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Neutron-proton radii in $N \approx Z$ nuclei

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A simple formula is derived that describes how the Coulomb interaction affects the proton radius in nuclei. It determines the difference between neutron and proton radii in nuclei with $N \approx Z$. It also estimates the difference between the radii of the Z core neutrons and protons in nuclei with a large neutron excess. The results obtained from the derived formula are compared with radii calculated in a Skyrme Hartree-Fock calculation.

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There is an ongoing quest to determine the radius of the neutron distribution in nuclei. Recent advances in nuclear theory and some new experiments have given this field an additional impetus, but the experimental studies are plagued by model dependence when relating the observables to the values of neutron radii. A promising attempt to measure the neutron radius in Pb^{208} is the parity-violating electron-scattering experiment at the Thomas Jefferson National Accelerator Facility (JLab) [1]. The many theoretical studies involve the use of parameters that are not always well known, so the studies are hampered by the lack of reliable experimental data on neutron radii that would allow a calibration of some parameters.

The various studies are often expressed in terms of the difference between neutron and proton radii. Of course, knowing this difference enables one to determine the neutron radius because the proton radius is readily available experimentally.

In the present article, we study the difference of neutron-proton radii in nuclei with the number of neutrons N equal to the number of protons Z. In these nuclei, the symmetry energy does not play any significant role, the only part that affects protons differently than neutrons are the charge-asymmetric parts of the Hamiltonian, and of these the dominant one by far is the Coulomb interaction. It is, of course, expected that in such nuclei the protons will have a lager radius than the neutrons, forming a proton "skin." We will see that this effect is not negligible in nuclei with a charge Z. The number of stable nuclei with $N = Z$ is small; however, with the development of radioactive beams it will be possible to study unstable proton rich nuclei with larger values of Z and small $N - Z$.

Consider a nucleus with N neutrons and Z protons. Let H be the total Hamiltonian describing the nucleus:

$$H = H_0 + V, \tag{1}$$

where H_0 is the part of the Hamiltonian that conserves isospin symmetry and V contains all the parts of the total Hamiltonian that do not conserve this symmetry.

The ground state wave function of H is denoted by $|\tilde{0}\rangle$.

Let us treat now V in perturbation theory. We can than write

$$|\tilde{0}\rangle = |0\rangle + \sum_{n\neq 0} \varepsilon_n |n\rangle, \tag{2}$$

where $|0\rangle$ and $|n\rangle$ are the ground and excited states of the Hamiltonian H_0.

The major part of the isospin-violating interaction is the Coulomb force, and of it the dominant one is the one-body Coulomb potential [2]. Let us simplify this potential by using a potential derived from a homogenous spherical charge distribution with radius R. Inside the sphere ($r \leqslant R$),

$$V_{\mathrm{C}}(r) = -\frac{Ze^2}{R^3} \sum_i \left(\frac{1}{2} r_i^2 - \frac{3}{2} R^2\right) \left(\frac{1}{2} - t_z(i)\right), \tag{3}$$

We need only to deal with the isovector part $V_{\mathrm{C}}^{(1)}$ of this potential because the isoscalar part is assumed to be contained in H_0. The nonconstant part that contributes to the nondiagonal matrix element is

$$V_{\mathrm{C}}^{(1)} = \frac{Ze^2}{2R^3} \sum_i r_i^2 t_z(i). \tag{4}$$

Therefore,

$$\varepsilon_n = \frac{\langle 0|V_{\mathrm{C}}^{(1)}|n\rangle}{E_n - E_0}. \tag{5}$$

We note that by choosing a nucleus with $N \neq Z$ in the case of the Hamiltonian H_0 means that the isospin symmetry is spontaneously broken. A symmetry potential that is a result of the difference in the two-body interaction between two nucleons in the $T = 0$ and $T = 1$ states

$$V_{\mathrm{sym}} = \frac{U}{A}(N - Z)t_z \tag{6}$$

is isospin breaking even though its origin is an isospin-conserving part of the nucleon-nucleon interaction. Therefore, even in the absence of real isospin-breaking parts in H_0, one should expect some small differences in the wave functions of the core protons and neutrons. By "core" we mean the Z-neutrons occupying the orbits that the protons occupy. For $N = Z$ nuclei this isospin breaking is zero for a nucleus described by H_0. When the excess of neutrons is small, this holds to a good approximation.

We will now use the notion of the isovector giant monopole (IVGM) [2,3] in order to calculate the mixing coefficient ε_n in Eq. (2). The z component of the isovector monopole operator is

$$\hat{M}_0^{(1)} = \sum_i r_i^2 t_z(i). \tag{7}$$

For use with off-diagonal matrix elements we can write

$$V_{\mathrm{C}}^{(1)} = \frac{Ze^2}{2R^3}\hat{M}_0^{(1)}. \tag{8}$$

An "ideal" IVGM is

$$|M_0^{(1)}\rangle = \frac{\sum_i r_i^2 t_z(i)|0\rangle}{\Omega} \equiv \frac{\hat{M}_0^{(1)}|0\rangle}{\Omega}, \tag{9}$$

where Ω is a normalization constant.

The state above exhausts the entire isovector monopole strength $r^2 t_z$. Clearly, this state is not an eigenstate of the system. The strength is spread over several MeV; however, it is still concentrated in a relatively narrow energy region. In our approach the above state is treated as a doorway. For more discussion see Refs. [2,4].

Let us now use the energy-weighted sum rule (EWSR):

$$\frac{1}{2}\langle 0|[Q,[H,Q]]|0\rangle = \sum_n (E_n - E_0)\langle n|Q|0\rangle^2. \tag{10}$$

Applying this to the $\hat{M}_0^{(1)}$ operator we get

$$\frac{\hbar^2}{2m}A\langle r\rangle^2(1+\kappa) = \sum_n (E_n - E_0)\left|\langle n|\sum_i r_i^2 t_z(i)|0\rangle\right|^2, \tag{11}$$

which, for $V_{\mathrm{C}}^{(1)}$, becomes

$$\left(\frac{Ze^2}{2R^3}\right)^2 \frac{\hbar^2}{2m}A\langle r^2\rangle(1+\kappa) = \sum_n (E_n - E_0)\left|\langle 0|V_{\mathrm{C}}^{(1)}(r_i)|n\rangle\right|^2. \tag{12}$$

In the above equations, κ is the exchange correction (see [2,4,5]).

We now assume that the sum can be exhausted by the single state $|M_0^{(1)}\rangle$. Replacing the sum with a single term we can write

$$\left(\frac{Ze^2}{2R^3}\right)^2 \frac{\hbar^2}{2m}A\langle r^2\rangle(1+\kappa) = (E_M - E_0)\left|\langle 0|V_{\mathrm{C}}^{(1)}|M_0^{(1)}\rangle\right|^2. \tag{13}$$

Let us now calculate the expectation value of $\hat{M}_0^{(1)}$ with the wave function $|\tilde{0}\rangle$ to first order in ε_n, using Eq. (2):

$$\begin{aligned}\langle\tilde{0}|\sum_i r_i^2 t_z(i)|\tilde{0}\rangle &\equiv \frac{1}{2}\left[N\langle\tilde{r}_n^2\rangle - Z\langle\tilde{r}_p^2\rangle\right] \\ &= \langle 0|\sum_i r_i^2 t_z(i)|0\rangle + 2\sum_{n\neq 0}\varepsilon_n\langle n|\sum_i r_i^2 t_z(i)|0\rangle,\end{aligned} \tag{14}$$

where $\langle\tilde{r}_n^2\rangle$ and $\langle\tilde{r}_p^2\rangle$ are, respectively, the neutron and proton mean-square radii evaluated with the $|\tilde{0}\rangle$ wave function.

The right-hand side (rhs) of the above equation can be written as

$$\mathrm{rhs} = \frac{1}{2}\left[N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right] + \frac{Ze^2}{R^3}\sum_{n\neq 0}\frac{\left|\langle n|\hat{M}_0^{(1)}|0\rangle\right|^2}{E_0 - E_n}, \tag{15}$$

where $\langle r_n^2\rangle$ and $\langle r_p^2\rangle$ are the mean-square radii of neutrons and protons evaluated with $|0\rangle$.

We now will deal with the second term in the above equation. Using the EWSR and the doorway hypothesis we can write

$$\frac{Ze^2}{R^3}\sum_{n\neq 0}\frac{\left|\langle n|\hat{M}_0^{(1)}|0\rangle\right|^2}{E_0 - E_n} = \frac{Ze^2}{R^3}\frac{\hbar^2}{2m}A\frac{\langle r^2\rangle(1+\kappa)}{(E_0 - E_M)^2}. \tag{16}$$

We now take $R = 1.2A^{1/3}$ fm , for a homogenous charge distribution $\langle r^2\rangle = \frac{3}{5}R$, use $\kappa \approx 0.3$ as found in a number of calculations [4,5], and take $E_M - E_0 \approx 140A^{-1/3}$ MeV [5,6]. With these reasonable choices and for nuclei with $N \approx Z$ we find that the rhs of Eq. (16) simplifies to

$$\mathrm{rhs} = 2\times 10^{-3}Z^{7/3}\ \mathrm{fm}^2. \tag{17}$$

Therefore, we can write

$$\tfrac{1}{2}\left[N\left(\langle\tilde{r}_n^2\rangle - \langle r_n^2\rangle\right) - Z\left(\langle\tilde{r}_p^2\rangle - \langle r_p^2\rangle\right)\right] = -2\times 10^{-3}Z^{7/3}\ \mathrm{fm}^2, \tag{18}$$

where the radii are expressed in fm.

We now write $N\langle\tilde{r}_n^2\rangle = (N-Z)\langle\tilde{r}_{\mathrm{exc}}^2\rangle + Z\langle\tilde{r}_{n,c}^2\rangle$ and $N\langle r_n^2\rangle = (N-Z)\langle r_{\mathrm{exc}}^2\rangle + Z\langle r_{n,c}^2\rangle$, where $\langle\tilde{r}_{\mathrm{exc}}^2\rangle$ and $\langle\tilde{r}_{n,c}^2\rangle$ denote the mean-square radii of the excess neutrons and of the core neutrons, respectively (similarly for $\langle r_{\mathrm{exc}}^2\rangle$ and $\langle r_{n,c}^2\rangle$) [4]. We can now write the left-hand side (lhs) of Eq. (18):

$$\begin{aligned}\mathrm{lhs} = \frac{1}{2}Z\left[\langle\tilde{r}_{n,c}^2\rangle - \langle r_p^2\rangle\right] + \frac{N-Z}{2}\left[\langle\tilde{r}_{\mathrm{ex}}^2\rangle - \langle r_{\mathrm{ex}}^2\rangle\right] \\ - \frac{Z}{2}\left[\langle r_{n,c}^2\rangle - \langle r_p^2\rangle\right].\end{aligned} \tag{19}$$

We now drop the last term in this equation because, in the absence of Coulomb mixing and for nuclei with no (or small) neutron excess, the two mean-square radii are equal.

Denoting differences in the root mean square radii (rms) as

$$\delta r_{np} = \langle\tilde{r}_{n,c}^2\rangle^{1/2} - \langle\tilde{r}_p^2\rangle^{1/2} \quad \text{and} \quad \delta r_{\mathrm{ex}} = \langle\tilde{r}_{\mathrm{ex}}^2\rangle^{1/2} - \langle r_{\mathrm{ex}}^2\rangle^{1/2}, \tag{20}$$

we find

$$\delta r_{np} + \frac{(N-Z)}{Z}\frac{r_{\mathrm{exc}}}{r_p}\delta r_{\mathrm{exc}} = -1.6\times 10^{-3}Z\ \mathrm{fm}. \tag{21}$$

For $N = Z$ nuclei, or for nuclei with a small neutron excess, this formula reduces to

$$\delta r_{np} = -1.6\times 10^{-3}Z\ \mathrm{fm}. \tag{22}$$

The difference between the rms radii of neutrons and protons in such nuclei is negative—as one should expect, the Coulomb force pushes the protons away. The dependence on the charge of the nucleus is simple and the difference increases with Z.

For O^{16}, the neutron rms radius is smaller than that of the proton one by 0.01 fm; in Ca^{40}, this difference is -0.03 fm; in Ni^{56}, it is -0.04 fm; and in Sn^{100}, it becomes -0.08 fm. A Skyrme Hartree-Fock calculation for Ca^{40} [4] results in $\delta r_{np} = -0.04$ fm. In nuclei with a large neutron excess, one can use a realistic Skyrme HF to compute the rms for the Z core neutrons (that is, to exclude the excess neutrons) and Z protons and then find $\delta r_{np} = \langle\tilde{r}_{n,c}^2\rangle^{1/2} - \langle\tilde{r}_p^2\rangle^{1/2}$. The results are shown in Table I.

TABLE I. The values of $\delta r_{np} = \langle \tilde{r}^2_{n,c} \rangle^{1/2} - \langle \tilde{r}^2_p \rangle^{1/2}$ in fm.

Nucleus	Eq. (22)	HF
Ca^{40}	−0.03	−0.04
Sr^{88}	−0.06	−0.10
Ce^{140}	−0.09	−0.11
Pb^{208}	−0.13	−0.14

The results for nuclei with the large neutron excess are indicative of the effect Coulomb repulsion has on the difference between neutron and proton radii in the core. The neutron radii for the core nucleons are smaller than the proton radii. Of course, the excess neutrons occupy higher orbits with larger and larger radii, and the total neutron rms radius is larger that the proton rms radius. However, it is clear that the Coulomb repulsion mitigates the difference in the neutron and proton radii. Equation (22) is valid to a very good approximation for $N = Z$ nuclei, and to a good approximation for nuclei with a very small ratio $(N - Z)/Z$.

The proton skin in nuclei like Sn^{100} is significant; the proton radius is about 1.5% larger then the neutron radius.

Comparison to experiment is difficult. As already mentioned, there is little reliable data for neutron radii, especially for nuclei with a small neutron excess. In an experiment performed with antiprotonic atoms [7], δr_{np} is negative for Ca^{40}, Ni^{58}, and Ni^{60}. However, the error bars in this measurements are too large to make this comparison definite.

Even a small proton "skin" in $N = Z$ nuclei can make a difference in certain processes that occur at the surface. For example, the interaction of medium energy (∼180 MeV) pions with nuclei is an example of this. An article published 28 years ago [8] discussed the results of the observation of the two isospin components $\Delta T_z = \pm 1$ of the isovector dipole in Ca^{40} in the charge exchange reactions $(\pi^{\pm}, \pi^0)$ [9]. The energies of the pions were about 164 MeV, which is in the strongly absorbing regime. The cross section for the (π^-, π^0) reaction was 70% larger than in the (π^+, π^0) process. The transition strength $S_{\pm}$ of the two components of the isovector dipole satisfies the relationship [5,8]

$$(S_- - S_+) \sim \left(N\left\langle r_n^2\right\rangle - Z\left\langle r_p^2\right\rangle\right). \tag{23}$$

As we have seen in Ca^{40}, the proton radius is larger than the neutron radius by 0.03–0.04 fm. It was shown in [8] that this is enough to make a big difference in the pion cross-sections because of the surface nature of the two charge-exchange reactions.

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