New approach for α-decay calculations of deformed nuclei

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We present a new theoretical approach to evaluate α-decay properties of deformed nuclei, namely the multichannel cluster model (MCCM). The deformed α-nucleus potential is taken into full account, and the coupled-channel Schrödinger equation with outgoing wave boundary conditions is employed for quasibound states. Systematic calculations are carried out for well-deformed even-even nuclei with $Z \geqslant 98$ and isospin dependence of nuclear potentials is included in the calculations. Fine structure observed in α decay is well described by the four-channel microscopic calculation, which is performed for the first time in α-decay studies. The good agreement between experiment and theory is achieved for both total α-decay half-lives and branching ratios to the ground-state rotational band of daughter nuclei. Predictions on the branching ratios to high-spin daughter states are presented for superheavy nuclei, which may be important to interpret future observations.

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I. INTRODUCTION

Accurate calculations of absolute α-decay half-lives have been obtained with both phenomenological and microscopic methods, and some important ingredients such as various shell effects are revealed [1–14]. The main focus of these α-decay calculations is generally on favored α transitions (i.e., $\ell = 0$). Besides this, the renewed interest in α decay has recently been stimulated by the study of fine structure, which poses a tough test of traditional decay theories and a stringent challenge of experimental studies. In this case, the phenomenon of α decay is much more complicated since there exists significant coupling among different channel states. For even-even nuclei, the decay ends up in different daughter states for which the spin-parity is well known in experiment and the branching ratios can be measured with reasonable accuracy. Compared with favored transitions, such transitions involve not only the decay energy Q_α and the energy spectrum E_{J_d} in daughter nuclei but also the nonzero angular momentum $\ell \neq 0$. Moreover, the internal structure of nuclear states has some influence on these transitions as well.

There are some primary studies on α-decay fine structure of well-deformed α emitters [15–17]. They, working in the Wentzel-Kramers-Brillouin (WKB) framework, reduce α decay as a one-dimensional semiclassical problem. The decay widths for the transitions to various daughter states are evaluated at slightly different decay energies and various centrifugal barriers, ignoring the mixing of channel states during the tunneling. Within this framework, favored α transitions and, sometimes, $0^+ \to 2^+$ transitions can be described with satisfied accuracy. There is, however, a tendency toward overestimating the branching ratios for $0^+ \to 4^+$ transitions in the various WKB calculations, and the deviation of the branching ratios to 4^+ states is significantly larger than those of $0^+ \to 0^+, 2^+$ transitions. The apparent reason is that the coupling effect of various channel states is not included in these calculations. A remedy for this has been achieved by the coupled-channel approach [18–21], where α decay is understood as a three-dimensional quantum problem and the exact solution of the coupled-channel Schrödinger equation becomes necessary.

Recently we have developed the theoretical formalism based on the coupled-channel approach to study $0^+ \to 0^+, 2^+$ α transitions in deformed nuclei [20,21]. When we take into account more decay channels for α decay to the ground-state rotational band of daughter nuclei, a large increase of computational time has to be spent due to the complication of the numerical double-folding potential and also because very high precision is required to solve the coupled-channel Schrödinger equation. To overcome this difficulty, improvements have been made in two ways: On the one hand, instead of the numerical double-folding potential, we employ a simple nuclear potential of popular Woods-Saxon shape in our calculations. This potential not only has a simple and clear analytical expression but also achieves remarkable success in both nuclear structure and nuclear reactions. On the other hand, in contrast to the usual coupled-channel calculations where the multipole expansion is employed to deal with the deformed α-nucleus potential [18–21], the present study, based on an axially deformed Woods-Saxon potential, takes into full account the coupling potential between different channels using the matrix diagonalization instead. This allows one to perform a multichannel analysis of α decay in a straightforward and consistent manner. For the sake of clarity, we refer to this new approach as the multichannel cluster model (MCCM). In this work we report on a four-channel calculation of total α-decay half-lives and branching ratios to various daughter states, which is the first theoretical attempt to simultaneously describe α transitions to four daughter states within the coupled-channel framework.

This article is organized in the following way. In Sec. II, we present the detailed formulas of the calculation of α-decay half-lives and branching ratios within the MCCM, and the

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coupling α-nucleus potential connected with the deformation and orientation is discussed in detail. In Sec. III, the new theoretical results of our calculations are compared with the experimental data, and some predictions of the branching ratios to high-spin daughter states are also given for superheavy nuclei. A summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK FOR α DECAY OF DEFORMED NUCLEI

The study of α decay includes two distinct aspects. The first one is the α-formation problem closely related to nuclear structure. It concerns the probability that an α cluster is present in decaying nuclei. The other is the barrier penetration problem, which has been extensively studied since α decay was described in 1928 as a quantum mechanical tunneling effect [22,23]. Considering that the crucial role in the α-decay process is played by the quantum tunneling through the potential barrier, it is really applicable and convenient to make the analysis of α decay in the framework of the cluster model [1,2]. In the α-cluster representation of the decaying nucleus, the principal dynamical variables are, respectively, the α-core relative coordinate vector $\mathbf{r}$ and the daughter internal coordinate orientation Ω_d with respect to the laboratory reference system. The α-core Hamiltonian can be written as

$$H = -\frac{\hbar^2}{2\mu}\nabla^2_{\mathbf{r}} + H_d(\Omega_d) + V(\mathbf{r}, \Omega_d), \tag{1}$$

where μ is the reduced mass of the α-core system, $H_d(\Omega_d)$ is the intrinsic Hamiltonian of the core nucleus, describing the rotation of the core with excitation energies E_{J_d}, and $V(\mathbf{r}, \Omega_d)$ represents the interaction between the centers of mass of the core and the α particle. We restrict our attention to the situation of a spherical α cluster interacting with a reflection- and axial-symmetric core nucleus with quadrupole and hexadecapole deformations. In such a picture, one obtains the usual coupled-channel equations for radial components [20,21]

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{\ell_I(\ell_I+1)}{r^2}\right) - Q_{J_d}\right]u_I(r) + \sum_J V_{I,J}(r)u_J(r) = 0, \tag{2}$$

where the index I denotes the decay channel defined by the quantum numbers $n\ell j$ of α-core relative motion and the internal quantum numbers $J_d E_{J_d}$ of core nuclei, and the quantity Q_{J_d}, given by $Q_{J_d} = Q_0 - E_{J_d}$, is the released energy leaving the daughter nucleus in the state J_d. The interaction matrix is given by

$$V_{I,J}(r) = (\Theta_I(\hat{\mathbf{r}}, \Omega_d)||V(\mathbf{r}, \Omega_d)||\Theta_J(\hat{\mathbf{r}}, \Omega_d)), \tag{3}$$

where $\Theta_I(\hat{\mathbf{r}}, \Omega_d)$ stands for the angular component of the decay channel I, and the round brackets represent integration over all coordinates except the radial variable r.

In the microscopic theory of α decay, the basic quantity is the channel wave function describing the decay to a certain daughter state. Its eigencharacteristics $n\ell j$ are defined by the Wildermuth condition [24], which relates the α cluster to the shell model and accounts for the Pauli exclusion principle. The partial decay width is determined from the asymptotic behavior of the channel wave function. Initially, we need to deal with the interaction matrix elements consisting of nuclear and Coulomb components. The nuclear coupling component can be generated by changing the radius in the Woods-Saxon potential to a dynamical operator [25]

$$R \to R_0 + \hat{O} = R_0 + \beta_2 R_d Y_{20} + \beta_4 R_d Y_{40}, \tag{4}$$

where β_2 and β_4 are the quadrupole and hexadecapole deformation parameters of the daughter nucleus, respectively. And thus the deformed nuclear potential is written as

$$V_N(r, \hat{O}) = \frac{V_0}{1 + \exp[(r - R_0 - \hat{O})/a]}. \tag{5}$$

The parameter R_0 is well considered here and given by $R_0 = R_d + 1.17$ (fm), where the constant results from the finite size effect of the α cluster. The radius R_d and the diffuseness a have the following form, including the isospin-dependent term:

$$R_d = (1.00 + 0.39 I_d)A_d^{1/3}(\text{fm}), \tag{6a}$$

$$a = 0.50 + 0.33 I_d(\text{fm}), \tag{6b}$$

where $I_d = (N_d - Z_d)/A_d$. To evaluate the matrix elements of this deformed potential between the channel states $|I0\rangle$ and $|J0\rangle$, we first search for the eigenvalues λ_α and eigenvectors $|\alpha\rangle$ of the operator $\hat{O}$. They can be easily obtained by diagonalizing the matrix $\hat{O}$, whose elements are written in terms of the Clebsch-Gordan coefficient as follows [25]:

$$\hat{O}_{ij} = \sum_{\ell=2,4}\sqrt{\frac{(2\ell+1)(2I+1)}{4\pi(2J+1)}}\beta_\ell R_d \times [\langle I, 0, \ell, 0|J, 0\rangle]^2, \tag{7}$$

where $I = 2(i-1)$, $J = 2(j-1)$. Then, following the general quantum theories we obtain

$$V_{ij}^N(r) = \langle I0|V_N(r, \hat{O})|J0\rangle = \sum_\alpha \langle I0|\alpha\rangle\langle\alpha|J0\rangle V_N(r, \lambda_\alpha). \tag{8}$$

For the deformed Coulomb interaction $V^C(r)$, we express it to the first order in $\sum_\ell \beta_\ell Y_{\ell 0}$, and thus its matrix elements can be given by

$$\begin{aligned} V_{ij}^C(r) = {} & \left[\frac{Z_\alpha Z_d}{2R_C}(3 - r^2/R_C^2)\theta(R_C - r) + \frac{Z_\alpha Z_d}{r}\theta(r - R_C)\right]\delta_{ij} \\ & + \sum_\ell \frac{3Z_\alpha Z_d}{2\ell+1}\left[r^\ell R_C^{-(\ell+1)}\theta(R_C - r) + R_C^\ell r^{-(\ell+1)}\theta(r - R_C)\right] \\ & \times \beta_\ell\sqrt{\frac{(2\ell+1)(2I+1)}{4\pi(2J+1)}}[\langle I, 0, \ell, 0|J, 0\rangle]^2, \end{aligned} \tag{9}$$

where $R_C = 1.20(A_\alpha^{1/3} + A_d^{1/3})$ (fm), $\theta(r) = 1$ if $r \geqslant 0$ and $\theta(r) = 0$ if $r < 0$. In conclusion, the interaction matrix element is a sum of $V_{ij}^N(r)$ and $V_{ij}^C(r)$.

Next, we solve the coupled equation (2) describing the motion of the α particle in the deformed potential. The radial wave functions in Eq. (2) should be regular at the origin, $u_{n\ell j}(r \to 0) = 0$. Furthermore, they should be proportional to a purely outgoing Coulomb wave at a large distance R,

$$u_{n\ell j}(r) \simeq N_{\ell j}\left[G_\ell\left(k_{J_d} r\right) + i F_\ell\left(k_{J_d} r\right)\right], \tag{10}$$

where $N_{\ell j}$ are normalization constants determined by the boundary condition, $G_\ell(k_{J_d} r)$ and $F_\ell(k_{J_d} r)$ are, respectively, the irregular and regular Coulomb wave functions with $k_{J_d} = \sqrt{2\mu Q_{J_d}}/\hbar$. When integrating the coupled equations (2) numerically, following Refs. [20,21], we adjust the potential depth V_0 to make all channels simultaneously characterized by the experimental Q_{J_d} values. This means that the α-nucleus potential remains the same for all channels of a given α emitter. Then, we consider the Pauli exclusion principle for the nucleons in the preformed α cluster with respect to those in the core nucleus, which results in a nonlocal α-core interaction. With this in mind, the quantum number n (i.e., the number of internal nodes in the radial wave function) is chosen to satisfy the Wildermuth condition [24],

$$G = 2n + \ell = \sum_{i=1}^{4} g_i. \tag{11}$$

In this expression, g_i are the corresponding oscillator quantum numbers of the nucleons composing the cluster, whose values are required to guarantee the α cluster entirely outside of the shell occupied by the core nucleus. This is sufficient to account for the main effects of the Pauli principle, and the remaining effects are largely absorbed into the effective α-nucleus potential via the fit of the parameters. In the present study, the global quantum number G is taken as $G = 22$ for heavy and superheavy nuclei with $N > 126$. This agrees well with the previous calculations [1,6,14,20,21].

By calculating the decay probability current with the asymptotical behavior of the radial wave function, one can ultimately express the partial width of the channel ℓj in the following form [20,21]:

$$\Gamma_{\ell j}(R) = \frac{\hbar^2 k_{J_d}}{\mu} \frac{|u_{n\ell j}(R)|^2}{G_\ell\left(k_{J_d} R\right)^2 + F_\ell\left(k_{J_d} R\right)^2}. \tag{12}$$

It is worth stressing that the distance R should be large enough to exclude the coupling among various outgoing waves and the expression Eq. (12) is rather insensitive to the choice of R.

In order to compare the theoretical and experimental α-decay quantities, one may need to include the α-preformation probability which measures the extent to which the α cluster is formed at the nuclear surface. This can be taken into account by multiplying the partial decay width by a α-preformation factor P_α. This factor in principle can be expressed by the overlap integral between the wave function of the parent nucleus and the decaying-state wave function of the α-daughter system. However, it is extremely difficult to achieve these wave functions due to the insufficient knowledge of nuclear potentials involved, together with the additional complication of the nuclear many-body problem. This notwithstanding, the microscopic calculation of Varga *et al.* [26] indicates that the weight of α clustering is as high as 0.3 for the typical nucleus ^{212}Po with two protons and two neutrons outside the double closed shell. The analysis of Stewart *et al.* [27] shows that the internal amplitudes associated with the α-preformation factors are essentially constant over a wide range of even-even actinide nuclei. Experimentally, it is known that the preformation factor varies slowly in the open-shell region and has a value less than unity [28]. As before, we take the same preformation factor for all even-even nuclei. This means that there is a single free parameter for the preformation factor. In the present study, its value is taken as $P_\alpha = 0.39$, remaining the same as in the previous systematic calculations [20,21]. This procedure is consistent with Buck's model [1] and the value agrees well with both the microscopic calculations and the experimental results [26–28].

III. NUMERICAL RESULTS AND DISCUSSIONS

The main focus of our analysis is on even-even transfermium nuclei. These nuclei, typically with $Z \approx 102$ and $N \approx 150$–160, are at the gateway to the superheavy mass region, and their stability is attributed to shell effects, similar to predicted superheavy elements. Moreover, near their ground states, they are all characterized by similarly strong collective motion with regular ground-state rotational spectrum (i.e., $2^+, 4^+, 6^+, \ldots$). This affords us an excellent opportunity to test nuclear models. In a word, the α-decay study of these deformed transfermium nuclei may not only provide a stern test of α-decay theoretical models but also lay the groundwork for further researches on superheavy nuclei.

In α-decay studies, it is well known that the Q_0 value is the most crucial quantity in the evaluation of α-decay half-lives. And the excitation spectrum in the daughter nuclei, especially the energy of the 2^+ level, has a significant effect on α-decay fine structure [21]. Consequently, we use their experimental data in our calculations [29–31]. But in some α emitters under investigation, the spectroscopic information does not exist for highly excited states. For these nuclei, we parametrize the ground-state rotational band of the daughter nucleus as $E_{J_d} = \kappa J_d(J_d + 1)$, where κ is adjusted to the available experimental excitation energies. This is a good approximation, because the high-spin channels are strongly restrained by the large centrifugal barrier so that the exact location of highly excited states play a minor role in the four-channel calculations, and the results obtained from the above systematic trend are very close to the experimental values. The deformation parameters of daughter nuclei are taken from the theoretical values of Möller *et al.* [32].

Within the MCCM, we have performed a detailed α-decay study of highly deformed nuclei. Table I shows the numerical results of our evaluations for even-even nuclei. The first column of Table I denotes the decay from the ground state of the parent nucleus to the various final states of the daughter system. The second and third columns are, respectively, the decay energy Q and relative angular momentum ℓ of each

TABLE I. Comparison of the calculated α-decay half-lives and branching ratios to various daughter states (0^+, 2^+, 4^+, and 6^+) with the experimental data for well-deformed α emitters. As additional information, the deformation parameters β_2 and β_4 of daughter nuclei are also given in the table.

Decay	Q (MeV)	ℓ	b^{exp} (%)	b^{cal} (%)	$T_{1/2}^{\text{exp}}$ (s)	$T_{1/2}^{\text{cal}}$ (s)
		^{244}Cf $\to$ ^{240}Cm + α	($\beta_2 = 0.224$,	$\beta_4 = 0.087$)		
$0^+ \to 0^+$	7.329	0	75.00	73.17		
$0^+ \to 2^+$	7.294	2	25.00	25.24		
$0^+ \to 4^+$	7.203	4	–	1.59	1.66×10^3	1.47×10^3
$0^+ \to 6^+$	7.063	6	–	7.12×10^{-3}		
		^{246}Cf $\to$ ^{242}Cm + α	($\beta_2 = 0.224$,	$\beta_4 = 0.079$)		
$0^+ \to 0^+$	6.862	0	79.30	75.91		
$0^+ \to 2^+$	6.819	2	20.60	22.89		
$0^+ \to 4^+$	6.725	4	0.15	1.19	1.29×10^5	1.19×10^5
$0^+ \to 6^+$	6.574	6	1.60×10^{-2}	3.58×10^{-3}		
		^{248}Cf $\to$ ^{244}Cm + α	($\beta_2 = 0.234$,	$\beta_4 = 0.073$)		
$0^+ \to 0^+$	6.361	0	80.00	79.22		
$0^+ \to 2^+$	6.319	2	19.60	19.98		
$0^+ \to 4^+$	6.218	4	0.40	0.79	2.88×10^7	2.43×10^7
$0^+ \to 6^+$	6.064	6	–	4.58×10^{-3}		
		^{250}Cf $\to$ ^{246}Cm + α	($\beta_2 = 0.234$,	$\beta_4 = 0.057$)		
$0^+ \to 0^+$	6.128	0	84.70	80.74		
$0^+ \to 2^+$	6.086	2	15.00	18.56		
$0^+ \to 4^+$	5.987	4	0.30	0.70	4.13×10^8	3.29×10^8
$0^+ \to 6^+$	5.831	6	1.00×10^{-2}	3.91×10^{-3}		
		^{252}Cf $\to$ ^{248}Cm + α	($\beta_2 = 0.235$,	$\beta_4 = 0.040$)		
$0^+ \to 0^+$	6.217	0	84.20	82.85		
$0^+ \to 2^+$	6.174	2	15.70	16.45		
$0^+ \to 4^+$	6.073	4	0.24	0.70	8.61×10^7	1.05×10^8
$0^+ \to 6^+$	5.920	6	2.00×10^{-3}	4.92×10^{-3}		
		^{254}Cf $\to$ ^{250}Cm + α	($\beta_2 = 0.225$,	$\beta_4 = 0.030$)		
$0^+ \to 0^+$	5.926	0	83.00	85.62		
$0^+ \to 2^+$	5.884	2	17.00	13.76		
$0^+ \to 4^+$	5.783	4	–	0.62	1.69×10^9	3.17×10^9
$0^+ \to 6^+$	5.625	6	–	3.02×10^{-3}		
		^{246}Fm $\to$ ^{242}Cf + α	($\beta_2 = 0.224$,	$\beta_4 = 0.079$)		
$0^+ \to 0^+$	8.373	0	80.00	72.88		
$0^+ \to 2^+$	8.329	2	20.00	25.44		
$0^+ \to 4^+$	8.223	4	–	1.67	1.20×10^0	1.87×10^0
$0^+ \to 6^+$	8.058	6	–	1.06×10^{-2}		
		^{248}Fm $\to$ ^{244}Cf + α	($\beta_2 = 0.234$,	$\beta_4 = 0.073$)		
$0^+ \to 0^+$	7.999	0	80.00	73.90		
$0^+ \to 2^+$	7.958	2	20.00	24.49		
$0^+ \to 4^+$	7.862	4	–	1.59	3.87×10^1	3.17×10^1
$0^+ \to 6^+$	7.712	6	–	2.20×10^{-2}		
		^{250}Fm $\to$ ^{246}Cf + α	($\beta_2 = 0.234$,	$\beta_4 = 0.057$)		
$0^+ \to 0^+$	7.557	0	83.00	75.51		
$0^+ \to 2^+$	7.516	2	17.00	23.16		
$0^+ \to 4^+$	7.410	4	–	1.31	2.20×10^3	1.20×10^3
$0^+ \to 6^+$	7.249	6	–	1.49×10^{-2}		
		^{252}Fm $\to$ ^{248}Cf + α	($\beta_2 = 0.235$,	$\beta_4 = 0.040$)		
$0^+ \to 0^+$	7.153	0	84.00	79.22		
$0^+ \to 2^+$	7.111	2	15.00	19.62		
$0^+ \to 4^+$	7.015	4	0.97	1.15	9.14×10^4	4.82×10^4
$0^+ \to 6^+$	6.868	6	2.30×10^{-2}	1.37×10^{-2}		

TABLE I. *(Continued.)*

Decay	Q (MeV)	ℓ	b^{exp} (%)	b^{cal} (%)	$T_{1/2}^{\mathrm{exp}}$ (s)	$T_{1/2}^{\mathrm{cal}}$ (s)
			^{254}Fm $\to$ ^{250}Cf + α ($\beta_2 = 0.245$, $\beta_4 = 0.026$)			
$0^+ \to 0^+$	7.307	0	85.00	81.09		
$0^+ \to 2^+$	7.264	2	14.20	17.81		
$0^+ \to 4^+$	7.163	4	0.82	1.08	1.17×10^4	1.01×10^4
$0^+ \to 6^+$	7.008	6	6.60×10^{-3}	1.30×10^{-2}		
			^{256}Fm $\to$ ^{252}Cf + α ($\beta_2 = 0.236$, $\beta_4 = 0.015$)			
$0^+ \to 0^+$	7.027	0	85.00	84.44		
$0^+ \to 2^+$	6.981	2	15.00	14.66		
$0^+ \to 4^+$	6.875	4	–	0.89	1.17×10^5	1.33×10^5
$0^+ \to 6^+$	6.708	6	–	6.85×10^{-3}		
			^{252}No $\to$ ^{248}Fm + α ($\beta_2 = 0.235$, $\beta_4 = 0.049$)			
$0^+ \to 0^+$	8.551	0	75.00	75.43		
$0^+ \to 2^+$	8.507	2	25.00	22.96		
$0^+ \to 4^+$	8.404	4	–	1.59	3.91×10^0	2.68×10^0
$0^+ \to 6^+$	8.243	6	–	2.76×10^{-2}		
			^{254}No $\to$ ^{250}Fm + α ($\beta_2 = 0.235$, $\beta_4 = 0.033$)			
$0^+ \to 0^+$	8.223	0	–	77.87		
$0^+ \to 2^+$	8.179	2	–	20.63		
$0^+ \to 4^+$	8.076	4	–	1.48	5.67×10^1	3.04×10^1
$0^+ \to 6^+$	7.915	6	–	2.48×10^{-2}		
			^{256}No $\to$ ^{252}Fm + α ($\beta_2 = 0.245$, $\beta_4 = 0.018$)			
$0^+ \to 0^+$	8.582	0	87.00	80.35		
$0^+ \to 2^+$	8.535	2	13.00	18.32		
$0^+ \to 4^+$	8.427	4	–	1.31	2.93×10^0	1.77×10^0
$0^+ \to 6^+$	8.256	6	–	2.19×10^{-2}		
			^{256}Rf $\to$ ^{252}No + α ($\beta_2 = 0.236$, $\beta_4 = 0.024$)			
$0^+ \to 0^+$	8.930	0	–	78.89		
$0^+ \to 2^+$	8.883	2	–	19.56		
$0^+ \to 4^+$	8.776	4	–	1.52	2.00×10^0	9.89×10^{-1}
$0^+ \to 6^+$	8.609	6	–	2.65×10^{-2}		
			^{258}Rf $\to$ ^{254}No + α ($\beta_2 = 0.246$, $\beta_4 = 0.011$)			
$0^+ \to 0^+$	9.250	0	–	80.26		
$0^+ \to 2^+$	9.206	2	–	18.26		
$0^+ \to 4^+$	9.105	4	–	1.45	9.23×10^{-2}	9.43×10^{-2}
$0^+ \to 6^+$	8.946	6	–	2.71×10^{-2}		
			^{260}Sg $\to$ ^{256}Rf + α ($\beta_2 = 0.247$, $\beta_4 = -0.007$)			
$0^+ \to 0^+$	9.923	0	83.00	83.66		
$0^+ \to 2^+$	9.872	2	17.00	15.23		
$0^+ \to 4^+$	9.753	4	–	1.09	7.20×10^{-3}	6.88×10^{-3}
$0^+ \to 6^+$	9.566	6	–	1.65×10^{-2}		

decay channel. The experimental and theoretical values of the branching ratios to various daughter states are listed in columns 4 and 5. The experimental and theoretical α-decay half-lives are given in the last two columns. As additional information, the values of deformation parameters β_2 and β_4 are also listed in the table. As one can see, there is good agreement in both the branching ratios and the total α-decay half-lives. In particular, the experimental half-lives are well reproduced with a factor of less than 2. For the branching ratios, the most accurate results are those of the transitions from ground states to ground states and from ground states to first excited 2^+ states. The results for the transitions to second excited 4^+ states are less accurate. For the transitions to excited 6^+ states, the results are slightly worse but acceptable.

As one would expect, it is very difficult to calculate a small component accurately and safely because it is rather sensitive to various factors in a complicated system. It is known from available experimental cases that the branching ratio to the 4^+ state of the daughter nucleus is generally less than 1% and that to the 6^+ state is even less than 0.02%. Obviously, the branching ratios to highly excited states indeed belong to the category of small components. On the other hand, with the increasing of ℓ, the error bar of the experimental data becomes larger and larger, and the branching ratios of many nuclei need to be measured with improved accuracy. So it is not so surprising that there are slightly large deviations for the branching ratios to highly excited states.

Compared with the previous WKB studies, our results give a preferable description of α-decay fine structure, especially for the transitions to highly excited states. The reason for this is that the significant coupling of different decay channels is included in our calculations. Additionally, by extending the systematic trend of excitation energies E_{J_d} in daughter nuclei, we make some predictions of the α-decay branching ratios to high-spin daughter states for superheavy nuclei within the MCCM framework. This predictive power may be a very useful tool for estimating the α-decay properties of superheavy nuclei to be studied. It will be of great interest to compare these theoretical predictions with future experimental measurements.

IV. SUMMARY

In summary, we present in this paper a detailed α-decay study of well-deformed even-even nuclei by the multichannel cluster model (MCCM). Based on an axially deformed Woods-Saxon potential, the coupling potential is taken into full account in terms of the general quantum theories, and the decay width is computed by utilizing the quasibound solution of the coupled-channel Schrödinger equation. This is the first four-channel calculation of α decay within the framework of the coupled-channel approach. Our calculations take into account the coupled-channel effect resulting from nuclear deformation and the internal structure effect of nuclear states. The results shown in Table I are very satisfactory. Some predictions of the branching ratios to high-spin daughter states are made for superheavy nuclei. A precise measurement of these branching ratios is a good test of our calculations, and will give us valuable guidance to improve α-decay studies for the superheavy mass region. It will be interesting and desired to use general quantum theories and reasonable structure ideas to achieve more information on α decay.

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