Bremsstrahlung and pair production processes at low energies: Multidifferential cross section and polarization phenomena

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Radiative electron-proton scattering is studied in peripheral kinematics, where the scattered electron and photon move close to the direction of the initial electron. Even in the case of an unpolarized initial electron, the photon may have a definite polarization. The differential cross sections with longitudinally or transversity polarized initial electrons are calculated. The relevant effective degrees of polarization are explicitly derived. The same phenomena are considered for the production of an electron-positron pair by a photon, where the final positron (electron) can be also polarized. Differential distributions for the case of a polarized initial photon are given. We extend the results obtained for protons to atoms of any atomic number, including screening.

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I. INTRODUCTION

It was shown in the well-known articles of E. Haug [1,2] that the main contribution to the differential cross section for Bremsstrahlung induced by electrons and pair production induced by photons, is given by peripheral kinematics, which dominates starting from rather small energies of an initial electron or photon in the laboratory frame. For photon energies larger than 50 MeV, the contributions of kinematical regions outside the peripheral region are below 5%.

Peripheral kinematics occurs in Bremsstrahlung when the scattered electron and the photon, with small invariant mass (of the order of electron mass m), move close to the direction of the initial particle (hereafter we imply the laboratory reference frame). The contribution from peripheral kinematics to the cross section does not decrease when the energy of the initial particle increases.

The differential (and total) cross section of peripheral kinematics is of the order of α^3/m^2, whereas the contribution of nonperipheral kinematics is $\sim\alpha^3/s$, $s = 2M\omega$, or $s = 2EM$, with M being the target mass and ω of the order of E initial photon or electron energies. Omitting nonperipheral kinematics, the uncertainty (error) on the cross section is of the order of $1 + \mathcal{O}(m^2/s)$. Even for energies $\sim$10 MeV, for the scattering of electron on proton, this error does not exceed a fraction of a percent.

Therefore, peripheral kinematics represents the dominant contribution, starting almost from the threshold of the process.

Bremsstrahlung emission was the subject of intensive theoretical work in the 1960s [3,4]. Accurate calculations were done, including the radiative corrections owing to multiple virtual photons exchange with the target. Results were derived for the total cross section and single-parameter distributions. In Refs. [5,6], detailed distributions for the photon in the Bremsstrahlung processes was investigated. A more recent work [7] and references therein contains detailed discussion on these problems based on the formalism established in the previous works.

In modern experiments, owing to the high luminosity and the performances of the detector it is possible to achieve precise measurements of the multidifferential cross section, as function of the different observables that define completely the kinematics.

The motivation of our article is to reformulate the matrix element in the infinite momentum frame for Bremsstrahlung in ep and for pair production in γp interaction. Analytical formulas for the cross section and for the polarization observables as functions of the energy fraction and angle of the emitted photon or electron are given. The effects of finite electron mass are included. Explicit and transparent expressions are derived in the framework of a light-cone formalism, similar to Weinberg approach [8], which can be directly used as input for Monte Carlo simulations and analysis programs. The multidifferential cross section is integrated over the emission angle to compare the numerical results with previous calculations.

The accuracy of the calculated cross section can be estimated to be of the order

$$1 + \mathcal{O}\left(\frac{\alpha}{\pi}, \frac{m^2}{s}\right), \tag{1}$$

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where the first term refers to radiative corrections that were neglected and the second one is inherent to the method of calculation.

Our article is organized as follows. We start from considerations on the Bremsstrahlung process (Sec. II) in the collision of electrons with the target. Even in the case of an unpolarized initial electron, it is known that the emitted photon may acquire a nonvanishing polarization [9]. The relevant Stokes parameters related with its linear polarization are calculated. Corresponding results are obtained when the initial electron is longitudinally or transversely polarized with respect to the beam direction. The analysis is performed at the lowest order of perturbation theory.

Section III is devoted to the pair production by photons on a target. A linear polarization of the positron (electron) belonging to the the pair appears even in the case of the unpolarized photon. The cases of the linearly polarized as well as circularly polarized photons are considered. In Sec. IV we consider the distributions integrated on the momentum transferred to the target. Cases of both unscreened and completely screened atom-target are considered. In Sec. V the results of the numerical calculations for the relevant quantities are presented. The Conclusions section contains a short summary.

II. FORMALISM FOR A BREMSSTRAHLUNG PROCESS

Let us consider the process

$$e^-(p)+T(P)\to \gamma(k,e)+e^-(p')+T(P'), \qquad (2)$$

where T is a heavy target nucleus with electric charge Z. The particle four-momenta are indicated in brackets. Let us define $e=e(k)$ as the polarization four-vector of the photon. The relevant kinematical variables are

$$\begin{aligned} &s=2Pp, \quad p^2=p'^2=m^2, \quad P^2=P'^2=M^2,\\ &k^2=0, \quad p+q=p'+k, \quad P'+q=P, \end{aligned} \qquad (3)$$

where q is the momentum transferred to the target. For practical use, let us introduce dimensionless variables for the photon, such as the energy fraction $\bar{x}$ and the momentum fraction κ, which is function of the emission angle θ_γ,

$$\begin{aligned} &\kappa=\frac{|\vec{k}|}{m}=\frac{|\vec{k}|}{E\bar{x}}\frac{E\bar{x}}{m}=\theta_\gamma\cdot\gamma\cdot\bar{x}=\eta\bar{x}, \quad \gamma=E/m,\\ &\bar{x}=\frac{\omega}{E}=1-x, \end{aligned} \qquad (4)$$

where γ is the γ factor of the initial electron.

Using the advantages of the infinite momentum technique [10], the matrix element can be written as

$$\begin{aligned} M=&\frac{(4\pi\alpha)^{3/2}Z}{q^2}\cdot\frac{2}{s}[\bar{u}(P')\not{p}\ u(P)]\\ &\cdot[\bar{u}(p')O_{\mu\nu}u(p)]e^\nu(k)\tilde{P}^\mu, \end{aligned} \qquad (5)$$

$$O_{\mu\nu}e^\nu(k)\tilde{P}^\mu=\tilde{\not{P}}\frac{\not{p}\ -\not{k}\ +m}{-2pk}\not{e}+\not{e}\ \frac{\not{p}'\ +\not{k}\ +m}{2p'k}\tilde{\not{P}} \qquad (6)$$

(with $\not{p}\equiv\gamma_\mu p^\mu$), where $\tilde{P}=P-p\frac{M^2}{s}=(P_0,P_z,P_x,P_y)=\frac{M}{2}(1,-1,0,0)$ and $\tilde{p}=p-P\frac{m^2}{s}=E(1,1,0,0)$ are lightlike vectors in the light-cone decomposition (Sudakov parametrization) of vectors [10,11],

$$\begin{aligned} &p=\frac{m^2}{s}\tilde{P}+\tilde{p}, \quad q=\alpha\tilde{P}+\beta\tilde{p}+q_\perp,\\ &p'=\alpha'\tilde{P}+x\tilde{p}+p'_\perp, \quad k=\alpha_k\tilde{P}+\bar{x}\tilde{p}+k_\perp,\\ &\bar{x}=1-x, \quad e=\alpha_e\tilde{P}+e_\perp, \end{aligned} \qquad (7)$$

where $c_\perp P=c_\perp p=0$ for any vector $c_\perp$ and $\tilde{P}^2=\tilde{p}^2=0$. We use the following notation:

$$q_\perp^2=-\vec{q}^2, \quad p_\perp'^2=-\vec{p}^2, \quad k_\perp^2=-\vec{k}^2. \qquad (8)$$

The phase volume then reads

$$\begin{aligned} d\Gamma&=\frac{1}{(2\pi)^5}\delta^4(p+P-p'-k-P')\frac{d^3k}{2E_k}\frac{d^3p'}{2E_{p'}}\frac{d^3P'}{2E_{P'}}\\ &=\frac{1}{(2\pi)^5}\frac{1}{4s}\frac{dx}{x\bar{x}}d^2pd^2q, \quad d^4q=\frac{s}{2}d\alpha d\beta d^2q_\perp. \end{aligned} \qquad (9)$$

Using the on mass shell conditions $\alpha_k=\vec{k}^2/\bar{x}$; $\alpha'=(\vec{p}^2+m^2)/x$, we obtain

$$\begin{aligned} &2pk=\frac{D}{\bar{x}}, \quad D=\vec{k}^2+m^2\bar{x}^2, \quad \vec{k}=\vec{q}-\vec{p};\\ &2p'k=\frac{1}{x\bar{x}}D', \quad D'=\vec{r}^2+m^2\bar{x}^2, \quad \vec{r}=x\vec{q}-\vec{p}. \end{aligned} \qquad (10)$$

Let us note the useful relations

$$\begin{aligned} D-D'&=\bar{x}[\vec{q}^2(1+x)-2(\vec{p}\vec{q})],\\ D'-xD&=\bar{x}(\vec{p}^2+m^2\bar{x}^2-\vec{q}^2x). \end{aligned}$$

Using the Dirac equation for the spinors of the initial and the scattered electrons, one can write the expression for $O_{\mu\nu}\tilde{P}^\mu e^\nu$ as

$$\begin{aligned} &O_{\mu\nu}\tilde{P}^\mu e^\nu=As\not{e}+B\tilde{\not{P}}\not{q}\ \not{e}+C\not{e}\ \not{q}\ \tilde{\not{P}},\\ &A=x\bar{x}\left(\frac{1}{D'}-\frac{1}{D}\right), \quad B=\frac{\bar{x}}{D}, \quad C=\frac{x\bar{x}}{D'}. \end{aligned} \qquad (11)$$

Note that in the products of three-vectors, only the transversal component of q is relevant: $\not{q}\ \to\not{q}_\perp$. It is easy to show that omitting terms of the order m^2/s the longitudinal components cancel:

$$\tilde{\not{P}}\not{q}\ \not{e}=\tilde{\not{P}}(\beta\tilde{\not{p}}\ +\not{q}_\perp)\not{e}_\perp\approx\tilde{\not{P}}\not{q}_\perp\not{e}_\perp.$$

From the gauge condition $e(k)k=0$ we can express the light-cone component of e as $\alpha_e=2\vec{k}\vec{e}/(s\bar{x})$. At this point let us introduce the polarization density matrix of the photon:

$$e_ie_j^*=\frac{1}{2}\begin{pmatrix}1+\xi_3 & \xi_1-i\xi_2\\ \xi_1+i\xi_2 & 1-\xi_3\end{pmatrix}_{ij}, \quad i,j=x,y. \qquad (12)$$

The case of a polarized initial electron can be considered by introducing its density matrix:

$$u(p,a)\bar{u}(p,a)=(\not{p}+m)(1-\gamma_5\not{a}), \qquad (13)$$

where, for longitudinal electron polarization, the Sudakov decomposition for the polarization vector gives

$a = \lambda[(m/s)\tilde{P} - (1/m)\tilde{p}]$ and, in the case of transversal electron polarization, $a = a_\perp$.

The general expression for the cross section is

$$d\sigma^{eT\to e\gamma T} = d\sigma_0^{eT\to e\gamma T} P_e, \tag{14}$$

$$P_e = 1 + \lambda\xi_2 p_T + \lambda\xi_2 p_a + \tau_{pp} + \tau_{pq} + \tau_{qq}, \tag{15}$$

which becomes, in the unpolarized case,

$$d\sigma_0^{eT\to e\gamma T} = \frac{2\alpha^3 Z^2 d^2 q d^2 p R_p (1-x)dx}{\pi^2 (DD')^2 (q^2)^2}, \tag{16}$$

$$R_p = \vec{q}^2(1+x^2)DD' - 2xm^2(D-D')^2. \tag{17}$$

The effective degrees of polarization are

$$\begin{aligned} p_T &= \frac{1}{R_p}[-\vec{q}^2 DD'(1-x^2) + 2m^2 x\bar{x}(D-D')^2], \\ p_a &= \frac{2xm}{R_p}[(\vec{p}\vec{a})(D-D')^2 + (\vec{q}\vec{a})(D-D')(D'-xD)], \\ \tau_{pp} &= \frac{\vec{p}^2}{R_p}\frac{2x(D-D')^2}{\bar{x}^2}[\xi_3\cos(2\phi_p) + \xi_1\sin(2\phi_p)], \\ \tau_{qq} &= \frac{\vec{q}^2}{R_p}\frac{2x(xD-D')^2}{\bar{x}^2}[\xi_3\cos(2\phi_q) + \xi_1\sin(2\phi_q)], \\ \tau_{pq} &= \frac{1}{R_p}\frac{4x(D-D')(xD-D')}{\bar{x}^2}|\vec{q}||\vec{p}|[\xi_3\cos(\phi_p+\phi_q) \\ &\quad + \xi_1\sin(\phi_p+\phi_q)], \end{aligned} \tag{18}$$

where ϕ_q and ϕ_p are the azimuthal angles of vectors $\vec{q}$ and $\vec{p}$, respectively.

The momentum transfer squared, q^2, which enters in the definition of the unpolarized Bremsstrahlung cross section, must be understood as $q^2 = -(\vec{q}^2 + q^2_{\min})$ with $q^2_{\min} = (\vec{p}^2 + m^2\bar{x}^2)^2/[4E^2(x\bar{x})^2]$.

III. PAIR PRODUCTION

Let us consider the process

$$\gamma(k,e) + T(P) \to e^+(q_+) + e^-(q_-) + T(P'), \tag{19}$$

where T is a heavy target with electric charge Z. The kinematics is defined as

$$\begin{aligned} &s = 2Pk, \quad q_\pm^2 = m^2, \quad P^2 = P'^2 = M^2, \\ &k^2 = 0, \quad k + q = q_+ + q_-, \quad P' + q = P, \end{aligned} \tag{20}$$

where q is the four-momentum transferred to the target, and $e = e(k)$ is the polarization vector of the initial photon. The matrix element is written as

$$\begin{aligned} M = \frac{(4\pi\alpha)^{3/2}Z}{q^2}\cdot\frac{2}{s}[\bar{u}(P')\not{k}\;u(P)] \\ \cdot[\bar{u}(q_-)O_{\mu\nu}v(q_+)]e^\nu(k)\tilde{P}^\mu, \end{aligned} \tag{21}$$

with

$$O_{\mu\nu}e^\nu(k)\tilde{P}^\mu = \tilde{\not{P}}\frac{-\not{q}_+ + \not{k} + m}{2(kq_+)}\not{e} + \not{e}\frac{\not{q}_- - \not{k} + m}{2(kq_-)}\tilde{\not{P}}, \tag{22}$$

where we used again the light-cone decomposition of vectors

$$\begin{aligned} &\tilde{P} = P - k\frac{M^2}{2(pk)}, \quad q_\pm = \alpha_\pm\tilde{P} + x_\pm k + q_{\pm\perp}, \\ &x_+ + x_- = 1, \quad q = \alpha\tilde{P} + \beta k + q_\perp, \quad e = e_\perp. \end{aligned} \tag{23}$$

Similarly, the polarization vector a of the positron can be written in the form

$$a = \alpha_a\tilde{P} + a_\perp, \quad \alpha_a = \frac{2\vec{a}\vec{q}_+}{sx_+}, \tag{24}$$

where the condition $aq_+ = 0$ was applied, to find the expression for α_a. One finds

$$\begin{aligned} &2kq_\pm = \frac{1}{x_\pm}D_\pm, \quad D_\pm = \vec{q}_\pm^2 + m^2, \\ &(q_+ + q_-)^2 = \frac{1}{x_+x_-}[\vec{\rho}^2 + m^2], \quad \vec{q} = \vec{q}_+ + \vec{q}_-, \\ &\vec{\rho} = x_-\vec{q}_+ - x_+\vec{q}_-. \end{aligned} \tag{25}$$

The phase volume then reads

$$\begin{aligned} d\Gamma &= \frac{1}{(2\pi)^5}\delta^4(k + P - q_+ - q_- - P')\frac{d^3q_+}{2E_+}\frac{d^3q_-}{2E_-}\frac{d^3P'}{2E_{P'}} \\ &= \frac{1}{(2\pi)^5}\frac{1}{4s}\frac{dx_-}{x_-x_+}d^2q_+d^2q_-. \end{aligned} \tag{26}$$

The expression $O_{\mu\nu}\varepsilon^\nu(k)\tilde{P}^\mu$, applying the momentum conservation law $k + q = q_+ + q_-$, can be written in the form

$$\begin{aligned} &O_{\mu\nu}e^\nu(k)\tilde{P}^\mu = s\not{e}\;A^\gamma + B^\gamma\not{e}\;\not{q}\;\tilde{\not{P}} + C^\gamma\tilde{\not{P}}\not{q}\;\not{e}\;, \\ &A^\gamma = x_+x_-\left(\frac{1}{D_-} - \frac{1}{D_+}\right); \quad B^\gamma = -\frac{x_-}{D_-}; \\ &C^\gamma = \frac{x_+}{D_+}. \end{aligned} \tag{27}$$

The cross section becomes

$$\begin{aligned} d\sigma^{\gamma T\to e^+e^-T} &= d\sigma_0^{\gamma T\to e^+e^-T}P_\gamma, \\ d\sigma_0^{\gamma T\to e^+e^-T} &= \frac{2\alpha^3Z^2d^2q_-d^2q_+dx_-}{\pi^2(q^2)^2}\frac{R_\gamma}{(D_+D_-)^2}, \\ P_\gamma &= 1 + \xi_2p_T + \xi_2p_a + \tau_{q_+q_+} + \tau_{q_+q} + \tau_{qq}, \end{aligned} \tag{28}$$

with

$$R_\gamma = \vec{q}^2(x_+^2 + x_-^2)D_+D_- + 2m^2x_+x_-(D_+ - D_-)^2 \tag{29}$$

and

$$\begin{aligned} p_T &= \frac{(x_-^2 - x_+^2)}{R_\gamma}D_+D_-\vec{q}^2, \\ p_a &= \frac{2x_-m}{R_\gamma}[(\vec{a}\vec{q})D_+(D_- - D_+) + (\vec{q}_+\vec{a})(D_- - D_+)^2], \\ \tau_{q_+q_+} &= -\frac{2x_+x_-}{R_\gamma}(D_+ - D_-)^2\vec{q}_+^2[\xi_3\cos(2\phi_+) + \xi_1\sin(2\phi_+)], \\ \tau_{q_+q} &= \frac{4x_+x_-}{R_\gamma}(D_+ - D_-)D_+|\vec{q}_+||\vec{q}|[\xi_3\cos(\phi_+ + \phi_q) \\ &\quad + \xi_1\sin(\phi_+ + \phi_q)], \\ \tau_{qq} &= -\frac{2x_+x_-}{R_\gamma}D_+^2\vec{q}^2[\xi_3\cos(2\phi_q) + \xi_1\sin(2\phi_q)], \end{aligned}$$

where ϕ_q and ϕ_+ are the azimuthal angles of vectors $\vec{q}$ and $\vec{q}_+$ and $\xi_{1,2,3}$ are the Stokes polarization parameters of the initial photon. The final positron acquires the polarization a in the case when the initial photon is circularly polarized.

The square of the transferred momentum to the target, q^2, entering in the definition of the unpolarized photoproduction cross section must be understood as $q^2 = -(\vec{q}^{\,2} + q^2_{\min,\gamma})$, with $q^2_{\min,\gamma} = D_+^2/[4\omega^2(x_+x_-)^2]$.

IV. DISTRIBUTIONS WITH AND WITHOUT SCREENING

In an inclusive experimental setup, tagging one of the produced particles (a photon in the Bremsstrahlung process or a positron in the photoproduction process), the distributions obtained by integration on the momentum transferred to the target become important.

Performing the integration on the transversal component of the produced particles, we obtain the distributions on the energy fraction of one of them and on the momentum transferred to the target $\vec{q}^{\,2} = 4m^2 t$:

$$\begin{aligned}\frac{d\sigma^{eT\to e\gamma T}}{dx dt} &= \frac{\alpha^3 Z^2}{2m^2\bar{x}t^2}\Phi^e; \quad t \gg \frac{m^2\bar{x}^2}{16E^2x^2},\\ \frac{d\sigma^{\gamma T\to e^+e^-T}}{dx_+ dt} &= \frac{\alpha^3 Z^2}{2m^2 t^2}\Phi^\gamma, \quad t \gg \frac{m^2}{16\omega^2(x_+x_-)^2},\end{aligned} \tag{30}$$

with

$$\begin{aligned}\Phi^e &= \frac{2[t(1+x^2)+x]}{\sqrt{t(t+1)}}L - 4x;\\ \Phi^\gamma &= \frac{2[t(x_+^2+x_-^2) - x_+x_-]}{\sqrt{t(t+1)}}L + 4x_+x_-;\\ L &= \ln\frac{\sqrt{t+1}+\sqrt{t}}{\sqrt{t+1}-\sqrt{t}}.\end{aligned} \tag{31}$$

In the absence of screening, performing the integration on the transferred momentum and the transversal component of the produced particles, one obtains the spectral distributions for the unpolarized case [12]:

$$\begin{aligned}\frac{d\sigma_0^{eT\to eT\gamma}}{dx} &= \frac{2\alpha^3}{m^2\bar{x}}\left[\frac{4}{3}x + \bar{x}^2\right]\left[2\ln\frac{s}{m^2} + 2\ln\frac{x}{\bar{x}} - 1\right];\\ \bar{x} &= 1 - x = \frac{\omega}{E},\\ \frac{d\sigma_0^{\gamma T\to e^+e^-T}}{dx_+} &= \frac{2\alpha^3}{m^2}\left[1 - \frac{4}{3}x_+\bar{x}_+\right]\left[2\ln\frac{s x_+\bar{x}_+}{m^2} - 1\right].\end{aligned} \tag{32}$$

The spectra show a logarithmic enhancement (Weizsacker-Williams (WW) enhancement [13]). The different distributions for these processes, calculated within the Born approximation, can be found in Ref. [10].

The effect of complete screening can be taken into account in frame of the Molière model [14] by replacing

$$\frac{4\pi\alpha}{-q^2} \to \frac{4\pi\alpha[1 - F(-q^2)]}{-q^2}, \tag{33}$$

with

$$\begin{aligned}\frac{1 - F(\vec{q}^{\,2})}{\vec{q}^{\,2}} &= \sum_{i=1}^{3}\frac{\alpha_i}{\vec{q}^{\,2} + m^2\beta_i} = \frac{1}{4m^2}\sum_{i=1}^{3}\frac{\alpha_i}{t + \beta_i/4},\\ \alpha_1 &= 0.1, \quad \alpha_2 = 0.55, \quad \alpha_3 = 0.35;\\ \beta_i &= \left(\frac{Z^{1/3}}{121}\right)b_i, \quad b_1 = 6.0; \quad b_2 = 1.2; \quad b_3 = 0.3.\end{aligned} \tag{34}$$

The resulting spectral distributions have been derived in the preceding case without screening (30). For complete screening, we find, respectively,

$$\begin{aligned}\frac{d\sigma_{0\mathrm{scr}}^{eT\to eT\gamma}}{dx} &= \frac{\alpha^3 Z^2}{2m^2\bar{x}}\int_0^\infty\left(\sum_{i=1}^{3}\frac{\alpha_i}{t + \beta_i/4}\right)^2\Phi^e dt;\\ \frac{d\sigma_{0\mathrm{scr}}^{\gamma T\to e^+e^-T}}{dx} &= \frac{\alpha^3 Z^2}{2m^2}\int_0^\infty\left(\sum_{i=1}^{3}\frac{\alpha_i}{t + \beta_i/4}\right)^2\Phi^\gamma dt.\end{aligned}$$

The expressions for differential cross sections obey explicitly the gauge invariance requirement

$$(\vec{q}^{\,2})^2\left.\frac{d\sigma}{d^2q}\right|_{\vec{q}\to 0} = 0. \tag{35}$$

Keeping in mind the relation $\int_0^{2\pi} F(\cos\phi)\sin\phi d\phi = 0$ and introducing $\psi = \phi_p - \phi_q$, the differential distributions integrated on final-particle transversal momenta become

$$\begin{aligned}\frac{d\sigma^{eT\to e\gamma T}}{dt dx d\phi} &= \frac{\alpha^3 Z^2}{2\pi m^2}\int_0^\infty\frac{dy}{(t + t_{\min})^2}F^e(t, y, x),\\ t_{\min} &= \frac{m^2}{4E^2(x\bar{x})^2}\left(y + \frac{\bar{x}^2}{4}\right)^2,\end{aligned} \tag{36}$$

where we used the notation $\phi_q = \phi$, for simplicity. The result is

$$F^e = F^e_{\mathrm{unp}} + \lambda\xi_2\left(F^e_L + \vec{n}\vec{a}F^e_a\right) + [\xi_3\cos(2\phi) + \xi_1\sin(2\phi)]F^e_\tau, \tag{37}$$

with $\vec{n} = \vec{q}/|\vec{q}|$ and

$$\begin{aligned}F^e_{\mathrm{unp}} &= \frac{1}{\rho}[t(1+x^2)+x]\left[I_1^{(0)} - xJ_1^{(0)}\right] - \frac{x}{2}\left[I_2^{(0)} + J_2^{(0)}\right],\\ F^e_L &= -\frac{1}{\rho}[t(1-x^2)+x\bar{x}]\left[I_1^{(0)} - xJ_1^{(0)}\right] + \frac{x\bar{x}}{2}\left[I_2^{(0)} + J_2^{(0)}\right],\\ F^e_a &= \left\{-\frac{2}{\rho}\left[I_1^{(1)} - xJ_1^{(1)}\right] + I_2^{(1)} + J_2^{(1)}\right\}\sqrt{y}\\ &\quad + \left\{\frac{1+x}{\rho}\left[I_1^{(1)} - xJ_1^{(1)}\right] - I_2^{(1)} - xJ_2^{(1)}\right\}\sqrt{t},\\ F^e_\tau &= \frac{2x}{\bar{x}^2}\left(y\left\{2I_2^{(2)} - I_2^{(0)} + 2J_2^{(2)} - J_2^{(0)} - \frac{2}{\rho}\left[I_1^{(2)} - xJ_1^{(2)}\right]\right\}\right.\\ &\quad + t\left\{I_2^{(0)} + x^2J_2^{(0)} - \frac{2x}{\rho}\left[I_1^{(0)} - xJ_1^{(0)}\right]\right\}\\ &\quad \left. + 2\sqrt{yt}\left\{I_2^{(1)} + xJ_2^{(1)} - \frac{1+x}{\rho}\left[I_1^{(1)} - xJ_1^{(1)}\right]\right\}\right),\end{aligned} \tag{38}$$

with $\rho = \bar{x}[y - tx + \frac{\bar{x}^2}{4}]$. The quantities $I_k^{(i)}$ and $J_k^{(i)}$ are derived in the Appendix.

Similar calculations for the photoproduction process lead to

$$\frac{d\sigma^{\gamma T \to e^+e^- T}}{dt dx d\phi} = \frac{\alpha^3 Z^2}{2\pi m^2} \int_0^\infty \frac{dy}{(t + t_{\min,\gamma})^2} F^\gamma(t, y, x),$$
$$t_{\min,\gamma} = \frac{m^2}{4\omega^2(x_+x_-)^2} d_+^2, \quad d_+ = y + \frac{1}{4}, \tag{39}$$

with (see Appendix)

$$F^\gamma = \frac{1}{2}x_+x_-\left[K_2^{(0)} - \frac{2}{d_+}K_1^{(0)} + \frac{1}{d_+^2}\right] + t(x_+^2 + x_-^2)\frac{1}{d_+}K_1^{(0)} + \lambda\xi_2 x_- \vec{a}\vec{n}\left\{\sqrt{t}\left[\frac{1}{d_+}K_1^{(0)} - K_2^{(0)}\right] + \sqrt{y}\left[-\frac{2}{d_+}K_1^{(1)} + K_2^{(1)}\right]\right\} + 2x_+x_-[\xi_3\cos(2\phi) + \xi_1\sin(2\phi)]\left(-tK_2^{(0)} - y\left\{2K_2^{(2)} - K_2^{(0)} - \frac{2}{d_+}\left[2K_1^{(2)} - K_1^{(0)}\right] + 2\sqrt{ty}\left[K_2^{(1)} - \frac{1}{d_+}K_1^{(1)}\right]\right\}\right). \tag{40}$$

To take into account the screening effects, one must replace

$$\frac{1}{(t + t_{\min})^2} \to \left(\sum_{i=1}^{3} \frac{\alpha_i}{t + \beta_i/4}\right)^2, \tag{41}$$

following Eq. (33).

V. DISCUSSION AND RESULTS

In frame of the WW approximation one obtains the following from the differential distributions given previously.

(i) For the Bremsstrahlung process,

$$\frac{d\sigma^{eT \to e\gamma T}}{dp^2 dx d\phi_p} = \frac{\alpha^3 Z^2}{\pi d^4}\{(1 + x^2)d^2 - 4m^2p^2\bar{x}^2 + \lambda\xi_2[-d^2(1 - x^2) + 4m^2p^2x\bar{x}^3 + 2x\bar{x}^2 m\vec{p}\vec{a}(2p^2 - d)] + 4xp^2[\xi_3\cos(2\phi_p) + \xi_1\sin(2\phi_p)] \times [p^2 + d]\}\ln\frac{4E^2(x\bar{x})^2}{d}, \tag{42}$$

with $d = p^2 + m^2\bar{x}^2$.

(ii) For the pair production process,

$$\frac{d\sigma^{\gamma T \to e^+e^- T}}{dq_+^2 dx_+ d\phi_+} = \frac{\alpha^3 Z^2}{\pi c^4}\{(x_+^2 + x_-^2)c^2 + 4m^2q_+^2x_+x_- + \lambda\xi_2 m x_- \vec{q}_+\vec{a}(2q_+^2 - c) + 4x_+x_-q_+^2 \times [\xi_3\cos(2\phi_+) + \xi_1\sin(2\phi_+)] \times [-q_+^2 + c]\}\ln\frac{4\omega^2(x_+x_-)^2}{c}, \tag{43}$$

with $c = q_+^2 + m^2$.

Let us consider first the Bremsstrahlung process and calculate the differential distribution on the energy fraction of the photon. We integrate the multidifferential distributions [Eq. (42)] and obtain

$$\frac{d\sigma^{\rm unp}}{dx d\eta^2} = \frac{4\alpha^3 Z^2 \bar{x}^3}{m^2 d^2}\int_0^\infty \frac{q dq}{(q^2 + q_m^2)^2}\int_0^{2\pi}\frac{d\phi}{2\pi(d')^2} \times [q^2(1 + x^2)dd' - 2x(d - d')^2] = \frac{4\alpha^3 Z^2\bar{x}^3}{m^2 d^2} I_{\rm unp}; \tag{44}$$

$$\frac{(d\sigma^{\rm pol})^{\rm circ}_{\rm long}}{dx d\eta^2} = \frac{4\alpha^3 Z^2 \bar{x}^3}{m^2 d^2}\int_0^\infty \frac{q dq}{(q^2 + q_m^2)^2}\int_0^{2\pi}\frac{d\phi}{2\pi(d')^2} \times [-q^2(1 - x^2)dd' + 2x\bar{x}(d - d')^2] = \frac{4\alpha^3 Z^2\bar{x}^3}{m^2 d^2} I_{\rm pol}. \tag{45}$$

The degree of transverse polarization is calculated as the ratio

$$P_T = \frac{I_{\rm pol}}{I_{\rm unp}}. \tag{46}$$

We use the parametrization

$$d = \kappa^2 + \bar{x}^2 = \bar{x}^2(\eta^2 + 1), \quad d' = \alpha + \beta\cos\phi,$$
$$\alpha = \kappa^2 + \bar{x}^2 + \bar{x}^2 q^2, \quad \beta = -2\kappa q\bar{x}, \tag{47}$$
$$\alpha > \beta, \quad d - d' = -\bar{x}q[\bar{x}q - 2q\kappa\cos\phi].$$

The angular integration is analytically performed as explained in the Appendix. The remaining integration on the transverse momentum q, keeping in mind that

$$q_m^2 = \frac{(\kappa^2 + \bar{x}^2)^2}{4\gamma^2(x\bar{x})^2} \ll 1, \tag{48}$$

can be performed, in case of slow convergence, using an auxiliary numerically small parameter σ, which cancels in the final expression:

$$I = \int_0^\infty \frac{q^3 dq}{(q^2 + q_m^2)^2} f = \lim_{\sigma\to 0}\left[\int_0^\sigma \frac{q^3 dq}{(q^2 + q_m^2)^2} f(0) + \int_\sigma^\infty \frac{dq}{q} f(q^2)\right],$$
$$\gamma^{-2} \ll \sigma \ll 1. \tag{49}$$

In case of Bremsstrahlung, the explicit expressions for $f_{\rm unp}$, $f_{\rm pol}$ are

$$f_{\rm unp} = \frac{d(1 + x^2)}{R} - \frac{2x\bar{x}^2}{R^3}T; \quad f_{\rm pol} = -\frac{d(1 - x^2)}{R} + \frac{2x\bar{x}^3}{R^3}T;$$
$$T = \bar{x}^2 q^2\alpha + 4\bar{x}\kappa q\beta + \frac{4\kappa^2}{\beta^2}(R^3 - \alpha^3 + 2\alpha\beta^2); \tag{50}$$
$$R = \sqrt{\alpha^2 - \beta^2}.$$

For the case of pair production, we have

$$I_{\rm unp}^{\rm pair} = \int_0^\infty \frac{q^3 dq}{[q^2 + (q_m^{\rm pair})^2]^2} \left\{ \frac{d_-(x_+^2 + x_-^2)}{R} + 2\frac{x_+ x_-}{R^3} \times \left[q^2\alpha + 4q\kappa_-\beta + \frac{4\kappa_-^2}{\beta^2}(R^3 - \alpha^3 + 2\alpha\beta^2)\right]\right\}; \tag{51}$$

$$I_{\rm pol}^{\rm pair} = \int_0^\infty \frac{q^3 dq}{[q^2 + (q_m^{\rm pair})^2]^2} \frac{(x_-^2 - x_+^2)d_-}{R}; \tag{52}$$

$$\alpha = 1 + q^2 + \kappa_-^2, \quad \beta = -2q\kappa_-, \quad R = \sqrt{\alpha^2 - \beta^2},$$
$$d_- = \kappa_-^2 + 1, \quad \kappa_- = \theta_{e^-}\gamma x_-, \tag{53}$$
$$(q_m^{\rm pair})^2 = \frac{d_-^2}{4\gamma^2(x_+x_-)^2}, \quad x_+ + x_- = 1.$$

The relevant degree of longitudinal electron polarization is

$$P_L^{\rm pair} = \frac{I_{\rm pol}^{\rm pair}}{I_{\rm unp}^{\rm pair}}. \tag{54}$$

The case of complete screening can be obtained using similar calculations with the replacement [see Eq. (33)]

$$\frac{1}{(q^2+q_m^2)^2} \to \left(\sum_{i=1}^{3}\frac{\alpha_i}{q^2+\beta_i}\right)^2. \tag{55}$$

Performing the integration over $\vec{p}$ in Eq. (16), we can obtain the inclusive cross section over photon momenta $\vec{k}$ previously obtained in Ref. [15]:

$$\frac{d\sigma}{d^2k dx} = \frac{2\alpha^3(1-x)}{\pi m^4 c^2}\left\{2\left[1 + x^2 - \frac{4x(1-x)^2}{c} + \frac{4x(1-x)^4}{c^2}\right]\ln\left[\frac{sx}{m^2(1-x)}\right] - (1+x)^2 + \frac{16x(1-x)^2}{c} - \frac{16x(1-x)^4}{c^2}\right\}, \tag{56}$$

where $c = (1-x)^2 + \vec{k}^2/m^2$. Performing the integration over $\vec{k}$ in Eq. (16), we can obtain the cross section inclusive over scattered electron momenta $\vec{p}$ in agreement with Ref. [16]:

$$\frac{d\sigma}{d^2p dx} = \frac{4\alpha r_0^2(1-x)m^2}{\pi a^2}\left\{\left[1 + x^2 - \frac{4x(1-x)^2\vec{p}^2m^2}{a^2}\right] \times \ln\left(\frac{sx}{m^2}\right) + \frac{x\ln x}{1-x}\left[1 + x^2 + \frac{4x^2(3-x)m^2\vec{p}^2}{a^2}\right] - \frac{(1+x^2)^2}{2(1-x)^2} + \frac{8x(1-x+x^2)m^2\vec{p}^2}{a^2} - \frac{xa\left(1+x^2+\frac{4x^2m^2}{a}\right)}{(1-x)^2 r m^2}\ln\frac{x\left[2(1-x) - \frac{a}{m^2} + r\right]}{2x(1-x) + \frac{a}{m^2} + r}\right\}, \tag{57}$$

where $r = \sqrt{a/m^2}\sqrt{a/m^2 + 4x}$, $a = \vec{p}^2 + m^2(1-x)^2$, and $r_0 = \alpha/m$.

For large values of Z, Coulomb corrections owing to an arbitrary number of photons interacting with the charged leptons and with the nuclei have to be taken into account. This has been previously derived in the literature. For completeness, let us recall the results from Ref. [12], for the total cross sections of pair photoproduction in case of absence of screening,

$$\sigma^{\rm pair} = \frac{28}{9}\frac{Z^2\alpha^3}{m^2}\left[\ln\left(\frac{2\omega}{m}\right) - \frac{109}{42} - f(Z)\right], \tag{58}$$

and in case of complete screening,

$$\sigma^{\rm pair} = \frac{28}{9}\frac{Z^2\alpha^3}{m^2}\left[\ln(183Z^{-1/3}) - \frac{1}{42} - f(Z)\right], \tag{59}$$

where $f(Z)$ is the Bethe-Maximon-Olsen function

$$f(Z) = (Z\alpha)^2\sum_{n=1}^{\infty}\frac{1}{n[n^2 + (Z\alpha)^2]}. \tag{60}$$

The first term in brackets corresponds to WW approximation, the second one arises from terms beyond WW approximation, whereas the function $f(Z)$ contains higher-order corrections. They can be found in Ref. [12]. In frame of our formalism, they require accurate integration on the energy fraction x, as in Eq. (32).

The photon spectrum of the Bremsstrahlung process in case of complete screening has the form

$$d\sigma = \frac{4Z^2\alpha^3}{m^2}\frac{dx}{1-x}\left[\left(1 + x^2 - \frac{2}{3}x\right)\ln(183Z^{-1/3}) + \frac{x}{9}\right], \tag{61}$$

while in case of absence of screening, we have

$$d\sigma = \frac{4Z^2\alpha^3}{m^2}\frac{dx}{1-x}\left[\left(1 + x^2 - \frac{2}{3}x\right)\ln\left(\frac{2\omega}{m}\right) - \frac{1}{2} - f(Z)\right]. \tag{62}$$

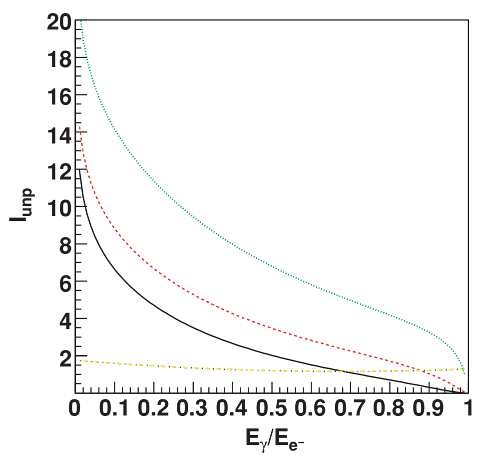

FIG. 1. (Color online) Unpolarized reduced cross section $I_{\rm unp}$ [see Eq. (44)] for the Bremsstrahlung process at $E_{e^-} = 3$ MeV (solid black line), $E_{e^-} = 10$ MeV (dashed red line), $E_{e^-} = 100$ MeV (dotted green line), and in the case of the fully screened process, at $E_{e^-} = 3$ MeV (dash-dotted blue line), $E_{e^-} = 10$ MeV (dash-double-dotted yellow line), $E_{e^-} = 100$ MeV (dash-triple-dotted magenta line). The fully screened distributions are essentially independent on energy.

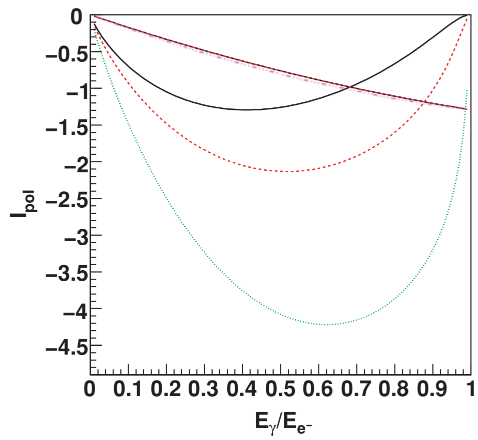

FIG. 2. (Color online) Polarized reduced cross section $I_{\rm pol}$ [see Eq. (45)], for the Bremsstrahlung process. Notations as in Fig. 1.

The relevant formula for pair photoproduction in the case of complete screening is

$$\frac{d\sigma^{\rm pair}}{dx_+} = \frac{4Z^2\alpha^3}{m^2}\left\{\left[1-\frac{4}{3}x_+(1-x_+)\right]\ln(183Z^{-1/3}) - \frac{1}{9}x_+(1-x_+) - \frac{7}{9}f(Z)\right\}. \quad (63)$$

The effects of Coulomb interaction for light nuclei ($Z < 10$) are of the order of 1%. They correspond to radiative corrections related to the lepton vertex, which are not discussed here.

In the case where the target (T) is a nucleon, the factor $D(\vec{q}^{\,2}) = F_1^2(-\vec{q}^{\,2}) + \frac{\vec{q}^{\,2}}{4M^2}F_2^2(-\vec{q}^{\,2})$ has to be taken into account. Such factor parametrizes the internal structure of the nucleon in terms of the Dirac and Pauli form factors F_1, F_2.

The distribution on the momentum transferred to the nuclei is equivalent to the distribution on the square of the three-momentum of the recoil proton $\vec{p}$:

$$\vec{p}^{\,2} = \vec{q}^{\,2}\left(1+\frac{\vec{q}^{\,2}}{4M^2}\right); \quad \frac{d\sigma}{d\vec{q}^{\,2}} = \frac{d\sigma}{2MdE'}, \quad (64)$$

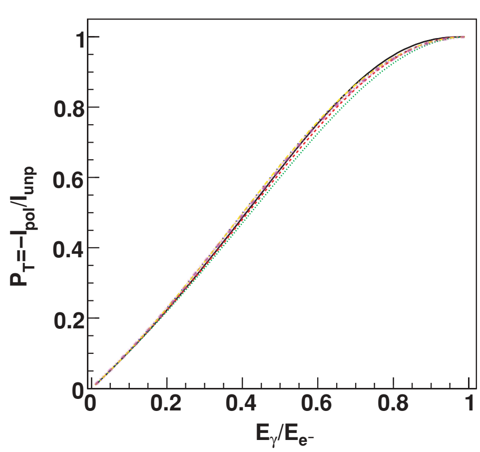

FIG. 3. (Color online) Degree of transverse polarization $P_T = I_{\rm pol}/I_{\rm unp}$ [see Eq. (46)] as a function of $x = E_\gamma/E_{e^-}$. The polarization is nearly independent on the energy, in all cases. Notations as in Fig. 1.

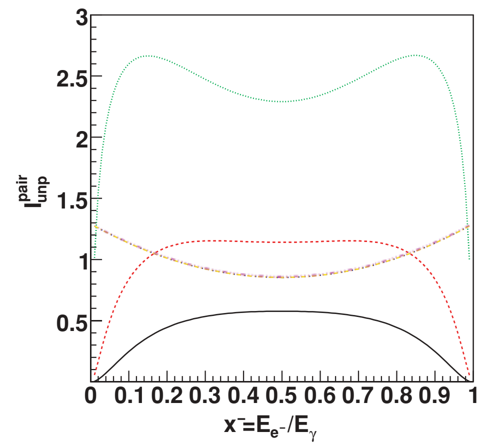

FIG. 4. (Color online) Unpolarized reduced cross section $I_{\rm unp}$ [see Eq. (51)] for the pair production process. Notations as in Fig. 1.

where $E' = \sqrt{p^2+M^2}$ is the energy of the recoil proton and M is its mass. It is useful to recall the relation between the recoil proton momentum p with the emission angle θ_p relative to the initial beam direction:

$$\frac{p}{2M} = \frac{\cos\theta_p}{\sin^2\theta_p}. \quad (65)$$

In practice, the ratio $p/(2M)$ ranges from 1 to 2, $\theta_p \simeq 60^\circ$. Therefore, approximate formulas for the emission angles of the produced particles can be used, as these angles are small compared with θ_p. For the Bremsstrahlung process, one has

$$\theta_e = \frac{|\vec{p}_e|}{Ex}; \quad \theta_\gamma = \frac{|\vec{k}|}{E(1-x)}; \quad \theta_e \sim \theta_\gamma \ll 1, \quad (66)$$

and for the pair production process,

$$\theta_+ = \frac{|\vec{q}_+|}{\omega x_+}; \quad \theta_- = \frac{|\vec{q}_-|}{\omega x_-}; \quad \theta_+ \sim \theta_- \ll 1. \quad (67)$$

The results are shown in Figs. 1–6. Three energies have been considered for the numerical applications, $E = 3$, 10, 100 MeV, in the cases of both no screening and complete screening. For the screening effect, the target is Au and the

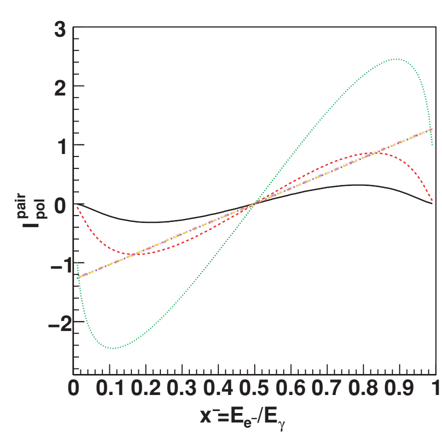

FIG. 5. (Color online) Polarized reduced cross section $I_{\rm pol}$ [see Eq. (52)], for the pair production process. Notations as in Fig. 1.

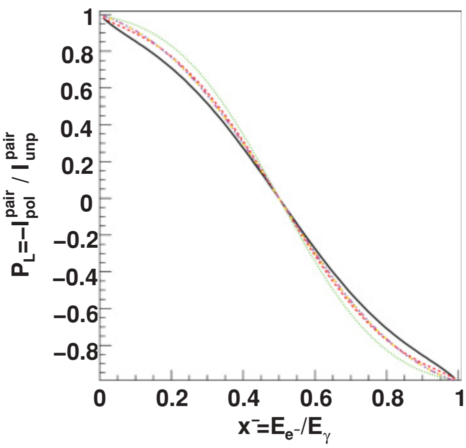

FIG. 6. (Color online) Degree of linear polarization $P_L = I^{\rm pair}_{\rm pol}/I^{\rm pair}_{\rm unp}$ [see Eq. (54)], as a function of $x^- = E_{e^-}/E_\gamma$, for the pair production process. Notations as in Fig. 1.

angle of the produced particles θ or of the electron in the pair is taken as 0.41 mrad.

For the Bremsstrahlung process, the functions $I_{\rm unp}$ and $I_{\rm pol}$, which correspond to the unpolarized and polarized cross sections omitting kinematical coefficients as defined in Eqs. (44) and (45), are shown in Figs. 1 and 2, as functions of the fraction of incident energy carried by the photon. The transverse polarization, $P^{\rm pair}$ [Eq. (46)], is built as their ratio and is shown in Fig. 3.

The comparison with the results of Ref. [3] show complete agreement. For example, Fig. 1 of Ref. [3] should be compared with Fig. 1 of the present work, in the same kinematics, after multiplying the quantity I by a factor of four, to match the definitions.

Similarly, in the case of pair production, the unpolarized and polarized functions from Eqs. (51) and (52) are shown in Figs. 4 and 5 as functions of the electron energy fraction, for the case of longitudinal electron polarization, when the initial photon is circularly polarized. The degree of polarization $P_L^{\rm pair}$ is shown in Fig. 6 [Eq. (54)].

In all cases, fully screened distributions are independent of energy. When screening is switched off, the polarized as well as the unpolarized distributions increase with energy (in absolute value). Also for pair production, the present results are fully consistent with the corresponding numerical results reported in Ref. [3].

VI. CONCLUSIONS

Multidifferential cross section for Bremsstrahlung and pair creation processes in electron proton scattering have been calculated using first-order perturbation theory. Considerations taking into account higher-order processes can be found in Ref. [17].

The calculation is done in frame of the light-cone parametrization of four-vectors, which is well suited to small-angle scattering.

General expressions for different observables have been given for unpolarized and polarized scattering in case of unscreened and fully screened atomic targets. The screened distributions are essentially independent of the energy.

For numerical applications, two cases have been illustrated: the transverse polarization of the photon, when the electron is longitudinally polarized in the Bremsstrahlung process and the longitudinal polarization of the electron, created using a pair production process by a circularly polarized photon.

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APPENDIX: THE QUANTITIES $I_k^{(i)}$, $J_k^{(i)}$

The quantities $I_k^{(i)}$ and $J_k^{(i)}$ are defined as

$$I_k^m = \frac{1}{2\pi}\int_0^{2\pi} d\phi \frac{(\cos\phi)^m}{(a-b\cos\phi)^k}. \tag{A1}$$

Their explicit expressions are (see tables in Gradshetyn and Ryzik [18])

$$\begin{aligned} I_1^{(0)} &= \frac{1}{d},\, d=\sqrt{a^2-b^2}; \quad I_1^{(1)} = \frac{1}{b}\left[\frac{a}{d}-1\right];\\ I_1^{(2)} &= \frac{a}{b^2}\left[\frac{a}{d}-1\right]; \quad I_2^{(0)} = \frac{a}{d^3}; \quad I_2^{(1)} = \frac{b}{d^3};\\ I_2^{(2)} &= \frac{1}{b^2}\left[1+\frac{a(2b^2-a^2)}{d^3}\right],\\ I_2^{(3)} &= \frac{a}{b^3}\left[2-\frac{3a}{d}+\frac{a^3}{d^3}\right]. \end{aligned} \tag{A2}$$

The relations with the other functions used in Sec. IV are

$$\begin{aligned} J_j^{(i)} &= I_j^{(i)}(a\to a_1, b\to b_1);\\ K_j^{(i)} &= I_j^{(i)}(a\to a_-, b\to b_-), \end{aligned} \tag{A3}$$

with

$$\begin{aligned} a &= y+t+\frac{\bar{x}^2}{4}; \quad b = 2\sqrt{yt};\\ a_1 &= y+tx^2+\frac{\bar{x}^2}{4}; \quad b_1 = 2x\sqrt{yt};\\ a_- &= t+y+\frac{1}{4}; \quad b_- = 2\sqrt{ty}. \end{aligned} \tag{A4}$$

[1] E. Haug, Z. Naturforsch. **30a**, 1099 (1975).
[2] E. Haug, Z. Naturforsch. **36a**, 413 (1981).
[3] H. Olsen and L. C. Maximon, Phys. Rev. **114**, 887 (1959).
[4] W. McMaster, Rev. Mod. Phys. **33**, 8 (1961).
[5] L. C. Maximon, A. De Miniac, T. Aniel, and E. Ganz, Phys. Rep. **147**, 189 (1987).
[6] J. Asai, H. S. Caplan, D. M. Skopik, W. Del Bianco, and L. C. Maximon, Can. J. Phys. **66**, 1079 (1988).

[7] S. Klein, Rev. Mod. Phys. **71**, 1501 (1999).
[8] S. Weinberg, Phys. Rev. **150**, 1313 (1966).
[9] M. May and G. C. Wick, Phys. Rev. **81**, 628 (1951).
[10] V. N. Baier, E. A. Kuraev, V. S. Fadin, and V. A. Khoze, Phys. Rep. **78**, 293 (1981).
[11] V. V. Sudakov, Sov. Phys. JETP **3**, 65 (1956) [Zh. Eksp. Teor. Fiz. **30**, 87 (1956)].
[12] A. I. Akhiezer and V. B. Berestetskij, *Quantum Electrodynamics* (Science, Moscow, 1981); V. B. Berestetskij, E. M. Lifshits, and L. P. Pitaevskij, *Relativistic Quantum Theory* (Science, Moscow, 1968); V. N. Baier, V. M. Katkov, and V. S. Fadin, *Radiation from Relativistic Electrons* (Atomizdat, Moscou, 1973) [in Russian].
[13] C. F. von Weizsacker, Z. Phys. **88**, 612 (1934); E. Williams, Phys. Rev. **45**, 729 (1939).
[14] G. Moliere, Z. Naturforsch. **2a**, 133 (1947).
[15] G. Altarelli and F. Buchella, Nuovo Cimento **34**, 1337 (1964).
[16] G. Altarelli and B. Stella, Nuovo Cimento Lett. **9**, 416 (1974).
[17] S. Bakmaev, E. A. Kuraev, I. Shapoval, and Y. P. Peresunko, Phys. Lett. B **660**, 494 (2008).
[18] I. S. Gradshetyn and I. M. Ryzik, in *Table of Integrals, Series, and Products*, edited by Alan Jeffrey and Daniel Zwillinger (Academic Press, London, UK, 2007).