Kaonic hydrogen atoms with realistic potentials

Y. Yan,[1,2,*] W. Poonsawat,[1,2] K. Khosonthongkee,[1,2] C. Kobdaj,[1,2] and P. Suebka[1,2]
[1]*School of Physics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, Thailand*
[2]*Center of Excellence in Physics, Ministry of Education, Bangkok, Thailand*
(Received 2 June 2009; revised manuscript received 25 November 2009; published 23 June 2010)

Kaonic hydrogen is studied with various realistic potentials in an accurate numerical approach based on Sturmian functions. It is found that the mass difference between the K^-p and $\bar{K}^0n$ channels has a considerable effect on theoretical results of the energy shift and decay width of kaonic hydrogen. On average, the theoretical result in the isospin symmetry limit is smaller by a factor of about 20% than the full result where the mass difference between the K^-p and $\bar{K}^0n$ channels is properly treated. The theoretical results based on realistic local potentials, which reproduce well scattering data, are inconsistent with the recent measurement of the energy shift and decay width of the $1s$ kaonic hydrogen state by the DEAR Collaboration.

DOI: 10.1103/PhysRevC.81.065208 PACS number(s): 36.10.Gv, 13.75.Jz

Kaonic hydrogen is mainly the Coulomb bound state of a K^- and a proton, but is affected by the strong interaction at small distances. The strong interaction couples the K^-p state to the $\bar{K}^0n$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$, and $\eta\Lambda$ channels and results in the $\pi\Sigma$ and $\pi\Lambda$ decaying modes. It is believed that the study of kaonic hydrogen effectively probes the low-energy, and especially zero-energy, strong kaon-nucleon interaction. Inspired by the recent precise determination of the energy and decay width by the DEAR Collaboration [1], kaonic hydrogen has been extensively studied in the theoretical sector, mainly in effective field theory [2–10].

The success of the effective field theory applied to kaonic hydrogen makes it possible to construct equivalent local $\overline{K}N$ potentials, which may be conveniently applied to computations of K-nuclear few-body systems and hyper-nucleus productions [11]. The solution of the Schrödinger or Lippmann-Schwinger equation with such an equivalent potential should approximate as closely as possible the scattering amplitude derived from the full coupled-channel calculation of the effective field theory.

In this work, we study kaonic hydrogen with local potentials, which are purely phenomenological or based on chiral SU(3) models. The $1s$ kaonic hydrogen energy shift and decay width are derived by solving the dynamical equation

$$\left[-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{l(l+1)}{r^2}-\mathbf{Q}^2+\mathbf{fV}\right]\mathbf{R}(r), \tag{1}$$

with

$$\mathbf{Q}^2=\begin{pmatrix} q_c^2 & 0\\ 0 & q_0^2\end{pmatrix},\quad \mathbf{f}=\begin{pmatrix} f_c & 0\\ 0 & f_0\end{pmatrix}, \tag{2}$$

$$\mathbf{V}=\mathbf{V}^{\rm em}+\mathbf{V}^h, \tag{3}$$

$$\mathbf{V}^{\rm em}=\begin{pmatrix} V^{\rm em} & 0\\ 0 & 0\end{pmatrix}, \tag{4}$$

$$\mathbf{V}^h=\begin{pmatrix} \frac{1}{2}(V_1^h+V_0^h) & \frac{1}{2}(V_1^h-V_0^h)\\ \frac{1}{2}(V_1^h-V_0^h) & \frac{1}{2}(V_1^h+V_0^h)\end{pmatrix}, \tag{5}$$

$$\mathbf{R}(r)=\begin{pmatrix} R_{K^-p}(r)\\ R_{\bar{K}^0n}(r)\end{pmatrix}, \tag{6}$$

$$q_c^2=\frac{[E^2-(M_p-M_{K^-})^2][E^2-(M_p+M_{K^-})^2]}{4E^2}, \tag{7}$$

$$q_0^2=\frac{[E^2-(M_n-M_{\bar{K}^0})^2][E^2-(M_n+M_{\bar{K}^0})^2]}{4E^2}, \tag{8}$$

$$f_c=\frac{E^2-M_p^2-M_{K^-}^2}{E}, \tag{9}$$

$$f_0=\frac{E^2-M_n^2-M_{\bar{K}^0}^2}{E}, \tag{10}$$

where $V^{\rm em}$ is the electromagnetic potential, V_0^h and V_1^h are respectively the isospin $I=0$ and 1 strong interactions of the $\overline{K}N$ system, and $R_{K^-p}(r)$ and $R_{\bar{K}^0n}(r)$ are respectively the K^-p and $\bar{K}^0n$ components of the radial wave function of the $\overline{K}N$ system. Equation (1) embeds into the Schrödinger equation the relativistic effect and the mass difference between the K^-p and $\bar{K}^0n$ components. The relativistic modification of the Schrödinger equation to Eq. (1) has been discussed in Refs. [12–17].

The local potentials considered herein are the phenomenological $\overline{K}N$ potential taken from Refs. [18,19] and the various effective potentials which are worked out in Ref. [20]. The interaction [18,19] is constructed by fitting the free $\bar{K}N$ scattering data [21], the KpX data of kaonic hydrogen by the KEK Collaboration [22] and the binding energy and decay width of $\Lambda(1405)$, which is regarded as an isospin $I=0$ bound state of $\bar{K}N$.

In Ref. [20], an effective local potential in coordinate space is constructed such as the solution of the Schrödinger or Lippmann-Schwinger equation with such a potential approximates as closely as possible the scattering amplitude derived from the full chiral coupled-channel calculation. Four versions of effective potentials referred to as ORB, HNJH, BNW, BMN have been constructed in Ref. [20], based respectively on the chiral SU(3) models [3,4,8,10].

The accurate evaluation of energy shifts, decay widths, and especially wave functions of exotic atoms has been a challenge to numerical methods [16,23]. An approach is required which is able to account accurately for both the strong short-range

*yupeng@sut.ac.th

TABLE I. $1s$ kaonic hydrogen energy shift ΔE_{1s} (ΔE^0_{1s}) and decay width Γ_{1s} (Γ^0_{1s}) derived by directly solving Eq. (1) with (without) the mass difference between the K^-p and $\bar{K}^0n$ states considered.

	ΔE^0_{1s} [eV]	Γ^0_{1s} [eV]	ΔE_{1s} [eV]	Γ_{1s} [eV]
AY [19]	−268	312	−384	288
ORB [20]	−255	534	−348	646
HNJH [20]	−248	527	−336	648
BNW [20]	−220	544	−288	674
BMN [20]	−197	517	−297	622

interaction and the long-range Coulomb force. The numerical approach based on Sturmian functions [24] has been found effective and accurate. In this work, we use the numerical method which has been carefully studied and discussed in Refs. [24–26] to study kaonic hydrogen.

The $1s$ kaonic hydrogen energy shift ΔE_{1s} and decay width Γ_{1s} shown in Table I are derived by solving Eq. (1) in the above-mentioned Sturmian function approach [24–26], with the mass difference between the K^-p and $\bar{K}^0n$ channels treated as shown in Eqs. (1) to (10). The negative energy shift in Table I means that the $1s$ energy level is effectively pushed up by the strong interaction since there exists one deep bound state, the $\Lambda(1405)$. Shown in Table I are also the energy shift ΔE^0_{1s} and decay width Γ^0_{1s} in the isospin symmetry limit, where the mass of proton is applied for both the proton and neutron and the mass of K^- for both the K^- and $\bar{K}^0$.

It is found from Table I that the theoretical results in the approximation of the isospin symmetry limit are rather different from the full results where the mass difference between the K^-p and $\bar{K}^0n$ channels is properly treated, as shown in Eqs. (1) to (10). Except for the decay width for the phenomenological $\overline{K}N$ potential [18,19], the isospin symmetry approximation largely underestimates both the energy shift and decay width of the $1s$ kaonic hydrogen. On average, the theoretical result for the energy shift in the isospin symmetry limit is smaller by a factor of about 28% than the full result. For the equivalent local potentials referred to as ORB, HNJH, BNW, BMN in Table I, which are constructed in Ref. [20], based respectively on the chiral SU(3) models [3,4,8,10], the isospin symmetry approximation for the decay width of the $1s$ kaonic hydrogen is about 18% smaller than the result where the mass difference between the K^-p and $\bar{K}^0n$ channels is considered.

The most recent experimental values on the energy shift and decay width of the ground state of kaonic hydrogen are, respectively,

$$\Delta E_{1s} = -193 \pm 37\ (\text{stat}) \pm 6\ (\text{syst})\ \ \text{eV} \tag{11}$$

and

$$\Gamma_{1s} = 249 \pm 111\ (\text{stat}) \pm 30\ (\text{syst})\ \ \text{eV} \tag{12}$$

obtained by the DEAR Collaboration [1]. These values are smaller by a factor of almost 2 than the experimental values measured by the KEK Collaboration [22], which are, respectively,

$$\Delta E_{1s} = -323 \pm 63\ (\text{stat}) \pm 11\ (\text{syst})\ \ \text{eV} \tag{13}$$

and

$$\Gamma_{1s} = 407 \pm 208\ (\text{stat}) \pm 100\ (\text{syst})\ \ \text{eV}. \tag{14}$$

It is clear that except for the decay width derived with the phenomenological $\overline{K}N$ potential referred to as AY in Table I, all other theoretical values are much larger than the DEAR data. However, the theoretical results shown in Table I for both the pure phenomenological potential and the chiral SU(3) symmetry-based potentials are fairly consistent with the KEK measurements, considering the large error of the KEK values of the $1s$ kaonic hydrogen decay width.

It is difficult to conclude whether the equivalent potentials based on chiral SU(3) models are reasonable since the KEK and DEAR data are so inconsistent with each other. One may have to wait for the more accurate measurement of the $1s$ kaonic hydrogen by the SIDDHARTA Collaboration.

One may argue that the theoretical results of the $1s$ kaonic hydrogen energy shifts and decay widths derived in the work with the the local, equivalent single-channel potentials may not reflect well the original equivalent interactions. To clear this issue, we compare the K^-p scattering lengths derived with the effective multichannel interactions [3,4,8,10,18] with the ones evaluated with the local single-channel potentials [19,20] using

$$a_{K^-p} = \tfrac{1}{2}(a_{I=0} + a_{I=1}) \tag{15}$$

in the isospin-symmetry limit, where the mass of proton is applied for both the proton and neutron and the mass of K^- for both the K^- and $\bar{K}^0$. Shown in Table II are the K^-p scattering lengths a_{K^-p} derived with the local single-channel potentials [19,20], which are employed here in this work to evaluate the energy shifts and decay widths of the $1s$ kaonic hydrogen and the K^-p scattering lengths $\tilde{a}_{K^-p}$ derived with the multichannel effective interactions [3,4,8,10,18]. It is found that the average discrepancy between a_{K^-p} and $\tilde{a}_{K^-p}$ is less than 5%, much smaller than the effect resulted from the mass difference between the K^-p and $\bar{K}^0n$ channels. One may conclude that the local single-channel potentials [19,20] applied to the KN system well approximate the original multichannel effective interactions.

TABLE II. K^-p scattering lengths a_{K^-p} derived with local single-channel potentials [19,20] compared with the K^-p scattering lengths $\tilde{a}_{K^-p}$ (taken from Refs. [18,20]) derived with the multichannel effective interactions [3,4,8,10,18].

	a_{K^-p} [fm]	$\tilde{a}_{K^-p}$ [fm]
AY [19]	$-0.678 + i0.506$	$-0.70 + i0.53$
ORB [3]	$-0.586 + i0.844$	$-0.617 + i0.861$
HNJH [4]	$-0.566 + i0.829$	$-0.608 + i0.835$
BNW [8]	$-0.487 + i0.838$	$-0.532 + i0.833$
BMN [10]	$-0.426 + i0.788$	$-0.410 + i0.824$

ACKNOWLEDGMENTS

The author would like to thank Th. Gutsche (Institute for Theoretical Physics, Tübingen University), M. Lutz (GSI)

and T. Hyodo (Department of Physics, Tokyo Institute of Technology) for fruitful discussions. This work is supported in part by the Commission on Higher Education, Thailand (CHE-RES-RG Theoretical Physics).

[1] G. Beer *et al.* (DEAR Collaboration), Phys. Rev. Lett. **94**, 212302 (2005); M. Gargnelli *et al.* (DEAR Collaboration), Int. J. Mod. Phys. A **20**, 341 (2005).
[2] J. A. Oller and U.-G. Meißner, Phys. Lett. B **500**, 263 (2001).
[3] E. Oset, A. Ramos, and C. Bennhold, Phys. Lett. B **527**, 99 (2002).
[4] T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Phys. Rev. C **68**, 018201 (2003).
[5] U. -G. Meißner, U. Raha, and A. Rusetsky, Eur. Phys. J. C **35**, 349 (2004); **41**, 213 (2005); **47**, 473 (2006).
[6] A. N. Ivanov, M. Cargnelli, M. Faber, J. Marton, N. I. Troitskaya, and J. Zmeskal, Eur. Phys. J. A **21**, 11 (2004).
[7] A. N. Ivanov, M. Cargnelli, M. Faber, H. Fuhrmann, V. A. Ivanova, J. Marton, N. I. Troitskaya, and J. Zmeskal, Eur. Phys. J. A **25**, 329 (2005).
[8] B. Borasoy, R. Nißler, and W. Weise, Phys. Rev. Lett. **94**, 213401 (2005); Eur. Phys. J. A **25**, 79 (2005).
[9] J. A. Oller, J. Prades, and M. Verbeni, Phys. Rev. Lett. **95**, 172502 (2005).
[10] B. Borasoy, U.-G. Meißner, and R. Nißler, Phys. Rev. C **74**, 055201 (2006).
[11] A. Limphirat, C. Kobdaj, M. Bleicher, Y. Yan, and H. Stoecker, J. Phys. G: Nucl. Part. Phys. **36**, 064049 (2009).
[12] P. R. Auvil, Phys. Rev. D **4**, 240 (1971).
[13] B. C. Pearce and B. K. Jennings, Nucl. Phys. A **528**, 655 (1991).
[14] W. R. Gibbs, Li Ai, and W. B. Kaufmann, Phys. Rev. C **57**, 784 (1998).
[15] A. Gashi, E. Matsinos, G. C. Oades, G. Rasche, and W. S. Woolcock, Nucl. Phys. A **686**, 463 (2001).
[16] A. Gashi, G. Rasche, and W. S. Woolcock, Phys. Lett. B **513**, 269 (2001).
[17] A. Gashi, G. C. Oades, G. Rasche, and W. S. Woolcock, Nucl. Phys. A **699**, 732 (2002).
[18] Y. Akaishi and T. Yamazaki, Phys. Rev. C **65**, 044005 (2002).
[19] T. Yamazaki and Y. Akaishi, Phys. Rev. C **76**, 045201 (2007).
[20] T. Hyodo and W. Weise, Phys. Rev. C **77**, 035204 (2008).
[21] A. D. Martin, Nucl. Phys. B **179**, 33 (1981).
[22] M. Iwasaki *et al.* (KEK Collaboration), Phys. Rev. Lett. **78**, 3067 (1997).
[23] I. Amirkhanov, I. Puzynin, A. Tarasov, O. Voskresenskaya, and O. Zeinalova, Phys. Lett. B **452**, 155 (1999).
[24] M. Rotenberg, Adv. At. Mol. Phys. **6**, 233 (1970).
[25] Y. Yan, R. Tegen, T. Gutsche, and A. Faessler, Phys. Rev. C **56**, 1596 (1997).
[26] P. Suebka and Y. Yan, Phys. Rev. C **70**, 034006 (2004).