## Understanding the major uncertainties in the nuclear symmetry energy at suprasaturation densities

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Within the interacting Fermi gas model for isospin asymmetric nuclear matter, effects of the in-medium three-body interaction and the two-body short-range tensor force owing to the  $\rho$  meson exchange, as well as the short-range nucleon correlation on the high-density behavior of the nuclear symmetry energy, are demonstrated respectively in a transparent way. Possible physics origins of the extremely uncertain nuclear symmetry energy at suprasaturation densities are discussed.

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### I. INTRODUCTION

The density dependence of nuclear symmetry energy  $E_{\text{sym}}(\rho)$  is currently a key issue in both nuclear physics and astrophysics (see, e.g., Refs. [1–11]). Despite much theoretical and experimental effort, our current knowledge about the  $E_{\text{sym}}(\rho)$  is still rather poor, especially at suprasaturation densities. Experimentally, some constraints on the  $E_{\text{sym}}(\rho)$ at subsaturation densities have been obtained recently from analyzing nuclear reaction data (see, e.g., Refs. [12–14]). At suprasaturation densities, however, the situation is much less clear because of the very limited data available and the few model analyses carried out so far, although some indications of a supersoft  $E_{\text{sym}}(\rho)$  at high densities have been obtained from analyzing the  $\pi^+/\pi^-$  ratio in relativistic heavy-ion collisions [15]. Theoretically, essentially all available many-body theories using various interactions have been used in calculating the  $E_{\text{sym}}(\rho)$ . Unfortunately, the predictions at suprasaturation densities are very diverse (for a recent review, see, e.g., Ref. [6]). Assuming all models are equally physical and noting that there is no first principle guiding its high-density limit, it is fair to state that the  $E_{\text{sym}}(\rho)$  at suprasaturation densities is currently still completely undetermined. So, why is the  $E_{\text{sym}}(\rho)$  so uncertain at suprasaturation densities? This is obviously an important question that should be addressed soon, especially because several dedicated experiments have now been planned to investigate the high-density behavior of the  $E_{\text{sym}}(\rho)$  at the CSR in China [16], GSI in Germany [17], MSU in the United States [18], and RIKEN in Japan [19]. Identifying the causes for the uncertain high-density  $E_{\text{sym}}(\rho)$ may help experimentalists to decide what experiments to do and what observables to measure. Though we cannot fully answer this question, we identify several important factors and demonstrate their effects on the high-density behavior of the  $E_{\text{sym}}(\rho)$  using probably the simplest manybody theory available, namely, the interacting Fermi gas model for isospin asymmetric nuclear matter (e.g., Ref. [20]). There are many long-standing physical issues on how to treat quantum many-body problems at high densities; the various techniques used in different many-body theories may

be among the possible origins of the very uncertain  $E_{\text{sym}}(\rho)$ at suprasaturation densities. Nevertheless, it is still very useful to examine effects of some common ingredients used in most many-body theories, such as the three-body and tensor forces, in the simplest model possible. While the interacting Fermi gas model cannot be expected to describe all properties of infinite nuclear matter and finite nuclei as accurately as those more advanced microscopic many-body theories, it does give an analytical expression for the  $E_{\text{sym}}(\rho)$  in terms of the isospin-dependent strong nucleon-nucleon (NN) interaction in a physically very transparent way [21]. Most importantly, the key underlying physics responsible for the uncertain  $E_{\text{sym}}(\rho)$  at suprasaturation densities can be clearly revealed. In particular, effects of the spin-isospin-dependent effective three-body force, the density dependence of the in-medium short-range tensor forces, and the short-range nucleon correlation can be demonstrated clearly. The results are expected to be useful for not only understanding predictions of the various manybody theories but also ultimately determining the  $E_{\text{sym}}(\rho)$  at suprasaturation densities.

# II. SYMMETRY ENERGY WITHIN THE INTERACTING FERMI GAS MODEL

According to the well-known Lane potential [22], the single-nucleon potential  $U_{n/p}$  can be well approximated by

$$U_{n/p}(\rho, k) \approx U_0(\rho, k) \pm U_{\text{sym}}(\rho, k)\delta,$$
 (1)

where the  $U_0(\rho, k)$  and  $U_{\text{sym}}(\rho, k)$  are, respectively, the isoscalar and isovector (symmetry) nucleon potentials. Within the interacting Fermi gas model for isospin asymmetric nuclear matter [20], the nuclear symmetry energy can be explicitly expressed as (for detailed derivation of this formula, please see Ref. [21] and references therein)

$$\begin{split} E_{\text{sym}}(\rho) &= E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{\text{pot1}} + E_{\text{sym}}^{\text{pot2}} \\ &= \frac{1}{6} \frac{\partial t}{\partial k} \bigg|_{k_F} \cdot k_F + \frac{1}{6} \frac{\partial U_0}{\partial k} \bigg|_{k_F} \cdot k_F + \frac{1}{2} U_{\text{sym}}(\rho, k_F), \end{split}$$
(2)

where  $t(k) = \hbar^2 k^2 / 2m$  is the kinetic energy, m is the average nucleon mass, and  $k_F = (3\pi^2 \rho / 2)^{1/3}$  (Thomas-Fermi approximation [23]) is the nucleon Fermi momentum in symmetric

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nuclear matter at density  $\rho$ . We notice here that Eq. (2) is identical to the one derived earlier by Brueckner et al. [24,25] using K matrices within the Brueckner theory. From Eq. (2), it is seen that the symmetry energy  $E_{\text{sym}}(\rho)$  is only dependent on the single-particle kinetic and potential energies at the Fermi momentum  $k_F$ . The first part  $E_{\text{sym}}^{\text{kin}} = \frac{\hbar^2}{6m} (\frac{3\pi^2}{2})^{\frac{2}{3}} \rho^{\frac{2}{3}}$  is the trivial kinetic contribution owing to the different Fermi momenta of neutrons and protons;  $E_{\mathrm{sym}}^{\mathrm{pot1}} = \frac{1}{6} \frac{\partial U_0}{\partial k}|_{k_F} \cdot k_F$  is attributable to the momentum dependence of the isoscalar potential and also to the fact that neutrons and protons have different Fermi momenta, while the  $E_{\text{sym}}^{\text{pot2}} = \frac{1}{2}U_{\text{sym}}(\rho, k_F)$  is attributable to the explicit isospin dependence of the nuclear strong interaction. The  $U_0(\rho_0, k)$  at normal nuclear density  $\rho_0$ is relatively well determined from the nucleon optical potential obtained from the Dirac phenomenological model analysis of nucleon-nucleus scattering data [26]. Moreover, interesting information about the  $U_0(\rho, k)$  at abnormal densities in a broad momentum range has been obtained from transport model analysis of nuclear collective flow in heavy-ion reactions [27]. For the momentum-dependent part of the isoscalar potential  $U_0(k, \rho)$ , we use here the well-known Gale-Bertsch-Das Gupta (GBD) parametrization [28],

$$U_{\text{GBD}}(\rho, k) = \frac{-75\rho/\rho_0}{1 + [k/(\Lambda k_F)]^2},$$
(3)

where  $\Lambda = 1.5$ . The  $E_{\text{sym}}^{\text{pot1}}$  is then given by

$$E_{\text{sym}}^{\text{pot1}} = 75\rho/\rho_0 \frac{1/(3\Lambda^2)}{[1 + (1/\Lambda^2)]^2}.$$
 (4)

The GBD potential describes reasonably well the nucleon-nucleus optical potential and has been widely used in transport model simulations of heavy-ion reactions [29]. Similar to the  $E_{\text{sym}}^{\text{kin}}$ , the  $E_{\text{sym}}^{\text{pot1}}$  always increases with density. However, the  $U_{\text{sym}}(\rho,k)$  is very poorly known, especially at high densities or momenta [6]. To reveal the fundamental physics responsible for the uncertain high-density behavior of the  $E_{\text{sym}}(\rho)$ , we denote  $u_{T0} = u'_{np}$  as the n-p interaction in momentum-space in the isosinglet (T=0) channel, while  $u_{T1} = u_{nn} = u_{pp} = u_{np}$  is the nuclear strong interaction in the isotriplet (T=1) channel. In the latter, the charge independence of strong interaction has been assumed. Then the single-nucleon meanfield potentials are [30,31]

$$U_{n}(\rho, k) = u_{nn} \frac{\rho_{n}}{\rho} + u_{np} \frac{\rho_{p}}{\rho} = u_{T1} \frac{\rho_{n}}{\rho} + u_{T1} \frac{\rho_{p}}{2\rho} + u_{T0} \frac{\rho_{p}}{2\rho},$$

$$U_{p}(\rho, k) = u_{pp} \frac{\rho_{p}}{\rho} + u_{pn} \frac{\rho_{n}}{\rho} = u_{T1} \frac{\rho_{p}}{\rho} + u_{T1} \frac{\rho_{n}}{2\rho} + u_{T0} \frac{\rho_{n}}{2\rho}.$$
(5)

Therefore,

$$U_0(\rho, k) = \frac{1}{2}(U_n + U_p) = \frac{1}{4}(3u_{T1} + u_{T0}),$$

$$U_{\text{sym}}(\rho, k) = \frac{1}{2\delta}(U_n - U_p) = \frac{1}{4}(u_{T1} - u_{T0}).$$
(6)

Thus, the  $U_{\rm sym}(\rho,k)$  measures the explicit isospin dependence of the nuclear strong interaction; namely, if the n-p interactions were the same in the isosinglet and isotriplet channels, then

the  $U_{\text{sym}}(\rho, k)$  would vanish. Currently, the calculation of  $U_{\text{sym}}(\rho, k)$  is rather model dependent [31,32]. In fact, its value can be either positive or negative at high densities or momenta [33–35], leading to the dramatically different predictions on the high-density behavior of the  $E_{\text{sym}}(\rho)$ . In coordinate space, in terms of the two-body NN interactions  $V_{T0}(r_{ij})$  and  $V_{T1}(r_{ij})$  for the isosinglet T=0 and isotriplet T=1 channels, respectively, we have [20]

$$E_{\text{sym}}^{\text{pot2}} = \frac{1}{2} U_{\text{sym}}(\rho, k_F) = \frac{1}{4} (\widetilde{V}_{T1} - \widetilde{V}_{T0}),$$
 (7)

where

$$\widetilde{V_{T0}} = \frac{1}{2}\rho \int V_{T0}(r_{ij}) d^3 r_{ij}, \quad \widetilde{V_{T1}} = \frac{1}{2}\rho \int V_{T1}(r_{ij}) d^3 r_{ij}.$$
(8)

It is clear that  $E_{\text{sym}}^{\text{pot2}}$  is determined by the competition between the  $\widetilde{V}_{T1}$  and the  $\widetilde{V}_{T0}$  and thus by the isospin dependence of the in-medium NN interactions. The latter is largely unknown and is actually a major thrust of research at various radioactive beam facilities around the world. In fact, how the nuclear medium may modify the bare NN interaction and properties of hadrons has long been one of the most critical issues in nuclear physics. For instance, it is very difficult to obtain the empirical saturation properties of symmetric nuclear matter by using the bare NN interactions within nonrelativistic many-body theories. Various in-medium effects, such as the many-body force [36,37], the relativistic effect relating the in-medium interactions to free space NN scattering [38,39], or the in-medium tensor force owing to both the  $\pi$  meson and  $\rho$  meson exchanges [40–42], have to be considered to obtain a reasonably good description of saturation properties of symmetric nuclear matter. How these effects manifest themselves in dense neutron-rich matter may affect the highdensity behavior of the nuclear symmetry energy.

## III. EFFECTS OF THE SPIN-ISOSPIN-DEPENDENT THREE-BODY FORCE

In this section, we examine effects of the spin-isospindependent three-body force on the symmetry energy at suprasaturation densities. As it has been shown repeatedly in the literature (see, e.g., Refs. [36,43–45]) that a zero-range three-body force can be reduced to an effective two-body force,

$$V_d = t_0 (1 + x_0 P_\sigma) \rho^\alpha \delta(r), \tag{9}$$

where  $t_0$ ,  $\alpha$ , and  $x_0$  are parameters and  $P_{\sigma}$  is the spin-exchange operator. Depending on how many and which properties of finite nuclei and nuclear matter are used in reducing the three-body force, various values have been used for both the  $x_0$  and  $t_0$  parameters. Moreover, the density-dependence  $\rho^{\alpha}$  is often used to mimic additional in-medium effects such as the many-body force. For instance, in relativistic approaches, the dressing of the in-medium spinors will also introduce a density-dependence to the interaction, which can lead to similar effects as the many-body force. The parameter  $\alpha$  can thus take on any value between 0 and 1. The parameter  $x_0$  controls the relative contributions to the three-body force from the isosinglet and isotriplet NN interactions. The three-body term has been included in many effective interactions, such as

the Skyrme and Gogny interactions, in both phenomenological (e.g., Refs. [46,47]) and microscopic (see, e.g., Ref. [48]) many-body theories for isospin asymmetric nuclear matter. Noticing that the potential energies owing to the three-body force in the T=1 and T=0 channels are, respectively (for details, see Table II and the corresponding equation  $E^{ST}$  in Ref. [45]),

$$V_d^{T1} = \frac{1 - x_0}{2} \frac{3t_0}{8} \rho^{\alpha + 1}, \quad V_d^{T0} = \frac{1 + x_0}{2} \frac{3t_0}{8} \rho^{\alpha + 1}, \quad (10)$$

one sees immediately that the terms containing  $x_0$  cancel out in calculating the equation of state (EOS) of symmetric nuclear matter. In fact, among the normally 13 parameters used in the Hartree-Fock calculations with the Gogny force,  $x_0$  is the only one having this special feature. To evaluate quantitatively the symmetry energy, we use the Gogny central force [45],

$$V_c(r) = \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)_i e^{-r^2/\mu_i^2}, \quad (11)$$

where  $P_{\tau}$  is the spin exchange operator and the values of the parameters W, B, H, M, and  $\mu$  are taken directly from Ref. [45]. It is necessary to stress here that because we are aiming at a qualitative understanding of why the symmetry energy is so uncertain at suprasaturation densities, the model parameters are not retuned self-consistently to reproduce any existing constraint on the symmetry energy at and/or below the normal nuclear-matter density  $\rho_0$ . The resulting  $E_{\rm sym}^{\rm pot2}$  is

$$E_{\text{sym}}^{\text{pot2}} = -\sum_{i=1,2} \left( \frac{H_i}{4} + \frac{M_i}{8} \right) \pi^{\frac{3}{2}} \mu_i^3 \rho - (1 + 2x_0) \frac{t_0}{8} \rho^{\alpha+1}.$$
(12)

Shown in Fig. 1 are the symmetry energy functions obtained with different values for the  $x_0$  and  $\alpha$  parameters. It is clearly seen that for a given value of the  $\alpha$  parameter, the  $x_0$  controls the high-density behavior of the symmetry energy. By varying the  $x_0$ , one can easily cover the whole range of symmetry energy calculated within the Hartree-Fock approach using more than 100 Skyrme and Gogny forces [47] without changing anything in the EOS of symmetric nuclear matter. In particular, with  $x_0 = 1$  and  $\alpha = 1/3$  as in the original Gogny force [45], only the (S = 1, T = 0) spin-isospin n-p interaction contributes to

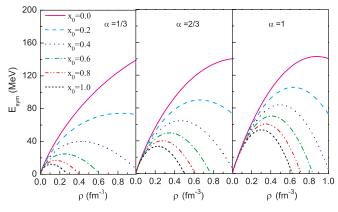


FIG. 1. (Color online) The symmetry energy obtained with different spin and density dependencies in the three-body force.

the three-body force and the symmetry energy; the  $E_{\rm sym}(\rho)$  drops quickly to zero above certain suprasaturation densities. We regard this kind of  $E_{\rm sym}(\rho)$  as being supersoft. Overall, depending on the value of the parameter  $x_0$ , the symmetry energy can be either stiff and keeps increasing with density or becomes supersoft above certain suprasaturation densities.

In the 2003 survey by J. R. Stone et al. [47] of 87 Skyrme interactions, the  $x_0$  ( $x_3$  in the notation of Ref. [47]) ranges between -1.56 and 1.92. As a special example, we notice that B. A. Brown used  $x_0$  between 0.03 and 0.9 with the SKX [49]. The values of  $x_0$  used in Fig. 1 are thus within the uncertainty range of  $x_0$  found in the literature. Moreover, we notice that the result here is consistent with the earlier finding that the EOS of pure neutron matter is very sensitive to the  $x_0$  parameter [49,50]. To the best of our knowledge, it is currently not clear how to fix the  $x_0$  alone experimentally. Nevertheless, it is worth noting that the cross section of pn charge exchange reactions and the symmetry potential  $U_{\rm sym}$ extracted from the isospin dependence of the nucleon optical potentials are all directly related to the spin-isospin-dependent nuclear interaction. They may thus be used to constrain the value of  $x_0$ . Moreover, the density dependence of the nuclear symmetry energy itself can be used to constrain the  $x_0$  should it be determined experimentally in the future. However, as we discuss in the next section, the in-medium tensor force may have similar effects on these observables.

### IV. EFFECTS OF THE IN-MEDIUM SHORT-RANGE TENSOR FORCE AND NUCLEON CORRELATION

Studies based on microscopic many-body theories indicate consistently that the symmetry energy is dominated by the isosinglet ( $S=1,\ T=0$ ) channel [30,51], while measured properties of deuterons indicate unambiguously that a tensor force is at work in the ( $S=1,\ T=0$ ) channel. It is also well known that the  $\pi$  and  $\rho$  meson exchanges contribute to the intermediate-range attractive and the short-range repulsive tensor forces, respectively, according to [40]

$$V_T^{\pi}(r) = \frac{f_{N_{\pi}}^2 m_{\pi}}{4\pi} \tau_1 \cdot \tau_2(-S_{12}) \left[ \frac{e^{-m_{\pi}r}}{(m_{\pi}r)^3} + \frac{e^{-m_{\pi}r}}{(m_{\pi}r)^2} + \frac{e^{-m_{\pi}r}}{3m_{\pi}r} \right]$$

and

$$V_T^{\rho}(r) = \frac{f_{N_{\rho}}^2 m_{\rho}}{4\pi} \tau_1 \cdot \tau_2(S_{12}) \left[ \frac{e^{-m_{\rho}r}}{(m_{\rho}r)^3} + \frac{e^{-m_{\rho}r}}{(m_{\rho}r)^2} + \frac{e^{-m_{\rho}r}}{3m_{\rho}r} \right], \tag{13}$$

where  $f_{N_{\pi}}^2/4\pi = 0.08$  and  $f_{N_{\rho}}^2/m_{\rho}^2 \simeq 2f_{N_{\pi}}^2/m_{\pi}^2$ . The  $S_{12} = 6(\vec{S} \cdot \vec{r})^2/r^2 - 2\vec{S}^2 = 4S^2P_2[\cos(\theta)]$  is the tensor operator. We notice that some effective interactions, such as the Paris force [52], only consider the tensor force owing to the  $\pi$  exchange and has a short-range cut-off. As shown in Fig. 3 of Ref. [53], the short-range behavior of the tensor forces used in popular effective interactions differ dramatically. Moreover, different short-range cutoffs are normally introduced, for example, in studying the single-particle energy levels in rare isotopes [53,54]. Unfortunately, the different behaviors of the tensor force at short distance being cut out, thus not probed from

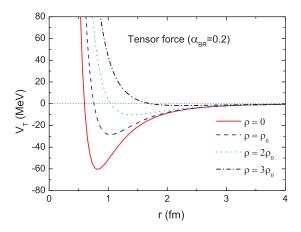


FIG. 2. (Color online) The radial part of the tensor force  $V_T = V_T^{\rho}(r) + V_T^{\pi}(r)$  at  $\rho = 0$ ,  $\rho_0$ ,  $2\rho_0$ , and  $3\rho_0$  with the BRS parameter  $\alpha_{\rm BR} = 0.2$ .

studying the energy levels of single particles, affect dramatically the high-density behavior of the symmetry energy. Furthermore, in-medium properties of the  $\rho$  meson may affect the strength of the short-range tensor force. Because the symmetry energy is very sensitive to the competition between the isosinglet  $(V_{T0})$  and the isotriplet  $(V_{T1})$  channels, the contribution of the in-medium short-range tensor force that exists only in the T=0 channel may thus affect significantly the high-density behavior of the  $E_{\text{sym}}(\rho)$ . To investigate effects of the in-medium tensor force, we use the Brown-Rho scaling (BRS) for the in-medium  $\rho$  meson mass according to  $m_{\rho}^{\star}/m_{\rho} = 1 - \alpha_{\rm BR} \cdot \rho/\rho_0$  [41]. Shown in Fig. 2 is the radial part of the total tensor force  $V_T = V_T^{\rho}(r) + V_T^{\pi}(r)$  at  $\rho = 0, \rho_0$ ,  $2\rho_0$ , and  $3\rho_0$ , respectively, with the BRS parameter  $\alpha_{BR}=0.2$ . As one expects, the total tensor force becomes more repulsive in denser matter when the  $\rho$  meson mass is reduced according to the BRS. While the experimental evidence for the BRS is still not very clear, we use it here as an effective way of adjusting the in-medium strength of the tensor force. This is useful for mimicking different ways of involving the tensor force in many-body calculations in the literature.

Noticing that the tensor force gives no contribution at the mean-field level to the potential energy with spherically symmetric tensor correlations [55], we assume here that the tensor force acting in the isosinglet n-p channels in nuclear matter behaves in a similar way to deuterons; namely, the tensor operator  $S_{12}$  is a constant of 2. Moreover, we introduce a two-step tensor correlation function, that is, f(r) = 0, for  $r < r_c$  and f(r) = 1, for  $r \ge r_c$ , where  $r_c = \eta (3/4\pi\rho)^{1/3}$  is the "healing distance" or short-range cutoff. Thus, the tensor contribution to the isospin T = 0 channel symmetry energy is  $V_T = \int f(r) [V_T^{\rho}(r_{ij}) + V_T^{\pi}(r_{ij})] d^3 r_{ij}$ . The parameter  $\eta$  is used to effectively vary the short-range cutoff. The density dependence in the  $r_c$  reflects the design that the size of nucleons shrinks as the density increases. With  $\eta = 2$ , the two nucleons will always keep in touch on their surfaces while their sizes decrease with increasing density.

Shown in Fig. 3 are the total symmetry energy functions obtained using three typical values for the  $\eta$  parameter. With  $\eta = 10$ , the  $r_c$  is so large that the tensor contribution to the

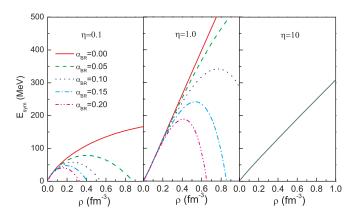


FIG. 3. (Color online) The symmetry energy with different values of the BRS parameter  $\alpha_{BR} = 0$ , 0.05, 0.10, 0.15, 0.20 using three values for the correlation parameter  $\eta$ .

 $E_{\mathrm{sym}}^{\mathrm{pot2}}$  is completely cut off. The  $E_{\mathrm{sym}}(\rho)$  thus keeps increasing with density owing to the  $E_{\rm sym}^{\rm kin}$  and the  $E_{\rm sym}^{\rm pot1}$  terms. With smaller  $\eta$  values, the high-density behavior of the  $E_{\rm sym}(\rho)$ is controlled by the BRS parameter  $\alpha_{BR}$ . Similar to varying the  $x_0$  parameter in the three-body force, by varying the  $\alpha_{\rm BR}$ parameter the tensor force can lead to  $E_{\text{sym}}(\rho)$  from stiff to supersoft. A recent study indicates that a value of  $\alpha_{BR} \approx 0.15$ is required to reproduce the measured lifetime of <sup>14</sup>C [56]. With such an  $\alpha_{BR}$ , the symmetry energy can easily become supersoft with  $\eta$  between 0.1 and 1. Results for the three typical cases shown in Fig. 3 clearly indicate that the high-density behavior of the symmetry energy is sensitive to both the short-range in-medium tensor force and the NN correlation function. The short-range repulsion generated by the  $\rho$ -meson exchange plays the key role in determining the symmetry energy at suprasaturation densities. In fact, the relationship between the short-range repulsive tensor force in the isosinglet n-p channel and the appearance of the supersoft symmetry energy was first noticed by Pandharipande et al. within variational many-body (VMB) theories [57,58]. Ultimately, because of the dominance of the repulsive n-p interaction in isospin symmetric nuclear matter at high densities, it is possible that pure neutron matter is energetically favored, leading to the negative symmetry energy at high densities. Of course, this can only occur if the  $\rho$  tensor contribution is sufficiently strong owing to, for example, its reduced mass in dense medium. It was also pointed out that the fundamental reason for the completely different high-density behaviors of the  $E_{\mathrm{sym}}$  predicted by the VMB and the relativistic mean field (RMF) models is the lack of the  $\rho$  tensor contribution to the energy in the RMF models [59].

To this end, it is interesting to note that the three-body force and the tensor force can affect similarly the high-density behavior of the symmetry energy. This is very similar to the situation in describing the saturation properties of symmetric nuclear matter. It has been shown that the saturation properties can be equally well described by including either the three-body force or the in-medium *NN* interactions based on the BRS [42,48]. Nevertheless, as was pointed out in Ref. [44], because the three-body force is essentially a convolution of two-body forces, a consistent three-body force should also include a tensor component. We have only

studied here separately effects of the three-body force and the two-body tensor force on the high-density behavior of the symmetry energy. Effects including both the two- and the three-body tensor forces simultaneously will be investigated in a forthcoming work. In particular, by varying parameters controlling both the three-body force and the tensor force together, we shall demonstrate how large the parameter space is in which the symmetry energy may become negative at suprasaturation densities.

#### V. SUMMARY

In summary, the high-density behavior of the symmetry energy  $E_{\rm sym}$  has long been regarded as the most uncertain property of dense neutron-rich nuclear matter because even its trend is still controversial. Within the interacting Fermi gas model, the  $E_{\rm sym}$  can be expressed in terms of the isospin-dependent NN strong interaction. The high-density behavior of the  $E_{\rm sym}$  is determined by the competition between the inmedium isosinglet and isotriplet nucleon-nucleon interactions. Respective effects of the in-medium three-body interaction and the short-range tensor force on the high-density behavior of the  $E_{\rm sym}$  are examined separately. It is found that the strength

of the spin (isospin) dependence of the three-body force and the in-medium  $\rho$  meson mass in the short-range tensor force are the key parameters controlling the high-density behavior of the  $E_{\rm sym}$ . These findings are useful for understanding why the nuclear symmetry energy is very uncertain at suprasaturation densities.

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