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π-η mixing and charge symmetry violating NN potential in matter

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We construct density-dependent class III charge symmetry violating (CSV) potential caused by the mixing of π-η mesons with off-shell corrections. The density dependence enters through the nonvanishing π-η mixing driven by both the neutron-proton mass difference and their asymmetric density distribution. The contribution of density-dependent mixing to the CSV potential is found to be appreciably larger than that of the vacuum part.

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I. INTRODUCTION

One of the interesting areas of research in nuclear physics is the study of symmetries and their violation. The general goal of the research in this area is to find small but observable effects of charge symmetry violation (CSV) that might provide significant insight into the dynamics of CSV interaction.

CSV of the NN interaction refers to a difference between proton-proton and neutron-neutron interactions only. It is most clearly manifested in $^1S_0$ scattering lengths; that is, the difference between pp and nn scattering lengths at $^1S_0$ state is nonzero [1–3]. Other convincing evidence of CSV comes from the binding energy difference of mirror nuclei, which is known as the Okamoto-Nolen-Schifer anomaly [4–6]. The modern manifestation of CSV includes the difference of neutron-proton form factors, hadronic correction to $g-2$ [7], the observation of the decay of $\Psi'(3686) \rightarrow (J/\Psi)\pi^0$, and so on [7].

The current understanding of CSV is that, at the fundamental level, it is caused by the finite mass difference between up (u) and down (d) quarks [1,8–12]. As a consequence, at the hadronic level, charge symmetry (CS) is violated because of the nondegenerate mass of nucleons.

Various mechanisms can lead to CSV in NN interaction. For example, neutral mesons with the same spin-parity but with different isospin can mix at the fundamental level because of quark mass difference (at the hadronic level owing to neutron-proton mass splitting). The most important is the ρ-ω mixing, which according to Refs. [10,13–17] is claimed to have successfully explained CSV observables. The other examples are π-η and π-η' mixing [18–20]. It is shown in Ref. [21] that π-η' mixing is important because it is of opposite sign to π-η mixing, where individual contribution is known to be small.

In previous work, the mixing amplitude was taken to be either constant or on-shell [15,16], which is not consistent for the construction of NN potential because the mixing amplitude has strong momentum dependence [22–28]. Even the ρ-ω mixing amplitude changes sign as one moves away from the $\rho(\omega)$ pole to the spacelike region. It is important to note that the mixing amplitude in the spacelike region is relevant for construction of the CSV potential.

Once the mixing amplitude is known, one can construct the CSV potential by evaluating a two-body NN scattering diagram involving mixed intermediate states such as π-η or ρ-ω. It is to be noted that external legs can also contribute to the CSV if one incorporates relativistic corrections, which was recently shown in Ref. [29].

In matter, there can be another source of symmetry breaking if the ground state contains an unequal number of neutrons (n) and protons (p) giving rise to ground-state-induced mixing of various charged states such as ρ-ω or π-η mesons, even in the limit $M_n = M_p$.

Such matter-induced mixing was studied in Refs. [30–35]. But none of these studies dealt with the construction of two-body potential except for Ref. [36], where density-dependent CSV potential was constructed considering only the effect of the scalar mean field on the nucleon mass and excluding the possibility of matter-driven mixing. Recently, the medium-dependent CSV potential due to ρ-ω mixing was constructed in Ref. [37].

The potential we construct here has two parts: one corresponds to vacuum mixing and the other involves the density-dependent mixing amplitude. The latter, as we discuss, vanishes only when both the interaction and the ground state respect isospin symmetry (i.e., only when both masses and the densities for the proton and neutron are taken to be equal). We would like to emphasize here that the density-dependent part remains nonzero even in symmetric nuclear matter if $M_n \neq M_p$ [see Eqs. (15)]. It should be noted, however, that the "charge symmetry breaking" refers only to the effects of the underlying interaction, in view of which the contribution driven by the asymmetric state can be interpreted as a correction to the CSV potential.

Although the vacuum contribution of π-η mixing to CSV has been shown to be negligible compared to that of ρ-ω mixing, in this work we explore the role of medium-dependent π-η mixing amplitude in constructing the CSV potential, taking into account the contribution of external legs.

Physically, in a dense system, intermediate mesons might be absorbed and reemitted from the Fermi spheres. In symmetric nuclear matter, the emission and absorption involving different isospin states such as π and η cancel when the contributions of both proton and neutron Fermi spheres are added, provided the nucleon masses are taken to be equal. In asymmetric nuclear matter, however, the unbalanced contributions coming from the scattering of neutron and proton Fermi spheres lead to the mixing that depends on both the density (ρ_B) and the asymmetry parameter [$\alpha = (\rho_n - \rho_p)/\rho_B$]. Inclusion of this process is discussed in Sec. II.

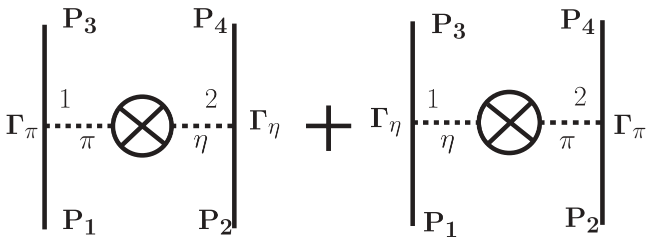

FIG. 1. Feynman diagrams that contribute to the construction of CSV NN potential in matter. Solid lines represent nucleons and dashed lines stand for mesons. The crossed circles indicate the symmetry-breaking piece.

We present the formalism in Sec. II, where the three-momentum-dependent π-η mixing amplitude is calculated to construct the CSV potential in matter. The numerical results are discussed in Sec. III. Finally, in Sec. IV, we summarize our results.

II. FORMALISM

The following matrix element, which is required to construct the CSV NN potential, is obtained from Fig. 1:

$$\begin{aligned}\mathcal{M}^{NN}_{\pi\eta}(q) = {} & [\bar{u}_N(p_3)\tau_3(1)\Gamma_\pi(q)u_N(p_1)]\Delta_\pi(q)\Pi_{\pi\eta}(q^2)\\ & \times \Delta_\eta(q)[\bar{u}_N(p_4)\Gamma_\eta(-q)u_N(p_2)]\\ & + [\bar{u}_N(p_3)\Gamma_\eta(q)u_N(p_1)]\Delta_\eta(q)\Pi_{\pi\eta}(q^2)\\ & \times \Delta_\pi(q)[\bar{u}_N(p_4)\tau_3(2)\Gamma_\pi(-q)u_N(p_2)], \end{aligned} \tag{1}$$

where u_N represents Dirac spinors; $\Pi_{\pi\eta}(q^2)$ is the π-η mixing amplitude; p_i $(i = 1, \ldots, 4)$ and q are the four-momenta of nucleons and mesons, respectively; and $\tau_3(1)$ and $\tau_3(2)$ are isospin operators at vertices 1 and 2 (see Fig. 1). The vertex factor is denoted by $\Gamma_j(q)$ $(j = \pi, \eta)$, and $\Delta_j(q)$ stands for meson propagator given by

$$\Delta_j^{-1}(q^2) = q^2 - m_j^2. \tag{2}$$

In the limit $q_0 \to 0$, Eq. (1) gives the momentum space CSV NN potential, $V^{NN}_{\rm CSV}(\mathbf{q})$.

In the present calculation, mixing is assumed to be generated by the $N\bar{N}$ loops, and the mixing amplitude $\Pi_{\pi\eta}(q^2)$ is generated by the difference between proton and neutron loop contributions, as shown in Fig. 2:

$$\Pi_{\pi\eta}(q^2) = \Pi^{(p)}_{\pi\eta}(q^2) - \Pi^{(n)}_{\pi\eta}(q^2), \tag{3}$$

where $\Pi^{(p)}_{\pi\eta}(q^2)$ or $\Pi^{(n)}_{\pi\eta}(q^2)$ is the π-η mixing self-energy. The origin of the relative sign between proton and neutron loops in Eq. (3) is related to the different signs involved in the coupling of π^0 and η to proton and neutron. The one loop contribution

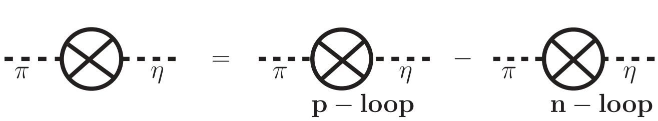

FIG. 2. The mixing amplitude is generated by the difference between proton and neutron loops.

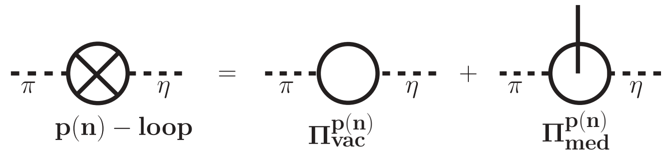

FIG. 3. The mixing self-energy contains a vacuum part and a medium part.

to the mixing self-energy is given by

$$i\Pi^{(N)}_{\pi\eta}(q^2) = \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\Gamma_\pi(q)G_N(k)\Gamma_\eta(-q)G_N(k+q)], \tag{4}$$

where the subscript N stands for the nucleon index (i.e., $N = p$ or n), and $k = (k_0, \mathbf{k})$ denotes the four-momentum of the nucleon in the loops. The main ingredient of our calculation is the in-medium nucleon propagator G_N, which consists of free (G^F_N) and density-dependent (G^D_N) parts [38]:

$$G^F_N(k) = \frac{\not{k} + M_N}{k^2 - M_N^2 + i\zeta}, \tag{5a}$$

$$G^D_N(k) = \frac{i\pi}{E_N}(\not{k} + M_N)\delta(k_0 - E_N)\theta(k_N - |\mathbf{k}|), \tag{5b}$$

where $E_N = \sqrt{M_N^2 + k_N^2}$ is the nucleon energy in which k_N and M_N denote the Fermi momentum and the mass of the nucleon, respectively. Note that the δ function in Eq. (5b) indicates the nucleons are on-shell, whereas $\theta(k_N - |\mathbf{k}|)$ ensures that propagating nucleons have momentum less than k_N.

Like the in-medium nucleon propagator G_N, the mixing self-energy $\Pi^{(N)}_{\pi\eta}(q^2)$ contains vacuum $[\Pi^{(N)}_{\rm vac}(q^2)]$ and density-dependent $[\Pi^{(N)}_{\rm med}(q^2)]$ parts as shown in Fig. 3. It should be noted that the density-dependent part is given by the combination of $G^F_N G^D_N + G^D_N G^F_N$, corresponding to scattering that we have discussed already, whereas the term proportional to $G^D_N G^D_N$ vanishes for low energy excitation [39]. The vacuum part, namely $[\Pi^{(N)}_{\rm vac}(q^2)]$, however, involves $G^F_N G^F_N$, which gives rise to the usual CSV part of the potential owing to the splitting of the neutron and proton mass.

The vacuum mixing contribution of the CSV NN potential can be used to calculate the difference between nn and pp scattering lengths, $\Delta a = a_{pp} - a_{nn}$, at ${}^1S_0$ state [29].

A. Pseudoscalar coupling

First we consider pseudoscalar (PS) coupling of nucleons to the mesons to describe πNN and ηNN interactions, which are represented by the following effective Lagrangians:

$$\mathcal{L}^{\rm PS}_{\pi NN} = -ig_\pi\bar{\Psi}\gamma_5\tau\cdot\Phi_\pi\Psi, \tag{6a}$$

$$\mathcal{L}^{\rm PS}_{\eta NN} = -ig_\eta\bar{\Psi}\gamma_5\Phi_\eta\Psi, \tag{6b}$$

where Ψ and Φ represent the nucleon and meson fields, respectively, and g_j stands for the meson-nucleon coupling constants. For PS coupling, the vertex factor $\Gamma_j = -ig_j\gamma_5$ $(j = \pi, \eta)$.

Now we proceed to calculate the CSV NN potential. For this purpose we must calculate the π-η mixing self-energy using Eq. (4). First we consider mixing in vacuum. After performing trace calculation, the vacuum contribution of π-η mixing self-energy is found to be

$$\Pi^{(N)}_{\rm vac}(q^2) = 4i g_\pi g_\eta \int \frac{d^4k}{(2\pi)^4}\left\{\frac{M_N^2 - k\cdot(k+q)}{(k^2-M_N^2)[(k+q)^2-M_N^2]}\right\}. \tag{7}$$

From dimensional counting it is found that the integral of Eq. (7) is divergent. We use dimensional regularization [40–42] to isolate the singularities in Eq. (7), which reduces to [20]

$$\begin{aligned}\Pi^{(N)}_{\rm vac}(q^2) = \frac{g_\pi g_\eta}{4\pi^2}&\left\{\frac{q^2}{3} + \left(M_N^2 - \frac{q^2}{2}\right)\right.\\ &\times\left[1+\frac{1}{\epsilon} - \gamma_E + \ln(4\pi\mu^2)\right]\\ &- \int_0^1 dx\,[M_N^2 - 3q^2x(1-x)]\\ &\left.\times \ln[M_N^2 - q^2x(1-x)]\right\}.\end{aligned} \tag{8}$$

In Eq. (8), μ is an arbitrary scale parameter, γ_E is the Euler-Mascheroni constant, and $\epsilon = 2 - D/2$, where D stands for the dimension of the integral. Notice that ϵ in Eq. (8) contains the singularity and it diverges as $D \to 4$. The divergences of Eq. (8) can be removed by adding appropriate counterterms [43].

It is clear from Eq. (8) that, unlike for the ρ-ω mixing amplitude, the singularities cannot be removed by simply subtracting the neutron loop contribution from the proton loop contribution. This is because of the singular term is proportional to the mass term. But one can eliminate this singular term by subtracting $\Pi^{(N)}_{\rm vac}(q^2=0)$ from $\Pi^{(N)}_{\rm vac}(q^2)$, which is

$$\begin{aligned}\hat\Pi^{(N)}_{\rm vac}(q^2) &= \Pi^{(N)}_{\rm vac}(q^2) - \Pi^{(N)}_{\rm vac}(q^2=0)\\ &= \frac{g_\pi g_\eta}{4\pi^2}\left\{\frac{q^2}{3} + M_N^2 \ln M_N^2\right.\\ &\quad - \frac{q^2}{2}\left[1+\frac{1}{\epsilon} - \gamma_E + \ln(4\pi\mu^2)\right]\\ &\quad - \int_0^1 dx\,[M_N^2 - 3q^2x(1-x)]\\ &\quad \left.\times \ln[M_N^2 - q^2x(1-x)]\right\}.\end{aligned} \tag{9}$$

Note that $\hat\Pi^{(N)}_{\rm vac}(q^2)$ is not finite but the divergent part proportional to the mass term has been removed. Now one can easily obtain a finite π-η mixing amplitude in vacuum by subtracting $\hat\Pi^{(n)}_{\rm vac}(q^2)$ from $\hat\Pi^{(p)}_{\rm vac}(q^2)$. In the following,

$$\begin{aligned}\Pi^{\rm PS}_{\rm vac}(q^2) = \frac{g_\pi g_\eta}{4\pi^2}&\left[q^2 \ln\left(\frac{M_p}{M_n}\right)\right.\\ &+ q\sqrt{4M_p^2 - q^2}\tan^{-1}\left(\frac{q}{\sqrt{4M_p^2-q^2}}\right)\\ &\left.- q\sqrt{4M_n^2 - q^2}\tan^{-1}\left(\frac{q}{\sqrt{4M_n^2-q^2}}\right)\right],\end{aligned} \tag{10}$$

gives the q^2-dependent π-η mixing amplitude. We obtain $\Pi^{\rm PS}_{\rm vac}(q^2 = m_\eta^2) = -1197\ {\rm MeV}^{-2}$, while experimentally it is found that $\Pi^{\rm PS}_{\rm vac}(q^2 = m_\eta^2) = -4200\ {\rm MeV}^{-2}$ [20]. In this equation, we substitute $q_0 = 0$ to obtain the three-momentum-dependent mixing amplitude $\Pi_{\rm vac}(\mathbf{q}^2)$, which is required for the construction of CSV potential. Now, expanding the mixing amplitude $\Pi^{\rm PS}_{\rm vac}(\mathbf{q}^2)$ in terms of $\mathbf{q}^2/M_N^2$ and keeping the lowest order, we obtain

$$\Pi^{\rm PS}_{\rm vac}(\mathbf{q}^2) = -a_1\mathbf{q}^2, \tag{11}$$

where $a_1 = \frac{g_\pi g_\eta}{4\pi^2}\ln(M_p/M_n)$ and, if $M_p = M_n$, the mixing amplitude in vacuum, that is, $\Pi^{\rm PS}_{\rm vac}(\mathbf{q}^2)$, vanishes. This result implies that CSV NN potential in vacuum does not exist for $M_p = M_n$.

Now we calculate the density-dependent part of the π-η mixing self-energy, which is denoted by $\Pi^{(N)}_{\rm med}(q^2)$. After performing trace calculation and k_0 integration, it reads

$$\begin{aligned}\Pi^{(N)}_{\rm med}(q^2) = -8g_\pi g_\eta \int_0^1 \frac{d^3\mathbf{k}}{(2\pi)^3 E_N}\left[\frac{(k\cdot q)^2}{q^4 - 4(k\cdot q)^2}\right]\\ \times\,\theta(k_N - |\mathbf{k}|).\end{aligned} \tag{12}$$

Substituting $E_N \simeq M_N$ and $q_0 = 0$ into Eq. (12) yields

$$\begin{aligned}\Pi^{(N)}_{\rm med}(\mathbf{q}^2) = \frac{g_\pi g_\eta}{\pi^2 M_N}&\left[\frac{k_N^3}{3} - \frac{\mathbf{q}^2 k_N}{8}\right.\\ &\left.- \frac{\mathbf{q}}{8}\left(k_N^2 - \frac{\mathbf{q}^2}{4}\right)\ln\left(\frac{\mathbf{q}+2k_N}{\mathbf{q}-2k_N}\right)\right].\end{aligned} \tag{13}$$

Equation (13) represents the three-momentum-dependent medium part of the π-η mixing self-energy. The mixing amplitude, as mentioned earlier, is again generated by the difference between the contributions from the proton and neutron loops. This medium part of the π-η mixing amplitude, after suitable expansion in the terms of $\mathbf{q}/k_N$, reads

$$\Pi^{\rm PS}_{\rm med}(\mathbf{q}^2) = a_1' - b_1'\mathbf{q}^2, \tag{14}$$

where the leading-order contribution is considered and

$$a_1' = \frac{g_\pi g_\eta}{3\pi^2}\left(\frac{k_p^3}{M_p} - \frac{k_n^3}{M_n}\right), \tag{15a}$$

$$b_1' = \frac{g_\pi g_\eta}{4\pi^2}\left(\frac{k_p}{M_p} - \frac{k_n}{M_n}\right). \tag{15b}$$

Following Eq. (1) and considering the contributions of external nucleon legs, one obtains the momentum space potential given as

$$\begin{aligned}V^{NN}_{\rm CSV}(\mathbf{q}^2) = T_3^+ \frac{g_\pi g_\eta}{4M_N^2}(\sigma_1\cdot\mathbf{q})(\sigma_2\cdot\mathbf{q})\frac{\Pi^{\rm PS}_{\pi\eta}(\mathbf{q}^2)}{(\mathbf{q}^2+m_\pi^2)(\mathbf{q}^2+m_\eta^2)}\\ \times\left[1 - \frac{\mathbf{q}^2}{8M_N^2} - \frac{\mathbf{P}^2}{2M_N^2}\right],\end{aligned} \tag{16}$$

where $T_3^+ = \tau_3(1) + \tau_3(2)$ and $\mathbf{P} = (\mathbf{p}_1 + \mathbf{p}_3)/2 = (\mathbf{p}_2 + \mathbf{p}_4)/2$. Equation (16) presents CSV class III potential in momentum space. In contrast to π-η mixing, ρ-ω mixing produces both class III and class IV NN interactions [7,20,23]. Note that the terms within the square bracket in Eq. (16) are the contributions from the external legs.

The coordinate space CSV NN potential is obtained by Fourier transformation of Eq. (16):

$$V_{\rm CSV}^{NN}(r) = V_{\rm vac}^{NN}(r) + V_{\rm med}^{NN}(r), \qquad (17)$$

where $V_{\rm vac}^{NN}(r)$ represents the CSV NN potential in vacuum and $V_{\rm med}^{NN}(r)$ is the correction owing to density-driven mixing. Explicitly $V_{\rm vac}^{NN}(r)$ and $V_{\rm med}^{NN}(r)$ are given by

$$V_{\rm vac}^{NN}(r) = -T_3^{+}\frac{g_\pi g_\eta a_1}{48\pi M_N^2}\left[\frac{m_\pi^5 U(x_\pi) - m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right], \qquad (18a)$$

$$V_{\rm med}^{NN}(r) = -T_3^{+}\frac{g_\pi g_\eta}{48\pi M_N^2}\left\{a_1'\left[\frac{m_\pi^3 U(x_\pi) - m_\eta^3 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right] + \left(\frac{a_1'}{8M_N^2} + b_1'\right)\left[\frac{m_\pi^5 U(x_\pi) - m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right]\right\}, \qquad (18b)$$

where

$$U(x_i) = Y_0(x_i)(\sigma_1\cdot\sigma_2) + S_{12}(\hat{\mathbf{r}})Y_2(x_i), \qquad (19a)$$

$$Y_2(x_i) = \left(1 + \frac{3}{x_i} + \frac{3}{x_i^2}\right)Y_0(x_i), \qquad (19b)$$

$$S_{12}(\hat{\mathbf{r}}) = 3(\sigma_1\cdot\hat{\mathbf{r}})(\sigma_2\cdot\hat{\mathbf{r}}) - (\sigma_1\cdot\sigma_2), \qquad (19c)$$

and where $x_i = m_i r$, $i = \pi, \eta$, and $Y_0(x_i) = \frac{e^{-x_i}}{x_i}$.

Because mesons and nucleons are not point particles and they have internal structures, one must incorporate vertex corrections which, in principle, can be calculated using renormalizable models based on hadronic degrees of freedom. In the present calculation, the following phenomenological form factors are used to incorporate the vertex corrections:

$$F_i(\mathbf{q}^2) = \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + \mathbf{q}^2}\right), \quad i = \pi, \eta, \qquad (20)$$

where Λ_i is the cutofff parameter. With the inclusion of form factors, Eqs. (18a) and (18b) reduce to

$$V_{\rm vac}^{NN}(r) = -T_3^{+}\frac{g_\pi g_\eta a_1}{48\pi M_N^2}\left\{\left[\frac{a_\pi m_\pi^5 U(x_\pi) - a_\eta m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right] - \lambda\left[\frac{b_\pi m_\pi^5 U(X_\pi) - b_\eta m_\eta^5 U(X_\eta)}{m_\eta^2 - m_\pi^2}\right]\right\}, \qquad (21)$$

and

$$V_{\rm med}^{NN}(r) = -T_3^{+}\frac{g_\pi g_\eta}{48\pi M_N^2}\Bigg(a_1'\left[\frac{a_\pi m_\pi^3 U(x_\pi) - a_\eta m_\eta^3 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right] + \left(\frac{a_1'}{8M_N^2} + b_1'\right)\left[\frac{a_\pi m_\pi^5 U(x_\pi) - a_\eta m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right] - \lambda\left\{a_1'\left[\frac{b_\pi m_\pi^3 U(X_\pi) - b_\eta m_\eta^3 U(X_\eta)}{m_\eta^2 - m_\pi^2}\right] + \left(\frac{a_1'}{8M_N^2} + b_1'\right)\left[\frac{b_\pi m_\pi^5 U(X_\pi) - b_\eta m_\eta^5 U(X_\eta)}{m_\eta^2 - m_\pi^2}\right]\right\}\Bigg), \qquad (22)$$

where $X_i = \Lambda_i r$ and

$$a_i = \left(\frac{\Lambda_j^2 - m_j^2}{\Lambda_j^2 - m_i^2}\right), \qquad (23a)$$

$$b_i = \left(\frac{\Lambda_j^2 - m_j^2}{m_j^2 - \Lambda_i^2}\right), \quad i \neq j, \qquad (23b)$$

$$\lambda = \left(\frac{m_\eta^2 - m_\pi^2}{\Lambda_\eta^2 - \Lambda_\pi^2}\right). \qquad (23c)$$

B. Pseudovector coupling

Now we consider pseudovector (PV) coupling of nucleons to the mesons to describe πNN and ηNN interactions, which are represented by the following effective Lagrangians:

$$\mathcal{L}_{\pi NN}^{\rm PV} = -\frac{g_\pi}{2M_N}\bar{\Psi}\gamma_5\gamma^\mu\partial_\mu\tau\cdot\Phi_\pi\Psi, \qquad (24a)$$

$$\mathcal{L}_{\eta NN}^{\rm PV} = -\frac{g_\eta}{2M_N}\bar{\Psi}\gamma_5\gamma^\mu\partial_\mu\Phi_\eta\Psi, \qquad (24b)$$

where Ψ, Φ, τ, and g are defined in Sec. II A, and the vertex factors $\Gamma_j = i g_j\gamma_5\gamma^\mu q_\mu$ $(j = \pi, \eta)$. The mixing self-energy in vacuum is given by

$$\Pi_{\rm vac}^{(N)}(q^2) = 4i\left(\frac{g_\pi}{2M_N}\right)\left(\frac{g_\eta}{2M_N}\right)\int\frac{d^4k}{(2\pi)^4} \times\left\{\frac{q^2\left[M_N^2 - k\cdot(k+q)\right] - 2q\cdot(k+q)(k\cdot q)}{\left(k^2 - M_N^2\right)\left[(k+q)^2 - M_N^2\right]}\right\}. \qquad (25)$$

Note that the preceding integral is also divergent and we use dimensional regularization to isolate singularities, which reduces to

$$\Pi_{\rm vac}^{(N)}(q^2) = \frac{g_\pi g_\eta}{8\pi^2}\left\{-\frac{1}{\epsilon} + \gamma_E - \ln(4\pi\mu^2) + \int_0^1 dx\,\ln\left[M_N^2 - q^2x(1-x)\right]\right\}q^2, \qquad (26)$$

where ϵ, μ, and γ_E were discussed previously. It is to be noted that, unlike PS coupling, the singularity in Eq. (26) is not proportional to the mass term. Therefore, simple subtraction of the neutron loop contribution from the proton loop contribution, as for ρ-ω mixing in vacuum [23], removes the divergent parts. Thus, the finite π-η mixing amplitude in vacuum is found to be

$$\Pi_{\rm vac}^{\rm PV}(q^2) = \frac{g_\pi g_\eta}{4\pi^2}\left[q^2\ln\left(\frac{M_p}{M_n}\right) + q\sqrt{4M_p^2 - q^2}\tan^{-1}\left(\frac{q}{\sqrt{4M_p^2 - q^2}}\right) - q\sqrt{4M_n^2 - q^2}\tan^{-1}\left(\frac{q}{\sqrt{4M_n^2 - q^2}}\right)\right]. \qquad (27)$$

Similar to PS coupling, the leading-order contribution of the mixing amplitude in a vacuum is given by

$$\Pi_{\rm vac}^{\rm PV}(\mathbf{q}^2) = -a_2\mathbf{q}^2, \qquad (28)$$

where $a_2 = \frac{g_\pi g_\eta}{4\pi^2}\ln(M_p/M_n)$. Notice, that the leading-order contributions of π-η mixing amplitudes in vacuum are the same for both PS and PV coupling.

The three-momentum-dependent medium part of π-η mixing self-energy reads

$$\Pi^{(N)}_{\rm med}(\mathbf{q}^2) = -\frac{g_\pi g_\eta}{8\pi^2 M_N} \times \left[\mathbf{q}^2 k_N + \mathbf{q}\left(k_N^2 - \frac{\mathbf{q}^2}{4}\right)\ln\left(\frac{\mathbf{q}+2k_N}{\mathbf{q}-2k_N}\right)\right]. \quad (29)$$

The leading-order contribution of the medium part of the mixing amplitude, generated by the difference between the proton and neutron loop contributions, is

$$\Pi^{\rm PV}_{\rm med}(\mathbf{q}^2) = -b_2'\mathbf{q}^2, \quad (30)$$

where

$$b_2' = \frac{g_\pi g_\eta}{4\pi^2}\left(\frac{k_p}{M_p} - \frac{k_n}{M_n}\right). \quad (31)$$

Notice that the density-dependent mixing amplitude in PV coupling differs from that of PS coupling only by the term a_1', which makes a difference between the medium part of the CSV potentials. The momentum space potential is given by

$$V^{NN}_{\rm CSV}(\mathbf{q}^2) = T_3^+ \frac{g_\pi g_\eta}{4M_N^2}(\sigma_1\cdot\mathbf{q})(\sigma_2\cdot\mathbf{q}) \times \frac{\Pi^{\rm PV}_{\pi\eta}(\mathbf{q}^2)}{(\mathbf{q}^2+m_\pi^2)(\mathbf{q}^2+m_\eta^2)}\left[1 - \frac{\mathbf{q}^2}{8M_N^2} - \frac{\mathbf{P}^2}{2M_N^2}\right]. \quad (32)$$

In deriving Eq. (32), contributions from the external leg are considered and given within the square brackets. These contributions are same as that of PS coupling. From this momentum space CSV NN potential, one can obtain the coordinate space potential:

$$V^{NN}_{\rm vac}(r) = -T_3^+ \frac{g_\pi g_\eta a_2}{48\pi M_N^2}\left[\frac{m_\pi^5 U(x_\pi) - m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right], \quad (33a)$$

$$V^{NN}_{\rm med}(r) = -T_3^+ \frac{g_\pi g_\eta b_2'}{48\pi M_N^2}\left[\frac{m_\pi^5 U(x_\pi) - m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right]. \quad (33b)$$

Equations (33a) and (33b) show the coordinate space CSV NN potential without form factors. It should be noted that the CSV potentials in vacuum are the same for both PS and PV couplings, whereas the density-dependent parts are different. With form factors, Eqs. (33a) and (33b) reduce to

$$V^{NN}_{\rm vac}(r) = -T_3^+ \frac{g_\pi g_\eta a_2}{48\pi M_N^2}\left\{\left[\frac{a_\pi m_\pi^5 U(x_\pi) - a_\eta m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right] - \lambda\left[\frac{b_\pi m_\pi^5 U(X_\pi) - b_\eta m_\eta^5 U(X_\eta)}{m_\eta^2 - m_\pi^2}\right]\right\}, \quad (34a)$$

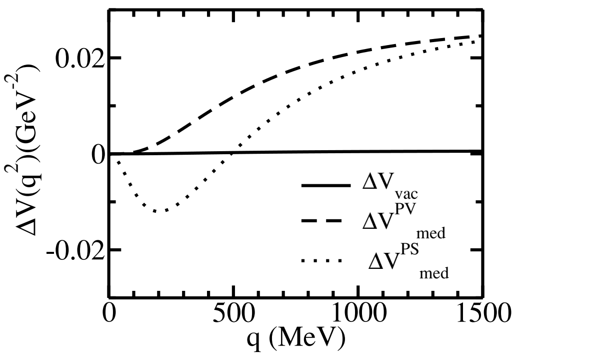

FIG. 4. Difference between CSV nn and pp potentials in momentum space.

$$V^{NN}_{\rm med}(r) = -T_3^+ \frac{g_\pi g_\eta b_2'}{48\pi M_N^2}\left\{\left[\frac{a_\pi m_\pi^5 U(x_\pi) - a_\eta m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right] - \lambda\left[\frac{b_\pi m_\pi^5 U(X_\pi) - b_\eta m_\eta^5 U(X_\eta)}{m_\eta^2 - m_\pi^2}\right]\right\}. \quad (34b)$$

III. RESULTS

In this section we present numerical results. All the figures show the difference between CSV nn and pp potentials in $^1S_0$ state. To obtain the density-dependent CSV potential, we consider the nuclear matter density $\rho_B = 0.148\ \text{fm}^{-3}$ and the asymmetry parameter $\alpha = 1/3$.

Figure 4 shows the difference between CSV nn and pp potentials in momentum space. The dotted and dashed curves represent density-dependent contributions of PS and PV couplings, respectively. The difference in the contributions of the density-dependent part of the CSV potential for these two types of coupling arises because of the term a_1'. The vacuum contribution of CSV potentials for both PS and PV couplings are the same, which is shown by the solid curve in Fig. 4.

The CSV potential in coordinate space is presented in Fig. 5. This figure shows the vacuum and medium contribution of the CSV potential without form factors. The same contributions

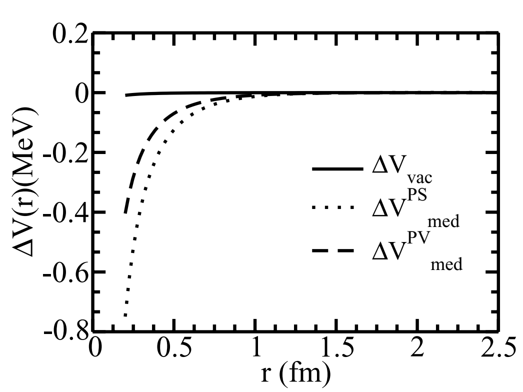

FIG. 5. Difference between CSV nn and pp potentials in coordinate space without form factors.

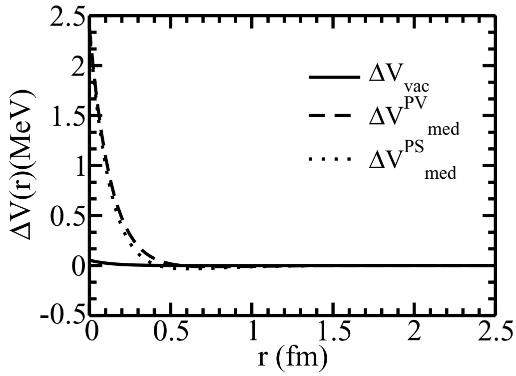

FIG. 6. Difference between CSV *nn* and *pp* potentials in coordinate space with form factors.

with the form factors are demonstrated in Fig. 6. Both the vacuum and the medium parts contribute with the same sign. Note that CSV potentials change sign with the inclusion of form factors. Figures 5 and 6 show that the medium contribution near the core region is much larger than the vacuum contribution.

The difference between *nn* and *pp* scattering lengths, Δa, was computed using the vacuum CSV potential constructed here using π-η mixing. It is found that $\Delta a =$ 0.00082 fm without form factors and -0.0001 fm with form factors.

IV. SUMMARY AND DISCUSSION

In the present work we constructed CS-violating two-body potential driven by the mixing of π-η states. In particular we discuss how such potential gets modified in the matter because of matter-induced effects. It is observed that the density-dependent contribution is larger than the vacuum contribution near the core region. This density-dependent part might contribute significantly to the CSV observables. We estimate the contribution of π-η mixing to the difference of *pp* and *nn* scattering lengths, Δa, where only the vacuum part contributes. For both the density-dependent and the vacuum parts, we find that the role of π-η mixing is smaller than that of ρ-ω mixing [29,37].

We restrict ourselves to the hadronic model, which has reasonable phenomenological success. In principle, such mixings should be derived from quantum chromodynamics (QCD). It would be interesting to compare the present estimates of the mixing amplitude with calculations from other models, for example, QCD in a large-N_c limit or the QCD sum rule. We leave this for future investigation.

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