Phase ambiguity of the threshold amplitude in $pp \to pp\pi^0$

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Measurements of spin observables in $pp \to \vec{p}\vec{p}\pi^0$ are suggested to remove the phase ambiguity of the threshold amplitude. The suggested measurements complement the Indiana University Cyclotron Facility data on $\vec{p}\vec{p} \to pp\pi^0$ to completely determine all the 12 partial wave amplitudes taken into consideration by Meyer *et al.* [Phys. Rev. C **63**, 064002 (2001)] and Deepak *et al.* [Phys. Rev. C **72**, 024004 (2005)].

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Introduction. Meson production in NN collisions has continued to excite considerable interest [1–4] since total cross-section measurements [5] for $pp \to pp\pi^0$ in the early 1990s were found to be more than a factor of 5 larger than the then available theoretical predictions [6]. To bridge the gap between experiment and theory, several mechanisms, like exchange of heavy mesons, two-pion exchange, off-shell extrapolation of the vertex form factor, final-state interactions, and contributions owing to Δ resonance and of low-lying nucleon resonances, were proposed. Hanhart *et al.* [7] in 2000 observed: "As far as microscopic model calculations of the reaction $NN \to NN\pi$ are concerned, one has to concede that theory is definitely lagging behind the development of the experimental sector . . . Further more they take into account only the lowest partial wave(s). Therefore, it is not possible to confront these models with the wealth of experimental information available nowadays specifically with differential cross-sections and with spin dependent observables." The Julich model, on the other hand, takes into consideration higher partial waves as well.

In contrast to elastic NN scattering, where channel spin is conserved, the $pp \to pp\pi^0$ transition at threshold to the final Ss state is a triplet to singlet. Next in order are the transitions to Ps states, which are singlet to triplet. As the energy is increased, transitions to Pp states are also expected to contribute, which are, however, triplet to triplet. Pionic d-wave effects were reported [8] even at a beam energy of 310 MeV. Measurements up to 425 MeV have also been reported [9], where evidence for a Ds state was seen even at 310 MeV. Advances in storage ring technology [10] led to detailed experimental studies, including measurements of spin observables employing polarized beams of protons on polarized proton targets. Of the two existing models [11,12] which include higher partial waves, the Julich meson exchange model [7,11] was thoroughly confronted with these data. The model was comparatively more successful with the less complete data on $\vec{p}\vec{p} \to d\pi^+$ [13] and $\vec{p}\vec{p} \to pn\pi^+$ [14] but failed to provide an overall satisfactory reproduction of the complete set of polarization observables in the case of $\vec{p}\vec{p} \to pp\pi^0$ [15]. In this context, a model-independent approach [16,17] was developed using irreducible tensor techniques [18]. The reaction is characterized, in this formalism, by irreductible tensor amplitudes $M^{\lambda}_{\mu}(s_f, s_i)$ of rank $\lambda = |s_f - s_i|, \ldots, (s_f + s_i)$, where s_i, s_f denote the initial and final spin states of the two protons. Each of these amplitudes is expressible in terms of partial wave amplitudes $M^{j}_{l(l_f s_f)j_f;l_i s_i}$, which are functions of the c.m. energy E and invariant mass W of the two-proton system in the final state. The relative orbital angular momenta between the two protons in the initial and final states are denoted by l_i and l_f, respectively, and l denotes the pion orbital angular momentum in the c.m. frame. The threshold amplitude $M^{0}_{0(00)0;11}$ contributes to $M^{1}_{0}(0,1)$, and an empirical estimate of the integrated $|M^{1}_{0}(0,1)|^2$ was presented in Ref. [16], based on the then existing data [5]. The same approach was employed subsequently to analyze [19] the Indiana University Cyclotron Facility data on $\vec{p}\vec{p} \to pp\pi^0$ [15] immediately after its publication. The 16 partial waves listed by Meyer *et al.* [15] covered the Ss, Ps, Pp, Sd, and Ds channels. Here, the capital letters denote l_f while the lower case indicate l. In Ref. [20], the same set of partial waves were listed, of which, the last four, covering Sd and Ds, were ignored following Ref. [15]. Because the final spin-singlet and spin-triplet states do not mix in any of the spin observables measured in Ref. [15], the Ss amplitude and the larger of the Ps amplitudes were both chosen to be real in Ref. [20]. This implies that the phase of the Ss amplitude remained ambiguous but chosen to be zero with respect to the larger Ps amplitude. The comparison of the empirically extracted amplitudes with the Julich model predictions revealed that (i) the Δ contributions are important and (ii) the model deviated very strongly in the case of ${}^3P_1 \to {}^3P_0 p$ and to a lesser extent in ${}^3F_3 \to {}^3P_2 p$, which "will guide the search for the possible shortcomings" [20].

The purpose of the present Brief Report is to extend the model-independent theoretical discussion to the spin polarization of the protons in the final state and to examine how the additional experimental measurements regarding the final spin state can be used to determine empirically the strengths of all these amplitudes and the ambiguous relative phase of the threshold Ss amplitude with respect to the 11 near-threshold Ps and Pp amplitudes considered in Refs. [15,19,20] for $pp \to pp\pi^0$. We mention that the p-wave charged pion production in $pn \to pp\pi^-$, $pp \to pn\pi^+$, $pp \to d\pi^+$ has more recently been discussed [21] using effective field theory and it was proposed earlier [4,12] to pin down the phase of the Ss partial wave amplitude with reference to the isospin-0-to-isospin-1 Sp amplitude by looking at the forward-backward assymetry in $pn \to pp\pi^-$.

Theoretical formalism. We consider the reaction $pp \to pp\pi^0$ at c.m. energy E and initial c.m. momentum $\mathbf{p_i} = p_i\hat{\mathbf{p}}_\mathbf{i}$, which may be chosen to be along the z axis. Let $\mathbf{q} = q\hat{\mathbf{q}} = -(\mathbf{p_1} + \mathbf{p_2})$ denote the pion momentum in the c.m. frame and let $\mathbf{p_f} = p_f\hat{\mathbf{p}}_\mathbf{f} = \frac{1}{2}(\mathbf{p_1} - \mathbf{p_2})$ in terms of the c.m. momenta $\mathbf{p_1}$ and $\mathbf{p_2}$ of the two protons in the final state.

Following Ref. [16], we write the matrix M in spin space for the reaction $pp \to pp\pi^0$ in the form

$$M = \sum_{s_i, s_f = 0}^{1} \sum_{\lambda = |s_i - s_f|}^{s_i + s_f} (S^\lambda(s_f, s_i) \cdot M^\lambda(s_f, s_i)), \tag{1}$$

where s_i and s_f denote the initial and final channel spins, respectively. The irreducible tensor operators $S^\lambda_\mu(s_f, s_i)$ of rank λ, with μ taking values $\mu = \lambda, \lambda - 1, \dots, -\lambda$, are defined in Ref. [18]. The irreducible tensor amplitudes $M^\lambda_\mu(s_f, s_i)$ in Eq. (1) are expressible as

$$M^\lambda_\mu(s_f, s_i) = \sum_{\mathcal{L}, j} W(l_i s_i L_f s_f; j\lambda)\, Z(s_f, s_i, \mathcal{L}, j) A^\lambda_\mu(\mathcal{L}), \tag{2}$$

where

$$A^\lambda_\mu(\mathcal{L}) = \left(\left(Y_{l_f}(\hat{\mathbf{p}}_f) \otimes Y_l(\hat{\mathbf{q}})\right)^{L_f} \otimes Y_{l_i}(\hat{\mathbf{p}}_i)\right)^\lambda_\mu, \tag{3}$$

and the symbol $\mathcal{L}$ is used to collectively denote $\mathcal{L} \equiv \{l_f, l, L_f, l_i\}$. It may be noted that $(-1)^{l_f + l + l_i} = -1$ owing to parity conservation. The complex numbers $Z(s_f, s_i, \mathcal{L}, j)$ are given by

$$Z(s_f, s_i, \mathcal{L}, j) = \frac{[L_f][j]^2}{[s_f]} (-1)^{j - s_i + 1} \times \sum_{j_f} [j_f] W(s_f l_f j l; j_f L_f) M^j_{l(l_f s_f) j_f; l_i s_i} \tag{4}$$

in terms of the 16 partial wave reaction amplitudes

$$M^j_{l(l_f s_f) j_f; l_i s_i} = F\, \langle (l(l_f s_f) j_f) j || T || (l_i s_i) j \rangle, \tag{5}$$

proportional to the reduced on-energy-shell T-matrix elements $\langle (l(l_f s_f) j_f) j || T || (l_i s_i) j \rangle$ for the reaction. The purely kinematical factor

$$F = (-i)^{l_i - l - l_f} 4(2\pi)^{1/2} \sqrt{W\omega(E - \omega) q p_f / p_i} \tag{6}$$

is introduced explicitly in Eq. (5) so that the dependence on E and W is seen to be completely taken care of by the $M^j_{l(l_f s_f) j_f; l_i s_i}$. They are identical to the amplitudes denoted as T in Ref. [20]. We may, following Refs. [15,20], neglect the last 4 amplitudes, which are Sd and Ds, and consider the first 12 amplitudes, which are, for simplicity, enumerated as $f_1, \dots, f_{12}$ in Table I.

The unpolarized double-differential cross section may now be written as

$$\frac{d^2\sigma_0}{dW\, d\Omega_f\, d\Omega} = \frac{1}{4} \mathrm{Tr}[M\, M^\dagger], \tag{7}$$

where $M^\dagger$ denotes the Hermitian conjugate of M given by Eq. (1). The invariant mass W of the two protons in the final state is given by

TABLE I. List of the partial wave amplitudes for the reaction $pp \to pp\pi^0$.

Initial pp state	Type	Final $pp\pi^0$ state	Partial wave amplitudes
$^3P_0$	Ss	$^1S_0, s$	$M^0_{0(00)0;11} = f_1$
$^1S_0$	Ps	$^3P_0, s$	$M^0_{0(11)0;00} = f_2$
$^1D_2$		$^3P_2, s$	$M^2_{0(11)2;20} = f_3$
$^3P_0$	Pp	$^3P_1, p$	$M^0_{1(11)1;11} = f_4$
$^3P_2$		$^3P_1, p$	$M^2_{1(11)1;11} = f_5$
$^3P_2$		$^3P_2, p$	$M^2_{1(11)2;11} = f_6$
$^3F_2$		$^3P_1, p$	$M^2_{1(11)1;31} = f_7$
$^3F_2$		$^3P_2, p$	$M^2_{1(11)2;31} = f_8$
$^3P_1$		$^3P_0, p$	$M^1_{1(11)0;11} = f_9$
$^3P_1$		$^3P_1, p$	$M^1_{1(11)1;11} = f_{10}$
$^3P_1$		$^3P_2, p$	$M^1_{1(11)2;11} = f_{11}$
$^3F_3$		$^3P_2, p$	$M^3_{1(11)2;31} = f_{12}$

$$W = \sqrt{\left(E^2 + m_\pi^2 - 2E\omega\right)}, \tag{8}$$

where m_π denotes the pion mass and ω denotes the c.m. energy of pion. It may be noted that

$$\frac{d^2\sigma_0}{d^3 p_f d\Omega} = \frac{W}{p_f} \frac{d^2\sigma_0}{dW d\Omega_f d\Omega}. \tag{9}$$

It is worth noting that the threshold Ss amplitude f_1 alone contributes to

$$M^1_\mu(0, 1) = \frac{1}{4\sqrt{3}\pi} f_1 Y_{1\mu}(\hat{\mathbf{p}}_\mathbf{i}), \tag{10}$$

which is spherically symmetric both with respect to $\hat{\mathbf{p}}_\mathbf{f}$ as well as to $\hat{\mathbf{q}}$ in the final state, while all the other irreducible tensor amplitudes are independent of f_1.

Final-state polarization with initially unpolarized protons. If the colliding protons are unpolarized, the spin density matrix ρ^f characterizing the two protons in the final state is given by

$$\rho^f = \tfrac{1}{4} M M^\dagger, \tag{11}$$

so that Eq. (7) is identical to $\mathrm{Tr}[\rho^f]$.

The final spin state is completely determined through measurements of the polarizations

$$\mathbf{P}_i = \frac{\mathrm{Tr}[\boldsymbol{\sigma}_i\, \rho^f]}{\mathrm{Tr}[\rho^f]}, \quad i = 1, 2 \tag{12}$$

of the two protons and their spin correlations

$$C_{\alpha\beta} = \frac{\mathrm{Tr}[\boldsymbol{\sigma}_{1\alpha}\, \boldsymbol{\sigma}_{2\beta}\, \rho^f]}{\mathrm{Tr}[\rho^f]}, \quad \alpha, \beta = x, y, z. \tag{13}$$

All these spin observables may elegantly be calculated by considering

$$P^k_\mu(s'_f, s_f) = \mathrm{Tr}\left[S^k_\mu(s'_f, s_f)\, \rho^f\right], \tag{14}$$

where $S^k_\mu(s'_f, s_f)$ are given in terms of the Pauli spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ of the two protons in the final state through

$$S^0_0(0,0) = \frac{1}{4}(1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \tag{15}$$

$$S^0_0(1,1) = \frac{1}{4}(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \tag{16}$$

$$S^1_\mu(1,1) = \frac{\sqrt{3}}{2\sqrt{2}}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^1_\mu, \tag{17}$$

$$S^2_\mu(1,1) = \frac{\sqrt{3}}{2}(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)^2_\mu, \tag{18}$$

$$S^1_\mu(0,1) = \frac{1}{2\sqrt{2}}(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)^1_\mu - \frac{1}{4}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)^1_\mu, \tag{19}$$

$$S^1_\mu(1,0) = \frac{\sqrt{3}}{2\sqrt{2}}(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)^1_\mu + \frac{\sqrt{3}}{4}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)^1_\mu. \tag{20}$$

Thus, the double-differential cross section is given by

$$\frac{d^2\sigma_0}{dW\, d\Omega_f\, d\Omega} = \mathrm{Tr}[\rho^f] = P^0_0(0,0) + P^0_0(1,1), \tag{21}$$

in terms of the double-differential cross sections, $P^0_0(0,0)$ leading to the final singlet state and $P^0_0(1,1)$ leading to the final triplet state of the two protons. If we use the notations $(\mathbf{P_i})_\mu$ to denote the spherical components, that is,

$$(\mathbf{P_i})_0 = P_{iz}; (\mathbf{P_i})_{\pm 1} = \mp\frac{1}{\sqrt{2}}(P_{ix} \pm P_{iy}), \tag{22}$$

it follows from Eqs. (19) and (20) that

$$P^1_\mu(1,0) - \sqrt{3}P^1_\mu(0,1) = \frac{\sqrt{3}}{2}\mathrm{Tr}[\rho^f](\mathbf{P}_1 - \mathbf{P}_2)_\mu, \tag{23}$$

whereas it follows from Eq. (17) that

$$P^1_\mu(1,1) = \frac{\sqrt{3}}{2\sqrt{2}}\mathrm{Tr}[\rho^f](\mathbf{P}_1 + \mathbf{P}_2)_\mu, \tag{24}$$

which together determine $\mathbf{P}_1$ and $\mathbf{P}_2$ individually. Finally, the spin correlations $C_{\alpha\beta}$ defined in Eq. (13) may likewise be related to Eq. (14) using

$$P^0_0(1,1) - 3P^0_0(0,0) = \mathrm{Tr}[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\rho^f], \tag{25}$$

$$P^1_\mu(1,0) + \sqrt{3}P^1_\mu(0,1) = \frac{\sqrt{3}i}{2}\mathrm{Tr}[\rho^f(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)]_\mu, \tag{26}$$

$$P^2_\mu(1,1) = \frac{\sqrt{3}}{2}\mathrm{Tr}\big[\rho^f(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)^2_\mu\big]. \tag{27}$$

Using the known properties [18] of the spin operators S^λ_μ and standard Racah techniques, we may obtain a master formula for all the final-state spin observables, which is given by

$$P^k_\mu(s'_f, s_f) = \frac{1}{4}\sum_{s_i,\lambda,\lambda'}(-1)^{s_f - s_i}[s_f]\,[s'_f]^2[\lambda][\lambda'] \times W(s'_f\lambda' s_f\lambda; s_i k)[M^\lambda(s_f, s_i) \otimes M^{\dagger\lambda'}(s'_f, s_i)]^k_\mu, \tag{28}$$

where $M^{\dagger\lambda}_\mu(s_f, s_i)$ are defined in terms of the complex conjugates $M^\lambda_\mu(s_f, s_i)^*$ of $M^\lambda_\mu(s_f, s_i)$ given by Eq. (2) through

$$M^{\dagger\lambda}_\mu(s_f, s_i) = (-1)^\mu M^\lambda_{-\mu}(s_f, s_i)^*. \tag{29}$$

Noting once again that $(-1)^{l_f + l + l_i} = -1$, owing to parity conservation, we may express

$$M^{\dagger\lambda}_\mu(s_f, s_i) = (-1)^{1-\lambda}\sum_{\mathcal{L}} W(l_i s_i L_f s_f; j\lambda) \times Z^*(s_f, s_i, j, \mathcal{L})\, A^\lambda_\mu(\mathcal{L}), \tag{30}$$

where $Z^*(s_f, s_i, j, \mathcal{L})$ denote the complex conjugates of $Z(s_f, s_i, j, \mathcal{L})$ given by Eq. (4).

Relative phase of the threshold amplitude. We may now take advantage of the fact that $M^1_0(0,1)$ given by Eq. (10) is spherically symmetric with respect to $\hat{\mathbf{p}}_f$ and $\hat{\mathbf{q}}$ and involves only the threshold amplitude f_1. Moreover, $M^\lambda_\mu(1,1)$ are independent of f_1 and depend only on the Pp amplitudes $f_4, \ldots, f_{12}$. Therefore, we focus attention on Eqs. (26) and (23), which involve

$$[M^\lambda(1,1) \otimes M^{\dagger 1}(0,1)]^1_\mu = \sum_{\mathcal{L},j} Z(1,1,j,\mathcal{L}) f_1^* \mathcal{A}^1_\mu \tag{31}$$

$$[M^1(0,1) \otimes M^{\dagger\lambda}(1,1)]^1_\mu = -\sum_{\mathcal{L},j} Z^*(1,1,j,\mathcal{L}) f_1\, \mathcal{A}^1_\mu, \tag{32}$$

where

$$\mathcal{A}^1_\mu = \frac{1}{4\sqrt{3}\pi} W(l_i 1 L_f 1; j\lambda)[A^\lambda(\mathcal{L}) \otimes Y_1(\hat{\mathbf{p}}_\mathbf{i})]^1_\mu. \tag{33}$$

Expressing

$$[A^\lambda(\mathcal{L}) \otimes Y_1(\hat{\mathbf{p}}_\mathbf{i})]^1_\mu = \frac{\sqrt{3}}{4\pi}\sum_{L_i} W(L_f l_i 11; \lambda L_i)[\lambda][L_i][l_i] \times C(l_i 1 L_i, 000) A^1_\mu(l_f l L_f L_i) \tag{34}$$

and carrying out the summation over $\mathcal{L}$ and j, we obtain

$$P^1_\mu(1,0) = f_1^*[F_1 A^1_\mu(1110) + F_2 A^1_\mu(1112) + F_3 A^1_\mu(1122)], \tag{35}$$

$$P^1_\mu(0,1) = \frac{-1}{\sqrt{3}} f_1\big[F_1^* A^1_\mu(1110) + F_2^* A^1_\mu(1112) + F_3^* A^1_\mu(1122)\big], \tag{36}$$

where $F_i, i = 1,2,3$ are well-defined linear combinations of the Pp amplitudes given by

$$F_1 = \frac{1}{32\pi^{3/2}}\left[f_4 - \frac{5}{6}f_5 + \frac{5}{2\sqrt{3}}f_6 + \frac{1}{3\sqrt{3}}f_9 - \frac{1}{6}f_{10} - \frac{\sqrt{5}}{6\sqrt{3}}f_{11}\right], \tag{37}$$

$$F_2 = \frac{1}{32\sqrt{2}\pi^{3/2}}\left[f_5 - \sqrt{3}f_6 + \sqrt{\frac{3}{2}}f_7 + \frac{3}{\sqrt{2}}f_8\right], \tag{38}$$

$$F_3 = \frac{-1}{32\sqrt{2}\pi^{3/2}}\left[\sqrt{3}f_5 + f_6 + \sqrt{7}f_7 + \sqrt{\frac{7}{3}}f_8\right], \tag{39}$$

Because the Pp amplitudes have been determined both in magnitude and in relative phase with respect to f_2 in Ref. [20], we may express $F_\alpha = |F_\alpha| \exp[i\Delta_\alpha]$, $\alpha = 1, 2, 3$ and treat $|F_\alpha|$ and Δ_α as known. In Ref. [20], f_2 was assumed to be real. Because the relative phase between f_1 and f_2 could not be ascertained from the measurements of Meyer *et al.* [15], f_1 was also assumed to be real, although only one of the amplitudes can be taken as real. Therefore, we choose f_2 to be real and express $f_1 = |f_1| \exp[i\delta_1]$. This leads to

$$P^1_\mu(1,0) - \sqrt{3}P^1_\mu(0,1) = 2\sum_{\alpha=1}^{3} R_\alpha \cos(\Delta_\alpha - \delta_1) A^1_\mu(\alpha), \tag{40}$$

$$P^1_\mu(1,0) + \sqrt{3}P^1_\mu(0,1) = 2i\sum_{\alpha=1}^{3} R_\alpha \sin(\Delta_\alpha - \delta_1) A^1_\mu(\alpha), \tag{41}$$

where $R_\alpha = |F_\alpha||f_1|$ and $A^1_\mu(\alpha)$ for $\alpha = 1, 2, 3$ denote $A^1_\mu(1110)$, $A^1_\mu(1112)$, $A^1_\mu(1122)$, respectively.

It is seen from Eq. (11) that measuring the double-differential cross section (7) yields $\mathrm{Tr}[\rho^f]$. Measurements of $(\mathbf{P}_1 - \mathbf{P}_2)_\mu$ given by Eq. (23) then lead to empirical determination of Eq. (40), while measurements of spin correlations $C_{xy} - C_{yx}, C_{yz} - C_{zy}, C_{zx} - C_{xz}$, where $C_{\alpha\beta}$ are given by Eq. (13) lead to empirical determination of Eq. (41) using Eq. (26).

Thus, we find that it is possible to determine empirically the relative phase δ_1 of f_1, without any trigonometric ambiguities, because R_α and Δ_α are known from Ref. [20]. We therefore advocate measurement of these pp spin observables in the final state, employing simply an unpolarized beam and unpolarized target initially, to complement the spin observables measured by Meyer *et al.* [15], so that the amplitudes $f_1, f_2, \ldots, f_{12}$ may be determined empirically without any phase ambiguity.

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