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Importance of momentum dependent interactions at the energy of vanishing flow

Rajiv Chugh and Rajeev K. Puri*

Department of Physics, Panjab University, Chandigarh-160 014, India

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We study the balance energy (E_{bal}) as a function of combined system mass for different colliding geometries, which range from central to semiperipheral ones. We find that E_{bal} follows a power law behavior ($\propto A^{\tau}$) at all colliding geometries. We also study the effect of momentum dependent interactions on E_{bal} as well as on its mass dependence. We find that the inclusion of momentum dependent interactions changes the value of τ drastically at peripheral geometries in agreement with other calculations.

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I. INTRODUCTION

The collective transverse in-plane flow [1–3] has been used extensively over the past three decades to study the properties of hot and dense nuclear matter [i.e., the nuclear matter equation of state (EOS) as well as the in-medium nucleon-nucleon (nn) cross section]. This has been reported to be highly sensitive toward the above-mentioned properties as well as toward the entrance channel parameters such as combined mass of the system [4], colliding geometries [5], and incident energy of the projectile [6]. The dependence of the collective flow on the above-mentioned parameters has revealed very interesting physics, especially the beam-energy dependence, which has also led to its disappearance. At lower incident energies, the dominance of the attractive mean field prompts the scattering of the particles into negative deflection angles, thus, by producing negative flow, whereas frequent nn collisions and repulsive mean field at higher incident energies result in the emission of particles into positive deflection angles and, hence, yield positive flow. While going through the incident energies, collective transverse in-plane flow disappears at a particular incident energy termed the *balance energy* (E_{bal}) or *energy of vanishing flow* (EVF) [7]. The E_{bal} has been studied experimentally as well as theoretically over a wide range of mass, which ranges from $^{12}C + {}^{12}C$ to $^{238}U + {}^{238}U$ at different colliding geometries and has been found to vary drastically as a function of combined mass of the system [8–13] as well as a function of the impact parameter [9,14–18].

A power law mass dependence ($\propto A^{\tau}$) of E_{bal} was also reported in the literature [8–13]. Earlier power law parameter τ was supposed to be close to $-1/3$ (which results from the interplay between the attractive mean field and repulsive nn collisions) [8], whereas recent studies showed a deviation from the above mentioned power law [9–13], where τ was close to -0.45. Recently, Sood and Puri [12], reported a power law mass dependence $\propto 1/\sqrt{A}$ for heavier nuclei, which suggested the increasing importance of the Coulomb interactions.

It is well established that both collective flow as well as its disappearance depend crucially on the colliding geometry [9,14–19]. The energy of vanishing flow increases linearly with the increase in the impact parameter [9,16,17]. This effect, however, depends on the mass of the system [9]. As a consequence, the mass dependence of E_{bal} is also expected to vary strongly with the colliding geometry. The momentum dependent interactions (MDI) are also known to have sizable effects on the collective flow as well as on its disappearance [13,14,17,18,20–22] in addition to multifragmentation [23–25] and particle production [26]. Recently, Sood and Puri [13] presented a systematic study for understanding the role of MDI in transverse flow as well as on its disappearance for the full range of mass (from $^{12}C + {}^{12}C$ to $^{197}Au + {}^{197}Au$). They found that for a given incident energy, the impact of the MDI is different in different system masses. Here, we plan to extend that study to peripheral colliding geometries.

Our aim is to study the mass dependence of E_{bal} over the range of colliding geometries, which range from the central to the semiperipheral ones and to study the impact of MDI on E_{bal} as well as on its mass dependence. For the present study, we use the quantum molecular dynamics (QMD) model [27–29].

II. THE MODEL

In the QMD model, each nucleon propagates under the influence of mutual interactions. The propagation is governed by the classical equations of motion:

$$\dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}; \quad \dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}, \tag{1}$$

where H stands for the Hamiltonian, which is given by

$$H = \sum_i^A \frac{\mathbf{p}_i^2}{2m_i} + \sum_i^A \left(V_i^{\text{Skyrme}} + V_i^{\text{Yuk}} + V_i^{\text{Coul}} + V_i^{\text{mdi}}\right). \tag{2}$$

The V_i^{Skyrme}, V_i^{Yuk}, V_i^{Coul}, and V_i^{mdi} in Eq. (2) are, respectively, the Skyrme, Yukawa, Coulomb, and momentum dependent potentials. The MDI are obtained by parametrizing the term taken from the measured energy dependence of the nucleon-nucleus optical potential. It can be parametrized as

$$V_{ij}^{\text{MDI}} = t_4 \ln^2[t_5(\mathbf{p}_i - \mathbf{p}_j)^2 + 1]\, \delta(\mathbf{r}_i - \mathbf{r}_j). \tag{3}$$

Here, $t_4 = 1.57$ MeV and $t_5 = 5 \times 10^{-4}$ MeV^{-2}. The final form of the momentum dependent potential reads as

$$U^{\text{MDI}} = \delta \cdot \ln^2[\epsilon(\rho/\rho_0)^{2/3} + 1]\rho/\rho_0. \tag{4}$$

*rkpuri@pu.ac.in

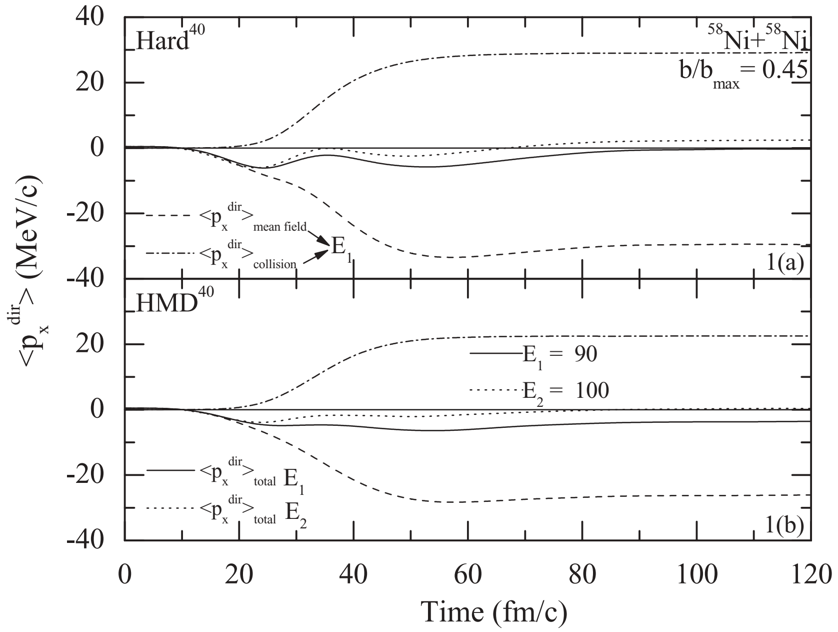

FIG. 1. The time evolution of $\langle p_x^{\rm dir}\rangle$ as well as its decomposition into $\langle p_x^{\rm dir}\rangle_{\text{mean field}}$ and $\langle p_x^{\rm dir}\rangle_{\text{collision}}$. The results in 1(a) (1(b)) is for Hard^{40} (HMD^{40}) EOS. Solid (dotted) lines represent $\langle p_x^{\rm dir}\rangle$ below (above) $E_{\rm bal}$.

A parametrized form of the local plus momentum dependent potential is given by

$$U = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma} + \delta \ln^2[\epsilon(\rho/\rho_0)^{2/3} + 1]\rho/\rho_0. \quad (5)$$

III. RESULTS AND DISCUSSION

For the present study, we simulated events for the reactions of ^{20}Ne + ^{20}Ne, ^{40}Ca + ^{40}Ca, ^{58}Ni + ^{58}Ni, ^{131}Xe + ^{131}Xe, and ^{197}Au + ^{197}Au over the range of colliding geometries, which vary from central to semiperipheral ones. We use hard (labeled as Hard) and hard with MDI (HMD) equations of state (EOS). We use a constant isotropic cross section of 40 mb strength. There are several methods in the literature to define the nuclear transverse in-plane flow. In most of the studies, the $E_{\rm bal}$ is extracted from (p_x/A) plots, where one plots (p_x/A) as a function of $Y_{\rm c.m.}/Y_{\rm beam}$. By using the linear fit to the slope, one can find the so-called reduced flow. Naturally, the energy at which the reduced flow passes through zero is called the $E_{\rm bal}$. Alternatively, one can also use a more integrated quantity *directed transverse momentum* $\langle p_x^{\rm dir}\rangle$, which is defined as [11,27,29,30]

$$\langle p_x^{\rm dir}\rangle = \frac{1}{A}\sum_{i=1}^{A} \text{sgn}\{y(i)\} p_x(i), \quad (6)$$

where $y(i)$ and $p_x(i)$ are, respectively, the rapidity and the momentum of the ith particle. The rapidity is defined as

$$Y(i) = \frac{1}{2}\ln\frac{\vec{E}(i) + \vec{p}_z(i)}{\vec{E}(i) - \vec{p}_z(i)}, \quad (7)$$

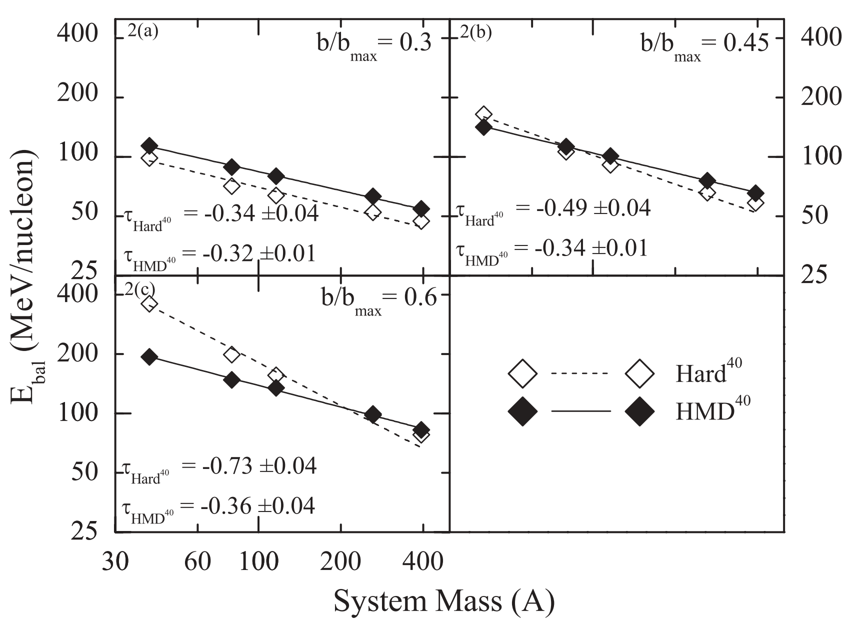

FIG. 2. $E_{\rm bal}$ as a function of combined mass of system. Solid (open) diamonds are for HMD (Hard) EOS. Lines are power law fit $\propto A^{\tau}$.

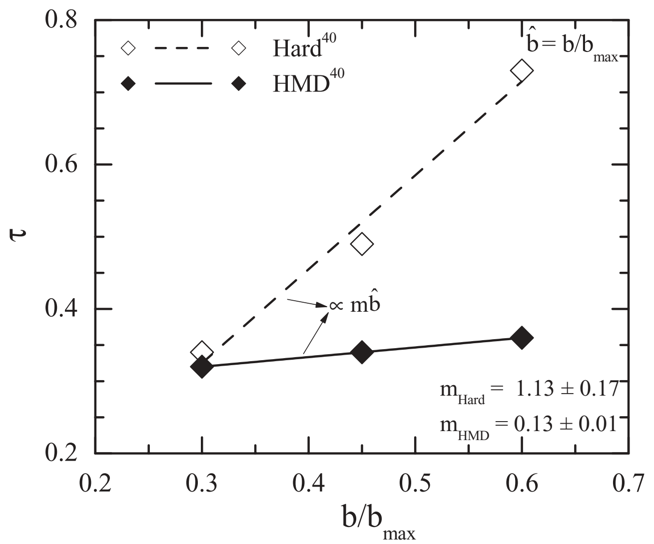

FIG. 3. τ as a function of the reduced impact parameter. Symbols have the same meaning as in Fig. 2. Lines are of linear fit $\propto m(b/b_{\max})$.

where $\vec{E}(i)$ and $\vec{p}_z(i)$ are, respectively, the energy and the longitudinal momentum of the ith particle. In this definition, all the rapidity bins are taken into account. It, therefore, presents an easier way to measure the in-plane flow rather than complicated functions such as (p_x/A) plots. It has been shown in Ref. [11] that the disappearance of flow occurs at the same incident energy in both cases by showing the equivalence between p_x/A and $\langle p_x^{\rm dir}\rangle$ as far as $E_{\rm bal}$ is concerned. It is worth mentioning that $E_{\rm bal}$ has the same value for all fragment types [16,31–33]. Furthermore, the apparatus corrections and acceptance do not play any role in the calculation of the EVF [4,33].

In Fig. 1, we display $\langle p_x^{\rm dir}\rangle$ as a function of the reaction time for the collisions of ^{58}Ni + ^{58}Ni by using Hard (top panel) and HMD EOS (bottom panel) at energies around $E_{\rm bal}$ for semicentral colliding geometry. The lines are explained in the caption for Fig. 1. It has been argued in Ref. [30] that the flow at any point during the reaction can be divided into the parts that emerge from the (attractive) mean field potential and the (repulsive) nn collision contributions. We have also decomposed the transverse momentum ($\langle p_x^{\rm dir}\rangle$) into the contributions created by the mean field [$\langle p_x^{\rm dir}\rangle_{\rm mean\ field}$ (dashed line)] and binary nn collisions [$\langle p_x^{\rm dir}\rangle_{\rm collision}$ (dashed-dotted line)]. This extraction, which is developed from the simulations of the QMD model, is performed as follows [30]: Here at each time step during the collision, momentum transferred by the mean field and the binary collision is analyzed separately. Naturally, we get two values at each time step, which can be followed throughout the reaction. The total transverse momentum can be obtained by adding both these contributions. From Fig. 1, we see that $\langle p_x^{\rm dir}\rangle$ saturates at around 100 fm/c. Also, the flow caused by the binary collisions is $+ve$, whereas the mean field is $-ve$. If we compare Figs. 1(a) and 1(b), we see the inclusion of MDI decreases the absolute values of flow caused by collisions as well as mean field. The suppression of nn collisions caused by MDI happens because the inclusion of MDI accelerates nucleons in the transverse direction during the initial phase of the reaction, which leads to the lower density that results in fewer nn collisions. Since MDI are attractive for lower relative momenta and are repulsive for higher ones, at energies around 100 MeV/nucleon, MDI changes sign (i.e., turns from attractive to repulsive), thus, reducing the absolute value of mean field flow (or increasing the flow because of mean field).

In Fig. 2, we display $E_{\rm bal}$ as a function of combined mass of the system for Hard and HMD EOS. Various symbols are explained in the caption. The lines are power law fit ($\propto A^\tau$). Figures 2(a)–2(c) are, respectively, for $b/b_{\max} =$ 0.3, 0.45, and 0.6. From the figure, we see that $E_{\rm bal}$ follows the power law behavior at all colliding geometries. The values of τ are -0.34 ± 0.04 (-0.32 ± 0.01), -0.49 ± 0.04 (-0.34 ± 0.01), and -0.76 ± 0.04 (-0.36 ± 0.04), respectively, at $b/b_{\max} = 0.3$, 0.45, and 0.6 for Hard^{40} (HMD^{40}) EOS. From the figure, we see that the value of τ for Hard^{40} increases drastically as one goes from central to semiperipheral colliding geometries. This is because $E_{\rm bal}$ increases approximately linearly with the increase in the impact parameter [9,14–16], which is caused by the decrease in the number of nn binary collisions. However, the increase in $E_{\rm bal}$ with impact parameter is different for different masses. The effect is stronger in lighter masses compared to heavier ones [9]. This changes the slope drastically. However interestingly, the value of τ for HMD^{40} almost remains constant as one goes from $b/b_{\max} = 0.3$ to 0.6. This is because of the fact that, in lighter systems, since $E_{\rm bal}$ is large, the effect of MDI is large and because of its repulsive nature, it suppresses the $E_{\rm bal}$ by a large amount. Whereas in heavier systems, its effect is reversed because of the small value of $E_{\rm bal}$, where it enhances $E_{\rm bal}$. It is worth mentioning here that, at a fixed incident energy, the effect of MDI is different for different system masses as shown by Sood and Puri [13]. In Fig. 3, we display τ as a function of colliding geometry. Symbols have the same meaning as in Fig. 2. Lines represent linear fit $\propto m\frac{b}{b_{\max}}$. From the figure, we see drastic changes in the values of m for $\tau_{\rm Hard^{40}}$, whereas for $\tau_{\rm HMD^{40}}$, m remains almost constant.

IV. SUMMARY

In summary, we have studied $E_{\rm bal}$ as a function of combined mass of the system for different colliding geometries, which range from central to semiperipheral. We find that $E_{\rm bal}$ follows a power law behavior ($\propto A^\tau$) at all colliding geometries. We have also studied the effect of MDI on $E_{\rm bal}$ as well as its mass dependence. We find that the inclusion of MDI changes the value of τ drastically at higher colliding geometries.

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[1] W. Scheid, H. Müller, and W. Greiner, Phys. Rev. Lett. **32**, 741 (1974).
[2] H. A. Gustafsson *et al.*, Phys. Rev. Lett. **52**, 1590 (1984).
[3] K. G. R. Doss *et al.*, Phys. Rev. Lett. **57**, 302 (1986).
[4] C. A. Ogilvie *et al.*, Phys. Rev. C **40**, 2592 (1989).
[5] Q. Pan and P. Danielewicz, Phys. Rev. Lett. **70** 2062 (1993); J. Lukasik *et al.*, Phys. Lett. B **608**, 223 (2005).
[6] J. Lukasik and W. Trautmann, arXiv:0708.2821v1 (2008); B. Hong *et al.*, Phys. Rev. C **66**, 034901 (2000).
[7] D. Krofcheck *et al.*, Phys. Rev. Lett. **63**, 2028 (1989).
[8] V. de la Mota, F. Sebille, M. Farine, B. Remaud, and P. Schuck, Phys. Rev. C **46**, 677 (1992); G. D. Westfall *et al.*, Phys. Rev. Lett. **71**, 1986 (1993); H. Zhou, Z. Li, and Y. Zhuo, Phys. Rev. C **50**, R2664 (1994); H. Zhou, Z. Li, Y. Zhuo, and G. Mao, Nucl. Phys. A **580**, 627 (1994).
[9] D. J. Magestro *et al.*, Phys. Rev. C **61**, 021602(R) (2000).
[10] D. J. Magestro, W. Bauer, and G. D. Westfall, Phys. Rev. C **62**, 041603(R) (2000).
[11] A. D. Sood and R. K. Puri, Phys. Rev. C **69**, 054612 (2004); **73**, 067602 (2006).
[12] A. D. Sood and R. K. Puri, Phys. Lett. B **594**, 260 (2004).
[13] A. D. Sood and R. K. Puri, Eur. Phys. J. A **30**, 571 (2006).
[14] J. P. Sullivan *et al.*, Phys. Lett. B **249**, 8 (1990).
[15] A. Buta *et al.*, Nucl. Phys. A **584**, 397 (1995).
[16] R. Pak *et al.*, Phys. Rev. C **53**, R1469 (1996).
[17] S. Soff, C. Hartnack, H. Stöcker, and W. Greiner, Phys. Rev. C **51**, 3320 (1995).
[18] Z. Y. He *et al.*, Nucl. Phys. A **598**, 248 (1996).
[19] M. B. Tsang, G. F. Bertsch, W. G. Lynch, and M. Tohyama, Phys. Rev. C **40**, 1685 (1989).
[20] J. Zhang, S. Das Gupta, and C. Gale, Phys. Rev. C **50**, 1617 (1994).
[21] G. Peilert, H. Stöcker, W. Greiner, A. Rosenhauer, A. Bohnet, and J. Aichelin, Phys. Rev. C **39**, 1402 (1989).
[22] D. Krofcheck *et al.*, Phys. Rev. C **43**, 350 (1991).
[23] Y. K. Vermani, S. Goyal, and R. K. Puri, Phys. Rev. C **79**, 064613 (2009).
[24] S. Kumar, S. Kumar, and R. K. Puri, Phys. Rev. C **78**, 064602 (2008).
[25] L.-W. Chen, C. M. Ko., and B.-A. Li, Phys. Rev. C **69**, 054606 (2004).
[26] J. Aichelin *et al.*, Phys. Rev. Lett. **62**, 1461 (1989).
[27] J. Aichelin, Phys. Rep. **202**, 233 (1991); D. T. Khoa *et al.*, Nucl. Phys. A **548**, 102 (1992); Y. K. Vermani and R. K. Puri, J. Phys. G. **36**, 105103 (2009); E. Lehmann, R. K. Puri, A. Faessler, G. Batko, and S. W. Huang, Phys. Rev. C **51**, 2113 (1995).
[28] J. Singh, S. Kumar, and R. K. Puri, Phys. Rev. C **62**, 044617 (2000); R. K. Puri and J. Aichelin, J. Comput. Phys. **162**, 245 (2000); Y. K. Vermani, J. K. Dhawan, S. Goyal, R. K. Puri, and J. Aichelin, J. Phys. G **37**, 015105 (2010); Y. K. Vermani and R. K. Puri, Europhys. Lett. **85**, 62001 (2009); A. D. Sood and R. K. Puri, Phys. Rev. C **79**, 064618 (2009).
[29] C. Hartnack *et al.*, Eur. Phys. J. A **1**, 151 (1998); C. Hartnack and J. Aichelin, Phys. Rev. C **49**, 2801 (1994); C. Hartnack, J. Aichelin, H. Stöcker, and W. Greiner, Mod. Phys. Lett. A **9**, 1151 (1994).
[30] E. Lehmann, A. Faessler, J. Zipprich, R. K. Puri, and S. W. Huang, Z. Phys. A **355**, 55 (1996).
[31] R. Pak *et al.*, Phys. Rev. Lett. **78**, 1022 (1997); **78**, 1026 (1997).
[32] G. D. Westfall, Nucl. Phys. A **630**, 27c (1998).
[33] D. Cussol *et al.*, Phys. Rev. C **65**, 044604 (2002).