Strong $M1$ components in $3_i^- \rightarrow 3_1^-$ transitions in nearly spherical nuclei: Evidence for isovector-octupole excitations

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An evaluation of data obtained in $(n,n'\gamma)$ experiments reveals strong $M1$ $3_i^- \rightarrow 3_1^-$ transitions in nuclei near the $N = 50$ (^{92}Zr, ^{94}Mo, and ^{96}Mo), $Z = 50$ (^{112}Cd and ^{114}Cd), and $N = 82$ (^{144}Nd) shell closures. The observed $\langle 3_1^- || M1 || 3_i^- \rangle$ matrix elements scale with the $\langle 2_1^+ || M1 || 2_{ms}^+ \rangle$ matrix elements connecting the mixed-symmetric and symmetric quadrupole excitations. In accordance with a picture of a mixed two-component quantum system, the energy difference between the initial 3_i^- state and the 3_1^- octupole phonon is proportional to the $|\langle 3_1^- || E3 || 0_{gs}^+ \rangle|$ matrix element. The possibility of assigning the 3^- states of interest as octupole isovector states is discussed.

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I. INTRODUCTION

The nucleus, a two-component quantum system, is a unique laboratory for the investigation of single-particle behavior and collective degrees of freedom. For the latter, the two-component nature results in the possibility of coupling the eigenvectors of the proton and neutron entities in a symmetric and an antisymmetric manner [1]. The antisymmetric coupling is also referred to as isovector or mixed symmetric (ms). Mixed-symmetry excitations [2] were predicted within the interacting boson approximation (IBA) [3,4], a radical truncation of the shell model. Within the sdf-IBA-2, pairs of nucleons are coupled to form s bosons ($J^\pi = 0^+$), d bosons ($J^\pi = 2^+$), and negative-parity f bosons ($J^\pi = 3^-$). In this extension of the IBA, proton and neutron bosons are distinguished by the F-spin quantum number, which is the boson analog of isospin for fermions. States with maximum F spin are fully symmetric to the exchange of protons and neutrons, while states with an F spin less than F_{max} are of ms character.

Since their prediction and the first experimental observations [5–7] more than 20 years ago, a large number of examples of the 1_{ms}^+ scissors mode [8,9] and 2_{ms}^+ states in spherical nuclei [10] have been observed. The experimental signature for the 2_{ms}^+ state is a strong $M1$ decay to the symmetric one-phonon 2_1^+ state and a weakly collective $E2$ decay to the ground state. Strong evidence for these states as building blocks of a second vibrational structure was provided by the identification of $[2^+ \otimes 2_{ms}^+]$-multiphonon states in several nuclei, for example, ^{92}Zr [11], ^{94}Mo [12], and ^{136}Ba [13]. The evolution of symmetric and ms states with the underlying shell structure and valence particle number is determined by the quadrupole-quadrupole residual interaction and the isovector quadrupole-quadrupole proton-neutron interaction.

The symmetric octupole phonon, the next multipole order, is well established in spherical nuclei [14]. The excitation energies and the excitation strengths are dependent on the shell structure. As long as particle-hole excitations across a shell closure can be excluded, negative-parity demands the involvement of the unique parity subshell in the two-quasiparticle components of the collective wave function. In the case of a large configuration space for $\Delta j = 3$ and $\Delta l = 3$ particle-hole excitations, strong octupole correlations driven by the long-range octupole-octupole interaction are observed.

A candidate for the fourth building block of vibrational structures in spherical nuclei, the octupole ms phonon, 3_{ms}^-, has been proposed for ^{94}Mo [12]. In this nucleus, the 3^- state at 3011 keV exhibits a strong $M1$ component in its decay to the first excited 3^- state (2534 keV). Within the $U_{\pi\nu}(1) \otimes U_{\pi\nu}(5) \otimes U_{\pi\nu}(7)$ limit of the sdf-IBA-2 [15], the decay strength of an octupole ms state to the symmetric octupole state has been predicted to be

$$B(M1, 3_{ms}^- \rightarrow 3_1^-) = \frac{9}{\pi}(g_\pi - g_\nu)^2 \frac{N_\pi N_\nu}{N^2}, \tag{1}$$

where g_ρ ($\rho = \pi$ or ν) are the proton (π) and neutron (ν) g factors, N_ρ are the boson numbers, and N the total boson number. As the predicted isovector excitation must have contributions from proton and neutron components in its wave function, the number of nuclei in which it can be expected to be observed is limited. Promising parts of the nuclear landscape are the mass regions where the Fermi level is below or within the unique-parity subshell for protons and neutrons. Because of blocking, only a limited number of configurations contribute to an octupole excitation when, for at least one type of nucleons, the unique-parity subshell is completely filled.

For isovector octupole excitations strong $E1$ transitions to the symmetric and ms 2^+ states are predicted [15], in addition to the $M1$ fingerprint. These transitions are in agreement with the two-body nature of the $E1$ operator [10,15,16] derived in the sdf-IBA-2. $E1$ transitions, as, for example, those observed for ^{94}Mo, connecting a 3^- state to states of higher phonon order (e.g. $[2_1^+ \otimes 2_1^+]$-coupled two-phonon states) would demand even a three-body term in the $E1$ operator.

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TABLE I. Excitation energies of the initial 3_i^- and the first excited 3_1^- states, $M1$ strengths of the connecting transitions, $M1$ strengths of the transition(s) connecting the mixed-symmetric and symmetric quadrupole phonons, and ratios of transition matrix elements.

Nucleus	$E_{3_i^-}$ (keV)	$E_{3_1^-}$ (keV)	$B(M1, 3_i^- \to 3_1^-)$ (μ_N^2)	$B(M1, 2_{ms}^+ \to 2_1^+)$ (μ_N^2)	$\frac{\lvert\langle 3_1^- \lvert\lvert M1 \rvert\rvert 3_i^- \rangle\rvert}{\lvert\langle 2_1^+ \lvert\lvert M1 \rvert\rvert 2_{ms}^+ \rangle\rvert}$	Ref. No.
^{92}Zr	3039.8	2339.6	0.25(5)	0.37(4)	0.96(11)	[11]
^{94}Mo	3011.5	2533.8	0.39(10)	0.56(5)	0.99(17)	[12]
^{96}Mo	3178.7	2234.6	0.04(1)	0.17(2)	0.57(11)	[21]
^{112}Cd	2416.0[a]	2005.2	0.25(8)	0.099(7)	1.87(36)	[22]
	2866.7		0.06(4)		0.92(35)	
^{114}Cd	2384.6[a]	1958.0	0.076(19)	0.089(9)	1.09(19)	[23]
	(2871.6)		<0.06		<1.02	
^{144}Nd	2606.0[a]	1551	0.008(2)	0.32(5)		[24]
	2779.0		0.09(6)		0.65(26)[b]	
	(2868.2)		0.017^{+46}_{-17}			

[a]$[2_1^+ \otimes 3_1^-]_{3^-}$ state, for a discussion see text.
[b]The value contains the sum of the $M1$ strength from the 2606- and 2779-keV levels.

The quasiparticle phonon model [17] has proven to be able to describe low-lying collective excitations and the electromagnetic transitions connecting them with great accuracy (e.g., see Ref. [18] and references therein). Within this model, a boson mapping is done using the phonons obtained by solving the equations of the random phase approximation. This model is particularly successful in describing $E1$ transitions, which violate the ideal boson picture, for example, the previously discussed two- or three-phonon exchanging transitions. This results because the $E1$ operator splits into boson-allowed and, owing to the fermionic structure of the bosonic phonons, boson-forbidden parts [19].

In this work, octupole states in different nuclei are presented that exhibit strong $M1$ components in their decays to the first excited octupole states. Experimental difficulties are discussed and uncertainties in the data sets are noted (Sec. II). The possibilities of $M1$ spin-flip contributions and the mixing with 3^- states originating from other excitations (e.g., quadrupole-octupole $[2_1^+ \otimes 3_1^-]_{3^-}$, two-quasiparticle states) are discussed. Finally, an interpretation of systematic trends of the observed $M1$ transitions as fingerprints of octupole isovector excitations is given within the picture of an interacting two-component quantum system (Sec. III).

II. EVALUATED DATA

For the identification of candidates for octupole isovector excitations, absolute transition rates are required for decays from 3^- states. The $(n,n'\gamma)$ reaction [20] has been shown to populate the states of interest and can provide the required data, that is, level lifetimes, multipole-mixing ratios, and branching ratios. Consequently, an evaluation of data sets obtained with the inelastic neutron scattering technique using monoenergetic neutrons in the entrance channel was performed. This evaluation revealed corresponding $M1$ transitions between 3^- states in several other nuclei, besides the already mentioned ^{94}Mo [12], near the $N = 50$ (^{92}Zr [11], ^{96}Mo [21]), $Z = 50$ (^{112,114}Cd [22,23]), and $N = 82$ (^{144}Nd [24]) shell closures. Here a systematic overview of several 3_i^- ($i > 1$) states showing enhanced $3_i^- \to 3_1^-$ $M1$ transitions is given. The experimental data, together with the references, are listed in Table I and examples of the observed decay behavior are shown in Figs. 1–3. For completeness it must be mentioned that other data sets for spherical nuclei obtained with the same method did not contain obvious candidates (e.g., see Refs. [25] and [26]).

The states under investigation are found in the energy region (>2.7 MeV) where the level density is already high. This complexity inflicts experimental difficulties that lead to ambiguities in the evaluated data sets or make an identification impossible. The ambiguities in the data sets for which a candidate could be assigned shall be acknowledged. For ^{92}Zr [11] the parity of the candidate is not firmly assigned; however, the evidence for negative parity is that the two observed multipole-mixing parameters tending toward $\delta = 0$. The multipole-mixing parameter $\delta = 0.02^{+0.03}_{-0.02}$ for the 2105.2-keV transition to the first excited 2^+ state would fit well for an $E1$ transition. Additionally, the short level lifetime supports a negative-parity assignment. In ^{112}Cd [22] the 3_2^- (2416 keV) and 3_3^- (2867 keV) states show enhanced $M1$ transitions to the 3_1^- octupole phonon. For the decay from the 3_2^- state (2385 keV) to the 3_1^- state in ^{114}Cd, the multipole-mixing parameter δ is not uniquely determined. From a comparison to ^{112}Cd [23], the $\delta = 1.2(3)$ solution is more likely, as it results in a similar strength for the $E2$ component. The state at 2871 keV also decays to the first excited 3^- state. The spin of this state is limited to $J = 2$ or 3. No multipole-mixing ratio is known for the $(2, 3) \to 3_1^-$ transition, but for pure $M1$ multipolarity the decay to the octupole phonon has an upper limit of $B(M1) = 0.06\,\mu_N^2$. The consideration of this state is critical and is based on the comparison to ^{112}Cd. For ^{144}Nd [24] the transition connecting the first and third excited 3^- states is multiply placed in the level scheme and its strength is divided. Furthermore, a second candidate can be found at 2868 keV. For this state the spin is ambiguously assigned as (2,3), but the

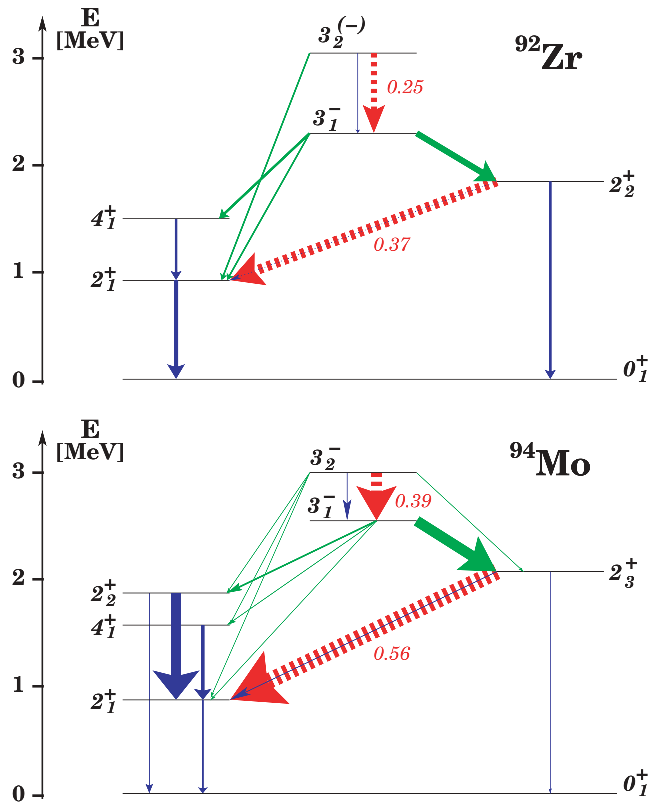

FIG. 1. (Color online) Partial level schemes of ^{92}Zr and ^{94}Mo, including the octupole excitations and their observed decay channels. Isovector $M1$ components of transitions are shown by dashed (red) arrows. The observed $B(M1)$ strength (μ_N^2) is given adjacent to the arrows. Additional $E2$ strength is indicated by a second (blue) arrow. $E1$ transitions (green arrows) and $E2$ transitions (blue arrows) are shown. The widths of the arrows are proportional to the observed transition strengths.

negative parity is firm. The $M1$ strength of the decay to the 3_1^- state is $B(M1) = 0.017^{+46}_{-17}\,\mu_N^2$. The aforenoted uncertainties underline the spectroscopic difficulties caused by the high level density.

III. DISCUSSION

In this section the origin of the observed strong $3_i^- \rightarrow 3_1^-$ $M1$ transitions is discussed. Considerations including the microscopic structure of octupole excitations show that, within a given shell, an alternative explanation of the observed $M1$ strength as $[(n+1)l'_{l'+1/2}, nl_{l+1/2}] \rightarrow [(n+1)l'_{l'+1/2}, nl_{l-1/2}]$ spin-flip transition strength is not possible. As a negative-parity excitation has to include the unique-parity subshell, the minimum angular momentum that the second combination can couple to is $J^\pi = 4^-$. For example, in the $N = 50$ region neither the neutron $[1h_{11/2}, 2d_{3/2}]$ configuration nor the proton $[1g_{9/2}, 2p_{1/2}]$ configuration can contribute to a 3^- state. This consideration is of particular importance as $\Delta j = 3$, $\Delta l = 3$: $\nu[1h_{11/2}, 2d_{5/2}]_{3^-}$ and $\pi[1g_{9/2}, 2p_{3/2}]_{3^-}$ configurations are known to be the dominant components of the octupole excitation [14]. However, when particle-hole

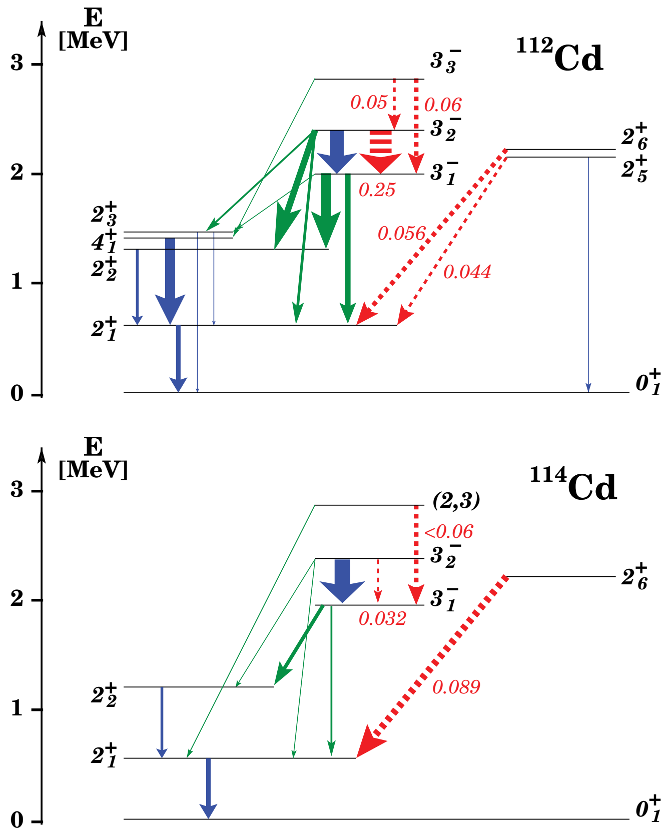

FIG. 2. (Color online) Partial level schemes of ^{112}Cd and ^{114}Cd, including the octupole excitations and their observed decay channels. $M1$ components of transitions are given as dashed (red) arrows. The observed $B(M1)$ strength (μ_N^2) is given adjacent to the arrows. Additional $E2$ strength is indicated by a second (blue) arrow. For the $3_2^- \rightarrow 3_1^-$ transition in ^{114}Cd the multipole-mixing ratio ($\delta = 1.2$) was used. The assignment of the 2871-keV state is critical. Furthermore, $E1$ transitions (green arrows) and $E2$ transitions (blue arrows) are shown. The widths of the arrows are proportional to the observed transition strengths. A detailed discussion is given in the text.

excitations across the shell closure cannot be excluded, spin-flip excitations including the unique-parity subshell can arise. In the mass regions where the candidates are observed, this can be the case for protons ($1g_{9/2} \rightarrow 1g_{7/2}$) and neutrons ($1g_{9/2} \rightarrow 1g_{7/2}$, $1h_{11/2} \rightarrow 1h_{9/2}$). The $M1$ spin-flip strength between the spin-orbit partners can contribute to the total transition strength. Evidence against $M1$ spin-flip excitations across the $N = 82$ shell closure is provided by recent (γ, γ') experiments [27]. For ^{138}Ba the 1^+ spin-flip excitations were found above 6 MeV in excitation energy. Similar behavior can be expected for the Zr and Mo isotopes as, in their mass region, no neutron intruder states have been observed. Consequently, for these nuclei the observed $M1$ strength can be attributed to a change in orbital angular momentum. However, spin-flip influences can be expected for the two Cd isotopes, as they are situated in a mass region that is well known for intruder configurations [28,29] at low energies. This becomes even more apparent in the context of recent measurements that showed enhanced transition rates in the light Sn isotopes

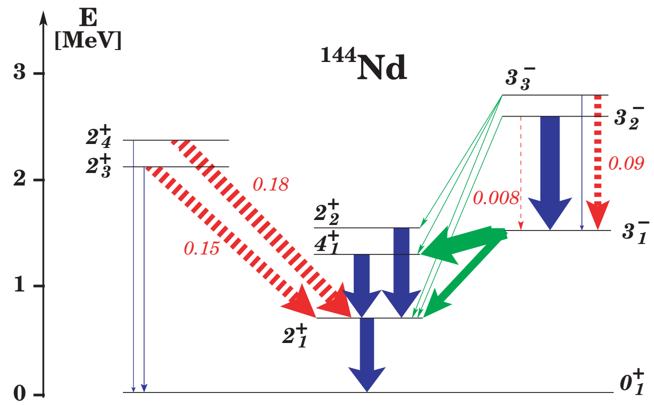

FIG. 3. (Color online) Partial level scheme of ^{144}Nd [24], including the octupole excitations and their observed decays. The isovector $M1$ components of transitions are given as dashed (red) arrows. The observed $B(M1)$ strength (μ_N^2) is given adjacent to the arrows. Additional $E2$ strength is indicated by a second (blue) arrow. Furthermore, $E1$ transitions (green arrows) and $E2$ transitions (blue arrows) are shown. The widths of the arrows are proportional to the observed transition strengths. A detailed discussion is given in the text.

(see Refs. [30] and [31]). The enhanced transition strength is explained by proton excitations across the $Z = 50$ shell closure contributing to the wave functions of the 2_1^+ and 3_1^- states.

The close proximity of the states of interest to the excitation energy of the quadrupole-octupole coupled $[2_1^+ \otimes 3_1^-]_{1^-}$ state [32,33], led to them being considered as candidates for the 3^- member of this quintuplet. In a pure phonon picture, a quadrupole-octupole coupled state is expected to decay with pure $E2$ multipolarity to the 3_1^- state, with an $E2$ strength corresponding to the annihilation of the quadrupole phonon. Owing to the previous considerations, which exclude the explanation of the observed $M1$ strength as spin-flip, for most of the states listed in Table I an interpretation of the observed 3^- states as quadrupole-octupole coupled $[2_1^+ \otimes 3_1^-]_{3^-}$ states can be excluded. A mixture of the states presented in Table I with the quadrupole-octupole coupled 3^- state cannot be excluded, because, for some of the isovector-octupole candidates, the transition of interest contains additional $E2$ components. States for which an $E2$ strength comparable to or larger than the $B(E2, 2_1^+ \to 0_{gs}^+)$ strength have been observed are considered to have a dominant quadrupole-octupole component in their wave function. These states are indicated. Examples for which such behavior is observed are ^{94}Mo [12] and ^{96}Mo [21]. This finding raises, for these nuclei, the question of the energy at which the quadrupole-octupole coupled 3^- state is to be found.

For ^{144}Nd the state at 2779 keV was assigned as a member of the quadrupole-octupole quintuplet (Table VI in Ref. [24]). Of the observed electromagnetic decay properties published in that reference, it is the state at 2606 keV that exhibits the characteristic properties expected for a member of this two-phonon multiplet [34]. This assignment is confirmed, as K-conversion coefficient measurements following the thermal neutron capture by ^{143}Nd of the 1095-keV transitions indicate an $E2$ component of at least 75% [34]. This state decays with the $E2$ strength, $B(E2, 3_2^- \to 3_1^-) = 23(4)$ W.u. to the 3_1^- state [24], to be compared with the transition strengths from the annihilation of the quadrupole phonon, $B(E2, 2_1^+ \to 0^+) = 17(1)$ W.u. As shown in Fig. 3 the $3_3^- \to 3_1^-$ decay contains only a small $E2$ component, $B(E2, 3_3^- \to 3_1^-) = 2.4^{+14}_{-11}$ W.u. The observation that approximately 10% of the summed $M1$ strength from both the 3_2^- and the 3_3^- states is found in the $3_2^- \to 3_1^-$ transition and approximately 10% of the summed $E2$ strength in the $3_3^- \to 3_1^-$ transition indicates mixing of the two excitations. The existing data set does not contain information about the origin of the additional candidate at 2868 keV. At the observed excitation energy above the pairing gap, a noncollective two-quasiparticle structure is likely. The large error in the $M1$ transition strength from that state prevents any conclusions about its origin.

The picture is more complicated for the two Cd isotopes. The strong $E2$ component in the transitions to the octupole phonon identifies the 3^- states at 2416 keV (^{112}Cd) and 2384 keV (^{114}Cd) as candidates for quadrupole-octupole coupled states [35]. In addition to the expected $E2$ strength, strong $M1$ components are observed. As mentioned previously, in the neighboring Sn isotopes, strong evidence for proton excitations across the $Z = 50$ shell closure is observed. Consequently, the presence of strong $M1$ components can be attributed to $[qp, g_{7/2}] \to [qp, g_{9/2}]$ spin-flip transitions. This conclusion is supported, as the $M1$ strength in ^{112}Cd compared to the corresponding transition in ^{114}Cd is three times larger, and in the Sn isotopes the proton admixtures are confirmed to be stronger in the lighter isotopes. Thus, despite the large $M1$ components in the transitions to the first excited 3^- states, the quadrupole-octupole coupled structure of these states is firm. In ^{112}Cd the 3_3^- state at 2866 keV exhibits an $M1$ transition with a strength comparable to that observed for the $2_{ms}^+ \to 2_1^+$ transition. For ^{114}Cd the state at 2872 keV is the only state near the excitation energy of the 3_3^- state in ^{112}Cd for which a decay to the first 3_1^- state has been observed. Assuming pure $M1$ multipolarity, the upper limit for the transition strength is $0.06\,\mu_N^2$.

Despite the preceding considerations of (intruder) single-particle configurations that may mix into the observed 3^- states, the observation of unexpectedly strong $3_i^- \to 3_1^-$ $M1$ transitions in spherical nuclei in different mass regions indicates that they originate from a collective excitation, rather than just occurring randomly owing to large g factors for a given two-quasiparticle combination. Furthermore, as shown in Fig. 4 and the last column in Table I, the $|\langle 3_1^- || M1 || 3_i^- \rangle|$ transition matrix elements scale with the corresponding $|\langle 2_1^+ || M1 || 2_{ms}^+ \rangle|$. Given the collective nature of the 2_{ms}^+ state, this is a strong indicator of dominant collective octupole components in the discussed 3^- states as well. The $B(M1, 2_{ms}^+ \to 2_1^+)$ strength in the $U(5)$ limit of the sd-IBA-2 is predicted to be

$$B(M1, 2_{ms}^+ \to 2_1^+) = \frac{9}{2\pi}(g_\pi - g_\nu)^2 \frac{N_\pi N_\nu}{N^2}. \quad (2)$$

Consequently, the ratio

$$\frac{|\langle 3_1^- || M1 || 3_i^- \rangle|}{|\langle 2_1^+ || M1 || 2_{ms}^+ \rangle|} = \sqrt{\frac{14}{5}}\, \frac{g_\pi^{(3)} - g_\nu^{(3)}}{g_\pi^{(2)} - g_\nu^{(2)}} \quad (3)$$

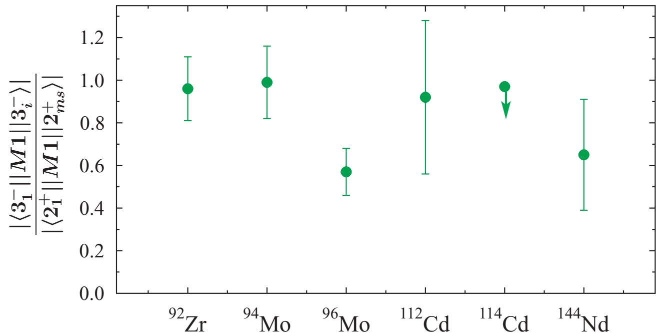

FIG. 4. (Color online) Ratio of the $|3_i^-\rangle \to |3_1^-\rangle$ and $|2_{\rm ms}^+\rangle \to |2_1^+\rangle$ $M1$ matrix elements. For a discussion see the text.

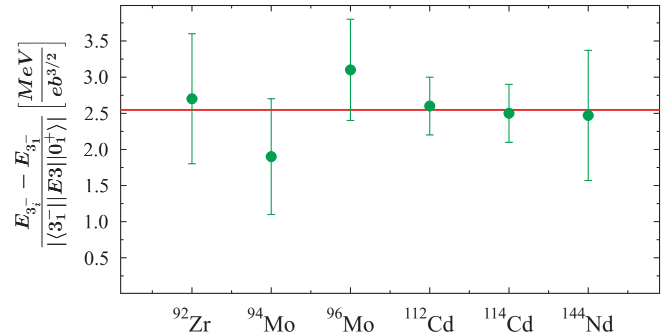

FIG. 5. (Color online) Ratio of the energy difference between the mixed-symmetry candidate and the first excited octupole state, $\Delta E = E_{3_i^-} - E_{3_1^-}$, and the $|\langle 3_1^-||E3||0_{\rm gs}^+\rangle|$ transition matrix element. The horizontal line gives the average value of 2.53 MeV/eb$^{3/2}$.

depends only on the $g^{(J)}$ factors ($J = 2$ or 3) of the involved states. The use of bare g factors ($g_\pi^{(J)} = 1$ and $g_\nu^{(J)} = 0$) simplifies the ratio in Eq. (3) to $\sqrt{14/5}$. The experimental values shown in Fig. 4 deviate from the absolute value but stay rather constant. However, missing, that is, unobserved, strength owing to fragmentation of collective 3^- states over several levels caused by the enhanced level density is likely. Except for ^{144}Nd, the possible fragmentation of the $3_i^- \to 3_1^-$ $M1$ strength into several transitions is not considered.

In a picture for which the isovector part of the residual octupole-octupole interaction is the driving force that mixes proton and neutron configurations to symmetric and antisymmetric combinations, the energy splitting between these configurations ($\Delta E = E_{3_i^-} - E_{3_1^-}$) is proportional to the strength of the driving force. A plausible approach for the octupole-octupole residual interaction, V_o, is a superposition of isocalar, $V_{o,\rm is}$, and isovector, $V_{o,\rm iv}$, parts of the form

$$V_o = V_{o,\rm is} + V_{o,\rm iv}. \tag{4}$$

Justification for this approach is provided by first-order meson exchange. The isoscalar part describes the exchange of uncharged mesons, and the isovector part the exchange of charged mesons. The two parts of the octupole interaction can be approached as $V_{o,\rm is} = \kappa_{\rm is}\hat{O}_\rho\hat{O}_\rho$ ($\rho = \pi$ or ν) and $V_{o,\rm iv} = \kappa_{\rm iv}\hat{O}_\rho\hat{O}_{\rho'}$, with ($\rho \neq \rho'$). For isovector states with $J > 2$, quasiparticle phonon model calculations have so far assumed a constant ratio ($\kappa_{\rm iv}/\kappa_{\rm is} = -1.5$) of isovector $\kappa_{\rm iv}$ and isoscalar $\kappa_{\rm is}$ coupling constants (see discussion in Ref. [36], chap. III, and references therein), because of the lack of available data.

Following that approach, the isovector part of the octupole-octupole residual interaction can be expected to scale with the total octupole-octupole residual interaction. A good measure for the latter is the $|\langle 3_1^-||E3||0_{\rm gs}^+\rangle|$ matrix element [14]. Thus, within the picture of an interacting two-component quantum system, the energy difference of symmetric and antisymmetric coupled states can be expected to scale with the $|\langle 3_1^-||E3||0_{\rm gs}^+\rangle|$ matrix element. The corresponding data are reported in Table II and shown in Fig. 5. An energy shift of the assumed isovector-octupole state owing to mixing with the quadrupole-octupole coupled 3^- state or two-quasiparticle excitations was not taken into account, because of the uncertainties in the evaluated data sets.

The ratio $\Delta E/\langle 3_1^-||E3||0_1^+\rangle$ is constant over all nuclei where data are available, with a fitted average value of

$$\frac{E_{3_i^-} - E_{3_1^-}}{|\langle 3_1^-||E3||0_1^+\rangle|} = 2.53\left[\frac{\rm MeV}{\rm eb^{3/2}}\right]. \tag{5}$$

Using that value and the matrix elements found in Ref. [37], an isovector octupole state can be predicted to be at an excitation energy of 2817^{+117}_{-140} keV for ^{94}Zr and at 2790(35) keV for ^{142}Ce. For the latter several spin-3 states are observed [26] in the energy region. However, the observed multipole-mixing ratios lead to assignments of positive parity for all of them. For ^{94}Zr [25], a negative-parity state is observed at 3089 keV. The decay of this state to the first excited 3_1^- state exhibits an $M1$ strength of $B(M1, (4)^- \to 3_1^-) = 0.11\,\mu_N^2$. The tentative

TABLE II. The energy splitting ΔE between the mixed-symmetric candidate and the symmetric octupole phonon, the $|\langle 3_1^-||E3||0_{\rm gs}^+\rangle|$ transition matrix element calculated from the $B(E3, 0_{\rm gs}^+ \to 3_1^-)$ strength found in Ref. [37], and the ratio of the energy splitting and the $|\langle 3_1^-||E3||0_{gs}^+\rangle|$ matrix element for each nucleus studied.

Nucleus	$E_{3_i^-}$ (keV)	$E_{3_1^-}$ (keV)	ΔE (keV)	$\lvert\langle 3_1^-\lVert E3\rVert 0_{\rm gs}^+\rangle\rvert$ (eb$^{3/2}$)	$\Delta E/\lvert\langle 3_1^-\lVert E3\rVert 0_{\rm gs}^+\rangle\rvert$ (MeV/eb$^{3/2}$)
^{92}Zr	3039.8	2339.6	700.2	0.26(14)	2.7(9)
^{94}Mo	3011.5	2533.8	477.7	0.25(10)	1.9(8)
^{96}Mo	3178.7	2234.6	944.1	0.30(7)	3.1(7)
^{112}Cd	2866.7	2005.2	861.5	0.34(4)	2.6(4)
^{114}Cd	2871.6	1958.0	913.6	0.36(6)	2.5(4)
^{144}Nd	2779.0	1551	1268.0	0.51(2)	2.47(9)

spin assignment $J^{\pi} = (4)^{-}$ is based on the excitation function of the state [20]. In the excitation function the yields of all transitions from a given state enter. Thus, the likely scenario of unobserved decay branches could change this assignment.

IV. CONCLUSIONS

The nature of strong $M1$ transitions connecting 3_i^- $(i > 1)$ states at an excitation energy of ~3 MeV, with the first excited 3_1^- states in nearly spherical nuclei, was investigated. The observed $M1$ transition strengths are of the same order of magnitude as the $2^+_{\rm ms} \to 2^+_1$ $M1$ transitions, which are the fingerprints of the low-lying quadrupole isovector excitation. Furthermore, it was shown that the energy splitting between the states of interest and the symmetric octupole phonon is proportional to the octupole collectivity of the respective nucleus. Within the considerable experimental uncertainties, the possibility of assigning the observed 3_i^- states as candidates for isovector octupole excitations was presented.

With their assignment as isovector octupole excitations, the data provide nuclear models with the long needed experimental information to fix the coupling constant of the isovector $(\pi\nu)$ part of the residual octupole-octupole interaction. However, further experimental work to remove the ambiguities in the data and theoretical investigations with microscopic models are necessary to confirm the isovector character of the proposed candidates.

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