Nonlinear waves in a quark gluon plasma

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We study the propagation of perturbations in the quark gluon plasma. This subject has been addressed in other works and in most of the theoretical descriptions of this phenomenon the hydrodynamic equations have been linearized for simplicity. We propose an alternative approach, also based on hydrodynamics but taking into account the nonlinear terms of the equations. We show that these terms may lead to localized waves or even solitons. We use a simple equation of state for the QGP and expand the hydrodynamic equations around equilibrium configurations. The resulting differential equations describe the propagation of perturbations in the energy density. We solve them numerically and find that localized perturbations can propagate for long distances in the plasma. Under certain conditions our solutions mimic the propagation of Korteweg-de Vries solitons.

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I. INTRODUCTION

The heavy-ion collisions performed at Brookhaven National Laboratory's Relativistic Heavy-Ion Collider (RHIC) create a hot and dense medium, which behaves as a perfect fluid. During the first years of the RHIC program, hydrodynamics was applied to describe the space-time evolution of the bulk of the fluid. In the last years hydrodynamics became relevant to study also the perturbations on the fluid, such as, for example, the waves generated by the passage of a supersonic parton. This field was opened by the observation of a broad structure in azimuthal di-hadron correlations [1,2]. This broad structure is called the "away-side jet" and recoils against the "near-side jet" (or "trigger jet"). In the framework of hydrodynamics, this observation could be explained by the conical shock waves generated by large energy deposition in the hydrodynamical medium [3–14]. Although quite elegant, this understanding of the away-side jet in terms of conical shock waves still needs confirmation. A very recent and improved analysis by the STAR collaboration has given further support to this picture [15]. A more solid evidence of this phenomenon may come from the study of jets at the Large Hadron Collider (LHC), where the energy released by the nuclear projectiles in the central rapidity region will be larger [16] and so the formed fireball will be larger and live longer, allowing for a more complete study of waves.

In this work we discuss another possible mechanism for the formation of broad structures in the away-side jet. In the limit where the jet loses most of its energy, which is rapidly thermalized and incorporated to fluid, a pulse is formed, which propagates through the fluid. During its motion this energy density pulse spreads both in the longitudinal and transverse direction. After hadronization this traveling and expanding "hot spot" will form particles with a broader angular distribution than those coming from the near-side jet. This is depicted in Fig. 1. Notice that in this process there is no Mach cone formation. During the motion of the energy density pulse, the medium undergoes an expansion leading to a spread of this pulse. A further spreading will occur during the hadronization and final particle formation. Therefore, in this picture it is essential that the initial perturbation remains localized to a good extent. Otherwise, it will spread too much and destroy the jet-like topology, which is compatible with data. Highly localized perturbations can exist and propagate through a fluid. The most famous are the Korteweg-de Vries (KdV) solitons, which are solutions of the KdV equation. This equation may be derived from the equations of hydrodynamics under certain conditions. One of them is to preserve the nonlinear terms of the Euler and continuity equations. The other one is to have a third-order spatial derivative term. This term comes from the equation of state of the fluid and it appears because the Lagrangian density contains higher derivative couplings [17–19] or because of the Laplacians appearing in the equations of motion of the fields of the theory [20]. This happens, for example, in the nonlinear Walecka model of nuclear matter at zero and finite temperature. For a quark gluon plasma (QGP) it depends on the coupling regime and on the properties of the QCD vacuum. As it will be seen in this work, if we consider the simplest case of a free gas of massless quarks and gluons, the hydrodynamical equations do not give origin to the KdV equation. Instead they generate a nonlinear differential equation for the perturbation which has no third-order stabilizing term. This equation is called breaking wave equation and is also very well known in the literature. The numerical solution of this equation shows that an initial Gaussian-like perturbation in the energy density evolves creating a vertical "wall" in its front, which breaks and loses localization. In our case, surprisingly enough, this same phenomenon happens but it takes a very long time and long distances, compared to the nuclear scales. So, from the practical point of view, there is no distinction between a breaking pulse and a soliton. This persistence of localization in the breaking wave is the main result of our article and gives support to the process shown in Fig. 1. However, from this finding to a realistic calculation and a serious attempt to describe the data there is still a long way. The next step now will be to quantify the broadening of the moving bubble in Fig. 1, which will be directly reflected in the angular distribution of the fragments. For this we need to extend our formalism to two spatial dimensions (longitudinal x and radial r). This is a heavily numerical project and it is still in progress. Based on previous works with the analogous nonrelativistic problem

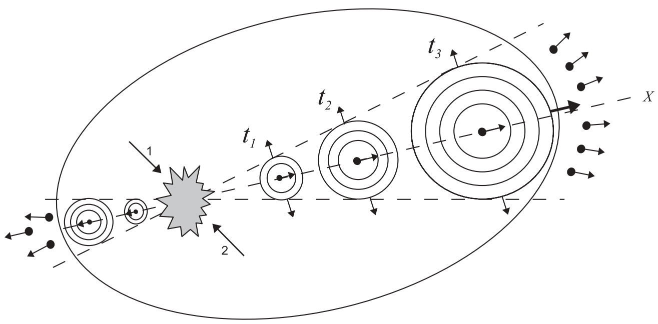

FIG. 1. Parton-parton collision forming two back-to-back jets, which evolve in a hot quark gluon plasma. The circles represent localized (soliton-like) energy density perturbations which traverse the fluid and suffer expansion, forming a narrow near-side jet and a broad away-side jet.

for nuclear matter, discussed in Ref. [21], we have reasons to expect a soliton-like evolution along the x direction with a "leakage" to the radial direction, which would cause the angular broadening in the final matter distribution.

In the theoretical description of these perturbations [4,10,11], very often the hydrodynamic equations are linearized for simplicity. As it is usually done in nonrelativistic hydrodynamics, linearization consists [22] in considering only first-order terms in the velocity and in the energy and pressure perturbations and neglecting higher-order terms and derivatives involving them. In this work we revisit the relativistic hydrodynamic equations expanding them in a different way, in terms of a small expansion parameter (σ) closely following what is done in magnetohydrodynamics of plasmas [23] and keeping the nonlinear features of the problem. Techniques of plasma physics started to be applied to nuclear hydrodynamics long ago [24,25] to study perturbations in the cold nucleus, treated as a fluid. We extended those pioneering studies to relativistic and warm nuclear matter [17–20] and now to the quark gluon plasma (QGP).

The most interesting aspect of [17–20,24,25] was to find at some point of the development, the (KdV) equation for the perturbation in the nuclear matter density. This is the "nuclear soliton." Our main contribution was to establish a connection between the KdV equation (and the properties of its solitonic solutions) and a modern underlying nuclear matter theory (which in our case was a variant of the nonlinear Walecka model) and then to show that the soliton solution exists even in relativistic hydrodynamics [17,18].

In the next section we review the main formulas of relativistic hydrodynamics. In Sec. III we discuss the quark gluon plasma equation of state. In Secs. IV and V we show how to derive the differential equations which govern the time evolution of perturbations at zero and finite temperature, respectively. In Sec. VI we present the numerical solutions of the obtained differential equations and in Sec. VII we present some conclusions.

II. RELATIVISTIC FLUID DYNAMICS

In this section we review the main expressions of one-dimensional relativistic hydrodynamics, which we are going to apply to study QPG both at zero and finite temperature. In the case of a cold QGP we might be concerned about quantum effects. Indeed, in (cold) nuclear physics, in the study of the nucleus or of compound nuclei produced in reactions at low energy, it is necessary to include, for example, Pauli blocking effects. In our case, the cold QGP may exist in the core of compact stars. Compared to cold nuclei, these stars are different in two main aspects: They are infinite (to a good approximation) and the density is much higher than the normal nuclear density. In bound systems, the boundary is a source of discretization of the energy levels. If the boundary is very far away the levels tend to form a continuum. At the same time, at higher densities hadronic matter is believed to be in a deconfined phase, where the coupling constant goes asymptotically to zero. However, in the core of neutron stars this coupling is probably not yet so small and one might have to worry about nonperturbative effects. Moreover, it is also believed that the matter in the core of neutron stars is in a color superconducting phase and, more precisely, in a color-flavor locked (CFL) phase. In this phase the quarks form a difermion condensate and this is the condition for the existence of superfluidity. Therefore, CFL matter is a superfluid! Superfluidity is a property of quantum systems, which have to be treated with an extension of classical hydrodynamics. The equations of the relativistic superfluid have been studied in recent years (see, for example, Refs. [26] and [27]) and they require the knowledge of many unknowns such as some new conductivity and viscosity coefficients. In this context, our formalism based on classical hydrodynamics is admittedly crude because it does not take into account quantum effects. The correct quantum description should be based on the hydrodynamics of relativistic colored superfluids, which is very complicated and still in its infancy. Therefore, we believe that our treatment is the best tool to a first study of the propagation of nonlinear waves in cold and dense quark matter. This confidence in perfect fluid hydrodynamics as a tool to study the core of neutron stars comes also from some recent studies published in Refs. [28] and [29]. In these articles, hydrodynamics has been successfully used to describe the motion of layers in compact stars, with an equation of state derived from the MIT bag model at zero (or very low) temperature.

Throughout this work we employ natural units $c = 1$, $\hbar = 1$, and (Boltzmann's constant) $k_B = 1$. The velocity four-vector u^ν is defined as $u^0 = \gamma$, $\vec{u} = \gamma\vec{v}$, where γ is the Lorentz factor given by $\gamma = (1 - v^2)^{-1/2}$ and thus $u^\nu u_\nu = 1$. The velocity field of the matter is $\vec{v} = \vec{v}(t, x, y, z)$. The energy-momentum tensor is, as usual, given:

$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - p g_{\mu\nu}, \tag{1}$$

where ε and p are the energy density and pressure, respectively. Energy-momentum conservation is ensured by

$$\partial_\nu T_\mu{}^\nu = 0. \tag{2}$$

The projection of (2) onto a direction perpendicular to u^μ gives the relativistic version of the Euler equation [30,31]:

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{(\varepsilon + p)\gamma^2}\left(\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right). \tag{3}$$

The relativistic version of the continuity equation for the baryon density is [30]

$$\partial_\nu {j_B}^\nu = 0. \tag{4}$$

Because ${j_B}^\nu = u^\nu \rho_B$, the above equation can be rewritten as

$$\frac{\partial \rho_B}{\partial t} + \gamma^2 v \rho_B \left(\frac{\partial v}{\partial t} + \vec{v} \cdot \vec{\nabla} v \right) + \vec{\nabla} \cdot (\rho_B \vec{v}) = 0. \tag{5}$$

The relativistic version of the continuity equation for the entropy density is given by the projection of (2) onto the direction of u^ν [31]:

$$(\varepsilon + p)\partial_\mu u^\mu + u^\mu \partial_\mu \varepsilon = 0. \tag{6}$$

At this point we recall the Gibbs relation:

$$\varepsilon + p = \mu_B \rho_B + T s, \tag{7}$$

and the first law of thermodynamics:

$$d\varepsilon = T ds + \mu_B d\rho_B. \tag{8}$$

We will later consider a hot gas of quarks and gluons, where the net baryon density is zero, [i.e., $\rho_B = 0$ ($d\rho_B = 0$) at $T \neq 0$]. Using this last relation in (8) and then inserting (8) and (7) in (6) we arrive at

$$T s(\partial_\mu u^\mu) + T u^\mu(\partial_\mu s) = 0,$$

and finally at

$$\partial_\nu (s u^\nu) = 0, \tag{9}$$

which was expected for a perfect fluid. For future use, the above formula will be expanded as

$$\frac{\partial s}{\partial t} + \gamma^2 v s \left(\frac{\partial v}{\partial t} + \vec{v} \cdot \vec{\nabla} v \right) + \vec{\nabla} \cdot (s \vec{v}) = 0, \tag{10}$$

which is quite similar to (5).

III. THE QGP EQUATION OF STATE

We are going to use the ideal gas equation of state (EOS) for the QGP because it is a simple starting point. Moreover, it seems to produce a sensible phenomenology of compact stars, as shown in Refs. [32] and [33]. In this EOS the pressure and energy density are functions of the quark Fermi momentum [i.e., $p = p(k_F)$ and $\epsilon = \epsilon(k_F)$]. Because k_F is related to the baryon density, ρ_B, we can also write $p = p(\rho_B)$, $\epsilon = \epsilon(\rho_B)$. Finally we substitute the intermediate variables and connect directly p and ϵ. Indeed, for an ideal gas we have $p = \epsilon/3$.

One can improve the EOS introducing perturbative interactions among the constituents. This was done, for example, in Ref. [34]. In this case the pressure and energy density depend explicitly on the chemical potential and on powers of the coupling constant α_s. Because it is a running coupling it depends on the QCD scale $\Lambda_{\rm QCD}$. We can go further and try to introduce nonperturbative interactions. In the high-temperature and low-density region we can just use results from lattice QCD. In the region of low (or zero) T and high baryon density (or chemical potential), lattice calculations are not yet reliable and we have to use models. One of them is the Polyakov loop extended Nambu-Jona-Lasinio model (PNJL) [35]. In all these EOS new parameters characterizing the interaction appear but we still have $p = p(\epsilon)$, where p and ϵ may depend on the coordinates *but not on their derivatives*.

All these improved equations of state, when plugged into our hydrodynamical equations would yield breaking wave equations for the baryon density perturbations, with different coefficients. The only ingredients which might lead us to an equation with a dispersive term (KdV-like) are (i) Laplacian terms from equations of motion of the vector fields treated in an improved mean-field approximation (MFA), where derivatives of the fields are not completely neglected [20]; (ii) higher-order derivative terms in the Lagrangian (and therefore also in p and ϵ also treated in a more flexible version of the mean-field approximation [17,18].

In QCD, option (i) is not possible. We need to make a mean-field approximation. Everything is similar to the traditional MFA used in Walecka-type models, except that the derivatives of the fields are not neglected. In QCD the MFA washes out the gluon field because of color. A nonvanishing gluon field would "choose" some direction in the adjoint color space, without a good reason for that. Option (ii) suffers from the same problem mentioned above. Moreover, there are no higher-order derivative terms in the QCD Lagrangian. They might be added by hand to mimic some nonperturbative effect caused by the nontrivial vacuum. We refrain from doing this here, because this would be a significant departure from standard QCD.

Our equation of state is derived from the MIT bag model. It describes an ideal gas of quarks and gluons and takes into account the effects of confinement through the bag constant $\mathcal{B}$. This constant is interpreted as the energy needed to create a bubble or bag in the vacuum (in which the noninteracting quarks and gluons are confined) and it can be extracted from hadron spectroscopy or from lattice QCD calculations. The baryon density is given by

$$\rho_B = \frac{1}{3} \frac{\gamma_Q}{(2\pi)^3} \int d^3k \;\; [n_{\vec{k}} - \bar{n}_{\vec{k}}], \tag{11}$$

where

$$n_{\vec{k}} \equiv n_{\vec{k}}(T) = \frac{1}{1 + e^{(k - \frac{1}{3}\mu)/T}}, \tag{12}$$

and

$$\bar{n}_{\vec{k}} \equiv \bar{n}_{\vec{k}}(T) = \frac{1}{1 + e^{(k + \frac{1}{3}\mu)/T}}, \tag{13}$$

where from now on μ is the baryon chemical potential. At zero temperature the expression for the baryon density reduces to

$$\rho_B = \frac{2}{3\pi^2} {k_F}^3, \tag{14}$$

where k_F is the highest occupied level. The energy density and the pressure are given by

$$\varepsilon = \mathcal{B} + \frac{\gamma_G}{(2\pi)^3} \int d^3k \;\; k \;\; (e^{k/T} - 1)^{-1} + \frac{\gamma_Q}{(2\pi)^3} \int d^3k \;\; k \;\; [n_{\vec{k}} + \bar{n}_{\vec{k}}], \tag{15}$$

and

$$p = -\mathcal{B} + \frac{1}{3}\left\{\frac{\gamma_G}{(2\pi)^3}\int d^3k\ \ k\ \ (e^{k/T}-1)^{-1} + \frac{\gamma_Q}{(2\pi)^3}\int d^3k\ k[n_{\vec{k}} + \bar{n}_{\vec{k}}]\right\}. \tag{16}$$

The statistical factors are $\gamma_G = 2(\text{polarizations}) \times 8(\text{colors}) = 16$ for gluons and $\gamma_Q = 2(\text{spins}) \times 2(\text{flavors}) \times 3(\text{colors}) = 12$ for quarks. From the above expressions we derive the useful formulas:

$$3(p+\mathcal{B}) = \varepsilon - \mathcal{B} = \frac{8\pi^2}{15}\, T^4 + \frac{6}{\pi^2}\int_0^\infty dk\ \ k^3[n_{\vec{k}} + \bar{n}_{\vec{k}}], \tag{17}$$

and

$$p = \tfrac{1}{3}\varepsilon - \tfrac{4}{3}\mathcal{B}. \tag{18}$$

The speed of sound, c_s, is given by

$${c_s}^2 = \frac{\partial p}{\partial \varepsilon} = \frac{1}{3}. \tag{19}$$

IV. WAVE EQUATION AT ZERO TEMPERATURE

In the core of a dense star the temperature is close to zero and the baryon density is very high. The quark distribution function becomes the step function. Using (14) in (15) and (16) we find

$$\varepsilon(\rho_B) = \left(\tfrac{3}{2}\right)^{7/3}\pi^{2/3}{\rho_B}^{4/3} + \mathcal{B}, \tag{20}$$

and

$$p(\rho_B) = \tfrac{1}{3}\left(\tfrac{3}{2}\right)^{7/3}\pi^{2/3}{\rho_B}^{4/3} - \mathcal{B}. \tag{21}$$

From (18) we have $\vec{\nabla}p = \frac{1}{3}\vec{\nabla}\varepsilon$ and also $\frac{\partial p}{\partial t} = \frac{1}{3}\frac{\partial \varepsilon}{\partial t}$. Combining these expressions with (20) and (21) we find

$$\vec{\nabla}p = \tfrac{4}{9}\left(\tfrac{3}{2}\right)^{7/3}\pi^{2/3}{\rho_B}^{1/3}\ \vec{\nabla}\rho_B, \tag{22}$$

and

$$\frac{\partial p}{\partial t} = \frac{4}{9}\left(\frac{3}{2}\right)^{7/3}\pi^{2/3}{\rho_B}^{1/3}\,\frac{\partial \rho_B}{\partial t}. \tag{23}$$

Finally, substituting (20), (21), (22), and (23) into (3) we obtain

$$\rho_B\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = \frac{(v^2-1)}{3}\left[\vec{\nabla}\rho_B + \vec{v}\,\frac{\partial \rho_B}{\partial t}\right], \tag{24}$$

which is the relativistic version of the Euler equation for the QGP at $T = 0$.

Following the same formalism already used for nuclear matter in Refs. [17–20] we will now expand both (5) and (24) in powers of a small parameter σ and combine these two equations to find one single differential equation which governs the space-time evolution of the perturbation in the baryon density. We write (5) and (24) in one Cartesian dimension (x) in terms of the dimensionless variables:

$$\hat{\rho} = \frac{\rho_B}{\rho_0}, \quad \hat{v} = \frac{v}{c_s}, \tag{25}$$

where ρ_0 is an equilibrium (or reference) density, upon which perturbations may be generated. Next, we introduce the ξ and τ "stretched" coordinates [23–25]:

$$\xi = \sigma^{1/2}\frac{(x-c_s t)}{R}, \quad \tau = \sigma^{3/2}\frac{c_s t}{R}. \tag{26}$$

After this change of variables we expand (25) as

$$\hat{\rho} = 1 + \sigma\rho_1 + \sigma^2\rho_2 + \cdots \tag{27}$$

$$\hat{v} = \sigma v_1 + \sigma^2 v_2 + \cdots \tag{28}$$

Neglecting terms proportional to σ^n for $n \geqslant 3$ and organizing the equations as a series in powers of σ, (5) and (24) acquire the form:

$$\sigma\left\{\frac{\partial \rho_1}{\partial \xi} - \frac{\partial v_1}{\partial \xi}\right\} + \sigma^2\left\{\frac{\partial v_2}{\partial \xi} - \frac{\partial \rho_2}{\partial \xi} + \frac{\partial \rho_1}{\partial \tau} + \rho_1\frac{\partial v_1}{\partial \xi} + v_1\frac{\partial \rho_1}{\partial \xi} - {c_s}^2 v_1\frac{\partial v_1}{\partial \xi}\right\} = 0,$$

and

$$\sigma\left\{\frac{1}{3{c_s}^2}\frac{\partial \rho_1}{\partial \xi} - \frac{\partial v_1}{\partial \xi}\right\} + \sigma^2\left\{-\frac{\partial v_2}{\partial \xi} + \frac{1}{3{c_s}^2}\frac{\partial \rho_2}{\partial \xi} + \frac{\partial v_1}{\partial \tau} + v_1\frac{\partial v_1}{\partial \xi} - 2\rho_1\frac{\partial v_1}{\partial \xi} - \frac{v_1}{3}\frac{\partial \rho_1}{\partial \xi} + \frac{\rho_1}{3{c_s}^2}\frac{\partial \rho_1}{\partial \xi}\right\} = 0,$$

respectively. In these equations each bracket must vanish independently, that is, $\{\cdots\} = 0$. From the terms proportional to σ we obtain ${c_s}^2 = 1/3$ and $\rho_1 = v_1$, which are then inserted into the terms proportional to σ^2 giving after some algebra:

$$\frac{\partial \rho_1}{\partial \tau} + \frac{2}{3}\rho_1\frac{\partial \rho_1}{\partial \xi} = 0. \tag{29}$$

Returning to the $x - t$ space the above equation reads

$$\frac{\partial \hat{\rho}_1}{\partial t} + c_s\frac{\partial \hat{\rho}_1}{\partial x} + \frac{2}{3}c_s\hat{\rho}_1\frac{\partial \hat{\rho}_1}{\partial x} = 0, \tag{30}$$

where we have used the notation $\hat{\rho}_1 \equiv \sigma\rho_1$, which is a small perturbation in the baryon density. Equation (30) is the so-called breaking wave equation for $\hat{\rho}_1$ at zero temperature in the QGP.

V. WAVE EQUATION AT FINITE TEMPERATURE

In the central rapidity region of a typical heavy-ion collision at RHIC we have a vanishing net baryon number (i.e., $\rho_B = 0$). The energy is mostly stored in the gluon field, which forms the hot and dense medium. We will now apply hydrodynamics to study this medium and focus on perturbations in the energy density and their propagation. Following the formalism developed in the previous section we will expand and combine the Euler equation given by (3) and the continuity equation for the entropy density given by (10).

As $\rho_B = 0$, the baryon chemical potential is zero ($\mu = 0$) and so the distribution functions given by (12) and (13) are the same [i.e., $n_{\vec{k}} = \bar{n}_{\vec{k}} = 1/(1+e^{k/T})$]. In this case the integral in (17) can be easily performed and we obtain

$$3(p+\mathcal{B}) = \varepsilon - \mathcal{B} = \tfrac{37}{30}\pi^2 T^4. \tag{31}$$

Solving the first identity for the pressure and recalling [36] that $s = (\partial p/\partial T)_V$ we arrive at

$$s = \frac{\partial}{\partial T}\left(-\mathcal{B} + \frac{37}{90}\pi^2 T^4\right) = 4\frac{37}{90}\pi^2 T^3. \tag{32}$$

The "bag constant" parameter, $\mathcal{B}$, is chosen to be $\mathcal{B}^{1/4} = 170$ MeV, which was used in Ref. [37], in the context of dense star physics. For simplicity of notation, we will rewrite $\mathcal{B}$ as

$$\mathcal{B} = \tfrac{37}{30}\pi^2 (T_B)^4, \tag{33}$$

where T_B is the appropriate number to reproduce the chosen value of $\mathcal{B}$. Inserting (33) into the second identity of (31) we have the following expression for $\varepsilon(T)$:

$$\varepsilon = \tfrac{37}{30}\pi^2 \big(T^4 + T_B{}^4\big). \tag{34}$$

Solving the second identity of (31) for the temperature we obtain

$$T = \left[\frac{30}{37\pi^2}(\varepsilon - \mathcal{B})\right]^{1/4}, \tag{35}$$

which, inserted into (32) yields

$$s = s(\varepsilon) = 4\frac{37}{90}\pi^2 \left[\frac{30}{37\pi^2}(\varepsilon - \mathcal{B})\right]^{3/4}. \tag{36}$$

Substituting, then, (36) in (10) in the one-dimensional case and using (31) to write $(\varepsilon - \mathcal{B})$ in terms of the temperature we have finally

$$(1-v^2)\left[\left(\frac{90}{148\pi^2 T^4}\right)\frac{\partial \varepsilon}{\partial t} + \frac{\partial v}{\partial x} + \left(\frac{90 v}{148\pi^2 T^4}\right)\frac{\partial \varepsilon}{\partial x}\right] + v\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = 0. \tag{37}$$

Also, from (31) we have

$$\varepsilon + p = \tfrac{148}{90}\pi^2 T^4. \tag{38}$$

Inserting the above equation into (3) and using $\vec{\nabla} p = \frac{1}{3}\vec{\nabla}\varepsilon$ and also $\frac{\partial p}{\partial t} = \frac{1}{3}\frac{\partial \varepsilon}{\partial t}$ we find

$$\frac{148}{30}\pi^2 T^4 \left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = (v^2 - 1)\left(\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial t}\right). \tag{39}$$

We now rewrite (37) and (39) in dimensionless variables:

$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0}, \quad \hat{v} = \frac{v}{c_s}, \tag{40}$$

where ε_0 is the reference energy density. Expanding (40) in powers of σ we have

$$\hat{\varepsilon} = 1 + \sigma\varepsilon_1 + \sigma^2\varepsilon_2 + \cdots \tag{41}$$

and

$$\hat{v} = \sigma v_1 + \sigma^2 v_2 + \cdots \tag{42}$$

Neglecting higher-order terms in σ and changing variables to the $(\xi - \tau)$ space, Eqs. (37) and (39) become

$$\sigma\left\{-\frac{90\,\varepsilon_0}{148\pi^2 T^4}\frac{\partial \varepsilon_1}{\partial \xi} + \frac{\partial v_1}{\partial \xi}\right\} + \sigma^2\left\{\frac{90\,\varepsilon_0}{148\pi^2 T^4}\left(-\frac{\partial \varepsilon_2}{\partial \xi} + \frac{\partial \varepsilon_1}{\partial \tau} + v_1\frac{\partial \varepsilon_1}{\partial \xi}\right) + \frac{\partial v_2}{\partial \xi} - c_s{}^2 v_1 \frac{\partial v_1}{\partial \xi}\right\} = 0, \tag{43}$$

and

$$\sigma\left\{-\frac{148\pi^2 T^4 c_s}{30}\frac{\partial v_1}{\partial \xi} + \frac{\varepsilon_0}{c_s}\frac{\partial \varepsilon_1}{\partial \xi}\right\} + \sigma^2\left\{\frac{148\pi^2 T^4 c_s}{30}\left(-\frac{\partial v_2}{\partial \xi} + \frac{\partial v_1}{\partial \tau} + v_1\frac{\partial v_1}{\partial \xi}\right) + \frac{\varepsilon_0}{c_s}\frac{\partial \varepsilon_2}{\partial \xi} - \varepsilon_0 c_s v_1 \frac{\partial \varepsilon_1}{\partial \xi}\right\} = 0. \tag{44}$$

As before, in the above equations, each bracket must vanish independently. From the first bracket of (43) we have

$$v_1 = \frac{90\varepsilon_0}{148\pi^2 T^4}\varepsilon_1, \tag{45}$$

which, inserted into the terms proportional to σ^2, yields

$$\frac{\partial \varepsilon_1}{\partial \tau} + \left(\frac{90\varepsilon_0}{148\pi^2 T^4}\right)\frac{2}{3}\varepsilon_1\frac{\partial \varepsilon_1}{\partial \xi} = 0. \tag{46}$$

Coming back to the $x - t$ space the above equation becomes

$$\frac{\partial \hat{\varepsilon}_1}{\partial t} + c_s\frac{\partial \hat{\varepsilon}_1}{\partial x} + \left(\frac{90\varepsilon_0}{148\pi^2 T^4}\right)\frac{2}{3}c_s\hat{\varepsilon}_1\frac{\partial \hat{\varepsilon}_1}{\partial x} = 0, \tag{47}$$

where $\hat{\varepsilon}_1 \equiv \sigma\varepsilon_1$ is a small perturbation in the energy density. Equation (47) is the breaking wave equation for $\hat{\varepsilon}_1$ in a QGP at finite temperature. In this equation T is the temperature of the background (i.e., $T = T_0$) and it is related to the energy density through (34). Using (34) and the relations deduced in the previous section, Eq. (47) becomes, finally,

$$\frac{\partial \hat{\varepsilon}_1}{\partial t} + c_s\frac{\partial \hat{\varepsilon}_1}{\partial x} + \left[1 + \left(\frac{T_B}{T_0}\right)^4\right]\frac{c_s}{2}\hat{\varepsilon}_1\frac{\partial \hat{\varepsilon}_1}{\partial x} = 0, \tag{48}$$

where $T_0 > T_B$.

VI. NUMERICAL ANALYSIS AND DISCUSSION

Equations (30) and (48) have the form,

$$\frac{\partial f}{\partial t} + c_s\frac{\partial f}{\partial x} + \alpha f\frac{\partial f}{\partial x} = 0, \tag{49}$$

which is a particular case of the equation:

$$\frac{\partial f}{\partial t} + c_s\frac{\partial f}{\partial x} + \alpha f\frac{\partial f}{\partial x} + C\frac{\partial^3 f}{\partial x^3} = 0, \tag{50}$$

when $C = 0$. The last equation is the famous Korteweg-de Vries equation, which has an analytical soliton solution given [38]:

$$f(x,t) = \frac{3(u - c_s)}{\alpha}\,\mathrm{sech}^2\left[\sqrt{\frac{(u-c_s)}{4B}}(x - ut)\right], \tag{51}$$

where u is an arbitrary supersonic velocity.

A soliton is a localized pulse that propagates without change in shape. On the other hand, the solutions of (30) and (48) will break, that is, they will acquire an oscillating behavior and will be spread out, losing localization. Whether or not a given physical system will support soliton propagation depends ultimately on its equation of state [in our case, on the function $\varepsilon = \varepsilon(\rho_B)$ or $\varepsilon = \varepsilon(p)$]. If the EOS takes into account the inhomogeneities in the system, the energy density will, in general, be a function of gradients and/or Laplacians. When used as input in hydrodynamical equations, these higher-order derivatives will lead to the KdV equation. In a hadronic phase, where the degrees of freedom are baryons and mesons, we have shown [17–20] that the hydrodynamical equations will indeed give origin to the KdV equation. In the present case, for this simple model of the quark gluon plasma, this was not the case and we could only obtain the breaking wave equation.

A. Zero temperature

Although the main focus of this work are the perturbations in a hot QGP formed in heavy-ion collisions, for completeness, we discuss in this subsection the zero temperature case, which might be relevant for astrophysics.

We will present numerical solutions of (30) with the following initial condition, inspired by (51)

$$\hat{\rho}_1(x, t_0) = A\ \mathrm{sech}^2\left[\frac{x}{B}\right], \tag{52}$$

where A and B represent the amplitude and width (of the initial baryon density pulse), respectively. In Fig. 2 we show the numerical solution of (30) for $A = 0.075$ and $B = 1$ fm for different times. We can observe the evolution of the initial Gaussian-like pulse and the formation of a "wall" on the right side. Figure 3 shows the numerical solution of (30) for $A = 0.35$ and $B = 1$ fm. The time evolution of the pulse is similar to the one found in Fig. 2 but the "wall" formation and dispersion occurs much earlier. In Fig. 4 we present another solution of (30) for $A = 0.075$ and $B = 0.5$ fm. We can see that the initial pulse starts to develop small secondary peaks, which are called "radiation" in the literature. Further time evolution would increase the strength of these peaks until the complete loss of localization.

From these figures we learn how the solution depends on the initial amplitude and width: It lives longer as a compact pulse for smaller amplitudes and larger widths. Changes in one quantity may compensate the changes in the other, creating a very stable moving object. In fact, the most striking conclusion to be drawn here is that for a wide variety of choices in the initial conditions the solution remains stable and localized for distances much larger than the nuclear size.

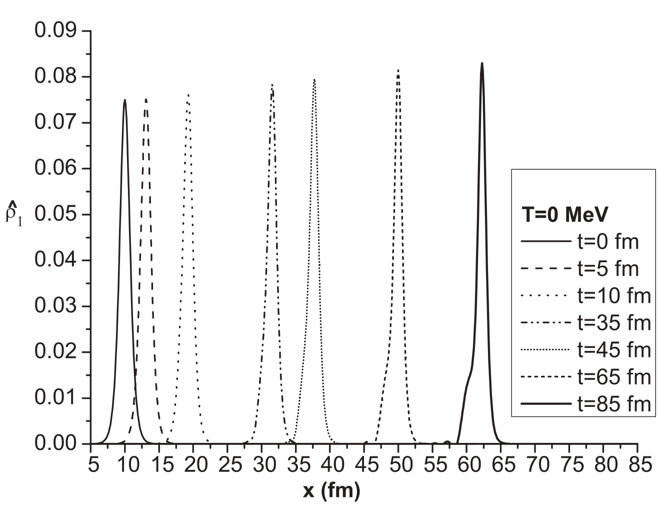

FIG. 2. Time evolution of a baryon density pulse at zero temperature.

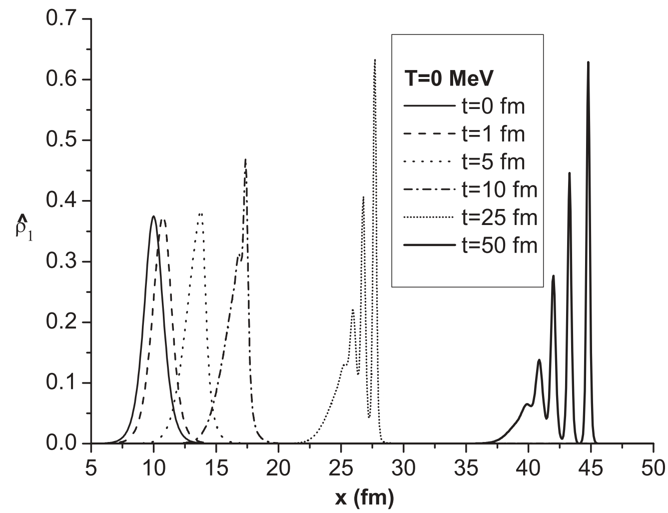

FIG. 3. The same as Fig. 2 for a larger amplitude.

B. Finite temperature

We now turn to the study of the solutions of (48) for initial conditions given by (52) (replacing $\hat{\rho}_1$ by $\hat{\varepsilon}_1$). Now, beside the amplitude and width, the solution will depend also on the temperature. When $T_0 = T_B$, Eq. (48) reduces to

$$\frac{\partial \hat{\varepsilon}_1}{\partial t} + c_s \frac{\partial \hat{\varepsilon}_1}{\partial x} + c_s \hat{\varepsilon}_1 \frac{\partial \hat{\varepsilon}_1}{\partial x} = 0. \tag{53}$$

When $T_0 \gg T_B$, Eq. (48) reduces to

$$\frac{\partial \hat{\varepsilon}_1}{\partial t} + c_s \frac{\partial \hat{\varepsilon}_1}{\partial x} + \frac{c_s}{2} \hat{\varepsilon}_1 \frac{\partial \hat{\varepsilon}_1}{\partial x} = 0. \tag{54}$$

Observing these two formulas we can see that, because $c_s = 1/3$ is fixed, the only change in the differential equation

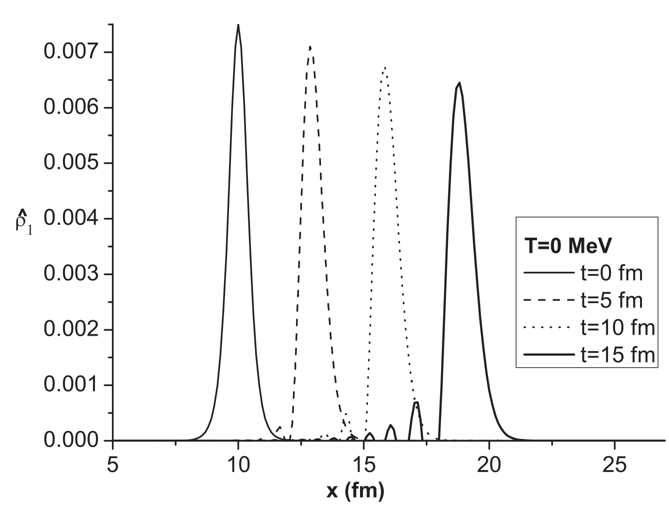

FIG. 4. The same as Fig. 2 for a smaller width.

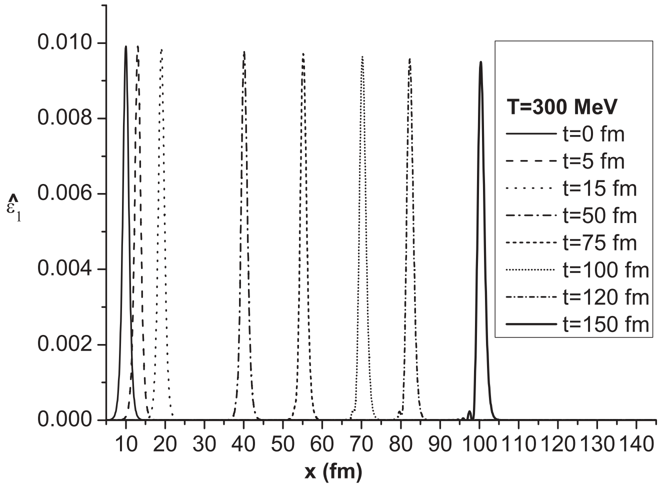

FIG. 5. Time evolution of an energy density pulse at $T = 300$ MeV.

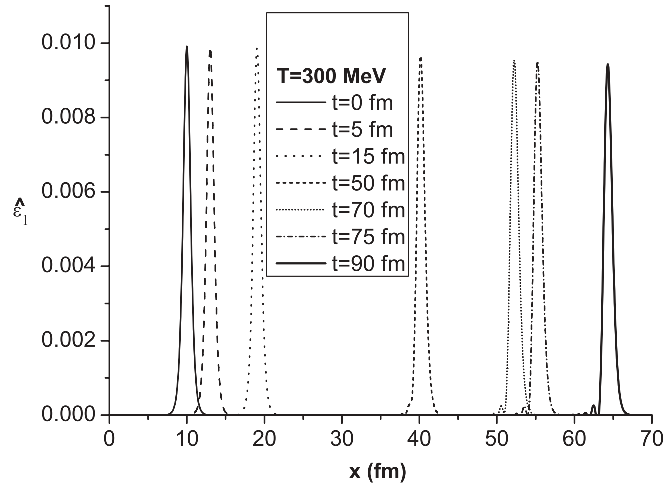

FIG. 7. The same as Fig. 5 for a smaller width.

with temperature happens in the numerical coefficient of the last term which goes from 0.5 to 1. Therefore, our results depend very weakly on the temperature. A stronger dependence on T would appear if c_s was allowed to change with temperature. This would correspond to having a different and more complicated equation of state for the quark gluon plasma.

In Fig. 5 we show the solution of (48) with the initial condition given by (52) with $A = 0.01$, $B = 1$ fm, and $T = 300$ MeV. Figure 6 shows the same as Fig. 5 but with $A = 0.1$ and $B = 1$ fm. As in the zero-temperature case, we observe that increasing the initial amplitude the breaking process develops earlier. In Fig. 7 we show the same as Fig. 5 but with $A = 0.01$ and $B = 0.2$ fm. Figures 8 and 9 show the time evolution of a pulse with the same initial amplitude ($A = 0.5$) and width ($B = 1$ fm) but different temperatures. Even though one temperature is $T = 150$ MeV (Fig. 8) and the other is $T = 300$ MeV (Fig. 9) we can hardly notice any difference.

We would like to emphasize that our calculation is still preliminary. However, any calculation of a new or unexplored effect has to start from some simplified study, which is later refined. The realistic solution of the three-dimensional relativistic hydrodynamics equations is a formidable numerical task. Before embarking in such heavy numerical calculations it is interesting to make exploratory studies. Their outcome might be completely disappointing or, on the contrary, might encourage us to proceed further with more involved calculations. This strategy is common and a recent example is the study of Mach cone formation. The preliminary studies considered a fast parton crossing a static medium and loosing energy to this medium, which was subsequently propagated in the form of conical waves. This static approximation was performed in Ref. [4]. In subsequent articles, such as Ref. [13] the expansion of the medium was introduced. In the present work, we find that the localized pulses stay localized for several tens of fm. This means that they last longer than the plasma, they survive, they may exist in the final hadronic later stage of the collisions and be observed. This is very interesting and

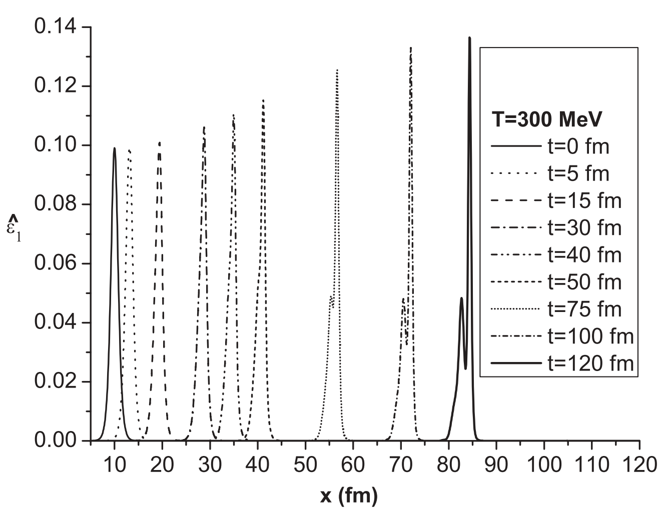

FIG. 6. The same as Fig. 5 for a larger amplitude.

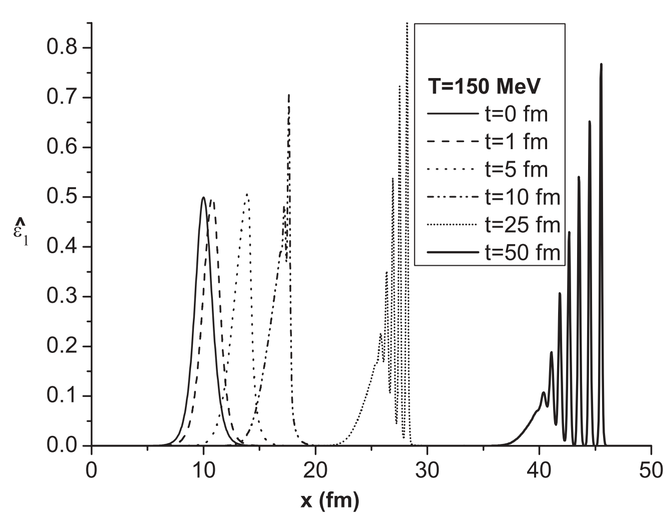

FIG. 8. Evolution of the energy density pulse at $T = 150$ MeV.

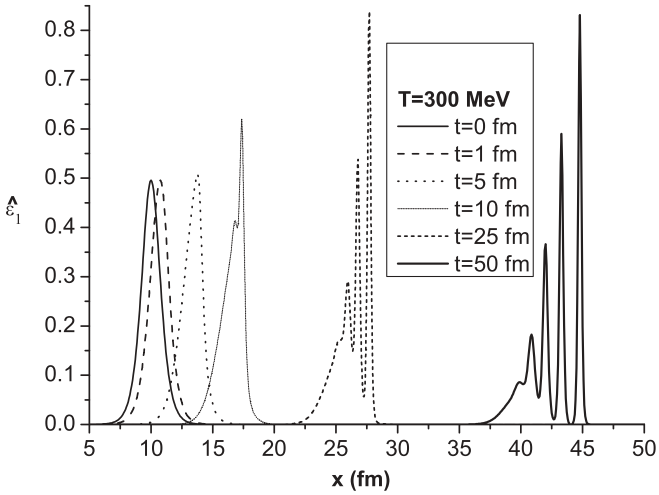

FIG. 9. The same as Fig. 8 for $T = 300$ MeV.

encourages us to include the expansion of the medium and check if the localization of these pulses persists even in the presence of expansion. The long timescale found by us is, in fact, the most interesting and promising result of the article. A very negative and discouraging result would have been to find out that our pulses break and lose localization in less than the typical QGP lifetime (i.e., less than 10 fm).

VII. CONCLUSIONS

We have proposed an alternative explanation for the observed broadening of the away-side peak. It is based on the hydrodynamical treatment of energy perturbations. In contrast to other approaches we went beyond linearization of the fundamental equations and did not neglect the nonlinear terms. We used a simple equation of state for the QGP and expanded the hydrodynamic equations around equilibrium configurations. The resulting differential equations describe the propagation of perturbations in the energy density. We solved them numerically and found that localized perturbations can propagate for long distances in the plasma. Under certain conditions our solutions mimic the propagation of Korteweg-de Vries solitons. However, as said before, from this finding to a realistic calculation and a serious attempt to describe the data there is still a long way. The main result found in this work, namely, the persistence of soliton-like configurations, is very promising and encourages us to extend our formalism to two spatial dimensions. This project is in progress.

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