Fine structure of α decay to rotational states of heavy nuclei

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To gain a better insight into α-decay fine structure, we calculate the relative intensities of α decay to 2^+ and 4^+ rotational states in the framework of the generalized liquid drop model (GLDM) and improved Royer's formula. The calculated relative intensities of α decay to 2^+ states are in good agreement with the experimental data. For the relative intensities of α decay to 4^+ states, a good agreement with experimental data is achieved for Th and U isotopes. The formula we obtain is useful for the analysis of experimental data of α-decay fine structure. In addition, some predicted relative intensities which are still not measured are provided for future experiments.

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α decay is one of the most important decay modes for unstable nuclei and has become a powerful tool to investigate the nuclear structure and identify new isotopes or new elements. The α radioactivity has been explained successfully by Gamow [1] and by Condon and Gurney [2] as a quantum tunneling effect. On the basis of Gamow's theory, the experimental α-decay half-lives of nuclei can be explained by both phenomenological and microscopic models [3–9]. In recent years, there has been an increased interest in the α decay to excited states of daughter nuclei from both experimental and theoretical sides [10,11]. Such α transitions belong to the unfavored cases, which are strongly hindered compared to the ground state ones. It is important to study such transitions experimentally to build the energy-level schemes of daughter nuclei [12]. Theoretically, the hindered α transition is an effective tool to study the properties of α emitters because it is closely related to the internal structure of nuclei [13–25]. However, it is difficult to describe quantitatively the unfavored α transitions because of the influence of both the nonzero angular momenta and the excitations of nucleons. It is well known that the GLDM is one of the most successful models for describing the processes of fusion, fission, cluster emission, proton emission, and α decay. The proximity energy term has been introduced in the GLDM including an accurate radius and the mass asymmetry can reproduce the reasonable potential barrier heights and positions [6]. In our previous work, we investigated the branching ratios of α decay to the excited states of daughter nuclei in the framework of the GLDM by taking into account the angular momenta of the α particles and the excitation probabilities of the daughter nuclei. Our theoretical results reproduced the experimental data successfully [23,24]. Additionally, the Royer's formula can well reproduce the experimental half-lives of the favored α decay [6,7]. We improved the Royer's formula by taking into account the role of the angular momentum of the α particle so that it can be used to study the half-lives and branching ratios of unfavored α decay [24,26]. Although we can obtain some knowledge about the α-decay fine structure by analyzing the branching ratios of α decay to excited states of daughter nuclei, it is not enough. Recently, Peltonen *et al.* performed systematic calculations for the relative intensities of α decay to 2^+ and 4^+ rotational states of daughter nuclei by using the stationary coupled channels approach. A good agreement between theoretical results and experimental data was obtained [21]. Thus it is interesting to extend the GLDM and improved Royer's formula to investigate the relative intensities of α decay to rotational states. This is our motivation for this study. In this Brief Report, we compute the relative intensities of α decay to 2^+ and 4^+ rotational states in the framework of the GLDM and improved Royer's formula in order to gain a better insight into the α-decay fine structure.

The GLDM has been successfully used to calculate the half-lives of the favored and unfavored α decay [6,23,24,26]. The most attractive feature of the GLDM is that it can describe the process of the shape evolution from one body to two separated fragments in a unified way. For the unfavored α decay, the centrifugal potential energy can no longer be neglected. The macroscopic energy is determined within the GLDM, including the volume, surface, Coulomb, proximity, and centrifugal potential energy

$$E = E_V + E_S + E_C + E_{\rm Prox} + E_{\rm cen}(r). \tag{1}$$

The centrifugal potential energy $E_{\rm cen}(r)$ is adopted by the following form:

$$E_{\rm cen}(r) = \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}, \tag{2}$$

where r and μ are the distance between the two fragments and the reduced mass of the α-daughter system, respectively.

The energy released from the ground state of a parent nucleus into the ith excited state of the daughter nucleus with excitation energy E_i^* is

$$Q_{0\longrightarrow i} = Q_{\rm g.s.\longrightarrow g.s.} - E_i^*. \tag{3}$$

The penetration probability $P(Q_i, l)$ of a parent nucleus decaying via α emission is calculated using the WKB

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approximation, which is written by the following formula:

$$P(Q_i, l) = \exp\left[-\frac{2}{\hbar}\int_{R_{\rm in}}^{R_{\rm out}} \sqrt{2B(r)(E(r)-E_{\rm sph})}\,dr\right]. \quad (4)$$

The deformation energy (relative to the sphere) is small up to the rupture point between the fragments. $R_{\rm in}$ and $R_{\rm out}$ are the two turning points of the WKB action integral.

Usually the branching ratio of α decay from the ground state of the parent nucleus into the level i of the daughter nucleus is determined as

$$b_i\% = \frac{\Gamma(Q_i, l_i)}{\sum_m \Gamma(Q_m, l_m)} \times 100\% = \frac{P(Q_i, l_i)}{\sum_m P(Q_m, l_m)} \times 100\%, \quad (5)$$

where the sum m is going over all states, which can be populated during the α transition from the ground state of the parent nucleus.

To gain a better insight into fine structure in α decay, Delion *et al.* characterized a quantity describing the fine structure, which is written as [20]

$$I_i = \log_{10}\frac{\Gamma_0}{\Gamma_i} = \log_{10}\frac{T_i}{T_0} = \log_{10}\frac{P_0}{P_i}, \quad (6)$$

which represents the relative intensity of different channels with respect to the favored channel and is only related to the penetration probabilities.

It is well known that Royer's formula can well reproduce the experimental half-lives of α decay in the favored cases. For the unfavored α decay, the effect of centrifugal barrier needs to be considered. In Ref. [26], we obtained the improved Royer's formula by taking into account the contribution of centrifugal barrier, which can be written as

$$\log_{10} T_{1/2}(Q_i, l) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_i}} + \frac{1.0l(l+1)}{\sqrt{(A-4)(Z-2)A^{-2/3}}}. \quad (7)$$

By combining Eqs. (7) and (8), we can extract the α-decay relative intensity

$$I_i = \log_{10}\frac{\Gamma_0}{\Gamma_i} = \log_{10}\frac{T_0}{T_i} = \frac{cZ(\sqrt{Q_\alpha}-\sqrt{Q_i})}{\sqrt{Q_\alpha Q_i}} + \frac{1.0l(l+1)}{\sqrt{(A-4)(Z-2)A^{-2/3}}}. \quad (8)$$

In this case, the α-decay relative intensity can be evaluated by the analytical formula including just one fitting coefficient c. In our calculation, we use the new coefficients derived by Schubert *et al.* very recently. The new values of the coefficients for e-e nuclei are ($a = -25.2505$, $b = -1.191$, $c = 1.5526$) [7].

We have performed systematic calculations on relative intensities of α decay to 2^+ and 4^+ rotational states for 52 even-even heavy nuclei. The experimental values, together with the calculated results are given in Table I. The first column of Table I denotes the parent nuclei. The α-decay energies Q_α between the ground states of parent and daughter nuclei and the excitation energies of the daughter nuclei in 2^+ and 4^+ states are listed in columns 2, 3, and 4, respectively. In columns 6, 7, and 8, the experimental and calculated relative intensities of α decay to 2^+ states are presented. In the last three columns, the experimental and calculated relative intensities of α decay to 4^+ states are shown. The experimental data, including the Q_α, excitation energies, and relative intensities, are taken from Ref. [21]. Note that the excitation energies which have not been measured are derived from a simple rigid rotor model. We marked them with symbol "a". From Table I, one can see that our calculated relative intensities of α decay to 2^+ states are in good agreement with the experimental data. Concerning I_4, a good agreement is achieved only for Th and U isotopes. For other isotope chains, the calculated values deviate from the experimental data. The largest difference between experiment and theory is about 2.40. A similar phenomenon also exists in some previous works. For example, the branching ratios of α decay to 4^+ excited states cannot be reproduced well [23,25,29]. In addition, the theoretical predicted I_2 and I_4 are made by our method for the unavailable experimental data. These predictions could be useful for future experiments.

In fact, the deformation effects have some influence on α decay half-lives. Recent study suggests that the calculated half-lives decrease by a factor of 2–3 compared to the spherical calculations [27]. However, for the relative intensity, which is defined as the ratio of the decay widths between the ground and excited states. It is observed experimentally that the deformation parameter in the ground state is the same as that in the excited state approximately [28]. Thus the influence of deformation effects can be canceled in our calculations. In the previous works of α-decay fine structure [16,29], the authors also made a similar assumption: the nuclear potential vanishes outside the radius of the daughter nucleus by using a square well potential and the quantities such as relative intensities, branching ratios describing the α-decay fine structure are not very sensitive to the potential form. In other words, these quantities are not very sensitive to the nuclear deformations. This is different from the studies of α-decay half-lives.

To show the agreement degree more clearly between different theoretical models and experimental data, the relative intensities I_2 and I_4 from experimental data and different theoretical results, including predictions for some light nuclei, versus the number "n" are plotted in Fig. 1. Note that the input parameters of our method are the same as those of the stationary coupled channels approach. From Fig. 1, we can see that our predictions of light nuclei have slight differences between our method and the stationary coupled channels approach for I_2 and I_4. For I_2, the deviation degree becomes larger and larger with the increase of charge number Z for each model. But our calculated results are closer to the experimental data than those with the stationary coupled channels approach. This indicates that our method works better than the model of Peltonen *et al.* For I_4, it is found that the obtained agreement numbers of our method are less than those of the stationary coupled channels approach, which means that the stationary coupled channels approach works better than our method. In a word, the three approaches are all not enough to reproduce the absolute experimental data of α-decay fine structure. Therefore it is important to develop the theoretical models of α decay by taking into account more reasonable physical factors to further

TABLE I. Comparison between the calculated and experimental relative intensities of α decay to 2^+ and 4^+ for even-even heavy nuclei. The symbol "a" represents the energies not measured, which are calculated within a simple rotor approximation.

n	Nuclei	Q_α (MeV)	E_2^* (KeV)	E_4^* (KeV)	$I_2^{\text{Expt.}}$	I_2^{GLDM}	I_2^{Formula}	$I_4^{\text{Expt.}}$	I_4^{GLDM}	I_4^{Formula}
1	$^{172}_{76}$Os	5.254	199.300	562.300	–	1.288	1.285	–	3.934	3.937
2	$^{174}_{76}$Os	4.900	156.720	462.300	–	1.157	1.155	–	3.649	3.651
3	$^{186}_{76}$Os	2.846	100.106	329.427	–	1.590	1.534	–	5.535	5.335
4	$^{180}_{78}$Pt	5.285	135.100	395.500	–	0.955	0.966	–	2.978	3.025
5	$^{182}_{78}$Pt	4.977	131.600	397.700	–	1.007	1.010	–	3.223	3.238
6	$^{184}_{78}$Pt	4.618	132.110	408.620	–	1.105	1.100	–	3.611	3.593
7	$^{186}_{78}$Pt	4.352	127.000	400.400	–	1.151	1.141	–	3.820	3.785
8	$^{188}_{78}$Pt	4.033	119.800	383.770	–	1.208	1.189	–	4.071	4.001
9	$^{190}_{78}$Pt	3.272	137.159	434.087	–	1.776	1.712	–	6.039	5.810
10	$^{186}_{80}$Hg	5.236	154.900	419.080	–	1.092	1.093	–	3.202	3.226
11	$^{188}_{80}$Hg	4.740	162.970	435.960	–	1.292	1.274	–	3.752	3.722
12	$^{228}_{90}$Th	5.520	84.373	250.783	0.424	0.693	0.712	2.503	2.181	2.242
13	$^{230}_{90}$Th	4.770	67.670	211.540	0.513	0.703	0.711	2.803	2.276	2.308
14	$^{232}_{90}$Th	4.083	63.823	204.680	0.557	0.801	0.796	3.054	2.646	2.634
15	$^{230}_{92}$U	5.993	72.200	226.430	0.324	0.580	0.605	2.249	1.886	1.970
16	$^{232}_{92}$U	5.414	57.762	186.828	0.334	0.562	0.581	2.356	1.854	1.924
17	$^{234}_{92}$U	4.859	53.200	174.100	0.400	0.592	0.608	2.553	1.977	2.027
18	$^{236}_{92}$U	4.573	49.460	162.250	0.455	0.597	0.614	2.692	2.002	2.050
19	$^{238}_{92}$U	4.270	49.550	163.000	0.577	0.649	0.653	3.006	2.161	2.186
20	$^{232}_{94}$Pu	6.716	59.000	196.666[a]	0.308	0.466	0.499	–	1.567	1.676
21	$^{234}_{94}$Pu	6.310	51.720	169.500	0.327	0.460	0.489	2.230	1.524	1.628
22	$^{236}_{94}$Pu	5.867	47.580	156.540	0.351	0.468	0.495	2.478	1.556	1.650
23	$^{238}_{94}$Pu	5.593	43.498	143.352	0.389	0.461	0.490	2.830	1.543	1.636
24	$^{240}_{94}$Pu	5.256	45.244	149.478	0.429	0.500	0.523	2.938	1.674	1.751
25	$^{242}_{94}$Pu	4.984	44.915	148.390	0.513	0.526	0.543	3.396	1.754	1.819
26	$^{244}_{94}$Pu	4.666	45.000	151.000	0.619	0.559	0.574	–	1.891	1.940
27	$^{238}_{96}$Cm	6.620	46.000	153.333[a]	0.358	0.415	0.449	–	1.389	1.504
28	$^{240}_{96}$Cm	6.398	44.630	147.450	0.391	0.423	0.453	3.136	1.401	1.512
29	$^{242}_{96}$Cm	6.216	44.080	146.000	0.456	0.428	0.459	3.326	1.432	1.534
30	$^{244}_{96}$Cm	5.902	42.824	141.690	0.510	0.442	0.469	3.541	1.473	1.568
31	$^{246}_{96}$Cm	5.475	44.540	147.300[a]	0.664	0.484	0.505	–	1.607	1.689
32	$^{248}_{96}$Cm	5.162	44.200	155.000	0.657	0.505	0.526	3.032	1.756	1.818
33	$^{250}_{96}$Cm	5.208	46.000	155.000	–	0.513	0.533	–	1.736	1.803
34	$^{240}_{98}$Cm	7.719	45.000	150.000[a]	0.327	0.364	0.405	–	1.218	1.355
35	$^{242}_{98}$Cm	7.516	35.000	116.666[a]	0.602	0.334	0.375	–	1.116	1.252
36	$^{244}_{98}$Cm	7.329	38.000	126.666[a]	0.477	0.350	0.390	–	1.172	1.306
37	$^{246}_{98}$Cm	6.862	42.130	137.000	0.585	0.386	0.422	2.723	1.279	1.399
38	$^{248}_{98}$Cm	6.361	42.965	142.348	0.611	0.416	0.447	2.301	1.389	1.494
39	$^{250}_{98}$Cm	6.128	42.852	142.010	0.752	0.429	0.458	2.451	1.430	1.531
40	$^{252}_{98}$Cm	6.217	43.400	143.600	0.729	0.423	0.456	2.545	1.423	1.522
41	$^{254}_{98}$Cm	5.926	43.000	143.333[a]	0.689	0.442	0.469	–	1.479	1.571
42	$^{246}_{100}$Fm	8.374	45.000	150.000[a]	0.602	0.341	0.386	–	1.142	1.291
43	$^{248}_{100}$Fm	8.002	41.000	136.666[a]	0.602	0.340	0.382	–	1.138	1.278
44	$^{250}_{100}$Fm	7.557	44.000	146.666[a]	0.689	0.366	0.405	–	1.225	1.357
45	$^{252}_{100}$Fm	7.153	41.530	137.810	0.748	0.374	0.409	1.938	1.244	1.367
46	$^{254}_{100}$Fm	7.307	42.721	141.875	0.777	0.371	0.408	2.016	1.238	1.364

TABLE I. *(Continued.)*

n	Nuclei	Q_α (MeV)	E_2^* (KeV)	E_4^* (KeV)	$I_2^{\text{Expt.}}$	I_2^{GLDM}	I_2^{Formula}	$I_4^{\text{Expt.}}$	I_4^{GLDM}	I_4^{Formula}
47	$^{256}_{100}$Fm	7.027	45.720	151.740	0.753	0.397	0.430	–	1.324	1.438
48	$^{252}_{102}$No	8.549	44.000	146.666[a]	0.477	0.334	0.378	–	1.117	1.264
49	$^{254}_{102}$No	8.226	44.000	145.000[a]	–	0.343	0.386	–	1.142	1.285
50	$^{256}_{102}$No	8.581	46.600	155.333[a]	0.826	0.342	0.385	–	1.139	1.286
51	$^{256}_{104}$Rf	8.995	46.400	153.800	–	0.329	0.374	–	1.100	1.249
52	$^{260}_{106}$Sg	9.923	51.000	170.000[a]	0.689	0.318	0.367	–	1.065	1.227

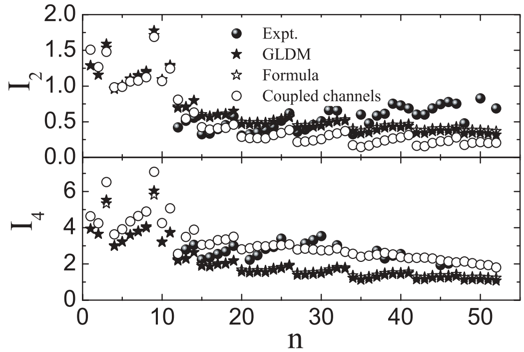

FIG. 1. Comparison between experimental data and different models about the relative intensities I_2 and I_4.

improve the agreement between theory and experiment and make more precise predictions for future experiments.

In summary, the GLDM and improved Royer's formula have been used to investigate the relative intensities of α decay to 2^+ and 4^+ rotational states of heavy nuclei. The calculated results of I_2 are in good agreement with the experimental data. For I_4 our calculated results are lower than the experimental data and our method does not work as well as the stationary coupled channels approach. However, we can obtain a good agreement with experimental data for Th and U isotopes and an agreement for other isotope chains qualitatively. The formula we obtained in this work is useful for the analysis of experimental data of α-decay fine structure. In addition, some predicted values of I_2 and I_4 for the cases of the experimental values are unavailable. These theoretical predictions are useful for future experiments.

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[1] G. Gamow, Z. Phys. **51**, 204 (1928).
[2] E. U. Condon and R. W. Gurney, Nature (London) **122**, 439 (1928).
[3] P. E. Hodgson and E. Béták, Phys. Rep. **374**, 1 (2003).
[4] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. C **45**, 2247 (1992).
[5] J. Wauters, P. Dendooven, M. Huyse *et al.*, Phys. Rev. C **47**, 1447 (1993).
[6] G. Royer, J. Phys. G: Nucl. Part. Phys. **26**, 1149 (2000).
[7] N. D. Schubert, M. A. Reyes, and V. A. Tamez, Eur. Phys. J. A **42**, 121 (2009).
[8] Y. K. Gambhir, A. Bhagwat, and M. Gupta, Phys. Rev. C **71**, 037301 (2005).
[9] P. R. Chowdhury, C. Samanta, and D. N. Basu, Phys. Rev. C **77**, 044603 (2008).
[10] P. Möller *et al.*, At. Data Nucl. Data Tables **66**, 131 (1997).
[11] R. B. Firestone, V. S. Shirley, C. M. Baglin *et al.*, *Table of Isotopes*, 8th ed. (Wiley Interscience, New York, 1996).
[12] J. Gasparro, G. Ardisson, V. Barci, and R. K. Sheline, Phys. Rev. C **62**, 064305 (2000).
[13] J. Wauters, N. Bijnens, P. Dendooven *et al.*, Phys. Rev. C **50**, 2768 (1994).
[14] A. N. Andreyev, M. Huyse, P. Van Duppen *et al.*, Nature (London) **405**, 430 (2000).
[15] J. D. Richards *et al.*, Phys. Rev. C **54**, 2041 (1996).
[16] A. Sobiczewski, I. Muntian, and Z. Patyk, Phys. Rev. C **63**, 034306 (2001).
[17] I. Muntian, Z. Patyk, and A. Sobiczewski, Phys. Lett. B **500**, 241 (2001).
[18] D. Karlgren, R. J. Liotta, R. Wyss *et al.*, Phys. Rev. C **73**, 064304 (2006).
[19] M. Asai *et al.*, Phys. Rev. C **73**, 067301 (2006).
[20] D. S. Delion, S. Peltonen, and J. Suhonen, Phys. Rev. C **73**, 014315 (2006).
[21] S. Peltonen, D. S. Delion, and J. Suhonen, Phys. Rev. C **78**, 034608 (2008).
[22] C. Xu and Z. Ren, Phys. Rev. C **75**, 044301 (2007).
[23] Y. Z. Wang, H. F. Zhang, J. M. Dong, and G. Royer, Phys. Rev. C **79**, 014316 (2009).
[24] Y. Z. Wang *et al.*, Eur. Phys. J. A **44**, 287 (2010).
[25] V. Yu. Denisov and A. A. Khudenko, Phys. Rev. C **80**, 034603 (2009).
[26] J. M. Dong *et al.*, Nucl. Phys. A **832**, 198 (2010).
[27] M. Ismail, A. Y. Ellithi, M. M. Botros, and A. Adel, Phys. Rev. C **81**, 024602 (2010).
[28] T. Grahn *et al.*, Phys. Rev. C **80**, 014323 (2009).
[29] C. Xu and Z. Ren, Nucl. Phys. A **778**, 1 (2006).