Cluster radioactive decay within the preformed cluster model using relativistic mean-field theory densities

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We have studied the (ground-state) cluster radioactive decays within the preformed cluster model (PCM) of Gupta and collaborators [R. K. Gupta, in *Proceedings of the 5th International Conference on Nuclear Reaction Mechanisms*, Varenna, edited by E. Gadioli (Ricerca Scientifica ed Educazione Permanente, Milano, 1988), p. 416; S. S. Malik and R. K. Gupta, Phys. Rev. C **39**, 1992 (1989)]. The relativistic mean-field (RMF) theory is used to obtain the nuclear matter densities for the double folding procedure used to construct the cluster-daughter potential with M3Y nucleon-nucleon interaction including exchange effects. Following the PCM approach, we have deduced empirically the preformation probability ${P_0}^{\mathrm{emp}}$ from the experimental data on both the α- and exotic cluster-decays, specifically of parents in the trans-lead region having doubly magic ^{208}Pb or its neighboring nuclei as daughters. Interestingly, the RMF-densities-based nuclear potential supports the concept of preformation for both the α and heavier clusters in radioactive nuclei. ${P_0}^{\alpha(\mathrm{emp})}$ for α decays is almost constant ($\sim 10^{-2}$–10^{-3}) for all the parent nuclei considered here, and ${P_0}^{c(\mathrm{emp})}$ for cluster decays of the same parents decrease with the size of clusters emitted from different parents. The results obtained for ${P_0}^{c(\mathrm{emp})}$ are reasonable and are within two to three orders of magnitude of the well-accepted phenomenological model of Blendowske-Walliser for light clusters.

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I. INTRODUCTION

Cluster radioactivity (CR) is the spontaneous emission of clusters heavier than the α particle. Since its first theoretical prediction [1] in 1980 and experimental confirmation [2] in 1984, this phenomenon is now established for some 26 decays with light to heavy (^{14}C to ^{34}Si) clusters from various actinides (^{221}Fr to ^{242}Cm). Theoretically, two types of models have been advanced, namely, (i) the unified fission models (UFM), such as the analytic super-asymmetric fission model (ASAFM) of Săndulescu, Poenaru, and Greiner [1], and (ii) the preformed cluster models (PCM), like that of Gupta and collaborators [3–7] based on collective potential energy surfaces. Some effort has also gone into understanding it within the Hatree-Fock-Bogoliubov mean-field theory [8] and the relativistic mean-field (RMF) theory [9], treating it as an asymmetric fission process (see, however, later a further discussion of the work of Ref. [9]). Besides these two model approaches (the UFM and PCM), semiempirical formulas have also been proposed by different authors [10–13] for calculating the half-life times $T_{1/2}$ of exotic cluster decays, together with a unified formula of $T_{1/2}$ for both the α and cluster radioactivity [14].

Whereas Gamow theory for α decay uses the square well potential, the UFM and PCM, advanced for the processes of CR as well as α decay, use the more realistic nuclear interaction potentials [15]. Furthermore, the UFM and PCM differ from each other for their noninclusion or inclusion of preformation or spectroscopic factors of the clusters being formed (or born) before penetrating the confining interaction barriers. Thus, in UFM, the preformation factor is taken to be unity, whereas in PCM, Gupta and collaborators [3–7] assumed the clusters to be preborn in the parent nucleus with certain probabilities ${P_0}^{\mathrm{theo}}$, calculated theoretically by solving the stationary Schrödinger equation for the dynamical flow of mass and charge. In another preformed-cluster-based calculation, Blendowske and Walliser [16] proposed for P_0 a simple phenomenological formula for light clusters ($A_c \leqslant 28$) as

$$ {P_0}^{c} = ({P_0}^{\alpha})^{(A_c-1)/3}, \quad (1) $$

with the α preformation factor ${P_0}^{\alpha}$, for decay from an even or odd parent, estimated as

$$ ({P_0}^{\alpha})^{\mathrm{even}} = 6.3 \times 10^{-3} \quad \text{and} \quad ({P_0}^{\alpha})^{\mathrm{odd}} = 3.2 \times 10^{-3}, \quad (2) $$

respectively. These studies show that ${P_0}^{c}$ almost decreases linearly with increase in the size or mass of the cluster. Furthermore, the ratio ${P_0}^{c}/{P_0}^{\alpha}$ is shown to be very small, indicating that ${P_0}^{\alpha}$ is very large in comparison to ${P_0}^{c}$. A recent study [17] has shown that the scaling law given by Eq. (1) between the preformation probability (spectroscopic) factor and mass of the emitted cluster can be understood in terms of the (Coulomb barrier and Q-value-based) fragmentation potential, a fact well understood and very well exploited for quite some time by Gupta and collaborators for calculating the theoretical preformation probabilities ${P_0}^{\mathrm{theo}}$ in the PCM used here, which also includes the nuclear potential [3–7,15], as already mentioned. Yet in another microscopic density-dependent cluster-daughter potential, calculated with the renormalized M3Y nucleon-nucleon interaction plus exchange term, using Fermi densities, the authors of Ref. [13] use for preformation probability of clusters an exponential function of the product of cluster and daughter charge numbers ($P_0 = 10^{-(aZ_cZ_d-b)}$), or alone the mass number of the cluster ($P_0 = 10^{-(cA_c-2)}$), with parameters a, b, or c fitted to match the experimental data.

The cluster preformation probability can also be estimated empirically. The first empirical estimate of ${P_0}^c/{P_0}^\alpha$ was given by Rose and Jones [2], ranging from 7×10^{-5} to 4×10^{-7}, in their pioneering experiment for the ground-state decay of ^{14}C from ^{223}Ra. In UFM, as already stated, the preformation probability for α as well as cluster decay is assumed to be one (i.e., ${P_0}^\alpha = {P_0}^c = 1$), while using a more realistic interaction potential (Coulomb + nuclear) and thus, in a way, treating the CR process just as the barrier penetration process. In PCM, together with the realistic interaction potential, the cluster-preformation probability is also associated, thus including the important nuclear structure information of the parent nucleus and daughter products in the model. Within the UFM also, the P_0 has been shown to be formally the penetrability of the internal part of the barrier, corresponding to still overlapping fragments [18].

Because of the very importance of preformation (spectroscopic) factor from the point of view of including nuclear structure information, in this paper, based on PCM, we use the RMF theory whose calculated mass and charge densities have already been shown by some of us and collaborators [19] to support the clustering effects in various heavy parents with observed cluster decays, along with the α-nucleus structure of all the α nuclei (^{8}Be to ^{32}S) together with the halo structure of neutron- and proton-rich isotopes of $^{6-14}$Be and odd-A $^{11-19}$B nuclei [20]. In the following, for studying the ground-state cluster radioactive decays, we have chosen the same nuclei as in Ref. [19], and many more parents, from the trans-lead region. The experimental data on decay constants for all the parents considered here are taken from [15] for both the α and cluster decays, supplemented by the recent review [21] for cluster decays, with the additional data for ^{34}Si decay of ^{242}Cm taken from [22], and the very recent data for ^{14}C and ^{15}N decays of ^{223}Ac from [23] and the ^{34}Si decay of ^{238}U from [24].

Our methodology for the present study is to use the RMF theory and the PCM, described briefly in Sec. II. The cluster and daughter densities are calculated by using the RMF formalism for spherical nuclei. Then, the M3Y effective nucleon-nucleon interaction, supplemented by a zero-range pseudopotential for exchange effects (M3Y + EX), is folded [25,26] with the RMF calculated cluster and daughter densities ρ_c and ρ_d, to obtain the nuclear interaction potential V_n, to which is added the Coulomb potential V_C for getting the total interaction potential $V(R) = V_n(R) + V_C(R)$ between the cluster and daughter nuclei. The Coulomb potential V_C is calculated here within the pointlike approximation [$V_C(R) = Z_c Z_d e^2/R$], though various prescriptions are available for the finite-size effects of one or both nuclei, such as the one in [13,27] for one spherical nucleus and that in [28] for two spheres. However, the finite-size effects in the Coulomb potential for spherical nuclei are found to be small [5,27], and these are neglected here for simplicity. The WKB penetration probability P is then calculated, to be used for obtaining the decay constant λ within the PCM. In this way, we determine the empirical preformation factor ${P_0}^{\text{emp}}$ for the cluster decays. Apparently, it would be interesting to determine P_0 theoretically within the RMF theory itself, a step to be carried out next. Moreover, by keeping in view the latest cluster decay studies [6,7], the deformation effects of the nuclei are also to be added in RMF densities. The calculated results of our present study are presented in Sec. III, and a summary of our work is given in Sec. IV.

II. THE RELATIVISTIC MEAN-FIELD THEORY AND THE PREFORMED CLUSTER MODEL

In the RMF model [9,19], an effective Lagrangian is taken to describe the nucleons interacting through the effective meson and electromagnetic (e.m.) fields. The equations of motion are obtained using the Euler-Lagrange variational principle. A set of coupled equations result from replacing the field operators by their expectation values. A set of Klein-Gordon-type equations is yielded for mesons and photons with sources having nucleonic currents and densities, and the Dirac equation describing the nucleon dynamics is yielded with potential terms having the e.m. and meson fields. This set of RMF-generated equations is then solved self-consistently to obtain the matter (neutron + proton) densities for the cluster and daughter nuclei, to treat them further for obtaining the nuclear interaction potential between them.

The Lagrangian density for a nucleon-meson many-body system [19] is

$$\begin{aligned}\mathcal{L} = {} & \overline{\psi_i}\{i\gamma^\mu\partial_\mu - M\}\psi_i + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 \\ & - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_s\overline{\psi_i}\psi_i\sigma - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} \\ & + \frac{1}{2}m_w^2 V^\mu V_\mu + \frac{1}{4}c_3(V_\mu V^\mu)^2 - g_w\overline{\psi}_i\gamma^\mu\psi_i V_\mu \\ & - \frac{1}{4}\vec{B}^{\mu\nu}\,.\,\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}^\mu\,.\,\vec{R}_\mu - g_\rho\overline{\psi}_i\gamma^\mu\vec{\tau}\psi_i\,.\,\vec{R}^\mu \\ & - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\psi}_i\gamma^\mu\frac{(1-\tau_{3i})}{2}\psi_i A_\mu. \end{aligned} \tag{3}$$

Here, the field for the σ meson is denoted by σ, that for the ω meson by V_μ, and that for the isovector ρ meson by $\vec{R}_\mu$. A_μ denotes the electromagnetic field. The ψ_i are the Dirac spinors for the nucleons whose third component of isospin is denoted by τ_{3i}. Here g_s, g_ω, and g_ρ and $e^2/4\pi = 1/137$ are the coupling constants for σ, ω, and ρ mesons and photons, respectively. g_2, g_3, and c_3 are the parameters for the nonlinear terms of σ and ω mesons, respectively. M is the mass of the nucleon and m_σ, m_ω, and m_ρ are the masses of the σ, ω, and ρ mesons, respectively. $\Omega^{\mu\nu}$, $\vec{B}_{\mu\nu}$, and $F^{\mu\nu}$ are the field tensors for the V^μ, $\vec{R}_\mu$, and the photon fields, respectively.

From the relativistic Lagrangian, we obtain the field equations for the nucleons and mesons. The set of coupled equations is solved numerically by a self-consistent iteration method using the NL3 parameter set [29], which is found to be successful in giving the cluster structure of both the heavy and light nuclei [19,20]. The scalar, baryon (vector), isovector, and proton densities are, respectively,

$$\rho_s(r) = \sum_{i=1}^{A}\bar{\psi}_i(r)\psi_i(r),$$

$$\rho(r) = \sum_{i=1}^{A}\psi_i^\dagger(r)\psi_i(r),$$

$$\rho_3(r) = \sum_{i=1}^{A} \psi_i^\dagger(r)\tau_{3i}\psi_i(r),$$
$$\rho_p(r) = \sum_{i=1}^{A} \psi_i^\dagger(r)\left(\frac{1-\tau_{3i}}{2}\right)\psi_i(r). \quad (4)$$

The center-of-mass motion is estimated by the usual harmonic oscillator formula $E_{\text{c.m.}} = 3/4(41A^{-1/3})$. The root-mean-square (rms) matter radius is defined as

$$\langle r_m{}^2\rangle = \frac{1}{A}\int \rho(r)r^2 d\tau, \quad (5)$$

where A is the mass number and ρ(r) is the spherical density. We follow the prescription of Refs. [30,31] to take care of the pairing interaction.

The nuclear interaction potential, $V_n(R)$, between the cluster and daughter nuclei, with the respective RMF calculated nuclear matter densities ρ_c and ρ_d, is

$$V_n(\vec{R}) = \int \rho_c(\vec{r}_c)\rho_d(\vec{r}_d)v(|\vec{r}_c - \vec{r}_d + \vec{R}| \equiv s)d^3r_c d^3r_d, \quad (6)$$

obtained by using the well-known double folding procedure [25,26] to the M3Y interaction, supplemented by a zero-range pseudopotential representing the single-nucleon exchange effects,

$$v(s) = 7999\frac{e^{-4s}}{4s} - 2134\frac{e^{-2.5s}}{2.5s} + J_{00}(E)\delta(s). \quad (7a)$$

This is denoted as M3Y + EX, with the exchange term given as

$$J_{00}(E) = -276(1 - 0.005E/A_{\alpha(c)}) \quad \text{MeV fm}^3. \quad (7b)$$

Here $A_{\alpha(c)}$ is the α-particle (or cluster) mass, and E, the energy measured in the center-of-mass of the α- or cluster-daughter nucleus system, is equal to the released Q value. Compared to, say, the energies involved in high-energy α scattering, $J_{00}(E)$ is practically independent of energy for the α- or cluster-decay process and hence can be taken as -276 MeV fm^3, an approximation also used in the work of Ref. [27].

To account for the Pauli blocking effect, the density dependence in the M3Y + EX interaction of Eq. (7a) was introduced for the first time in Ref. [32] (where it is called DDM3Y). More recently, in Ref. [33], the density dependence in the M3Y + EX interaction is used in the following factorized form, based on a functional form first proposed by Myers [34]:

$$v(s) = \left(7999\frac{e^{-4s}}{4s} - 2134\frac{e^{-2.5s}}{2.5s} + J_{00}(E)\delta(s)\right)$$
$$\times\left((1-\rho_c{}^{2/3})(1-\rho_d{}^{2/3})\right), \quad (7c)$$

which is shown in [33] to give the α-nucleus interaction potential similar to that obtained for the original density-dependent (DDM3Y) potential of Ref. [32] with exchange term and Pauli blocking effects included, but with a normalization factor $C = 1.3$. In a later work [35], such a normalization factor is shown to vary in the range 1.0–1.5 for various α-nucleus systems. In the present study, we set this normalization constant to 1.0 since it will make a difference of only a small factor to our deduced $P_0{}^{\text{emp}}$ values. It may be relevant to mention here that the authors of Ref. [9] use Eq. (7c) with the J_{00} term set to zero (i.e., DDM3Y without exchange effects) for their cluster-decay studies within the UFM. Furthermore, for α-decay studies, the work of Ref. [27] has shown that neither the density dependence nor the exchange effects play much of a role such that the results obtained with DDM3Y and M3Y + EX or only M3Y are almost identical for α decays. In our calculations, however, we have added the exchange effects and use the M3Y + EX interaction. It may be noted here that in our earlier brief reports of this work, made at some conferences [36], we have stated by mistake that the interaction used is DDM3Y, instead of M3Y + EX.

The decay constant λ in PCM is defined as [4]

$$\lambda_{\text{PCM}} = \frac{\ln 2}{T_{1/2}} = \nu_0 P_0 P, \quad (8)$$

and in UFM is simply given by [5]

$$\lambda_{\text{UFM}} = \nu_0 P. \quad (9)$$

Here, P_0 is the preformation probability of the cluster formed in the parent nucleus, ν_0 is the assault frequency, and P is the WKB penetrability in relative separation R coordinate. Within the PCM, an empirical estimate of the preformation factor can be defined [5] as

$$P_0{}^{\text{emp}} = \frac{\lambda_{\text{Expt}}}{\nu_0 P} = \frac{\lambda_{\text{Expt}}}{\lambda_{\text{UFM}}}, \quad (10)$$

the ratio between the experimental λ_{Expt} and calculated $\nu_0 P$ or the λ_{UFM}.

The WKB penetration probability P of the cluster tunneling through the interaction potential $V(R)$ [$=V_n(R) + V_C(R)$], as shown in Fig. 1, having energy equal to the Q value of the decay, is given by

$$P = \exp\left(-\frac{2}{\hbar}\int_{R_a}^{R_b} \{2\mu[V(R) - Q]\}^{1/2} dR\right), \quad (11)$$

with R_a, R_b as the turning points, satisfying $V(R_a) = V(R_b) = Q$, $\mu = A_d A_c/(A_d + A_c)$ as the reduced mass, and $Q = BE_p - (BE_d + BE_c)$. Here BE_p, BE_c, and BE_d are the experimental ground-state (g.s.) binding energies of the parent, cluster, and daughter nuclei, respectively, taken from Audi *et al.* [37]. For some nuclei, where experimental g.s. binding energy is not available (e.g., for ^{204}Pt, the daughter nucleus in the ^{34}Si decay of ^{238}U), we have used the RMF calculated g.s. binding energy (=1604.32 MeV, which is in close agreement with the value given by Möller *et al.* [38], i.e., 1604.08 MeV).

In Eq. (8), ν_0 is the impinging frequency with which the cluster hits the barrier, and it is given by

$$\nu_0 = \frac{\text{velocity}}{R_0} = \frac{(2E_2/\mu)^{1/2}}{R_0}, \quad (12)$$

where R_0 is the radius of parent nucleus and E_2 is the kinetic energy of the emitted cluster. The impinging frequency ν_0 is nearly constant $\sim 10^{21}\text{s}^{-1}$ for all the observed α as well as cluster decays. Since both the emitted cluster and daughter nuclei are produced in the ground state, the entire positive Q value of the decay is the total kinetic energy ($Q = E_1 + E_2$) available for the decay process, which is shared between the

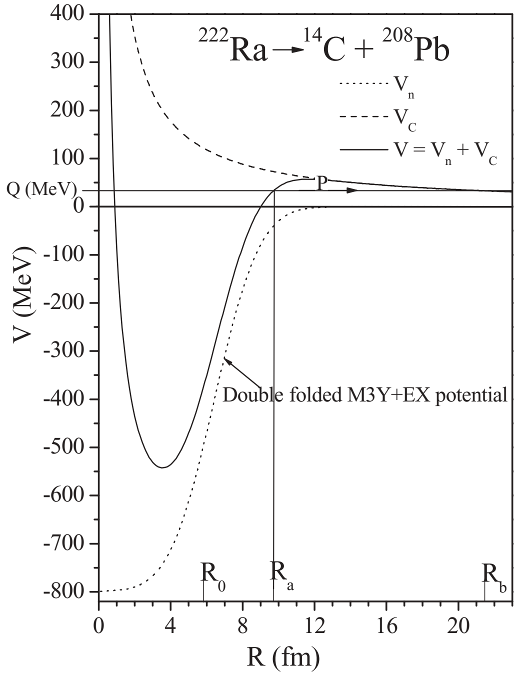

FIG. 1. The double folded M3Y + EX potential $V_n(R)$, the Coulomb $V_C(R)$ $(=Z_c Z_d e^2/R)$, and the total interaction potential $V(R)$ as a function of radial separation R between the cluster ^{14}C and daughter ^{208}Pb nuclei for decay of ^{222}Ra. The penetration path of the cluster with an energy equal to the Q value of the decay is also shown here.

two fragments, such that for the emitted cluster $E_2 = A_1 Q/A$ and E_1 $(=Q - E_2)$ is the recoil energy of the daughter nucleus. Note that the total kinetic energy available for the decay process is much less than the 20 MeV/nucleon limit discussed in [27] for ignoring the density dependence of the effective M3Y interactions without introducing much change in α-decay calculations. Also, it should be recalled here that in contrast to the present work based on the PCM, the RMF study in Ref. [9] is based on UFM, with the impinging frequency ν_0 taken as a variable parameter, accounting for the preformation factor P_0 in Eq. (8) (see Eq. (5) in [9], which depends on two adjustable parameters). However, as already noted, it is a well-known fact (see, e.g., Ref. [15]) that ν_0 is nearly constant for both the α and cluster decays and that inclusion of P_0 accounts for nuclear structure effects. In a more recent calculation [39], performed for a density-dependent M3Y nucleon-nucleon interaction with an improved WKB method, ν_0 is explicitly shown to be of the order of 10^{21} s^{-1} for the α decay of superheavy nuclei and is thus more like the standard result. Furthermore, as already mentioned, since the RMF density profiles support the cluster formations in heavy nuclei [19] (just as in α nuclei [20]), it is not justified to ignore the cluster preformation factor in any of the applications of the RMF method.

III. CALCULATIONS AND DISCUSSION

As already stated in Sec. I, the calculations involve four independent steps: First, the cluster and daughter matter densities are calculated within the RMF method. Using the double folding procedure, these are then folded with the M3Y + EX interaction potential to obtain the nucleus-nucleus potential. As a next step, the WKB penetration probability P is calculated by using the experimental Q values for α as well as cluster decays. Finally, the empirical estimates of the preformation factors are made for different parent nuclei chosen with doubly closed shell ^{208}Pb or its neighboring nuclei as the daughter products.

Figure 1 (solid line) illustrates the total interaction potential $V(R)$ for ^{14}C decay of ^{222}Ra, used to calculate the WKB penetration probability P. The calculated P values, together with the Q values and empirically estimated preformation probabilities P_0^{emp}, respectively, for α and cluster decays from various parent nuclei, are given in Tables I and II. The experimental data for the decay constant λ_{Expt} are also given for each case. For some cases, only the upper limits are known, which are used as such for the (upper limit) estimates of P_0. It is interesting to find in Table I that the empirically evaluated α preformation probabilities $P_0^{\alpha(emp)}$ are almost constant, of the order of 10^{-2}–10^{-3}, for all the parent nuclei studied here. This result suggests that a preformation factor P_0^{α} is required to match the experimental data on λ^{α}_{Expt}, and hence the nuclear structure information is important here in this study.

Table II shows that the cluster preformation factor $P_0^{c(emp)}$ decreases with increasing cluster size, in agreement with earlier studies discussed in Sec. I. It is relevant to note in Table I that, in comparison to $P_0^{c(emp)}$ for cluster decays from the same parent nuclei in Table II, we get a very large preformation factor $P_0^{\alpha(emp)}$ for α decays, such that the ratio $P_0^{c(emp)}/P_0^{\alpha(emp)}$, given in Table II, is very small, and it further decreases with the increase in size of the cluster from ^{14}C to ^{34}Si, as expected. The $P_0^{c(emp)}/P_0^{\alpha(emp)}$ value for ^{14}C decay from ^{222}Ra is quite close to the range of values obtained in the first experiments of Rose and Jones [2] $(=7 \times 10^{-5}$ to 4×10^{-7}, already mentioned in Sec. I). Furthermore, these results are also in line with the previous studies based on the PCM using the proximity potential [3–7].

Finally, in Fig. 2, we compare our calculated $-\log_{10} P_0^{c(emp)}$ as a function of cluster and parent mass (symbols) with the results of a phenomenological model calculation [Eqs. (1) and (2)] of Blendowske and Walliser [16]. Interestingly enough, the two calculations match within two to three orders of magnitude (compare symbols with dotted and dashed lines), which is quite reasonable. The important point is that the microscopic RMF formalism, combined with an effective M3Y nucleon-nucleon interaction, supports the concept of preformation of clusters in nuclei, introduced by Gupta and collaborators in PCM for cluster radioactive decays [3,4]. It is relevant to mention here that we have carried out our present study in the RMF formalism by assuming all the parent, daughter, and cluster nuclei to be spherical. The deformation effects in the RMF formalism for light nuclei are known to exist [40], and they should be taken up next. The deformation effects of nuclei have recently been shown to be important for α and cluster decays [6,7], and thus they could also influence our P_0^{emp} estimates. Furthermore, the improvements suggested in [39] for calculating the WKB penetration probability may also be of interest.

TABLE I. The WKB penetrability P, experimental decay constant $\lambda^{\alpha}_{\rm Expt}$, and the estimated $P_0^{\alpha(\rm emp)} = \lambda^{\alpha}_{\rm Expt}/\nu_0 P$ for α decays of various nuclei. The impinging frequency $\nu_0 \sim 10^{21}$ s^{-1} for each case. The Q values are calculated by using the experimental g.s. binding energies from [37].

Parent	Q (MeV)	P	$\lambda^{\alpha}_{\rm Expt}$ (s^{-1})	$P_0^{\alpha(\rm emp)}$
^{221}Fr	6.457	3.150×10^{-22}	2.406×10^{-02}	2.507×10^{-02}
^{221}Ra	6.881	5.190×10^{-21}	2.310×10^{-02}	1.412×10^{-03}
^{222}Ra	6.679	8.667×10^{-22}	1.824×10^{-02}	6.798×10^{-03}
^{223}Ra	5.979	8.479×10^{-25}	7.016×10^{-07}	2.831×10^{-04}
^{224}Ra	5.789	1.091×10^{-25}	2.189×10^{-06}	6.997×10^{-03}
^{226}Ra	4.871	6.691×10^{-31}	1.378×10^{-11}	7.863×10^{-03}
^{223}Ac	6.783	8.619×10^{-22}	5.440×10^{-03}	2.026×10^{-03}
^{225}Ac	5.935	1.844×10^{-25}	8.022×10^{-07}	1.500×10^{-03}
^{226}Th	6.451	1.479×10^{-23}	3.739×10^{-04}	8.368×10^{-03}
^{228}Th	5.520	4.427×10^{-28}	1.152×10^{-08}	9.353×10^{-03}
^{230}Th	4.770	1.228×10^{-32}	2.755×10^{-13}	8.708×10^{-03}
^{230}U	5.993	1.484×10^{-26}	3.857×10^{-07}	8.986×10^{-03}
^{231}Pa	5.150	1.078×10^{-30}	6.728×10^{-13}	2.334×10^{-04}
^{232}U	5.414	1.232×10^{-29}	3.074×10^{-10}	9.112×10^{-03}
^{233}U	4.909	9.197×10^{-33}	1.384×10^{-13}	5.782×10^{-03}
^{234}U	4.858	4.292×10^{-33}	9.011×10^{-14}	8.124×10^{-03}
^{235}U	4.678	2.552×10^{-34}	3.132×10^{-17}	4.848×10^{-05}
^{236}U	4.573	4.601×10^{-35}	9.142×10^{-16}	8.189×10^{-03}
^{238}U	4.270	2.023×10^{-37}	4.846×10^{-18}	9.957×10^{-03}
^{237}Np	4.958	6.823×10^{-33}	1.030×10^{-14}	5.807×10^{-04}
^{236}Pu	5.867	4.796×10^{-28}	7.730×10^{-09}	5.684×10^{-03}
^{238}Pu	5.593	1.743×10^{-29}	2.513×10^{-10}	5.225×10^{-03}
^{241}Am	5.638	1.156×10^{-29}	5.102×10^{-11}	1.600×10^{-03}
^{242}Cm	6.216	4.580×10^{-27}	4.195×10^{-08}	3.707×10^{-03}

IV. SUMMARY OF RESULTS

In the present work, we have studied the α and exotic cluster radioactive decay processes within the preformed cluster model using the RMF densities. The RMF densities are folded with the M3Y + EX interaction potential, using the double folding procedure to obtain the nuclear interaction potential. We have explored mainly the significance of the preformation or spectroscopic factor in the two processes. In line with previous studies, we find that the preformation factor

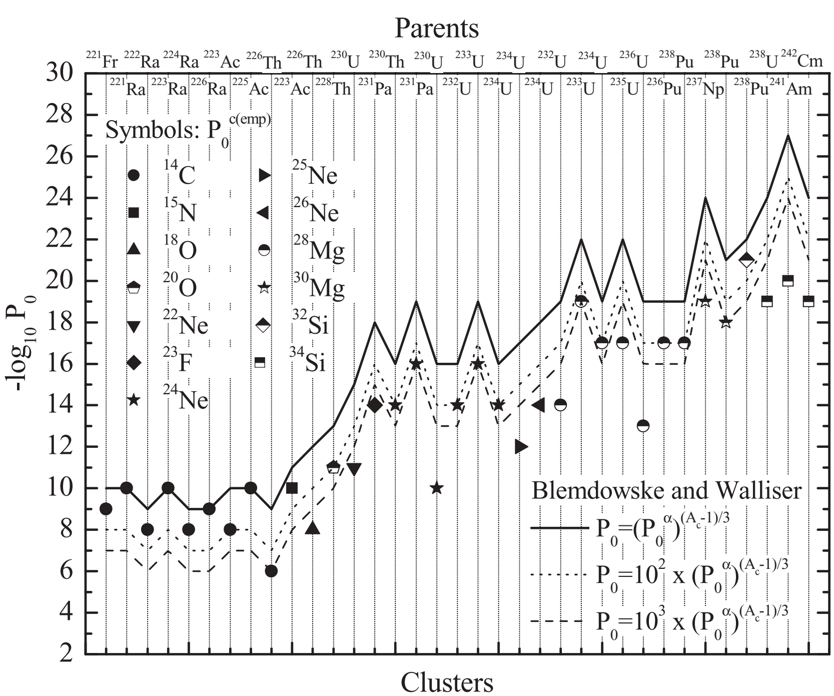

FIG. 2. The empirical preformation probability $P_0^{c(\rm emp)}$ for cluster decays from various parents (symbols) compared with the model calculations of Blendowske-Walliser [16] [solid line, Eqs. (1) and (2)] and ones raised by the factors of 10^2 and 10^3 (dot and dash lines, respectively).

TABLE II. Same as for Table I, but for cluster decays. Here, the ratio ${P_0}^{c(\mathrm{emp})}/{P_0}^{\alpha(\mathrm{emp})}$ is also given.

Decay	Q (MeV)	P	$\lambda^c_{\mathrm{Expt}}$ (s^{-1})	${P_0}^{c(\mathrm{emp})}$	${P_0}^{c(\mathrm{emp})}/{P_0}^{\alpha(\mathrm{emp})}$
$^{221}Fr \to {}^{14}C + {}^{207}Tl$	31.292	1.500×10^{-27}	1.203×10^{-15}	2.240×10^{-10}	8.938×10^{-09}
$^{221}Ra \to {}^{14}C + {}^{207}Pb$	32.396	3.990×10^{-26}	2.772×10^{-15}	1.902×10^{-11}	1.347×10^{-08}
$^{222}Ra \to {}^{14}C + {}^{208}Pb$	33.050	2.933×10^{-25}	6.749×10^{-12}	6.251×10^{-09}	9.196×10^{-07}
$^{223}Ra \to {}^{14}C + {}^{209}Pb$	31.829	2.112×10^{-27}	3.508×10^{-16}	4.609×10^{-11}	1.628×10^{-07}
$^{224}Ra \to {}^{14}C + {}^{210}Pb$	30.536	8.536×10^{-30}	9.413×10^{-17}	3.131×10^{-09}	4.476×10^{-07}
$^{226}Ra \to {}^{14}C + {}^{212}Pb$	28.197	1.373×10^{-34}	3.169×10^{-22}	6.853×10^{-10}	8.715×10^{-08}
$^{223}Ac \to {}^{14}C + {}^{209}Bi$	33.065	3.863×10^{-26}	1.741×10^{-13}	1.226×10^{-09}	6.051×10^{-07}
$^{223}Ac \to {}^{15}N + {}^{208}Pb$	39.473	6.778×10^{-27}	$<1.197 \times 10^{-15}$	$<4.549 \times 10^{-11}$	2.246×10^{-08}
$^{225}Ac \to {}^{14}C + {}^{211}Bi$	30.476	7.514×10^{-31}	1.043×10^{-19}	3.950×10^{-11}	2.633×10^{-08}
$^{226}Th \to {}^{14}C + {}^{212}Po$	30.547	1.230×10^{-31}	$<3.477 \times 10^{-16}$	$<8.072 \times 10^{-07}$	9.647×10^{-05}
$^{226}Th \to {}^{18}O + {}^{208}Pb$	45.727	2.540×10^{-29}	$<3.477 \times 10^{-16}$	$<3.611 \times 10^{-09}$	4.315×10^{-07}
$^{228}Th \to {}^{20}O + {}^{208}Pb$	44.723	1.541×10^{-31}	9.297×10^{-22}	1.703×10^{-12}	1.821×10^{-10}
$^{230}Th \to {}^{24}Ne + {}^{206}Hg$	57.762	3.231×10^{-32}	1.590×10^{-25}	1.344×10^{-15}	1.544×10^{-13}
$^{230}U \to {}^{22}Ne + {}^{208}Pb$	61.388	3.154×10^{-29}	4.243×10^{-19}	3.408×10^{-12}	3.792×10^{-10}
$^{231}Pa \to {}^{24}Ne + {}^{207}Tl$	60.411	1.090×10^{-29}	9.016×10^{-25}	2.211×10^{-17}	9.476×10^{-14}
$^{231}Pa \to {}^{23}F + {}^{208}Pb$	51.844	1.698×10^{-32}	1.682×10^{-25}	2.800×10^{-15}	1.200×10^{-11}
$^{230}U \to {}^{24}Ne + {}^{206}Pb$	61.357	9.107×10^{-30}	$<4.243 \times 10^{-19}$	$<1.238 \times 10^{-11}$	1.378×10^{-09}
$^{232}U \to {}^{24}Ne + {}^{208}Pb$	62.311	2.696×10^{-28}	2.720×10^{-21}	2.661×10^{-15}	2.920×10^{-13}
$^{232}U \to {}^{28}Mg + {}^{204}Hg$	74.320	1.951×10^{-30}	$<1.549 \times 10^{-23}$	$<2.079 \times 10^{-15}$	2.281×10^{-13}
$^{233}U \to {}^{24}Ne + {}^{209}Pb$	60.486	9.222×10^{-31}	9.965×10^{-26}	2.897×10^{-17}	5.011×10^{-15}
$^{233}U \to {}^{28}Mg + {}^{205}Hg$	74.226	1.833×10^{-30}	$<1.799 \times 10^{-28}$	$<2.566 \times 10^{-20}$	4.439×10^{-18}
$^{234}U \to {}^{24}Ne + {}^{210}Pb$	58.826	4.411×10^{-33}	5.956×10^{-26}	3.683×10^{-15}	4.534×10^{-13}
$^{234}U \to {}^{25}Ne + {}^{209}Pb$	57.869	3.919×10^{-35}	5.956×10^{-26}	4.266×10^{-13}	5.251×10^{-11}
$^{234}U \to {}^{26}Ne + {}^{208}Pb$	59.465	1.659×10^{-32}	5.956×10^{-26}	1.013×10^{-15}	1.246×10^{-13}
$^{234}U \to {}^{28}Mg + {}^{206}Hg$	74.111	1.488×10^{-30}	2.000×10^{-26}	3.529×10^{-18}	4.344×10^{-16}
$^{235}U \to {}^{28}Mg + {}^{207}Hg$	72.159	5.202×10^{-33}	$<2.443 \times 10^{-29}$	$<1.250 \times 10^{-18}$	2.577×10^{-14}
$^{236}U \to {}^{28}Mg + {}^{208}Hg$	70.565	4.177×10^{-35}	3.671×10^{-27}	2.369×10^{-14}	2.893×10^{-12}
$^{238}U \to {}^{34}Si + {}^{204}Pt$	86.055	1.397×10^{-31}	6.300×10^{-30}	1.218×10^{-20}	1.223×10^{-18}
$^{237}Np \to {}^{30}Mg + {}^{207}Tl$	74.818	5.030×10^{-31}	$<1.854 \times 10^{-28}$	$<9.996 \times 10^{-20}$	1.721×10^{-16}
$^{236}Pu \to {}^{28}Mg + {}^{208}Pb$	79.670	1.802×10^{-26}	1.469×10^{-22}	2.063×10^{-18}	3.630×10^{-16}
$^{238}Pu \to {}^{28}Mg + {}^{210}Pb$	75.912	7.928×10^{-31}	1.412×10^{-26}	4.659×10^{-18}	8.917×10^{-16}
$^{238}Pu \to {}^{30}Mg + {}^{208}Pb$	76.824	7.262×10^{-30}	1.412×10^{-26}	5.209×10^{-19}	9.970×10^{-17}
$^{238}Pu \to {}^{32}Si + {}^{206}Hg$	91.192	2.311×10^{-26}	3.468×10^{-26}	3.827×10^{-22}	7.325×10^{-20}
$^{241}Am \to {}^{34}Si + {}^{207}Tl$	93.927	4.745×10^{-27}	$<3.775 \times 10^{-26}$	$<2.060 \times 10^{-21}$	1.288×10^{-18}
$^{242}Cm \to {}^{34}Si + {}^{208}Pb$	96.511	1.090×10^{-25}	4.915×10^{-24}	1.149×10^{-20}	3.100×10^{-18}

${P_0}^{\alpha(\mathrm{emp})}$ for α decay is nearly constant, and for cluster decay ${P_0}^{c(\mathrm{emp})}$ decreases with the increase in size of the cluster, with the ratio ${P_0}^{c(\mathrm{emp})}/{P_0}^{\alpha(\mathrm{emp})}$ being small, as expected.

In conclusion, this study shows the importance of the preformation factor P_0 for the exotic cluster radioactive decays, introduced in the preformed cluster model of Gupta and collaborators based on quantum mechanical fragmentation theory. Apparently, it will be of great interest to see how this quantity could be treated theoretically within the RMF theory.

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