The propagator obtained has a pole associated with the spin-$\frac{5}{2}$ part and so-called off-shell nonpole contributions coming from the lower-spin components $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$.

V. COUPLING TO HIGHER-SPIN FERMIONS

In the case of the spin-$\frac{5}{2}$ field, in the spinor-tensor representation we deal with a system $(\psi_{\mu\nu}, \xi)$ that contains auxiliary degrees of freedom. One might raise the question whether the unphysical degrees of freedom could be eliminated from physical observables. Here we consider the simple case of the spin-$\frac{5}{2}$ resonance contribution to πN scattering, which is valid for applications in hadron physics. The corresponding $\pi N N^*_{\frac{5}{2}}$ coupling can be chosen as follows:

$$\begin{aligned}
\mathcal{L}_I = \frac{g_{\pi NN^*}}{4m_\pi^2}(\bar\psi_N(x), 0)\Gamma_{\mu\nu;\rho\sigma}\left[\hat P_{(\psi\psi)}\begin{pmatrix}\psi^{\rho\sigma}\\ \xi\end{pmatrix}\right]\partial^\mu\partial^\nu\pi(x)\\
+ \text{H.c.},
\end{aligned}$$

where the nucleon field is written as $(\bar\psi_N(x), 0)$, which implies the absence of auxiliary fields in the final state. The operator

$$\hat P_{(\psi\psi)} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

projects out the spin-$\frac{5}{2}$ field and ensures that there is no coupling to ξ. Hence, only the spin-$\frac{5}{2}$ component of the propagator $G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p)$, Eq. (20), contributes to physical observables at any order of perturbation theory. In Sec. III we demonstrated that the inclusion of auxiliary degrees of freedom in the vector field does not affect the physical observables. To our knowledge, this statement is not generally proven for the $(\psi_{\mu\nu}, \xi)$ system beyond the perturbation expansion. The reason is that the equation of motion for massive spin-$\frac{5}{2}$ fields in the spinor-tensor representation is defined only in the presence of an auxiliary field. This is unlike the case of the vector field, where auxiliary degrees of freedom can be removed by proper field transformations. Note that these degrees of freedom contribute because of $\psi_{\mu\nu} - \xi$ mixing. This mixing takes place only between the spin-$\frac{1}{2}$ sector of the spinor-tensor and the auxiliary spinor fields, as pointed out in the previous section. One may therefore hope that the use of a coupling that suppresses the spin-$\frac{1}{2}$ contributions would also prevent the appearance of the auxiliary degrees of freedom in the physical observables in the nonperturbative regime.

The possibility of removing unwanted degrees of freedom in the special case of spin-$\frac{3}{2}$ fields has been demonstrated by Pascalutsa in [3,13,14]. The idea of that approach is based on the observation that the Lagrangian of the Rarita-Schwinger fields for the specific choice $A = -1$ maintains gauge invariance in the massless limit. Therefore, the use of a gauge-invariant coupling suppresses the contribution from the lower-spin sector.

Guided by the results obtained in the spin-$\frac{3}{2}$ Rarita-Schwinger theory [3,13,14], one might expect that the lower-spin terms of the spin-$\frac{5}{2}$ propagator, Eq. (20), would not contribute to the physical observables as long as a gauge-invariant coupling is used. This, however, is not true for the spin-$\frac{5}{2}$ fields in the spinor-tensor representation. Such a conclusion can be drawn from the fact that the Lagrangian equation (2) is not invariant under the gauge transformations $\psi_{\mu\nu} \to \psi_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$ in the massless limit for any choice of the parameters a and b. We show this more explicitly by exploring the general structure of the gauge-invariant vertex function for the example of the πN scattering amplitude in the leading order of the perturbation expansion. The amplitude can in general be written in the form

$$\mathcal{M} \sim \bar{u}_N(p')\left[\Gamma^{\mu\nu;\rho\sigma}(q)\, G^{\frac{5}{2}}_{\rho\sigma;\alpha\beta}(q)\, \Gamma^{\dagger\,\alpha\beta;\lambda\tau}(q)\right] \times u_N(p)\, k'_\mu\, k'_\nu\, k_\lambda\, k_\tau, \tag{21}$$

where $p\,(k)$ and $p'(k')$ are the momenta of the initial and final nucleon (pion), respectively; q stands for the momentum of the resonance and depends on the channel (s or u) of interest. The gauge-invariant coupling to the spin-$\frac{5}{2}$ field imposes the following constraint on the vertex function $q_\rho\,\Gamma^{\mu\nu;\rho\sigma}(q) = q_\sigma\,\Gamma^{\mu\nu;\rho\sigma}(q) = 0$. For the transition matrix to be free from any contribution from the lower-spin sector, the expression in square brackets in Eq. (21) should be proportional to the spin-$\frac{5}{2}$ projection operator:

$$\left[\Gamma^{\mu\nu;\rho\sigma}(q)\, G^{\frac{5}{2}}_{\rho\sigma;\alpha\beta}(q)\, \Gamma^{\dagger\,\alpha\beta;\lambda\tau}(q)\right] \sim \mathcal{P}^{\frac{5}{2}\,\mu\nu;\lambda\tau}(q). \tag{22}$$

This gives an additional constraint $q_\mu\Gamma^{\mu\nu;\rho\sigma}(q) = q_\nu\Gamma^{\mu\nu;\rho\sigma}(q) = 0$. The vertex function can be decomposed in terms of the spin projection operators. There are only three operators, $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$, $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, that satisfy the properties $q^\mu\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) = 0$, $q^\nu\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q) = 0$, and so on. Hence, the decomposition can be written as

$$\Gamma_{\mu\nu;\rho\sigma}(q) = \alpha_1(\not{q})\,\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) + \alpha_2(\not{q})\,\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q) + \alpha_3(\not{q})\,\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q), \tag{23}$$

where the coefficients of the decomposition $\alpha_1(\not{q})$, $\alpha_2(\not{q})$, and $\alpha_3(\not{q})$ are polynomials of m and $\not{q}$. Note, that $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$, $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ commute with $\not{q}$. The spin-$\frac{5}{2}$ propagator can also be decomposed in terms of the spin projection operators. Because of the orthogonality properties of the projection operators, only those terms in $G^{\frac{5}{2}}_{\rho\sigma;\alpha\beta}(q)$ contribute to the matrix element Eq. (21) that contain $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$, $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ operators. If the parameters a and b in Eqs. (15) and (16) could be chosen in such a way that the propagator does not contain the $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ operators, the lower-spin contributions to the matrix element would be suppressed. This situation is realized in the spin-$\frac{3}{2}$ Rarita-Schwinger theory for the special choice of the free parameter $A = -1$ (see [4] for a discussion). In that case the use of a gauge-invariant coupling as suggested by Pascalutsa [3,13,14] suppresses the remaining spin-$\frac{1}{2}$ components, and the overall vertex has the desired projection properties.

For the spin-$\frac{5}{2}$ fields of Eq. (4), the contribution of the $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ projector can also be suppressed by choosing $a = -1$. As we already mentioned in Sec. II, this parameter is associated with both spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ degrees of freedom, whereas b regulates only the spin-$\frac{1}{2}$ ones. Indeed the expression Eq. (C1) derived for $a = -1$, $b = -1$ does not have the $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ projector. One might ask whether $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ can also be removed from the free propagator. The general conclusion is that the term $G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p)$, being a solution of Eq. (16), always has contributions from $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$. We have checked this by explicit calculation for arbitrary values of the parameter b.

This conclusion is ultimately linked to the fact that the Lagrangian of the free spin-$\frac{5}{2}$ fields (2) does not maintain gauge invariance in the massless limit. The same conclusion was also drawn in [7]. Therefore one can never remove the corresponding degrees of freedom from the transition matrix Eq. (21) provided the vertex function is written in the form of Eq. (23).

The solution to the problem was suggested in [4], where it was proposed to utilize the Rarita-Schwinger condition $\gamma_\mu\psi^{\mu\nu} = \gamma_\nu\psi^{\mu\nu} = 0$ to constrain the interaction vertex. As a result, the interaction vertex satisfies the condition $\gamma\cdot\Gamma = \Gamma\cdot\gamma = 0$. By applying this constraint to the decomposition Eq. (23), one can see that only the $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$ projector obeys the desired property and the decomposition reduces to

$$\Gamma_{\mu\nu;\rho\sigma}(q) = \alpha_1(\not{q})\,\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q). \tag{24}$$

Since the vertex function should be free from any singularities, the minimal power of $\not{q}$ in the function $\alpha_1(\not{q})$ should be of the fourth order. Then the simplest coupling can be written as follows:

$$\mathcal{L}^{\frac{5}{2}}_{\pi NN^*} = \frac{g_{\pi NN^*}}{m_\pi^2 m_R^4}\,\bar{\psi}_N(x)\left[\Box^2\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(\partial)\psi^{\rho\sigma}_{N^*}\right]\partial^\mu\partial^\nu\pi(x) + \text{H.c.} \tag{25}$$

The use of $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(\partial)$ ensures that only the spin-$\frac{5}{2}$ part of the propagator contributes, and the square of the d'Alembert operator guarantees that no other singularities except the mass pole term $(p^2 - m^2)^{-1}$ appear in the amplitude. As a result, the physical observables no longer depend on the arbitrary parameters a and b of the free Lagrangian. The πN scattering amplitude with intermediate $N^*(\frac{5}{2})$, Eq. (21), then reads

$$\mathcal{M} = \left(\frac{g_{\pi NN^*}}{m_\pi^2}\right)^2 \bar{u}_N(p')\left[\left(\frac{q^2}{m_R^2}\right)^4\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\lambda\tau}(q)\right] \times u_N(p)\, k'^\mu\, k'^\nu\, k^\lambda\, k^\tau. \tag{26}$$

The coupling in Eq. (25) can be generalized for the fields $\psi^{\{\rho\cdots\sigma\}}_{N^*}$ of the arbitrary spin J as

$$\mathcal{L}^J_{\pi NN^*} = \frac{g_{\pi NN^*}}{(m_\pi)^{\frac{1}{2}(J-1/2)}}\,\bar{\psi}_N(x) \times \left[\left(\frac{\Box}{m_R^2}\right)^{(J-\frac{1}{2})}\mathcal{P}^J_{\{\mu\cdots\delta;\rho\cdots\sigma\}}(\partial)\psi^{\{\rho\cdots\sigma\}}_{N^*}\right] \times \{\partial^\mu\}\cdots\{\partial^\delta\}\,\pi(x) + \text{H.c.}, \tag{27}$$

where the number of indices assigned to $\psi^{\{\rho\cdots\sigma\}}_{N^*}$ and $\mathcal{P}^J_{\{\mu\cdots\nu;\rho\cdots\sigma\}}(\partial)$ depends on the chosen representation. The coupling constructed in Eqs. (26) and (27) ensures that the physical observables do not depend on the free parameters of the theory.

VI. SUMMARY

In summary, we have investigated the general properties of free spin-$\frac{5}{2}$ fields in the spinor-tensor representation. The Lagrangian is written in terms of spin-$\frac{5}{2}$ and auxiliary fields $(\psi_{\mu\nu}, \xi)$ and coincides with that suggested in the literature for a specific choice of free parameters. We demonstrate that the Lagrangian in general depends on three arbitrary parameters: two of them are associated with the lower spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sector of the theory while the third one is linked to the auxiliary field ξ.

We deduce a free propagator of the system that is given by a 2×2 matrix in the $(\psi_{\mu\nu}, \xi)$ space. The diagonal elements stand for the propagation of the spin-$\frac{5}{2}$ and ξ fields, whereas the nondiagonal ones correspond to $\psi_{\mu\nu}$-ξ mixing. The mixing takes place between the spin-$\frac{1}{2}$ sector of the spinor-tensor representation and an auxiliary spinor field. While the free propagator includes auxiliary degrees of freedom, they do not contribute to the physical observables calculated within the perturbation theory provided there is no coupling to ξ.

As an application to hadron physics calculations, the interaction involving $(\psi_{\mu\nu}, \xi)$ is discussed for the example of $\pi N N^*_{\frac{5}{2}}$ coupling. The pure spin-$\frac{5}{2}$ propagator contains nonpole terms which contribute in the whole kinematical region. As we demonstrate, invariance under gauge transformations is not enough to remove these contributions. This is ultimately related to the fact that the free Lagrangian of the $(\psi_{\mu\nu}, \xi)$ system does not maintain gauge invariance in the massless limit for any choice of the free parameters. The desired result can, however, be obtained by construction of a coupling with higher-order derivatives. In that case, the amplitude of the πN scattering does not depend on the arbitrary parameters of the free Lagrangian. The suggested coupling is generalized to the Rarita-Schwinger fields of any half-integer spin.

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APPENDIX A: SPIN PROJECTION OPERATORS FOR THE SPINOR-TENSOR REPRESENTATION

The spin projection operators are taken from [7]. In the momentum space they are given by

$$\begin{aligned}
\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) &= \frac{1}{2}\left(\mathcal{P}^1_{\mu\rho}\mathcal{P}^1_{\nu\sigma} + \mathcal{P}^1_{\mu\sigma}\mathcal{P}^1_{\nu\rho}\right) - \frac{1}{5}\,\mathcal{P}^1_{\mu\nu}\mathcal{P}^1_{\rho\sigma} \\
&\quad - \frac{1}{10}\left(\not{\mathcal{P}}^1_{\mu}\not{\mathcal{P}}^1_{\rho}\mathcal{P}^1_{\nu\sigma} + \not{\mathcal{P}}^1_{\nu}\not{\mathcal{P}}^1_{\rho}\mathcal{P}^1_{\mu\sigma} + \not{\mathcal{P}}^1_{\mu}\not{\mathcal{P}}^1_{\sigma}\mathcal{P}^1_{\nu\rho} \right. \\
&\quad \left. + \not{\mathcal{P}}^1_{\nu}\not{\mathcal{P}}^1_{\sigma}\mathcal{P}^1_{\mu\rho}\right), \\
\mathcal{P}^{\frac{3}{2}}_{11;\mu\nu;\rho\sigma}(q) &= \frac{1}{2}\left(\mathcal{P}^1_{\mu\rho}\mathcal{P}^0_{\nu\sigma} + \mathcal{P}^1_{\nu\rho}\mathcal{P}^0_{\mu\sigma} + \mathcal{P}^1_{\mu\sigma}\mathcal{P}^0_{\nu\rho} + \mathcal{P}^1_{\nu\sigma}\mathcal{P}^0_{\mu\rho}\right) \\
&\quad - \frac{1}{6q^2}\,\mathcal{O}_{\mu\nu}\mathcal{O}_{\rho\sigma}, \\
\mathcal{P}^{\frac{3}{2}}_{22;\mu\nu;\rho\sigma}(q) &= \frac{1}{10}\left(\not{\mathcal{P}}^1_{\mu}\not{\mathcal{P}}^1_{\rho}\mathcal{P}^1_{\nu\sigma} + \not{\mathcal{P}}^1_{\nu}\not{\mathcal{P}}^1_{\rho}\mathcal{P}^1_{\mu\sigma} + \not{\mathcal{P}}^1_{\mu}\not{\mathcal{P}}^1_{\sigma}\mathcal{P}^1_{\nu\rho} \right. \\
&\quad \left. + \not{\mathcal{P}}^1_{\nu}\not{\mathcal{P}}^1_{\sigma}\mathcal{P}^1_{\mu\rho}\right) - \frac{2}{15}\,\mathcal{P}^1_{\mu\nu}\mathcal{P}^1_{\rho\sigma}, \\
\mathcal{P}^{\frac{3}{2}}_{21;\mu\nu;\rho\sigma}(q) &= -\mathcal{P}^{\frac{3}{2}}_{12;\rho\sigma;\mu\nu}(q) = \frac{1}{2\sqrt{5}\,q^2}\left(q_\rho\not{\mathcal{P}}^1_{\mu}\mathcal{P}^1_{\nu\sigma} \right. \\
&\quad \left. + q_\rho\not{\mathcal{P}}^1_{\nu}\mathcal{P}^1_{\mu\sigma} + q_\sigma\not{\mathcal{P}}^1_{\mu}\mathcal{P}^1_{\nu\rho} + q_\sigma\not{\mathcal{P}}^1_{\nu}\mathcal{P}^1_{\mu\rho}\right)\not{q} \\
&\quad - \frac{1}{3\sqrt{5}\,q^2}\,\mathcal{P}^1_{\mu\nu}\mathcal{O}_{\rho\sigma}\,\not{q}, \\
\mathcal{P}^{\frac{1}{2}}_{11;\mu\nu;\rho\sigma}(q) &= \mathcal{P}^0_{\mu\nu}\mathcal{P}^0_{\rho\sigma}, \\
\mathcal{P}^{\frac{1}{2}}_{22;\mu\nu;\rho\sigma}(q) &= \frac{1}{3}\,\mathcal{P}^1_{\mu\nu}\mathcal{P}^1_{\rho\sigma}, \\
\mathcal{P}^{\frac{1}{2}}_{33;\mu\nu;\rho\sigma}(q) &= \frac{1}{6\,q^2}\,\mathcal{O}_{\mu\nu}\mathcal{O}_{\rho\sigma}, \\
\mathcal{P}^{\frac{1}{2}}_{21;\mu\nu;\rho\sigma}(q) &= \mathcal{P}^{\frac{1}{2}}_{12;\rho\sigma;\mu\nu}(q) = \frac{1}{\sqrt{3}}\,\mathcal{P}^1_{\mu\nu}\mathcal{P}^0_{\rho\sigma}, \\
\mathcal{P}^{\frac{1}{2}}_{31;\mu\nu;\rho\sigma}(q) &= -\mathcal{P}^{\frac{1}{2}}_{13;\rho\sigma;\mu\nu}(q) = \frac{1}{\sqrt{6}\,q^2}\,\mathcal{O}_{\mu\nu}\mathcal{P}^0_{\rho\sigma}\,\not{q}, \\
\mathcal{P}^{\frac{1}{2}}_{23;\mu\nu;\rho\sigma}(q) &= -\mathcal{P}^{\frac{1}{2}}_{32;\rho\sigma;\mu\nu}(q) = \frac{-1}{3\sqrt{2}\,q^2}\,\mathcal{O}_{\rho\sigma}\mathcal{P}^1_{\mu\nu}\,\not{q}, \qquad \text{(A1)}
\end{aligned}$$

where the operators $\mathcal{P}^1_{\mu\nu}$, $\mathcal{P}^0_{\mu\nu}$, $\not{\mathcal{P}}^1_{\mu}$, and $\mathcal{O}_{\mu\nu}$ are defined as

$$\mathcal{P}^1_{\mu\nu} = g_{\mu\nu} - \tfrac{q_\mu q_\nu}{q^2}, \quad \not{\mathcal{P}}^1_{\mu} = \mathcal{P}^1_{\mu\nu}\gamma^\nu,$$
$$\mathcal{P}^0_{\mu\nu} = \tfrac{q_\mu q_\nu}{q^2}, \quad \mathcal{O}_{\mu\nu} = \not{\mathcal{P}}^1_{\mu}q_\nu + q_\mu\not{\mathcal{P}}^1_{\nu}.$$

The projection operators Eq. (A1) satisfy the following properties: They satisfy orthogonality conditions

$$\mathcal{P}^{J}_{ii;\mu\nu}{}^{;\tau\lambda}(q)\;\mathcal{P}^{J'}_{ii;\tau\lambda;\rho\sigma}(q) = \delta_{JJ'}\mathcal{P}^{J}_{ii;\mu\nu;\rho\sigma}(q) \qquad \text{(A2)}$$

and the sum rules

$$\begin{aligned}
&\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) + \sum_{i=1}^{2}\mathcal{P}^{\frac{3}{2}}_{ii;\mu\nu;\rho\sigma}(q) + \sum_{i=1}^{3}\mathcal{P}^{\frac{1}{2}}_{ii;\mu\nu;\rho\sigma}(q) \\
&\quad = \frac{1}{2}(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}). \qquad \text{(A3)}
\end{aligned}$$

APPENDIX B: LAGRANGIAN FOR THE FREE SPIN-$\frac{5}{2}$ FIELD

The functions $F_1(a)$, $F_2(a)$, $G_1(a, b)$, $G_2(a, b)$, and $B(a, b, c)$ of the free real parameters a, b, and c used in the definition of the Lagrangian equation (2) read

$$\begin{aligned}
F_1(a) &= \frac{1}{4}(5a^2+2a+1),\\
F_2(a) &= \frac{1}{8}(15a^2+10a+3),\\
G_1(a,b) &= \frac{5a^4-12a^3-20a^2-8a-4b^2-4b(7a^2+6a+1)-1}{2(3a+1)^2},\\
G_2(a,b) &= \frac{-15a^4+18a^2+8a+12b^2+6b(5a^2+6a+1)+1}{2(3a+1)^2},\\
B(a,b,c) &= -\frac{24c^2(3a+1)^2}{5(5a^2+6a+4b+1)^2}.
\end{aligned} \tag{B1}$$

Using the variational principle, one obtains two equations of motion (3) and (4). Here we show that all Rarita-Schwinger constraints [1] can be obtained from these equations. By multiplying Eq. (3) by $g^{\mu\nu}$, $\gamma^\mu p^\nu$, and $p^\mu p^\nu$ and carrying out the summation, we get

$$\begin{aligned}
&\big(g_{\rho\sigma}\{\not p[b-2F_1(a)+2G_1(a,b)+1]+m[2F_2(a)\\
&\quad+2G_2(a,b)-1]\}+2(\gamma_\sigma p_\rho+\gamma_\rho p_\sigma)[a+b\\
&\quad+F_1(a)]\big)\psi^{\rho\sigma}+2cm\xi=0,
\end{aligned} \tag{B2}$$

$$\begin{aligned}
&\big([2a+5b+2F_1(a)+G_1(a,b)]\,p^2 g_{\rho\sigma}+m\not p\,[G_2(a,b)\\
&\quad-2F_2(a)]g_{\rho\sigma}+\{-m+6mF_2(a)+[b-1+4F_1(a)]\,\not p\}\\
&\quad\times(\gamma_\rho p_\sigma+\gamma_\sigma p_\rho)+(12a+4)\,p^\rho p^\sigma\big)\psi^{\rho\sigma}+cm\not p\,\xi=0,
\end{aligned} \tag{B3}$$

$$\begin{aligned}
&\big(2[(2a+1)\,\not p-m]p^\rho p^\sigma+\{p^2[2a+b+2F_1(a)]\\
&\quad+2mF_2(a)\,\not p\,\}(\gamma^\rho p^\sigma+\gamma^\sigma p^\rho)+p^2\{[2b+G_1(a,b)]\not p\\
&\quad+mG_2(a,b)\}g^{\rho\sigma}\big)\psi_{\rho\sigma}+cmp^2\,\xi=0,
\end{aligned} \tag{B4}$$

respectively. Expressing $(\gamma_\sigma p_\rho+\gamma_\rho p_\sigma)\psi^{\rho\sigma}$ from the first equation (B2) and substituting it in Eqs. (B3) and (B4), we obtain

$$\begin{aligned}
&\big(4(3a+1)[a+b+F_1(a)]\,p^\rho p^\sigma+\tfrac{1}{2}\big(-[6F_2(a)-1]\\
&\quad\times[2F_2(a)+2G_2(a,b)-1]m^2+2\not p\,\{b+G_1(a,b)\\
&\quad-2F_2(a)\times[a+3b+3G_1(a,b)+1]+F_1(a)\\
&\quad\times[1-3G_2(a,b)]+[a+1]G_2(a,b)\}m\\
&\quad+p^2\{4a^2+14\,a\,b+9\,b^2+12F_1^2(a)\\
&\quad+2F_1(a)[4a+6b-3G_1(a,b)-3]\\
&\quad+2\,(a+1)\,G_1(a,b)+1\}\big)g^{\rho\sigma}\big)\psi_{\rho\sigma}+cm\,\{[1-6F_2(a)]\\
&\quad\times m+\not p\,[a-3F_1(a)+1]\}\xi=0,
\end{aligned} \tag{B5}$$

$$\begin{aligned}
&\big(2[a+b+F_1(a)][(2a+1)\not p-m]p^\rho p^\sigma+\tfrac{1}{2}[-2F_2(a)\\
&\quad\times[2F_2(a)+2G_2(a,b)-1]m^2\not p-p^2(\not p\{-3b^2-2ab\\
&\quad+b-4F_1^2(a)+2a+2a\,G_1(a,b)+F_1(a)[-4a-4b\\
&\quad+2G_1(a,b)+2]\}+m\{2G_2(a,b)a-2a-b+F_2(a)\\
&\quad\times[4a+4b+4G_1(a,b)+2]+2F_1(a)[G_2(a,b)\\
&\quad-1]\})]g^{\rho\sigma}\big)\psi_{\rho\sigma}+cm\{-[a+F_1(a)]\,p^2\\
&\quad-2mF_2(a)\,\not p\}\xi=0.
\end{aligned} \tag{B6}$$

Now, multiplying Eq. (B5) from the left by $(6a+2)^{-1}[(2a+1)\not p-m]$, subtracting it from Eq. (B6), and using the definitions (B1), we have

$$-\frac{cm(3a+1)}{8}\left((5m^2+3p^2)\,\xi+\frac{3cm}{B(a,b,c)}\times(\not p-3m)\,g^{\rho\sigma}\psi_{\rho\sigma}\right)=0. \tag{B7}$$

From Eqs. (4) and (B7), we obtain

$$(3a+1)\,cm^3\xi=0, \tag{B8}$$

which means that the auxiliary field vanishes provided $a\neq-\frac{1}{3}$. Having $\xi=0$, the remaining constraints

$$\begin{aligned}
(\gamma_\mu p_\nu+\gamma_\nu p_\mu)\,\psi^{\mu\nu}&=0,\\
p_\mu p_\nu\,\psi^{\mu\nu}&=0,\\
g_{\mu\nu}\,\psi^{\mu\nu}&=0
\end{aligned} \tag{B9}$$

can be easily derived from Eqs. (B2)–(B4) and Eq. (4).

Now multiplying Eq. (3) from the left by γ^ν and p^ν and using Eqs. (B9), we have two equations,

$$\begin{aligned}
&\{[a+6F_1(a)-1]\,\not p\,+[6F_2(a)-1]m\}(\gamma^\sigma g^{\mu\rho}+\gamma^\rho g^{\mu\sigma})\psi_{\rho\sigma}\\
&\quad+2(3a+1)(g^{\mu\rho}p^\sigma+p^\rho g^{\mu\sigma})\psi_{\rho\sigma}=0,
\end{aligned} \tag{B10}$$

$$\begin{aligned}
&[(a+1)\not p-m](p^\sigma g^{\mu\rho}+p^\rho g^{\mu\sigma})\psi_{\rho\sigma}+\{p^2[a+F_1(a)]\\
&\quad+m\not pF_2(a)\}(\gamma^\sigma g^{\mu\rho}+\gamma^\rho g^{\mu\sigma})\psi_{\rho\sigma}=0.
\end{aligned} \tag{B11}$$

Again, multiplying Eq. (B10) by $(6a+2)^{-1}[(a+1)\not p-m]$, subtracting the resulting equation from Eq. (B11), and using definitions (B1) we get

$$(\gamma^\sigma g^{\mu\rho}+\gamma^\rho g^{\mu\sigma})\,\psi_{\rho\sigma}=0, \tag{B12}$$

provided $a\neq-\frac{1}{3},-\frac{1}{2}$. Then the constraint

$$(p^\sigma g^{\mu\rho}+p^\rho g^{\mu\sigma})\,\psi_{\rho\sigma}=0 \tag{B13}$$

immediately follows from Eqs. (B10) and (B12). Having $\gamma^\rho\psi_{\rho\sigma}=0$, $p^\rho\psi_{\rho\sigma}=0$, $\xi=0$, and $g^{\rho\sigma}\psi_{\rho\sigma}=0$, Eq. (3) reduces to the Dirac equation $(\not p-m)\psi_{\rho\sigma}=0$. Finally, we have shown that all Rarita-Schwinger constraints can be obtained from Eqs. (3) and (4). Hence, the function $\psi_{\rho\sigma}$ obeying these equations describes the spin-$\frac{5}{2}$ field.

APPENDIX C: SPIN-$\frac{5}{2}$ PROPAGATOR

The solution of Eq. (19) can be written in the form

$$G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) = \frac{1}{p^2-m^2}\left((\not p+m)\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) + \frac{p^2-m^2}{m^2} \times \left[\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p) + \mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)\right]\right), \quad \text{(C1)}$$

$$G^{\rho\sigma}_{(\psi\xi)}(p) = \frac{1}{64\,m^3c}\{(\not p+m)[2(\gamma^\rho p^\sigma + \gamma^\sigma p^\rho) + 5m g^{\rho\sigma}] + 6(p^2 - m\not p)g^{\rho\sigma} - 16p^\rho p^\sigma\}, \quad \text{(C2)}$$

where the lower spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ parts $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$ are

$$\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p) = -\frac{4}{5}(\not p+m)\mathcal{P}^{\frac{3}{2}}_{11;\,\mu\nu;\rho\sigma}(p) + \frac{m}{\sqrt{5}}\left(\mathcal{P}^{\frac{3}{2}}_{12;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{3}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right), \quad \text{(C3)}$$

$$\begin{aligned}\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p) = \frac{1}{80\,m^2}\Big[&-\frac{3}{8}\big([(73m^2-12p^2)\not p + 3m(27m^2 - 8p^2)]\mathcal{P}^{\frac{1}{2}}_{11;\,\mu\nu;\rho\sigma}(p) + [(35m^2-36p^2)\not p - m(13m^2 + 96p^2)]\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(p) - \sqrt{3}[(43m^2-12p^2)\not p \\ &+ m(47m^2-28p^2)]\left[\mathcal{P}^{\frac{1}{2}}_{12;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{1}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right]\big) \\ &- [(16m^2+3p^2)\not p + m(16m^2-15p^2)]\mathcal{P}^{\frac{1}{2}}_{33;\,\mu\nu;\rho\sigma}(p) \\ &+ \frac{9}{2\sqrt{6}}(3m^2-2p^2)\,\not p\left[\mathcal{P}^{\frac{1}{2}}_{13;\,\mu\nu;\rho\sigma}(p) - \mathcal{P}^{\frac{1}{2}}_{31;\,\mu\nu;\rho\sigma}(p)\right] \\ &+ \frac{3m}{2\sqrt{6}}(64m^2-21p^2)\left[\mathcal{P}^{\frac{1}{2}}_{13;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{1}{2}}_{31;\,\mu\nu;\rho\sigma}(p)\right] \\ &+ \frac{9}{2\sqrt{2}}(3m^2+2p^2)\,\not p\left[\mathcal{P}^{\frac{1}{2}}_{23;\,\mu\nu;\rho\sigma}(p) - \mathcal{P}^{\frac{1}{2}}_{32;\,\mu\nu;\rho\sigma}(p)\right] \\ &- \frac{m}{2\sqrt{2}}(64m^2-69p^2)\left[\mathcal{P}^{\frac{1}{2}}_{23;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{1}{2}}_{32;\,\mu\nu;\rho\sigma}(p)\right]\Big]. \end{aligned} \quad \text{(C4)}$$

The solution with arbitrary values of parameters a and b is given in Appendix D.

APPENDIX D: SPIN-$\frac{5}{2}$ PROPAGATOR FOR ARBITRARY VALUES OF a AND b

The spin-$\frac{5}{2}$ propagator for arbitrary values of the parameters a and b can be written in the form of Eq. (C1), where the lower spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ parts $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$ are

$$\begin{aligned}\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p) = \frac{1}{(3a+1)^2}\Big[&-\left(\frac{(5a+1)^2}{5}\not p + \frac{75a^2+50a+7}{10}\,m\right)\mathcal{P}^{\frac{3}{2}}_{11;\,\mu\nu;\rho\sigma}(p) + \frac{(a+1)(5a+1)}{\sqrt{5}}\,\not p\left(\mathcal{P}^{\frac{3}{2}}_{12;\,\mu\nu;\rho\sigma}(p) - \mathcal{P}^{\frac{3}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right) \\ &+ \frac{15a^2+10a+3}{2\sqrt{5}}\,m\left(\mathcal{P}^{\frac{3}{2}}_{12;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{3}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right) + \left((a+1)^2\not p - \frac{3a^2+2a-1}{2}\,m\right)\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(p)\Big],\end{aligned}$$

$$\begin{aligned}\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p) = \frac{1}{20\,(3a+1)^2\,m^2}\Big[&-\frac{3}{8(5a^2+6a+4b+1)^2}(\{[2000a^6+4000a^5+5(640b+353)a^4+20(128b-25)a^3 \\ &+2(640b^2-444b-205)a^2-4(128b^2+148b+17)\,a-256b^2-56b-3]m^2-48(5a^3+5a^2 \\ &+4ba+a+b)^2p^2\}\not p+\{[6000a^6+18\,400a^5+25(384b+793)a^4+20(896b+479)a^3+2(1920b^2 \\ &+5120b+1043)a^2+4(640b^2+512b+43)a+256b^2+128b+1]m^2-24[150a^6+325a^5+5(48b+53)a^4 \\ &+20(17b+5)a^3+(96b^2+157b+17)a^2+(64b^2+26b+1)a+b\,(10b+1)]p^2\}m)\,\mathcal{P}^{\frac{1}{2}}_{11;\,\mu\nu;\rho\sigma}(p) \\ &-\frac{1}{8(5a^2+6a+4b+1)^2}(\{[2000a^6+4000a^5+5(640b-943)a^4+4(640b-2717)a^3+10(128b^2-348b \\ &-617)a^2-4(128b^2+580b+353)\,a-256b^2-344b-115]m^2-48[5a^3+14a^2+(4b+7)a+b+1]^2\,p^2\}\not p \\ &+\{[6000a^6+18\,400a^5+5(1920b+2021)a^4+20(896b-169)a^3+2(1920b^2+5120b-2197)a^2 \\ &+4(640b^2+512b-317)\,a+256b^2+128b-119]m^2-24[150a^6+865a^5+16(15b+106)a^4 \\ &+4(193b+340)a^3+(96b^2+589b+523)a^2+(64b^2+170b+97)a+10b^2+17b+7]p^2\}m)\,\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(p) \\ &+\{[4(5a^2+14a+5)m^2-3(2a+1)^2p^2]\not p-[4(15a^2+10a-1)m^2-3(12a^2+8a+1)p^2]m\}\,\mathcal{P}^{\frac{1}{2}}_{33;\,\mu\nu;\rho\sigma}(p) \\ &+\frac{\sqrt{3}}{8(5a^2+6a+4b+1)^2}(\{[2000a^6+4000a^5+5(640b+353)a^4+4(640b-341)a^3+2(640b^2-1092b \\ &-565)a^2-4\,(128b^2+364b+65)a-256b^2-200b-19]m^2-48[25a^6+95a^5+10(4b+11)a^4 \\ &+(86b+54)a^3+(16b^2+51b+12)a^2+(8b^2+12b+1)a+b(b+1)]p^2\}\not p+\{[6000a^6+18\,400a^5 \\ &+5(1920b+4937)a^4+20(896b+803)a^3+2(1920b^2+5120b+2663)a^2+4(640b^2+512b+223)a\end{aligned}$$

$$
\begin{aligned}
&+256b^2+128b+61]m^2-12[300a^6+1190a^5+5(96\,b+295)\,a^4+4(278b+203)\,a^3+2(96b^2+373b\\
&+108)a^2+2(64b^2+98b+13)\,a+20b^2+18b+1]p^2\}m)\left(\mathcal{P}^{\frac{1}{2}}_{12;\,\mu\nu;\rho\sigma}(p)+\mathcal{P}^{\frac{1}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right)\\
&+\frac{\sqrt{3}}{2\sqrt{2}(5a^2+6a+4b+1)}\{[200a^4+480a^3+(160b+287)a^2+6(32b+7)a+32b-1]m^2\\
&-12(2a+1)(5a^3+5a^2+4ba+a+b)p^2\}\,\not{p}\left(\mathcal{P}^{\frac{1}{2}}_{13;\,\mu\nu;\rho\sigma}(p)-\mathcal{P}^{\frac{1}{2}}_{31;\,\mu\nu;\rho\sigma}(p)\right)\\
&+\frac{\sqrt{3}}{2\sqrt{2}(5a^2+6a+4b+1)}\{8(15a^2+10a+3)(5a^2+6a+4b+1)m^2-3[120a^4+170a^3\\
&+(96b+81)\,a^2+16(4b+1)\,a+12b+1]p^2\}\,m\left(\mathcal{P}^{\frac{1}{2}}_{13;\,\mu\nu;\rho\sigma}(p)+\mathcal{P}^{\frac{1}{2}}_{31;\,\mu\nu;\rho\sigma}(p)\right)\\
&-\frac{1}{2\sqrt{2}(5a^2+6a+4b+1)}\{[200a^4+480a^3+(160b+611)a^2+6(32b+43)a+32b+35]m^2\\
&-12(2a+1)[5a^3+14a^2+(4b+7)a+b+1]p^2\}\,\not{p}\left(\mathcal{P}^{\frac{1}{2}}_{23;\,\mu\nu;\rho\sigma}(p)-\mathcal{P}^{\frac{1}{2}}_{32;\,\mu\nu;\rho\sigma}(p)\right)\\
&-\frac{1}{2\sqrt{2}(5a^2+6a+4b+1)}\{8(15a^2+10a+3)(5a^2+6a+4b+1)m^2-3[120a^4+386a^3+3(32b+99)a^2\\
&+8(8b+11)\,a+12b+9]p^2\}\,m\left(\mathcal{P}^{\frac{1}{2}}_{23;\,\mu\nu;\rho\sigma}(p)+\mathcal{P}^{\frac{1}{2}}_{32;\,\mu\nu;\rho\sigma}(p)\right)\Bigg].
\end{aligned}
$$

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