Abrupt changes in α-decay systematics as a manifestation of collective nuclear modes

C. Qi,[1,*] A. N. Andreyev,[2,3] M. Huyse,[2] R. J. Liotta,[1] P. Van Duppen,[2] and R. A. Wyss[1]

[1]*KTH, Alba Nova University Center, SE-10691 Stockholm, Sweden*

[2]*Instituut voor Kern-en Stralingsfysica, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium*

[3]*School of Engineering and Science, University of the West of Scotland, Paisley PA1 2BE, United Kingdom*

(Received 26 April 2010; published 22 June 2010)

An abrupt change in α-decay systematics around the $N = 126$ neutron shell closure is discussed. It is explained as a sudden hindrance of the clustering of the nucleons that eventually form the α particle. This is because the clustering induced by the pairing mode acting upon the four nucleons is inhibited if the configuration space does not allow a proper manifestation of the pairing collectivity.

DOI: 10.1103/PhysRevC.81.064319 PACS number(s): 21.10.Tg, 23.60.+e, 27.80.+w, 21.60.Cs

I. INTRODUCTION

It was nearly a century ago that the Geiger-Nuttall law, which was to revolutionize physics by its implications, was formulated based on α-decay systematics [1,2]. Indeed, its explanation by Gamow [3] and also by Gurney and Condon [4] required acceptance of the probabilistic interpretation of quantum mechanics. The extent to which this was revolutionary can perhaps best be gauged by noticing the multitude of models that have been put forward by outstanding physicists as an alternative to the probabilistic interpretation. This debate rages even at present [5].

The Gamow theory reproduced the Geiger-Nuttall law nicely. One can assert that this is an effective theory, where concepts like "frequency of escape attempts" have to be introduced. Yet Gamow's theory is so successful that even today it is applied, with minor changes, in studies of radioactive decays (e.g., Refs. [6–8]). In fact, a proper calculation of the decay process needs to address first the clustering of the nucleons at a certain distance outside the nuclear surface and, in a second step, the evaluation of the penetrability through the Coulomb and centrifugal barriers should be performed at the distance where the cluster was formed. The first step is a challenging undertaking because a proper description of the cluster in terms of its components requires a microscopic many-body framework that is very complicated. This is the reason why usually effective approaches are used when dealing with clusterization. That is, one evaluates the penetrability, which is an easy task especially if semiclassical approaches are applied, and free parameters are introduced for the clustering process trying to reproduce experimental data.

One may then wonder why effective approaches have been so successful. The reason is that the α-particle formation probability usually varies from nucleus to nucleus much less than the penetrability. On the logarithm scale of the Geiger-Nuttall law the differences in the formation probabilities are usually small fluctuations along the straight lines predicted by that law [9] for different isotopic chains. The importance of a proper treatment of α decay was attested by a recent calculation showing that the different lines can be merged in a single line. One thus obtained a generalization of the Geiger-Nuttall law that holds for all isotopic chains and all cluster radioactivities [10,11]. In this universal decay law (UDL) the penetrability is still a dominant quantity. By using three free parameters only, one finds that all known ground-state to ground-state radioactive decays are explained rather well. This good agreement is a consequence of the smooth transition in the nuclear structure that is often found when going from a nucleus to its neighboring nuclei. This is also the reason why, for example, the BCS approximation works so well in many nuclear regions.

In this paper we will show that, when a sudden transition occurs in a given chain of nuclei, departures from the UDL can be seen. Our aim is to understand why this difference appears. We will also try to discern whether one can, in general, obtain information about the structure of the nuclei involved in the decay. This would be an important task because many regions of the nuclidic chart now under scrutiny, especially superheavy nuclei, are radioactive and often α decay is the only tool that one has to explore their structure.

In Sec. II the formation amplitude is defined. In Sec. III α-formation amplitudes extracted from experimental data are presented and abrupt changes are noted. In Sec. IV the evaluation of the formation amplitudes and half-lives of Po isotopes, which do not follow the UDL, is performed. A summary and conclusions are in Sec. V.

II. THE FORMATION AMPLITUDE

After the seminal Gamow's paper, the first attempt to formulate a proper treatment of α decay was based on the compound system theory developed by Teichmann and Wigner [12]. Here the very complicated process occurring as the compound system decays is divided into an "internal region," where the compound state is restricted, and the complementary "external region." This division is such that in the external region only the Coulomb and centrifugal forces are important. Thus the decaying system behaves like a two-particle system. This formulation was applied by Thomas to α decay [13] to obtain the classical expression for the decay width Γ_l as

$$\Gamma_l(R) = 2\mathcal{P}_l(R)\frac{\hbar^2}{2\mu R}|\mathcal{F}_l(R)|^2, \quad (1)$$

*chongq@kth.se

where l is the angular momentum carried by the outgoing α particle, $\mathcal{P}$ is the penetration probability, μ is the reduced mass corresponding to the final system consisting of an α particle and a daughter nucleus, and R is the radius dividing the internal and external regions. At this point the wave function of the α particle already formed in the internal region is matched with the corresponding outgoing two-body wave function in the external region. The amplitude of the wave function in the internal region is the formation amplitude, that is,

$$\mathcal{F}_l(R) = \int d\mathbf{R} d\xi_d d\xi_\alpha [\Psi(\xi_d)\phi(\xi_\alpha)Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_\alpha, \mathbf{R}), \tag{2}$$

where d, α, and m label the daughter, α particle, and mother nuclei, respectively; Ψ are the intrinsic wave functions and ξ the corresponding intrinsic coordinates; and $\phi(\xi_\alpha)$ is a Gaussian function of the relative coordinates of the two neutrons and two protons that constitute the α particle, coupled to zero angular momentum [14,15]. The rest of the notation is standard.

One sees from Eq. (2) that $\mathcal{F}_l(R)$ would indeed be the wave function of the outgoing α particle, $\psi_\alpha(R)$, if the mother nucleus would behave at the point R as

$$\Psi_m(\xi_d, \xi_\alpha, \mathbf{R}) = [\Psi(\xi_d)\phi(\xi_\alpha)\psi_\alpha(R)Y_l(\mathbf{R})]_{J_m M_m}. \tag{3}$$

Since this is usually a small component of the mother nucleus wave function, the corresponding formation amplitude (2) is small, of the order of 10^{-2} [16]. The main problem in the evaluation of this quantity is the description of the clusterization of the four nucleons that eventually become the α particle. In pursuing this task one has found that the mode that determines clusterization is the pairing vibration [15,17]. In fact, the study of α clusterization gave rise to the realization that there should be a giant pairing vibration lying high in the nuclear spectra [18,19]. It is also interesting to notice that the α clusterization in α-decaying nuclei has triggered the appearance of effective models where the wave function of nuclei such as ^{212}Po is assumed to have the form (3). The spectra thus obtained agree well with the corresponding experimental data [20].

Returning to Eq. (1), one has that the wave function corresponding to the external region (i.e., to the outgoing channel), gives rise to the penetration probability $\mathcal{P}_l(R) = kR/(G_l^2 + F_l^2)$, where G_l and F_l are the irregular and regular Coulomb functions, respectively. From Eq. (1) it is straightforward to see that the width $\Gamma_l(R)$ cannot depend upon R, since outside the range of the nuclear interaction (i.e., just outside the nuclear surface) the internal and external wave functions are the same [21] [i.e., $\mathcal{F}_l(R) \propto G_L(R) + iF_l(R)$]. This is of course valid provided that the formation amplitude was evaluated properly. In fact, a way of probing the calculation is just by investigating whether the width is dependent upon R, and in such a case by how much [22].

The α-decay half-life can be written as

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_l} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi, \rho)}{R\mathcal{F}_l(R)} \right|^2, \tag{4}$$

where ν is the outgoing velocity of the emitted particle. The distance R will be taken as the touching point, that is, $R = R_0(A_d^{1/3} + 4^{1/3})$, with $R_0 = 1.2$ fm. The other quantities are standard [i.e., $H_l^+(\chi, \rho)$ is the Coulomb-Hankel function with arguments $\chi = 4Z_d e^2/\hbar\nu$ and $\rho = \mu\nu R/\hbar$].

In microscopic theories the formation amplitude is evaluated starting from the single-particle degrees of freedom of the neutrons and protons that eventually become the cluster. This requires advanced computing facilities as well as suitable theoretical schemes to describe the clustering process. It is therefore not surprising that the first calculations of absolute decay widths were performed after the appearance of the shell model. These calculations had limited success owing to the small shell-model spaces that could be included at that time [14]. Yet, in retrospect it is surprising to note the deep insight the pioneers in these shell-model calculations had on the role of configuration mixing to induce clustering [23]. That this was indeed the case was shown much later [15,17] in the case of the decay of the nucleus ^{212}Po with two protons and two neutrons outside the doubly magic core, ^{208}Pb, which has been considered as a textbook example in illustrating the clustering and decay of the α particle in heavy nuclei (see, e.g., Ref. [24] and references therein). In fact this case is very important for the present paper, since the most significant departure of the UDL from experimental data that we will investigate is in the ground-state to ground-state decays of Po isotopes.

III. THE EXPERIMENTAL DATA

Using the experimental decay half-lives [25] one can extract the formation amplitudes by applying Eq. (4). One thus obtains

$$\log_{10}|R\mathcal{F}_\alpha(R)| = -\frac{1}{2}\log_{10} T_{1/2}^{\mathrm{Exp}} + \frac{1}{2}\log_{10}\left[\frac{\ln 2}{\nu}|H_0^+(\chi, \rho)|^2\right]. \tag{5}$$

This is shown in Fig. 1 for different even-even isotopes as a function of the quantity $\rho' = \sqrt{2\mathcal{A}Z_d(A_d^{1/3} + 4^{1/3})}$, where $\mathcal{A} = 4A_d/A_m$. This is one of the two variables that defines the UDL [10].

One notices in Fig. 1 that at $\rho' \approx 70$ a division occurs between decays corresponding to $N < 126$ and $N > 126$. Perhaps even more important is that for most cases the UDL predicts the experimental values within a factor of 3, except for $N = 126$, where the difference becomes about one order of magnitude. This is so distinct that one may even suspect that the difference in the values of $\mathcal{F}_\alpha$ when going from one nucleus to its neighbors in the vicinity of $N = 126$ overruns the corresponding differences in the penetrability. If one understands the reason of this large variation, α decay may provide a powerful tool to study the structure of decaying nuclei. This point will be analyzed in the next section.

The case that shows the most significant hindrance corresponds to the α decay of the nucleus ^{210}Po, with $\log_{10}|R\mathcal{F}_\alpha(R)|^2 < -3$ fm^{-1}. The symbols with $\log_{10}|R\mathcal{F}_\alpha(R)|^2 \sim -2.7$ fm^{-1} correspond to the α decays of nuclei ^{208}Po ($N = 124$), ^{212}Rn ($N = 126$), and ^{194}Pb.

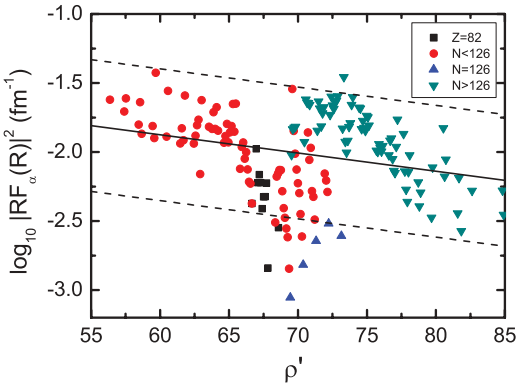

FIG. 1. (Color online) Plot of $\log_{10}|RF(R)|^2$ as a function of ρ'. The solid line denotes the smooth behavior of the UDL. The values between the two dashed lines differ from the corresponding UDL values by a factor of at most 3.

Finally, it is worthwhile to point out that this sudden change in α-decay systematics at $N = 126$ has also been noticed in Refs. [26–29]. Moreover, in the semiclassical approaches of Refs. [30–36] the decay half-lives of nuclei with $N = 126$ are significantly underestimated. In the α-decay formula of Refs. [34,35], an empirical correction term has been introduced to take into account the large underestimation around shell closures.

IV. EVALUATION OF THE HALF-LIVES IN Po ISOTOPES

In this section we will analyze, within a microscopic formalism, the half-lives of the isotopes that show the kink at $N = 126$ just discussed. We will take the decay of ^{210}Po as a typical example and compare it with that of ^{212}Po. To compare with experimental data, we extract the magnitude of the formation amplitudes from measured half-lives by using Eq. (5). One thus obtains the values $\mathcal{F}_\alpha(R) = 3.305 \times 10^{-3}$ fm$^{-3/2}$ in ^{210}Po and $\mathcal{F}_\alpha(R) = 1.082 \times 10^{-2}$ fm$^{-3/2}$ in ^{212}Po, where we used $R = 9.0$ fm. These correspond to a variation in the formation amplitudes by a factor of 3.28, that is a factor of 10.73 in the formation probabilities.

Within the shell model a four-particle state α_4 in ^{212}Po can be written as

$$|^{212}\mathrm{Po}(\alpha_4)\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\alpha_4)|^{210}\mathrm{Pb}(\alpha_2)\otimes{}^{210}\mathrm{Po}(\beta_2)\rangle, \quad (6)$$

where α_2 (β_2) labels two-neutron (two-proton) states. For the ground state of ^{212}Po, it was found [22] that $X[^{210}\mathrm{Pb(gs)}\otimes{}^{210}\mathrm{Po(gs)}] = 0.9$, whereas $X[^{210}\mathrm{Pb}(2_1^+)\otimes{}^{210}\mathrm{Po}(2_1^+)] = -0.3$.

Each of the terms in Eq. (6) corresponds to neutron-neutron (nn) or proton-proton (pp) states (i.e., states determined by the nn or pp interaction). The neutron-proton (np) interaction mixes those states. In other words, the amplitudes X are influenced by the np interaction. If this interaction is neglected, then only one of the configurations in Eq. (6) would appear. This is done, for instance, in cases where the correlated four-particle state is assumed to be provided by collective vibrational states. Rather typical examples of such states are $|^{210}\mathrm{Pb(gs)}\rangle$ and $|^{210}\mathrm{Po(gs)}\rangle$. It is therefore not surprising that calculations have been performed by assuming that $|^{212}\mathrm{Po(gs)}\rangle$ is a double pairing vibration [15,37], that is,

$$|^{212}\mathrm{Po(gs)}\rangle = |^{210}\mathrm{Pb(gs)}\otimes{}^{210}\mathrm{Po(gs)}\rangle. \quad (7)$$

The corresponding formation amplitude acquires the form

$$\mathcal{F}_\alpha[R;{}^{212}\mathrm{Po(gs)}] = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi[\mathbf{r_1},\mathbf{r_2};{}^{210}\mathrm{Pb(gs)}] \times \Psi[\mathbf{r_3},\mathbf{r_4};{}^{210}\mathrm{Po(gs)}], \quad (8)$$

where $\mathbf{r_1}$, $\mathbf{r_2}$ ($\mathbf{r_3}$, $\mathbf{r_4}$) are the neutron (proton) coordinates and $\mathbf{R}$ is the center of mass of the α particle.

With this expression for the formation amplitude the experimental half-life is reproduced rather well if a large number of high-lying configurations is included. These configurations are needed to describe the clusterization between the two neutrons and the two protons in the α particle. Yet the corresponding α-decay half-life is still too small by more than one order of magnitude. This is because the neutron-proton interaction is not included in Eq. (7). When this is done, and again a large configurations space is used, the neutrons and protons also become clustered, enhancing the value of the half-life. It is also important to underline that the inclusion of the large configuration space provides a half-life that is independent of the matching point R [22].

We reproduced these calculations by using a surface delta interaction and nine major shells of a harmonic oscillator (HO) representation. The decay of the nucleus ^{210}Po(gs) leads to the daughter nucleus ^{206}Pb(gs), which is a two-hole state. Here we used the five HO major shells corresponding to the single-hole states that describe the wave function of ^{206}Pb(gs) as

$$|^{206}\mathrm{Pb(gs)}\rangle = \sum_{h_1\leqslant h_2} X[h_1h_2;{}^{206}\mathrm{Pb(gs)}]\frac{(b^+_{h_1}b^+_{h_2})_{0^+}}{\sqrt{2}}|^{208}\mathrm{Pb(gs)}\rangle, \quad (9)$$

where h labels single-hole states and the hole creation operator is standard [i.e., $b^+_{jm} = (-1)^{j-m}c_{j-m}$]. The formation amplitude becomes

$$\mathcal{F}_\alpha[R;{}^{210}\mathrm{Po(gs)}] = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi^*[\mathbf{r_1},\mathbf{r_2};{}^{206}\mathrm{Pb(gs)}] \times [\mathbf{r_3},\mathbf{r_4};{}^{210}\mathrm{Po(gs)}]. \quad (10)$$

By comparing Eqs. (8) and (10) one sees that the only difference between the two expressions is the two-neutron wave function, which corresponds to the two-particle state ^{210}Pb(gs) in Eq. (8) and to the two-hole state ^{206}Pb(gs) in Eq. (10). Therefore the kink observed experimentally should be related to the difference in clusterization induced by the pairing force in these two cases. To analyze the clustering features we will consider only the spin-singlet component [i.e., $(\chi_1\chi_2)_0$] of the two-body wave function, since that is the only part entering the intrinsic α-particle wave function. This component has the form

$$\Psi_2(r_1,r_2;\theta_{12}) = \frac{1}{4\pi}\sum_{p\leqslant q}\sqrt{\frac{2j_p+1}{2}}X(pq;\mathrm{gs}) \times \varphi_p(r_1)\varphi_q(r_2)P_{l_p}(\cos\theta_{12}), \quad (11)$$

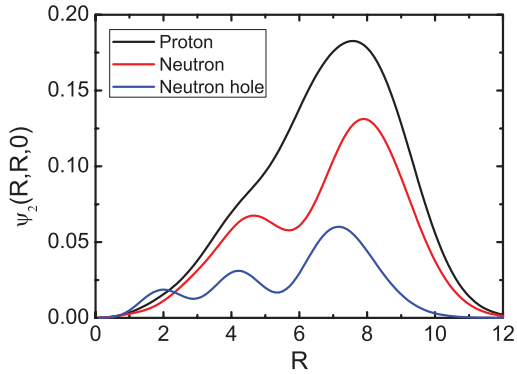

FIG. 2. (Color online) The two-body wave function $\Psi_2(R, R, 0)$ corresponding to the pairing vibrations in the two-neutron particle ^{210}Pb(gs), two-proton particle ^{210}Po(gs), and two-neutron hole ^{206}Pb(gs) cases.

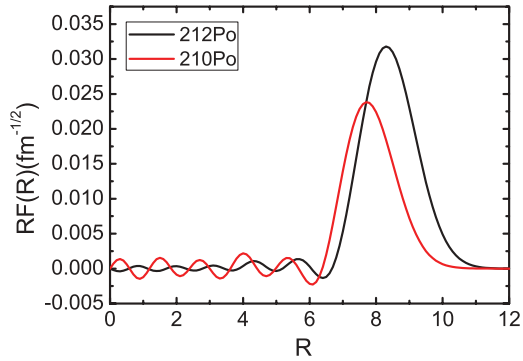

FIG. 3. (Color online) The α-formation amplitudes $R\mathcal{F}_\alpha(R)$ corresponding to the nuclei ^{212}Po(gs) and ^{210}Po(gs).

where φ is the single-particle wave function and P_l is the Legendre polynomial of order l satisfying $P_l(\cos 0) = 1$ (where for the ground states studied here $l_p = l_q$). As mentioned, the pairing vibrations show strong clustering features as the number of single-particle states is increased [17]. But another manifestation of the pairing collectivity is an enhancement of the wave function on the nuclear surface. The reason for this enhancement is that all configurations contribute with the same phase in the buildup of the two-particle wave function on the nuclear surface. The same mechanism increases the α-formation amplitude and, therefore, the relative values of the wave functions of ^{210}Pb(gs), ^{210}Po(gs), and ^{206}Pb(gs) on the nuclear surface give a measure of the importance of the corresponding formation amplitudes.

To study the behavior of the two-particle wave functions we will apply Eq. (11) with $r_1 = r_2$ and $\theta_{12} = 0$. This is reasonable since clustering causes the wave function to be strongly peaked at $\theta_{12} = 0$. Calling $R = r_1$, we have plotted in Fig. 2 $\Psi_2(R, R, 0)$ as a function of R. We see that the wave functions are indeed strongly enhanced at the nuclear surface, as expected. But the important feature for us is that the enhancement is strongest in ^{210}Pb(gs) and weakest in ^{206}Pb(gs). This is because there is a relatively small number of configurations in the hole-hole case. In addition, the radial wave functions corresponding to the high-lying particle states extend farther out in space with respect to the hole configurations.

With these two-body wave functions we proceeded to evaluate the α-formation amplitudes in ^{212}Po(gs) and ^{210}Po(gs). The results are shown in Fig. 3. From this figure we find that with $R = 9$ fm the observed ratio between the formation amplitudes in ^{212}Po and ^{210}Po can be reproduced nicely.

A. The neutron-proton interaction

We have assumed [Eq. (7)] that ^{212}Po(gs) is virtually a correlated two-neutron two-proton state. The same is valid for ^{210}Po(gs), although here the state is a correlated two-particle (proton) two-hole (neutron) state. This is a manifestation of the pairing vibrational character of two-particle states in the Pb region. That is, the correlated two-particle and two-hole states in the Pb region can be considered as boson degrees of freedom. This was one of the main assumptions in nuclear field theory [38] as well as in the original interacting boson model [39]. This assumption implies that the neutron-proton interaction does not play a very important role in the spectroscopy of the states. However, as we have seen, this interaction induces the clusterization of neutrons and protons. As pointed out in Ref. [15], in the Pb region low-lying neutron and proton single-particle states are very different from each other, or are particle-hole states. Therefore the neutron-proton interaction affects only slightly the ground states and the clusterization occurs through high-lying configurations. This point is supported by our shell-model calculations with the surface delta as well as realistic interactions.

Only when neutrons and protons move in the same orbits is it expected that the neutron-proton interaction would affect significantly the spectroscopic properties as well as the clusterization. We confirmed this by studying a model case in which the core consists of an equal number of neutrons and protons, namely the α decay of the fictional nucleus ^{168}Po(gs), with two neutrons and two protons outside the core ^{164}Pb(gs). We used, for neutrons as well as protons, the single-particle states corresponding to protons in the study performed here for ^{210}Po. We also used the same interaction. As expected, we again found that neutrons and protons are strongly clustered as a result of the corresponding pairing interaction, But also the proton-neutron clustering is significantly enhanced by the proton-neutron interaction, This indicates that in realistic $N = Z$ nuclear regions, for instance around ^{100}Sn, there should be a large probability to form an α particle (see, e.g., Ref. [40]). One can thus conclude that α-decay probes may be a powerful tool to get information about the structure of heavy $N \approx Z$ nuclei that, otherwise, would be difficult to reach.

V. SUMMARY AND CONCLUSIONS

In this paper we have applied the recently proposed universal decay law [10] to perform a systematic calculation of α-decay half-lives over all experimentally known cases. We found that although the UDL reproduces nicely most available experimental data, as expected, there is a case where it fails by a large factor. This corresponds to the α decays of nuclei

with neutron numbers equal to or just below $N = 126$. The reason for this large discrepancy is that in $N \leqslant 126$ nuclei the α-formation amplitudes are much smaller than the average quantity predicted by the UDL (Fig. 1). This is an indication that the α-decay transitions in these nuclei are hindered with respect to those in the open-shell region.

The case that shows the most significant hindrance corresponds to the α decay of the nucleus ^{210}Po, for which standard shell-model calculation is feasible. Starting from the formal definition of Eq. (2), we calculated the α-formation amplitude of ^{210}Po and compared it with that of ^{212}Po. In these two cases the formation amplitudes can be described by the simple expressions (8) and (10). We found that the formation amplitude in ^{210}Po is hindered with respect to the one in ^{212}Po owing to the hole character of the neutron states in the first case. This is a manifestation of the mechanism that induces clusterization, which is favored by the presence of high-lying configurations. Such configurations are more accessible in the neutron-particle case of ^{212}Po than in the neutron-hole case of ^{210}Po. This is a general feature in nuclei where neutrons and protons occupy different low-lying major shells. If instead both types of particles occupy the same shells, the neutron-proton interaction is very effective in inducing clustering and the formation amplitude increases strongly. This was the case in a calculation that we performed considering the fictitious $N = Z = 84$ ^{168}Po isotope as the mother nucleus, indicating that even in the physically meaningful $N = Z$ nuclear region α decay can be enhanced by large factors.

This allows one to assert that α decay is a powerful tool to investigate the shell structure of very unstable nuclei (including superheavy ones), where often only α-decay quantities can be measured.

ACKNOWLEDGMENTS

This work has been supported by the Swedish Research Council (VR), FWO—Vlaanderen (Belgium), GOA/2004/03 (BOF, K.U. Leuven), the IUAP–Belgian State–Belgian Science Policy (BriX network P6/23), and the European Commission within the Sixth Framework Programme through I3-EURONS (Contract No. RII3-CT-2004-506065).

[1] H. Geiger and J. M. Nuttall, Philos. Mag. **22**, 613 (1911).
[2] H. Geiger, Z. Phys. **8**, 45 (1922).
[3] G. Gamow, Z. Phys. **51**, 204 (1928).
[4] R. W. Gurney and E. U. Condon, Nature (London) **122**, 439 (1928).
[5] G. 't Hooft, arXiv:0908.3408, and references therein.
[6] S. K. Arun, R. K. Gupta, B. B. Singh, S. Kanwar, and M. K. Sharma, Phys. Rev. C **79**, 064616 (2009).
[7] H. F. Zhang, G. Royer, Y. J. Wang, J. M. Dong, W. Zuo, and J. Q. Li, Phys. Rev. C **80**, 057301 (2009).
[8] D. Ni and Z. Ren, Phys. Rev. C **80**, 051303(R) (2009).
[9] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. Lett. **65**, 2975 (1990).
[10] C. Qi, F. R. Xu, R. J. Liotta, and R. Wyss, Phys. Rev. Lett. **103**, 072501 (2009).
[11] C. Qi, F. R. Xu, R. J. Liotta, R. Wyss, M. Y. Zhang, C. Asawatangtrakuldee, and D. Hu, Phys. Rev. C **80**, 044326 (2009).
[12] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).
[13] R. G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).
[14] H. J. Mang, Phys. Rev. **119**, 1069 (1960).
[15] I. Tonozuka and A. Arima, Nucl. Phys. A **323**, 45 (1979).
[16] R. G. Lovas, R. J. Liotta, A. Insolia, K. Varga, and D. S. Delion, Phys. Rep. **294**, 265 (1998).
[17] F. A. Janouch and R. J. Liotta, Phys. Rev. C **27**, 896 (1983).
[18] M. W. Herzog, R. J. Liotta, and L. J. Sibanda, Phys. Rev. C **31**, 259 (1985).
[19] G. G. Dussel, R. Id Betan, R. J. Liotta, and T. Vertse, Phys. Rev. C **80**, 064311 (2009).
[20] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. C **51**, 559 (1995).
[21] E. Maglione, L. S. Ferreira, and R. J. Liotta, Phys. Rev. Lett. **81**, 538 (1998).
[22] G. Dodig-Crnković, F. A. Janouch, and R. J. Liotta, Nucl. Phys. A **501**, 533 (1989).
[23] J. O. Rasmussen, in *Alpha- Beta- and Gamma-Ray Spectroscopy*, Vol. 1, edited by K. Siegbahn (North-Holland, Amsterdam, 1965), p. 701.
[24] A. Astier, P. Petkov, M.-G. Porquet, D. S. Delion, and P. Schuck, Phys. Rev. Lett. **104**, 042701 (2010).
[25] G. Audi, O. Bersillon, J. Blachol, and A. H. Wapstra, Nucl. Phys. A **729**, 3 (2003).
[26] B. Al Bataina and J. Jänecke, Phys. Rev. C **37**, 1667 (1988).
[27] K. S. Toth *et al.*, Phys. Rev. C **60**, 011302(R) (1999).
[28] J. Wauters, P. Dendooven, M. Huyse, G. Reusen, P. Van Duppen, and P. Lievens (the ISOLDE Collaboration), Phys. Rev. C **47**, 1447 (1993).
[29] K. Van de Vel *et al.*, Phys. Rev. C **68**, 054311 (2003).
[30] B. A. Brown, Phys. Rev. C **46**, 811 (1992).
[31] D. N. Poenaru and M. Ivascu, J. Phys. **44**, 791 (1983).
[32] D. N. Poenaru and W. Greiner, Phys. Scr. **44**, 427 (1991).
[33] D. N. Poenaru, I. H. Plonski, and W. Greiner, Phys. Rev. C **74**, 014312 (2006).
[34] D. N. Poenaru, I. H. Plonski, R. A. Gherghescu, and W. Greiner, J. Phys. G **32**, 1223 (2006).
[35] D. N. Poenaru, R. A. Gherghescu, and N. Carjan, Europhys. Lett. **77**, 62001 (2007).
[36] M. Ismail, A. Y. Ellithi, M. M. Botros, and A. Adel, Phys. Rev. C **81**, 024602 (2010).
[37] F. A. Janouch and R. J. Liotta, Phys. Rev. C **25**, 2123 (1982).
[38] P. F. Bortignon, R. A. Broglia, D. R. Bes, and R. Liotta, Phys. Rep. **30**, 305 (1977).
[39] A. Arima and F. Iachello, Annu. Rev. Nucl. Part. Sci. **31**, 75 (1981).
[40] S. N. Liddick *et al.*, Phys. Rev. Lett. **97**, 082501 (2006).