Heavy quark production in deep inelastic electron-nucleus scattering

V. P. Gonçalves,[1] M. S. Kugeratski,[2] and F. S. Navarra[3]

[1]*Instituto de Física e Matemática, Universidade Federal de Pelotas Caixa Postal 354, CEP 96010-090, Pelotas, RS, Brazil*
[2]*Universidade Federal de Santa Catarina, Campus Universitário de Curitibanos, CEP 89520-000, Curitibanos, SC, Brazil*
[3]*Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil*

Heavy quark production has been very well studied over the last years both theoretically and experimentally. Theory has been used to study heavy quark production in *ep* collisions at HERA, in *pp* collisions at Tevatron and RHIC, in *pA* and *dA* collisions at RHIC, and in *AA* collisions at CERN-SPS and RHIC. However, to the best of our knowledge, heavy quark production in *eA* has received almost no attention. With the possible construction of a high energy electron-ion collider, updated estimates of heavy quark production are needed. We address the subject from the perspective of saturation physics and compute the heavy quark production cross section with the dipole model. We isolate shadowing and nonlinear effects, showing their impact on the charm structure function and on the transverse momentum spectrum.

I. INTRODUCTION

The construction of a high energy electron-ion collider (EIC) was proposed in 2005 [1] (see also Ref. [2]). During the subsequent years, several predictions for the inclusive and diffractive observables were made, especially in the context of saturation physics. One of the advantages of this new machine is that it will be possible to reach values of the saturation scale, Q_s, which are larger than those reached at HERA. A large saturation scale is crucial for the observation of most of the saturation effects. In particular, the collider environment is ideal for studying semi-inclusive and exclusive processes. In previous works [3–7], we made predictions for the inclusive nuclear structure function F_2 and F_L as well for the diffractive observables. In these works we have concluded that the nuclear structure function F_2^A is reduced up to 50% with respect to the case where saturation effects are not taken into account. We made estimates for the ratio between the nuclear diffractive and total cross sections and predicted that about 30% of the events at an EIC will be diffractive. We have also investigated the dependence on the β and $x_{I\!P}$ variables of the nuclear diffractive structure function $x_{I\!P} F_{2,A}^{D(3)}$. We showed that $x_{I\!P} F_{2,A}^{D(3)}$ becomes very flat in β and $x_{I\!P}$ when we increase the atomic number, A, and we found the same behavior for the ratio $R = F_{2,A1}^{D(3)}/F_{2,A2}^{D(3)}$ for two different nuclei. Concerning the exclusive vector meson production off nuclei, we showed that the coherent process (when the nucleus remains intact after the collision) for vector meson production will be much more important than the incoherent one (when the nucleus breaks up after the collision).

In this article we continue our study of quantities that could be measured in an electron-ion collider and calculate the cross section of heavy quark production using the dipole approach and a nuclear saturation model based on the physics of thecolor glass condensate (CGC). (For related studies see Refs. [8,9].) The main input of our calculation is the dipole-nucleus cross section, $\sigma_{dA}(x, \mathbf{r})$, which is determined by the quantum chromodynamics (QCD) at small x. In the

eikonal approximation it is given by

$$\sigma_{dA}(x, \mathbf{r}) = 2 \int d^2 \mathbf{b}\, \mathcal{N}^A(x, \mathbf{r}, \mathbf{b}), \tag{1}$$

where $\mathcal{N}^A(x, \mathbf{r}, \mathbf{b})$ is the forward dipole-target scattering amplitude for a dipole with size $\mathbf{r}$ and impact parameter $\mathbf{b}$ which encodes all the information about the hadronic scattering, and thus about the nonlinear and quantum effects in the hadron wave function (see, e.g., Ref. [10]). It can be obtained by solving the Balitsky-Kovchegov (BK) (JIMWLK) evolution equation in the rapidity $Y \equiv \ln(1/x)$ [11–13]. Many groups have studied the numerical solutions of the BK equation, but several improvements are still necessary before using the solution in the calculation of observables. In particular, one needs to include the next-to-leading order corrections into the evolution equation and perform a global analysis of all small x data. It is a program in progress (for recent results, see Refs. [14,15]). In the meantime it is necessary to use phenomenological models for $\mathcal{N}$ that capture the most essential properties of the solution. Following Ref. [6], we will use in our calculations the model proposed in Ref. [16], which describes the current experimental data on the nuclear structure function as well as includes the impact parameter dependence in the dipole nucleus cross section. In this model the forward dipole-nucleus amplitude is given by

$$\mathcal{N}^A(x, \mathbf{r}, \mathbf{b}) = 1 - \exp\left[-\tfrac{1}{2}\, \sigma_{\mathrm{dp}}(x, \mathbf{r}^2)\, T_A(\mathbf{b})\right], \tag{2}$$

where σ_{dp} is the dipole-proton cross section and $T_A(\mathbf{b})$ is the nuclear profile function, which is obtained from a three-parameter Fermi distribution for the nuclear density normalized to A (for details see, e.g., Ref. [9]). The above equation, based on the Glauber-Gribov formalism [17], sums up all the multiple elastic rescattering diagrams of the $q\bar{q}$ pair and is justified for large coherence length, where the transverse separation r of partons in the multiparton Fock state of the photon becomes a conserved quantity (i.e., the size of the pair r becomes eigenvalue of the scattering matrix). It is important to emphasize that for very small values of

x, other diagrams beyond the multiple Pomeron exchange considered here should contribute (e.g., Pomeron loops) and a more general approach for the high-density (saturation) regime must be considered. However, we believe that the present approach allows us to estimate the magnitude of the high-density effects in the kinematical range of the future eA colliders.

During the last years an intense activity in the area resulted in more sophisticated dipole proton cross sections, which had more theoretical constraints and which were able to give a better description of the more recent HERA and/or RHIC data [18–24]. In what follows we will use the bCGC model proposed in Ref. [18], which improves the IIM model [25] with the inclusion of the impact parameter dependence in the dipole proton cross sections. The parameters of this model were recently fitted to describe the current HERA data [20]. Following Ref. [18] we have that the dipole-proton cross section is given:

$$\sigma_{\rm dp}^{\rm bCGC}(x, \boldsymbol{r}^2) \equiv \int d^2\bar{\boldsymbol{b}} \, \frac{d\sigma_{\rm dp}}{d^2\bar{\boldsymbol{b}}}, \tag{3}$$

where

$$\frac{d\sigma_{\rm dp}}{d^2\bar{\boldsymbol{b}}} = 2\mathcal{N}^p(x, \boldsymbol{r}, \bar{\boldsymbol{b}})$$
$$= 2 \begin{cases} \mathcal{N}_0 \left(\frac{r\,Q_s}{2}\right)^{2\left(\gamma_s + \frac{\ln(2/rQ_s)}{\kappa\lambda Y}\right)} & rQ_s \leqslant 2 \\ 1 - \exp[-a\,\ln^2(b\,r\,Q_s)] & rQ_s > 2, \end{cases} \tag{4}$$

with $Y = \ln(1/x)$ and $\kappa = \chi''(\gamma_s)/\chi'(\gamma_s)$, where χ is the LO BFKL characteristic function. The coefficients a and b are determined uniquely from the condition that $\mathcal{N}^p(x, \boldsymbol{r})$ and its derivative with respect to rQ_s are continuous at $rQ_s = 2$. They are given by

$$a = -\frac{\mathcal{N}_0^2\gamma_s^2}{(1-\mathcal{N}_0)^2\ln(1-\mathcal{N}_0)} \quad \text{and} \quad b = \frac{1}{2}(1-\mathcal{N}_0)^{-\frac{(1-\mathcal{N}_0)}{\mathcal{N}_0\gamma_s}}. \tag{5}$$

In this model, the proton saturation scale Q_s now depends on the impact parameter:

$$Q_s \equiv Q_s(x, \bar{\boldsymbol{b}}) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{\bar{b}^2}{2B_{\rm CGC}}\right)\right]^{\frac{1}{2\gamma_s}}. \tag{6}$$

The parameter $B_{\rm CGC}$ was adjusted to give a good description of the t dependence of exclusive J/ψ photoproduction. Moreover, the factors $\mathcal{N}_0$ and γ_s were taken to be free. In this way a very good description of F_2 data was obtained. The parameter set which is going to be used here is the one presented in the second line of Table II of Ref. [20]: $\gamma_s = 0.46$, $B_{\rm CGC} = 7.5$ GeV^{-2}, $\mathcal{N}_0 = 0.558$, $x_0 = 1.84 \times 10^{-6}$, and $\lambda = 0.119$.

To estimate the magnitude of the saturation effects in heavy quark production it is important to compare the CGC predictions with those associated with linear QCD. As a model for the linear regime we consider the leading logarithmic approximation for the dipole-target cross section [26,27], where σ_{dA} is directly related to the nuclear gluon distribution xg_A as follows:

$$\sigma_{dA}(x, \boldsymbol{r}^2) = \frac{\pi^2}{3}\boldsymbol{r}^2\alpha_s xg_A(x, 10/\boldsymbol{r}^2). \tag{7}$$

The use of this cross section in the formulas given below will produce results that we denote color transparency (CT). In this limit we are disregarding multiple scatterings of the dipole with the nuclei and are assuming that the dipole interacts incoherently with the target. In what follows we consider two different models for the nuclear gluon distribution. In the first one we disregard the nuclear effects and assume that $xg_A(x, Q^2) = A.xg_N(x, Q^2)$, with xg_N being the gluon distribution in the proton and given by the GRV98 parametrization [28]. We will refer to this model as CT. In the second model we take into account the nuclear effects in the nuclear gluon distribution as described by the EKS98 parametrization [29]. In this case $xg_A(x, Q^2) = A.R_g(x, Q^2).xg_N(x, Q^2)$ with R_g given in Ref. [29]. We will call this model CT + Shad. In our calculations the charm quark mass is $m_c = 1.5$ GeV and the bottom quark mass is $m_b = 4.5$ GeV.

II. HEAVY QUARK PRODUCTION

The electron-proton (ep) collider at HERA has opened up a new kinematic regime in the study of the deep structure of the proton and, in general, of hadronic interactions, which is characterized by small values of the Bjorken variable $x = Q^2/s$, where Q^2 is the momentum transfer and $\sqrt{s}$ is the center-of-mass energy. In this regime we expect that the usual collinear approach [30] be replaced by a more general factorization scheme, as for example the $k_\perp$-factorization approach [31–33] or the quasi-multi-Regge-kinematics (QMRK) framework [34]. (For related studies see Refs. [35–37].) Let us present a brief review of these distinct approaches.

In the collinear factorization approach [30] all partons involved are assumed to be on mass shell, carrying only longitudinal momenta, and their transverse momenta are neglected in the QCD matrix elements. Moreover, the cross sections for the QCD subprocess are usually calculated in the leading order (LO), as well as in the next-to-leading order (NLO). In particular, the cross sections involving incoming hadrons are given, at all orders, by the convolution of intrinsically nonperturbative (but universal) quantities—the parton densities—with perturbatively calculable hard matrix elements, which are process dependent. The conventional gluon distribution $g(x, \mu^2)$, which drives the behavior of the observables at high energies, corresponds to the density of gluons in the proton having a longitudinal momentum fraction x at the factorization scale μ. This distribution satisfies the DGLAP evolution in μ^2 and does not contain information about the transverse momenta $k_\perp$ of the gluon. On the other hand, in the large energy (small-x) limit, we have that the characteristic scale μ of the hard subprocess of parton scattering is much less than $\sqrt{s}$, but greater than the $\Lambda_{\rm QCD}$ parameter. In this limit, the effects of the finite transverse momenta of the incoming partons become important, and the factorization must be generalized, implying that the cross sections are

now $k_\perp$ factorized into an off-shell partonic cross section and a $k_\perp$-unintegrated parton density function $\mathcal{F}(x, k_\perp)$, characterizing the $k_\perp$-factorization approach [31–33]. The function $\mathcal{F}$ is obtained as a solution of the evolution equation associated with the dynamics that governs the QCD at high energies. A sizable piece of the NLO and some of the NNLO corrections to the LO contributions on the collinear approach, related to the contribution of nonzero transverse momenta of the incident partons, are already included in the LO contribution within the $k_\perp$-factorization approach. Moreover, the coefficient functions and the splitting functions giving the collinear parton distributions are supplemented by all-order $\alpha_s \ln(1/x)$ resummation at high energies [38]. A detailed comparison between the predictions of the collinear and $k_\perp$-factorization approaches for the heavy quark photoproduction was performed in Refs. [9,39], which we indicate for more details of these two approaches.

In the last years, an alternative approach to calculating the heavy quark production at high energies was proposed considering the QMRK framework. It is based on an effective theory implemented with the non-Abelian gauge-invariant action obtained in Ref. [34]. In this approach the initial-state t partons are considered as Reggeons. In contrast to the $k_\perp$-factorization approach, the QMRK approach uses gauge-invariant amplitudes and is based on a factorization hypothesis that is proven in the leading logarithmic approximation. The phenomenological implications of this approach were discussed in detail in Refs. [40–42], which demonstrated that the QMRK approach is a powerful tool for the theoretical description of the high energy processes. In particular, in Ref. [42] the F_2^c and D-meson spectra are successfully described using the QMRK approach.

The heavy quark production can also be calculated using the color dipole approach [26]. This formalism can be obtained from the $k_\perp$-factorization approach after the Fourier transformation from the space of quark transverse momenta into the space of transverse coordinates (see, e.g., [43]). It is important to emphasize that this equivalence is only valid in the leading logarithmic approximation, being violated if the exact gluon kinematics is considered [44]. A detailed discussion of the equivalence or not between the dipole and the QMRK approaches still is an open question (see, however, Ref. [45]). The main advantage to use the color dipole formalism, is that it gives a simple unified picture of inclusive and diffractive processes and the saturation effects can be easily implemented in this approach. It is important to emphasize that phenomenological models based on the color glass condensate (see, e.g., [20]) or the solution of the running coupling BK equation [46–48] describe quite well the current experimental HERA data for inclusive and exclusive observables.

A. Charm structure function

In terms of virtual photon-target cross sections $\sigma_{T,L}$ for the transversely and longitudinally polarized photons, the nuclear F_2 structure function is given by

$$F_2^A(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \sigma_{\text{tot}}(x, Q^2),$$

with [26]

$$\sigma_{\text{tot}} = \sigma_T + \sigma_L$$

and

$$\sigma_{T,L} = \int d^2\boldsymbol{r}\, dz\, |\Psi_{T,L}(\boldsymbol{r}, z, Q^2)|^2\, \sigma_{dA}(x, \boldsymbol{r}), \qquad (8)$$

where $\Psi_{T,L}$ is the light-cone wave function of the virtual photon and σ_{dA} is the dipole nucleus cross section describing the interaction of the $q\bar{q}$ dipole with the nucleus target. In Eq. (8) $\boldsymbol{r}$ is the transverse separation of the $q\bar{q}$ pair and z is the photon momentum fraction carried by the quark (for details see, e.g., Ref. [43]). The charm component of the nuclear structure function $F_2^{c,A}(x, Q^2)$ is obtained directly from Eq. (8) isolating the charm flavor. In Fig. 1 we show $F_2^c(x, Q^2)/A$. As expected, in this kinematical domain it grows with Q^2 and falls with increasing x. What is really remarkable is the difference between the bCGC and the linear CT models, which can reach a factor up to 4! In previous estimates of this observable [49], nonlinear effects were found to be weaker. However, in Ref. [49] the input was different (unintegrated gluon distribution instead of a dipole cross section) and the procedure adopted to estimate the purely linear contribution was to switch off the nonlinear effects in the unintegrated gluon distribution of the proton. In some of our previous works (e.g., in Ref. [7]) we adopted an analogous procedure and tried to make this separation switching off the nonlinear component of the dipole-proton cross section. Although this method could give us some rough idea of the role played by some nonlinear effects, we were missing part of them associated with the fusion of gluons belonging to different nucleons. Therefore, we believe that the mentioned previous estimates have underestimated the importance of nonlinear effects.

B. Heavy quark spectrum

As discussed before, heavy quark production has been very well studied over the last years both theoretically and experimentally. The elementary cross sections have been calculated in perturbative QCD up to next-to-leading order and the parton densities have been extracted with the same degree of precision. Theory has been used to study heavy quark production in ep collisions at HERA, in pp collisions at Tevatron and RHIC, in pA and dA collisions at RHIC, and in AA collisions at CERN-SPS and RHIC. A recent and comprehensive survey of these advances can be found in Ref. [50]. However, to the best of our knowledge, heavy quark production in eA has received almost no attention. This is probably related to the fact that eA data are old and, until recently, there was no prospect of having high energy eA data. With the possible construction of a high energy EIC, updated estimates of heavy quark production are needed. We wish to address the subject from the perspective of saturation physics; thus, the best option to obtain the production cross section, isolating shadowing and non-linear effects, is to use the dipole model. The dipole approach is very natural for the study of exclusive hidden charm and beauty electro- and photoproduction especially in the vector meson channel. As it was shown in Ref. [51] the dipole formalism can be
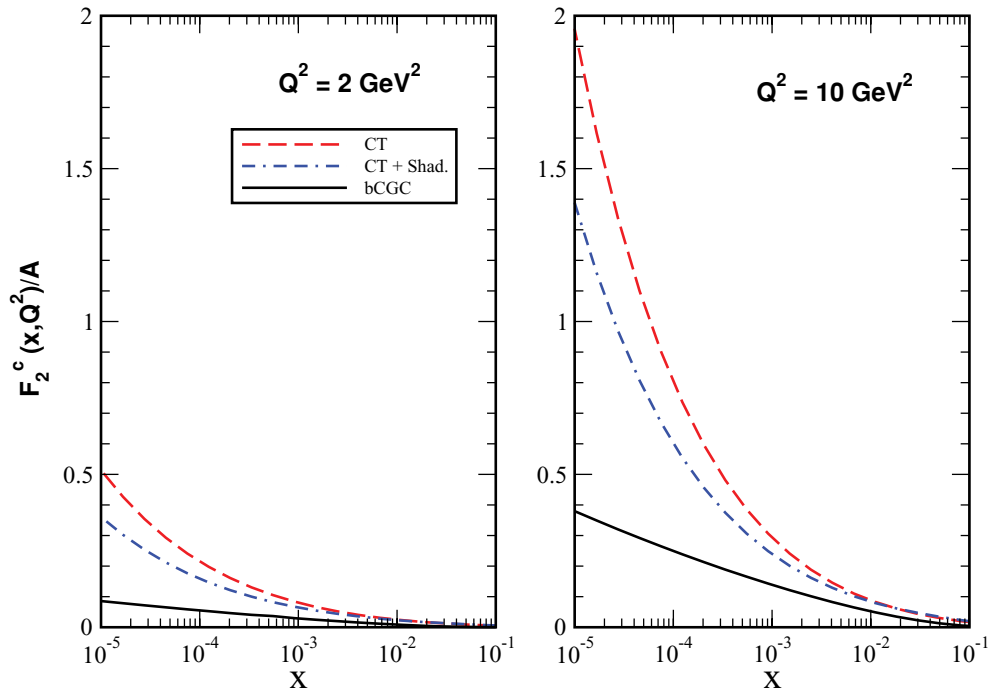
FIG. 1. (Color online) Nuclear charm structure functions for $A = Pb$ and different values of Q^2 considering different models for the dipole-nucleus cross section.

easily extended to open charm and beauty electroproduction obtaining a quite successful description of the HERA data for the F_2^c and D-meson spectra. Here, to calculate the differential heavy quark production cross section, $d\sigma^{T,L}/d^2 p_Q^\perp$, we have

extended the approach of Ref. [51], which was originally developed for ep scattering, to electron-ion collisions with the Glauber-Gribov formalism. In this extension we implicitly assume that the factorization of the cross section verified for
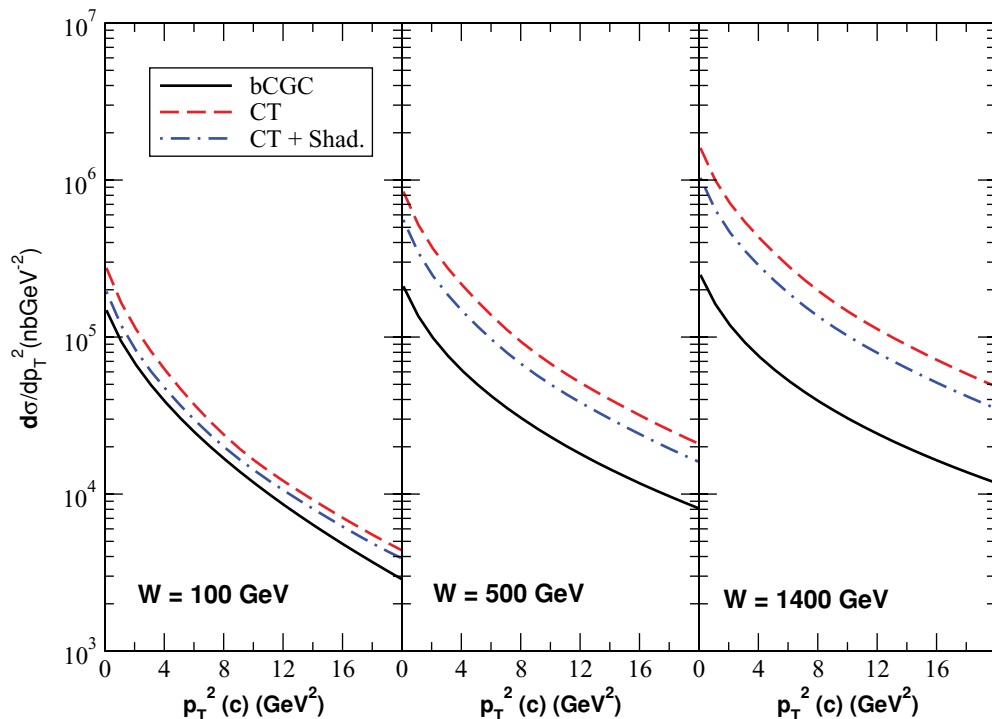

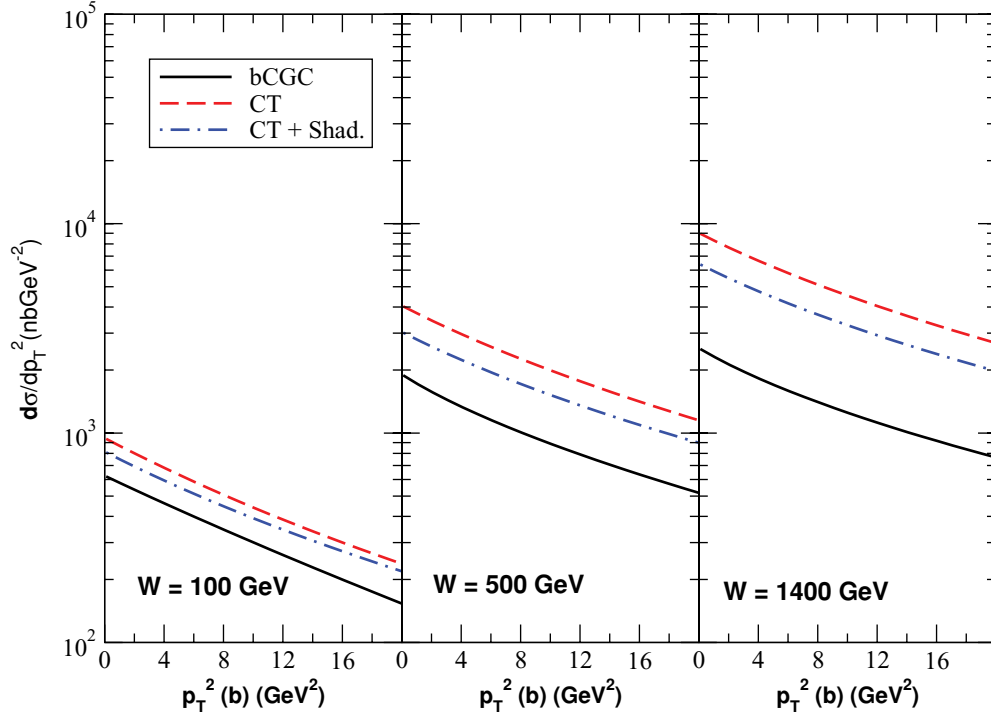FIG. 2. (Color online) Transverse momentum charm spectrum for $Q^2 = 2$ GeV^2 and different center-of-mass energies.

FIG. 3. (Color online) Transverse momentum bottom spectrum for $Q^2 = 2$ GeV^2 and different center-of-mass energies.

ep collisions remains valid in the nuclear case and make use of the dipole-nucleus cross section, which, in turn, depends on the dipole-nucleon cross section. For this last quantity we take the recent parametrization given by Eqs. (3) and (4). The resulting cross section reads

$$\frac{d\sigma(\gamma^* A \to Q X)}{d^2 p_Q^\perp}$$

$$= \frac{6 e_Q^2 \alpha_{em}}{(2\pi)^2} \int d\alpha \left\{ \left[m_Q^2 + 4 Q^2 \alpha^2 (1-\alpha)^2 \right] \right.$$

$$\times \left[\frac{I_1}{p_Q^{\perp 2} + \epsilon^2} - \frac{I_2}{4\epsilon} \right] + [\alpha^2 + (1-\alpha)^2]$$

$$\left. \times \left[\frac{p_Q^\perp \epsilon I_3}{p_Q^{\perp 2} + \epsilon^2} - \frac{I_1}{2} + \frac{\epsilon I_2}{4} \right] \right\}, \quad (9)$$

with

$$I_1 = \int dr\, r\, J_0(p_Q^\perp r)\, K_0(\epsilon r)\, \sigma_{dA}(\boldsymbol{r}),$$

$$I_2 = \int dr\, r^2\, J_0(p_Q^\perp r)\, K_1(\epsilon r)\, \sigma_{dA}(\boldsymbol{r}), \quad (10)$$

$$I_3 = \int dr\, r\, J_1(p_Q^\perp r)\, K_1(\epsilon r)\, \sigma_{dA}(\boldsymbol{r}),$$

where $J_{0,1}$ and $K_{0,1}$ are Bessel functions, $\epsilon = \alpha(1-\alpha)Q^2 + m^2$, and σ_{dA} is given by Eq. (1) or Eq. (7).

To calculate the D-meson production cross section we must let the charm quark fragment. Following [51] we convolute the charm quark production cross section [Eq. (9)] with the

nonperturbative fragmentation function:

$$\frac{d\sigma(\gamma^* A \to D X)}{dz\, d^2 p_D^\perp} = \int \frac{dp_c^\perp d\alpha}{\alpha} \frac{d\sigma(\gamma^* A \to c X)}{d^2 p_c^\perp d\alpha}$$

$$\times D_D^c \left(\frac{z}{\alpha} \right) \delta \left(p_D^\perp - \frac{z}{\alpha} p_c^\perp \right), \quad (11)$$

where $D_Q^h(z^*)$ is the well-known Peterson fragmentation function given by

$$D_Q^h(z^*) = \frac{n(h)}{z^* \left[1 - \frac{1}{z^*} - \frac{\epsilon_Q}{1-z^*} \right]^2}. \quad (12)$$

The fragmentation function gives the probability that the original charm quark with a momentum P fragments into a D meson with momentum fraction $z^* P$. There are more recent fragmentation functions but here we are only interested in checking if the differences between linear and nonlinear dynamics are affected by fragmentation. For this purpose the Peterson fragmentation function is adequate.

In Fig. 2 we show the transverse momentum spectrum of charm quarks. The main purpose of this figure is to show that the predictions of the linear physics (CT + Shad) differ from the total (i.e., bCGC) by a factor that increases with the energy W and goes from 1.5 ($W = 100$ GeV) to 4 ($W = 1400$ GeV). Moreover, this difference persists for a wide momentum window. At very large p_T we enter the deep linear regime and expect that the two curves coincide.

In Fig. 3 we show the transverse momentum spectrum of bottom quarks. As expected, we observe the same features of the charm distribution, except that now the nonlinear effects are weaker. Nevertheless, they are still noticeable. In Fig. 4 we show the Q^2 dependence of the p_T distribution at a fixed
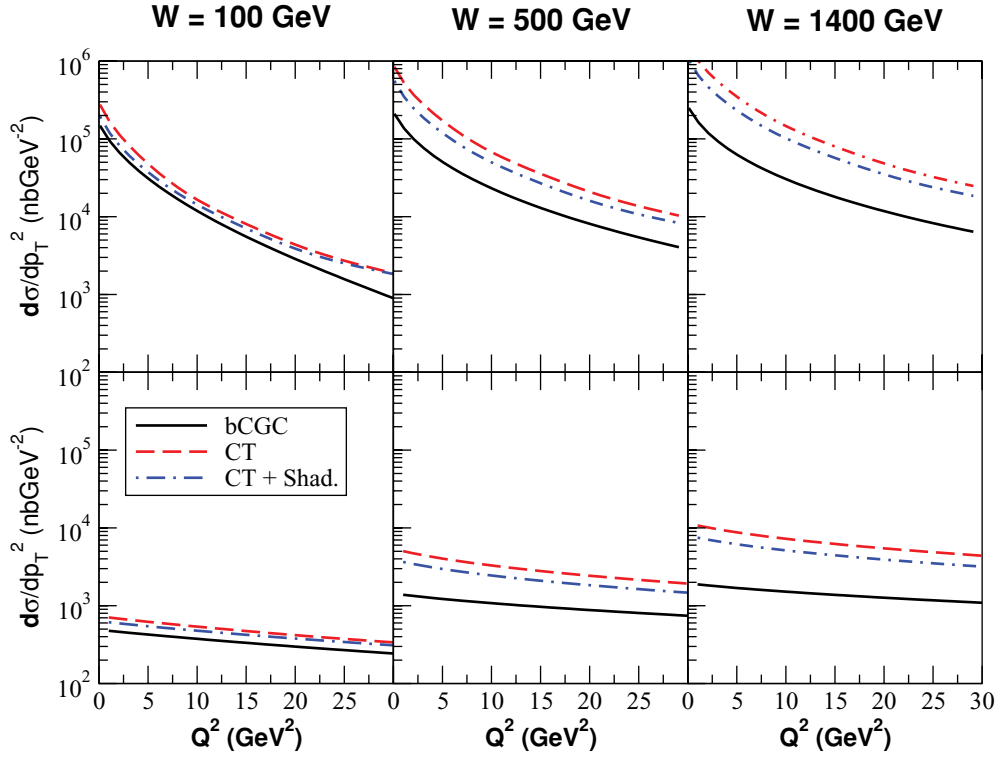
FIG. 4. (Color online) Dependence on the photon virtuality at $p_T^2 = 4\,\text{GeV}^2$.

value $p_T = 4\,\text{GeV}^2$ for different energies. The upper and lower panels show the charm and bottom distributions, respectively. Here again, we observe a remarkable strength and persistence up to large virtualities of the differences between CT + Shad and bCGC. In Fig. 5 we show the transverse momentum spectrum of D mesons for three energies $W = 200$, 500, and
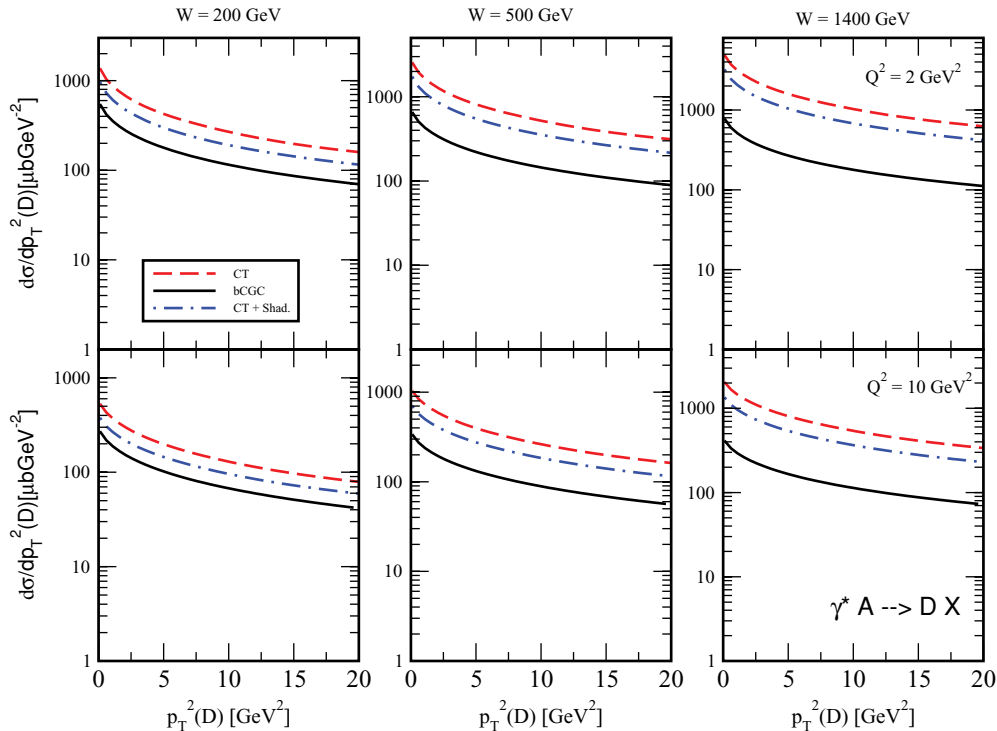

FIG. 5. (Color online) Transverse momentum spectra of D mesons.

1400 GeV and for two virtualities $Q^2 = 2$ GeV^2 (top panels) and $Q^2 = 10$ GeV^2 (bottom panels). As it can be seen, the differences between the curves CT, CT + Shad, and bCGC are the same as before.

III. CONCLUSIONS

In this work we have updated the calculations presented in Ref. [51] and extended them to electron-ion collisions. We compared the predictions of a saturation model (bCGC) with the predictions made with a linear model. The main conclusion was that it seems quite possible to observe the nonlinear effects both in the charm structure function $F_2^{c,A}(x, Q^2)$ and in the p_T distributions of the heavy quarks. For the energies considered, this difference is of a factor going from 1.5 to 4. As expected, the final state conversion of the heavy quarks into heavy mesons, performed through the convolution of our p_T distributions with the Peterson fragmentation function, does not change the difference between "full" ($=$ linear $+$ nonlinear) and linear predictions. In a future analysis of fragmentation we shall include the production of heavy mesons from light quarks. Although the fragmentation channel $q \rightarrow H$ (where q is a light quark and H is a D or B meson) is disfavored in comparison with $Q \rightarrow H$, the production of light quarks from the incoming photon is strongly enhanced by the photon wave function. One effect might compensate the other and, in the end, light quarks might play a significant role in heavy meson production [52]. If this would be the case, nonlinear effects would be even stronger.

Our results suggest that heavy quark production in high energy eA collisions is a promising signature of saturation. Previous estimates of this same observable were not so positive, probably because they addressed ep or pp collisions as in Ref. [53] or because the method employed to separate linear and nonlinear effects was not very accurate. From our figures we can also conclude that nonlinear dynamics, here as in several other contexts, leads to a depletion in the p_T spectra, in contrast to some early estimates [54].

A final comment is in order. As discussed in Sec. II, the heavy quark production at high energies can be calculated considering different approaches that are not equivalent in the full kinematic region. Consequently, a more detailed study of the saturation effects using these different approaches is important to estimate the theoretical uncertainty of our predictions. It is postponed for a future publication.

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